# Chapter 13
# Mathematics Education in the United Kingdom

**Geoffrey Howson and Leo Rogers**

## 1 Introduction

The United Kingdom comprises four countries: England, Wales, Scotland and Northern Ireland. Until 1922 (i.e. for much of the period with which this chapter is concerned) the whole of Ireland was included in the United Kingdom. The educational histories of these four countries differ, but the most significant differences, throughout history, have been between the educational systems of England and Scotland. Scotland has been an independent country since the Middle Ages and has maintained religious, legal and educational systems of its own. The Act of Union in 1707 brought Scotland and England under one king but maintained the Scottish traditions: the Scottish Education System has retained its independence and high quality status. Accordingly, the histories of these two countries are treated separately in this chapter. Some differences relating to the Irish and Welsh systems can be found in a footnote,[1] but, otherwise, little specific attention will be paid to these two countries.

[1] The population sizes of the four countries are given, in millions, in the following table.

| | England | Wales | Scotland | Ireland | Northern Ireland |
|---|---|---|---|---|---|
| 1800 | 8.3 | 0.6 | 1.6 | 5 | |
| 1901 | 30.5 | 2.0 | 4.5 | 4.5 | (1.2) |
| 2001 | 49.1 | 2.9 | 5.1 | | 1.7 |

These data show the greater proportional population growth in the more prosperous England and, in fact, conceal that in Ireland the population dropped significantly from 1845, when it was in excess of eight millions, because of deaths due to a great food famine between 1845 and 1848 (not eased by its partner countries) and large-scale emigration. The poverty which gave rise to this fall in population was reflected in the educational opportunities offered to the Irish people Thus, in Ireland (and later Northern Ireland) state education, as opposed to that provided by religious bodies, did not really commence until about 20 years after the 1922 partition of Ireland. The provision of education in Wales and Ireland was also complicated by religious issues: in Wales between those belonging to the Church of England and the Nonconformists and in Ireland between the Catholics and the Protestants. Certain differences still exist between the educational systems of England and those of Wales and Northern Ireland, for example, in the national curricula (for provision is made in the latter two countries for the teaching of national languages) and in the way that the system of frequent testing introduced in the Education Act of 1988 operates. The national curricula of the four constituent countries of the United Kingdom can be found on the ICMI web site.

G. Howson (✉)
University of Southampton, Southampton, UK
e-mail: geoffreyhowson@aol.com

L. Rogers
University of Oxford, Oxford, UK
e-mail: leorogers@appleinter.net

A. Karp and G. Schubring (eds.), *Handbook on the History of Mathematics Education*,
DOI 10.1007/978-1-4614-9155-2_13, © Springer Science+Business Media New York 2014

It will be noted that Scottish governments have traditionally tended to play a greater role in influencing education than their English counterparts. This difference and, in particular, the great diversity of provision in England both in the forms that school education took and also the curricula the schools followed make the description of the history of mathematics education in that country particularly complex. For, although in the nineteenth century, the United Kingdom occupied a leading position in industry and even in science and technology and possessed the largest world empire, it did not follow that it occupied a leading position in mathematics education. Nonetheless, the interconnection of the various aspects of the life of a society is evident. Below, much will be said about the legal and administrative sides of the changes that took place in the country – in particular, about various educational acts. They reflected the social processes unfolding in the United Kingdom and, in particular, the gradual governmental moves from a *laissez-faire* stance to a dictatorial one and, accordingly, from an educational system of great diversity to a more uniform one: moves with great implications for mathematics education at all levels.

The section on England of this chapter is written by Geoffrey Howson, and the section on Scotland is authored by Leo Rogers.

## 2 England

**Geoffrey Howson**
University of Southampton, Southampton, UK

### *2.1 Prelude*

As in many other countries, the first schools in England were provided by religious bodies and were primarily concerned with religious matters and needs (Howson 1982; Orme 2006). However, considerable consequences for education followed Henry VIII's break with the Church of Rome which led by 1540 to the dissolution of the monasteries and friaries. This had an immediate effect on the number and governance of schools. New grammar schools were established by Henry and his son, Edward VI, and throughout the following centuries by trade guilds, merchants and other wealthy persons (Watson 1908). Yet, there was little place for mathematics in any of them; their curricula reflected humanist thought. Pressures for change did, however, lead some schools in the seventeenth century, particularly in the seaports, to introduce a little mathematics, beyond basic arithmetic, into their curricula (Howson 1982). If one wished to learn mathematics, however, it was usually necessary to turn to a private tutor or, so-called mathematical practitioner (Taylor 1954, 1966; Howson 2011). The only two universities, Oxford and Cambridge, placed no pressure on schools to teach mathematics. For although both appointed professors of mathematics in the early seventeenth century, neither paid much attention to the teaching of the subject before the nineteenth (Warwick 2003; Howson 2010). Indeed, although the period up to 1800 witnessed a marked growth in the applications of mathematics and a need for knowledge of the subject, it did not lead to changes in the curricula of the vast majority of English schools and neither did it cause any government to demonstrate a marked interest in public education. The activity of mathematicians such as Briggs, Wallis, Gregory and Newton had a minimal impact on mathematics education in schools, and in general what was accomplished was due to the work of individuals rather than to that of authorities (Howson 1982, 2011).

## 2.2 *1800–1902 Elementary Education*

### 2.2.1 First Moves

By the beginning of the nineteenth century, England was clearly lagging behind several continental countries in its provision of elementary education and there were appeals for the state to help remedy this. However, the government did not wish to become involved and left education to other bodies. As a result, the Church of England in 1811 gathered together schools under its care in the so-called National Society, while the nonconformists in 1814 offered their own alternative, the schools of the British and Foreign School Society (Wardle 1976; Howson 1982). These offered a higher level of education than did the 'dame' schools, which had low educational aims, were run by ill-educated persons and were attended mainly by children of the working class. Some of the private schools for middle-class children run by clergymen or similarly well-educated ladies and gentlemen reached a much higher level. The government, however, still had worries concerning the education of the working class – would it lead them to be dissatisfied with their lot? Memories of the French revolution were still strong. Yet despite considerable opposition, Parliament agreed in 1833 to make a grant to further the work of these two societies. In 1839 an inspectorate was formed to ensure that the grant was being well spent. However, mathematics in such schools still meant, at best, arithmetic (see Yeldham 1936).

It is not known exactly what proportion of the population attended schools, but in 1840 it was believed that working class children attended school on average between 1 and 2 years, frequently interrupted, and that few stayed after the age of 10. By 1852, 33 % of pupils in inspected schools had been at school for less than 1 year and 73 % for less than two; by 1861 the respective percentages were 38 and 61 (quoted in Wardle 1976, p. 65). At both dates over half the pupils were under eight and less than 15 % over 11 (of whom many were 'pupil teachers'). Although some pupils progressed to become teachers through classroom experience, from the 1830s teacher training colleges began to be established by the two societies including one in 1842 specifically for women (Rich 1933; Howson 1982, 2010).

### 2.2.2 Growing Government Involvement

In 1858 the government established a public commission, chaired by the Duke of Newcastle, 'to inquire into the State of Popular Education in England and to consider and report what measures if any are required for the extension of sound and cheap elementary instruction to all classes of people'. The picture of mathematics education revealed in the Commission's Report is bleak indeed. Of the children attending the 1824 public weekday schools[2] visited, only 69.3 % were taught arithmetic, 0.6 % mechanics, 0.8 % algebra and 0.8 % Euclid. The corresponding data for private schools were 33.8 %, 1.29 %, 1.35 % and 1.15 %, respectively. However, such arithmetical instruction as was given appeared totally inadequate: 'in working sums explicitly stated the children were often successful enough, but they were usually quite ignorant of anything that required the simplest knowledge of a principle' (p. 47).

The Commission's recommendations were broadly to leave the system in place but to award grants to schools based on the pupils' attainments. This led to the system known as 'payment by results': the establishment of national 'standards' on which pupils would be examined and on the results of which payments would be made to their schools. The curriculum was set out for arithmetic (and there was no other mathematics) and the six standards consisted of no more than the four operations (up to short division) plus sums (exercises) on money, weights and common measures (reprinted in Howson 1982). By 1871 changes were made and a new Standard VI, 'proportion and vulgar and decimal fractions',

[2] Sunday schools were first created in the late eighteenth century, but apart from some attempts to teach literacy, usually concentrated on religious ends.

introduced. Teaching restricted to this unappetising diet and solely with the passing of tests in mind gave what the leading inspector, Matthew Arnold, called 'a mechanical turn to school teaching' that must be 'trying to the intellectual life of the school'. (*Minutes of the Committee of Council* 1869–1870, p. 291). Yet even these limited objectives were not attained, in 1873 only 15 % of pupils passed the examinations at the standard appropriate for their age, 57 % were a year, and 26 % 2 years, too old. This system¸ which helped raise standards in the weaker schools, but at the cost of inhibiting excellence and initiative among the better teachers, was abandoned in 1897 (Wardle 1976; Horn 2010).

### 2.2.3 State Elementary Education Introduced

Demands for greater, state-directed provision of elementary education grew, however, responding to data such as those disclosed by Arnold in 1868 (Howson 2010, pp. 49–51), for example, the rates of illiteracy in the English army (57 %) compared with those in the armies of France (27 %) and Prussia (2 %), and comparisons of economic growth. The result was the Education Act of 1870 which established school boards throughout the country with the duty to build schools where denominational schools did not already exist and to provide education up to the age of 13. In 1880 attendance became compulsory for all children, although many denominational schools still charged for attendance: not until 1918 was elementary education free for all.

## *2.3 1800–1902 Secondary Education*

### 2.3.1 Early Provision

By 1800 secondary education (i.e. post-elementary education, although that adjective was never used at the time) was supplied by three bodies. First were the nine great 'public schools', including Eton, Harrow and Winchester – largely attended only by the children of upper-class parents; second were the endowed grammar schools; and third were private schools both for boys and for girls (see De Bellaigue 2007 for the latter). Although the private schools might teach some mathematics other than arithmetic, the subject did not feature significantly in the curricula of the public and endowed schools although, for example, Winchester and Eton offered optional lectures in mathematics from visiting lecturers for additional fees (Howson 1974). The first move to change the *status quo* failed, for an appeal in 1805 by Leeds Grammar School to abandon the humanist curriculum set out by its founder and to allow it to teach mathematics and modern languages was rejected by the Lord Chancellor. Not until 1840 were schools given complete freedom to revise their curricula, although concessions were made which resulted in Winchester and Eton Colleges appointing their first mathematics masters in 1834 – several centuries after their foundation.

The curricula in those schools that opted to teach mathematics soon settled into a stable pattern of arithmetic, algebra and, most importantly, Euclid. The last was not only classical, and so more aligned to humanist aims than vocational ones, but, to quote De Morgan (1831), better served 'the great end [of mathematics teaching], the improvement of reasoning powers' (see Howson 1984). Nevertheless mathematics still did not enjoy esteem within the curriculum comparable to the classics (Howson 1974).

### 2.3.2 New Schools and New Demands

Matters changed, though, with the creation of a new type of school and with new demands from the armed services for would-be officers to demonstrate some mathematical ability. Upper middle-class parents who were unable to send their children to the great public schools wanted something better than was being provided by the endowed grammar schools. As a result, from the 1820s, 'proprietary' schools, funded, organised and managed by a committee, began to be established throughout the country. These were seen as worthy alternatives to the older public schools and many provided 'modern' curricular alternatives for older pupils, specifically designed to meet the demands of the armed forces. Their popularity led to a government commission being established in 1861, chaired by the Earl of Clarendon, to investigate the management and curricula of the nine public schools. The resulting report (1864) precipitated great changes in their curricula and in the way that mathematics was taught – changes that because of the prestige of the schools, the high quality of the mathematics teachers they attracted and the textbooks they produced had considerable effects upon mathematics teaching throughout the secondary school sector (Howson 1982, 2010).

### 2.3.3 More General Secondary Education

A further government commission, the Taunton Commission, was established in 1864 to consider the endowed grammar, private and proprietary schools. Its recommendations, published in 1868, were that three grades of secondary education should be established, one for each of the three social classes: thus emphasising the established view that nothing should be done that might disturb the prevalent social class system. Its report provided information concerning the varied treatment of mathematics in the grammar schools. There was a great divergence in aims: some schools offered wide-ranging mathematics courses, others aspired only to Euclid Book 1. While some schools studied Euclid, others used the practical geometry books written by practitioners rather than university fellows.

Yet already moves had begun that initiated a move towards a more standardised secondary mathematics school syllabus. Prompted by a local competition held in Devon in 1856, the universities of Oxford and Cambridge established school examinations set on syllabuses laid down by the two universities and awarded certificates to successful candidates.[3] The newer universities followed suit, and once entry to university began to depend upon success in such examinations, rather than recommendation by graduates, a rough (for there were differences in the syllabuses offered) national curriculum began to be formed. Euclid was still a major feature, but other topics were logarithms, algebra to simple equations (or quadratic for the more advanced), sequences, trigonometry, mensuration (calculation of lengths, areas, volumes) and some applied mathematics (mechanics, statics and dynamics) (Howson 1982, 2010).

### 2.3.4 The First Mathematics Teachers Association

The emphasis on Euclid, and a growing feeling that it was outdated and inappropriate, led in 1871 to the creation of the Association for the Improvement of Geometrical Teaching (AIGT), probably the world's first subject teachers association. As its name suggests, the AIGT argued for a replacement for Euclid, an aim already dismissed by a committee of the British Association set up in 1869 (including Cayley, Clifford and Sylvester) which thought nothing so far produced 'is fit to succeed Euclid'. The AIGT produced its own course, but it was not to prove a success or be widely welcomed by universities. Undeterred, the AIGT broadened its interests to other branches of mathematics teaching and in 1894 published the first number of its *Mathematics Gazette*, before changing its name to 'The Mathematical Association' (MA) in 1897 (see Price 1994).

[3]London University had, in 1838, shortly after its creation, established a matriculation examination that could be taken by students other than those attending the university or those who did not wish to proceed to a degree. The 1838 syllabus for mathematics and a specimen examination paper set that year can be found in Howson 1982, pp. 214–216.

### 2.3.5 A New Curriculum and Perry

An alternative to the classically dominated curriculum was offered by the Department of Science and Art (DSA), a governmental body established following the 1851 Great Exhibition. Grants were given to any school that would teach the more scientifically and technically oriented DSA curriculum, and further payments were made according to its pupils' results. Such classes were given mainly in private schools and in endowed schools experiencing financial difficulties. After 1870, publicly funded 'central' or 'higher grade' schools were also created in the larger cities intended to follow on from the elementary schools and with the children of the working class in mind (although they came to be dominated by those of the lower middle class) (Wardle 1976). They too made use of DSA funding but their number was small and their distribution throughout the country far from uniform (Horn 2010). Even more significantly for mathematics education, the DSA encouraged experimentation. In particular, in 1899 a new syllabus for evening courses was introduced, largely based on the work of John Perry who in 1900 published proposals for a new school syllabus in mathematics (reprinted in full in Howson 1982, pp. 222–224), the content of which was, in time, to be absorbed into school syllabuses. It included topics such as the slide rule and its underlying principles, the use of Simpson's and other rules for estimating the area of irregular figures, volumes of three-dimensional bodies, practical means of finding areas and volumes, using squared paper (displaying data, interpolation, Cartesian and polar coordinates, solving equations, finding maxima and minima, etc.) (see Brock and Price 1980), scalars and vectors, and differentiation. (See also Perry 1902; Price 1986)

### 2.3.6 Teacher Education

The last quarter of the century saw a growth in the provision of training and the establishment of Diplomas in Education for those who wished to become teachers in secondary schools (see Rich 1933; Howson 2010).

### 2.3.7 The Public Schools Set a New Pattern

By the late 1890s some students entering Cambridge had already covered so much mathematics at school (in G. H. Hardy's case, Winchester College) that they were allowed to sit the Mathematical Tripos (Cambridge examinations) at the end of six terms rather than the usual nine. Indeed the 1893 Cambridge *Student's Guide* warned prospective mathematics students that most students now arrived with 'some knowledge of Co-ordinate Geometry, Differential and Integral Calculus and Mechanics' (Warwick 2003, p. 260). This gives a clear indication of how the mathematics curriculum had developed in the public and proprietary schools from which, at that time, the vast majority of Cambridge students were drawn.

### 2.3.8 The Government Forced into Action

In 1899, an auditor disallowed public expenditure on 'higher elementary' classes on the grounds that such had never been approved by Parliament. This led to a court case that in 1902 effectively forced the government to take action: the outcome was the Education Act of 1902 abolishing school boards and replacing them with local education authorities (LEAs) which were given authority to establish new publicly funded secondary and technical schools and also to further the provision of elementary education.

## 2.4 *1902–1988: Changes in Elementary/Primary Education and Some General Patterns of Development*

### 2.4.1 Until the 1944 Education Act

Between 1902 and 1944, there were no major changes in the form of what was then called in England elementary education (other than the school leaving age being raised to 14 in 1918). The distinction between that and secondary education became much clearer – even in the salaries paid to teachers and the social status they enjoyed (Wardle 1976). Elementary school teachers were now better qualified and trained. Indeed, in 1905 the Board of Education issued a *Handbook of Suggestions for Teachers* containing in its Preface:

> The only uniformity of practice [sought] in Public Elementary Schools is that each teacher shall think for himself, and work out for himself such methods of teaching as may use his powers to advantage and be best suited to the particular needs and conditions of the school.

This rather remarkable tolerance of diversity is something which distinguished mathematics education in England, at both primary and secondary levels, from that in many other countries. Not until 1988 did a government take firm steps to change it. As a consequence the history of mathematics education in this period is one in which many individuals were able to have a considerable influence on teaching, although their names are not so well known internationally as those of, say, Rousseau, Pestalozzi, Montessori or Steiner.

Yet despite this apparent freedom, the tradition of payment by results was so deeply entrenched that many schools continued to be organised by standards and the curriculum, especially for the younger pupils, did not change significantly. (The Board of Education's Standards for 1905 are reprinted in Howson 1982, pp. 224–225.) The curriculum varied greatly in the senior grades, and in some parts of the country secondary education (post age 11), other than in grammar schools, became available to pupils. Some newly built elementary schools were also able to add subjects such as woodwork and domestic economy to the older pupils' curriculum but in country areas, however, such facilities were unlikely to exist and the curriculum remained very restricted. Teaching to the age of 11 became geared to the passing of the 'scholarship examination' that would provide pupils with free secondary education in a grammar school – for many such had been created, post 1902, by the new LEAs. The examination included a paper on 'Arithmetic' and so mathematical work was directed towards the attainment of a good mark in this paper. A typical paper from 1920 is reproduced in Howson 1982, pp. 229–230. Considerable technical facility is demanded and the questions are by no means straightforward. Drawing to scale (probably only with the use of squared paper), proportion and mensuration were expected. Relevance to practical matters is not very evident in questions such as 'Find the value of 13 2/3 – ¾ of 5 1/6′ and 'An iron article weighing 4 cwt. 7 lb is 16 times as heavy as a wooden model of it. The model consists of five parts, four of which weigh 8 lb. 7 oz., 5 lb. 3 oz., 6 lb. 11 oz. and 4 lb. 8 oz. respectively. Find the weight of the fifth part.' (For readers unfamiliar with imperial measures: 1 cwt. = 112 lb. and 1 lb = 16 oz. – a reason to be grateful for metric measures!). Some geometry was taught, but this related mainly to the names and properties of common two- and three-dimensional objects. This meagre diet led the Mathematical Association in 1938 to establish a committee 'to consider the teaching of mathematics at primary level'.

### 2.4.2 The 1944 Education Act and Primary Education

The war interrupted the work of the MA committee and when in 1946 a new committee was appointed it soon set aside the substantial pre-war work. By then, also, the 1944 Education Act had provided state secondary education for all children (see below) and so established a clear divide between primary (5–11) and secondary education (post-11): the term 'elementary education' fell into disuse. The new MA committee was much influenced by one of its members, Caleb Gattegno, who had just returned from working with Piaget and accordingly in its report (MA 1955) decided to deal with *mathematics*, not simply *arithmetic*, and to concentrate on the ways children think and the activities they find satisfying.

> Practice without the power of mathematical thinking leads nowhere; the power of mathematical thinking without practice is like knowing what to do but not having the skill or tools to do it; but the power of mathematical thinking supported by practice and rote learning will give the best opportunity for all children to enjoy and pursue mathematics as far as their individual abilities allow. (p. 4)

The document continues:

> We plead … for attempts to develop mathematical ideas through the study of broad environmental topics and through the investigation of situations and phenomena at first hand. (p. 20)

### 2.4.3 Change and Development

The MA report, but more effectively the many courses for teachers based on it, began to have great effect in primary schools and on teacher training. The 1950s were also to see the introduction of new materials into schools, Cuisenaire rods, Dienes' Multibase Arithmetical Blocks – and later his logic blocks, amongst others.

Even greater changes arose from the work of the Nuffield Primary Mathematics Project established in 1964 and the publication in 1965 by the newly established Schools Council for the Curriculum and Examinations[4] of *Mathematics in Primary Schools*, written by an outstanding schools inspector, Edith Biggs. Their treatment of certain mathematical topics differed, but both, along with the MA report and the Association of Teachers of Mathematics' (ATM)[5] *Notes on Mathematics in Primary Schools* (1967), only gave teachers advice, examples of lessons and guidance (in the form of multitudinous volumes in the case of the Nuffield Project). The design of an actual course was left to the teachers or, in practice, to textbook authors who supplied series 'based on' the advice and guidance offered and which, in general, left much to be desired. Later a scheme based on individual learning was produced for pupils aged 7–13 by the School Mathematics Project, but these left the teacher with too little to contribute to lesson planning as well as providing students with too unvaried a diet. Great improvements in the professional development of teachers came with the establishment by the Nuffield Project of teachers' centres (Corston 1969) and, following their success, the LEA local centres. Unfortunately, these, along with many mathematics adviser posts, were to disappear in the 1980s as LEA responsibilities and funds were cut. Television lessons, at primary and secondary level, did prove a continuing aid to teachers, particularly once they could be recorded and used when desired, rather than when classes had to be planned around the time of their transmission.

The effects of these initiatives on the actual curriculum were varied. Sets and multibase arithmetic came into many schools, but then gradually disappeared. Data gathering and display came in and stayed. More emphasis came to be placed on geometry and on number patterns in the hope that the

[4] A governmentally financed body, the Schools Council was established in 1964 as a consortium of interested bodies in which school teachers were dominant, mounted for 10 years various projects within the field of mathematics education. It was later reconstituted as a result of financial stringency and criticism before it was replaced in 1982 by separate examination and curriculum councils. These were later amalgamated and have since changed names and exact purposes at regular intervals.

[5] The ATM, created in 1962, evolved from the Association for Teaching Aids in Mathematics (ATAM 1952). Its journal *Mathematics Teaching* first appeared in 1955. The ATM grew rapidly in the 1960s and its appeal to primary and modern-school teachers led to its soon having more members than the older MA. The ATM web site contains a fascinating history of its early years and the mathematical and teaching concerns of its early leaders.

latter would facilitate the later learning of algebra. This naturally meant that less time was spent on the learning of arithmetic with the expected results and public reaction. Often, and particularly after the National Curriculum was established following the 1988 Education Act, primary school children in England tended to be introduced to concepts far earlier than were children in other countries (Howson et al. 1999).

#### 2.4.4 Teacher Training and the Beginnings of Research in Mathematics Education

The training of primary school teachers entered a new era in the mid-1960s: ‘Training Colleges’ became ‘Colleges of Education’, a 2-year course was replaced by a 3-year one, and gradually ‘certificate’ courses were phased out as students aspired to a B.Ed. degree and the teaching profession moved to becoming all graduate. Again, Open University television programmes also proved valuable in-service training. Primary education was the focus for much early research in mathematics education, and the need for, and the value of, educational research was recognised in 1946 by the establishment of the National Foundation for Educational Research. It was not, however, until the creation of the first two university chairs in mathematics education in the late 1960s that research in mathematics education really became established (Howson 2009).

### *2.5 1902–1988 Secondary Education*

#### 2.5.1 1902–1944

The establishment of state secondary education immediately raised the question of what form it should take. The answer was to base the new grammar school curriculum firmly on that of the old public and proprietary schools. The mathematics curriculum was still effectively determined by university entrance requirements and by the syllabuses of the various university examining boards. This meant that the differential calculus came to be taught in most grammar schools and high standards were expected of students. A typical 1910 ‘Analytical Geometry and Differential Calculus’ paper for 18-year-olds is reprinted in Howson 1982, pp. 225–226. The questions on conic sections would be meaningless to most 18-year-old students these days, and the technical demands of the calculus and the ability to ‘explain a method’ would also be excluded.[6] However, much has entered the mathematics syllabus of all 16–18 schools in the meantime, most obviously integral calculus, and, even more importantly, the examination is now taken by a far greater percentage of the population.

The mention of ‘analytical geometry’ indicates that the rule of Euclid had finally come to an end. This happened when Cambridge, in 1903, decided that it would accept any proof in geometry that appeared to form part of a systematic approach to the subject. Various courses in geometry were to appear in the next decade: the most influential being a rethought and pupil-centred, ‘watered-down’ Euclid by C. Godfrey and A. W. Siddons which continued to be used in some schools for the next 60 years. More surprising was a transformation geometry textbook published by W. J. Dobbs in 1913. An MA report on the teaching of geometry, published in 1923, was to consolidate the teaching of this topic in a ‘Godfrey and Siddons’ form for some 40 years. Indeed, as in other countries, the years after the First World War were times of retrenchment rather than innovation.

A report prepared by Godfrey for CIEM (ICMI) in 1912, showed that much new work had been introduced into schools in the past 10 years: about half of the 370 schools he surveyed (probably with

[6] The broadening of the curriculum at the loss of gaining technical fluency in a limited number of areas raises interesting questions as to which is easier to gain away from the classroom: knowledge of new areas or technical fluency and confidence?

the non-state schools over-represented) undertook out-of-door practical work on surveying and over half descriptive geometry (in the sense of Monge). Elementary statistical work was a common feature and about half claimed to use vectors in the teaching of mechanics and/or complex numbers. Godfrey had himself established a laboratory for practical mathematics at Winchester College where he taught, which was copied in several other public schools and which included many experiments on applied mathematics (Board of Education 1912, vol. 1, pp. 393–428 – a publication which together with Wolff 1915 are two key references for this period). By the 1920s, however, such work moved to the domain of the physics teachers. In fact, grammar school mathematics syllabuses were to show little change until the reform period of the 1960s. Students up to the age of 16 (when many in the grammar schools left after taking an examination known as the School Certificate) studied arithmetic, algebra up to quadratic and simultaneous equations, a geometry course loosely based on Euclid, logarithms and trigonometry. (A typical 1934 School Certificate paper is reprinted in Howson 1982 (pp. 232–233), along with proposals made in 1944 for a new School Certificate syllabus (pp. 234–237).

Beyond School Certificate (16–18), students began to specialise, taking usually no more than four subjects on which they were examined when seeking the Higher School Certificate. Those continuing to study mathematics, for its study was no longer compulsory, could take it as two separate subjects 'Pure' and 'Applied', or as one combined subject (but some universities demanded that students wanting to study for an honours degree in mathematics had to take 'double'-subject mathematics). The Pure syllabus typically comprised further algebra (e.g. properties of roots of polynomials), co-ordinate geometry, conic sections, the theorems of Ceva and Menelaus, further trigonometry and differential and integral calculus. The Applied papers tested dynamics, statics and, to a limited extent, hydrostatics (topics which in many countries were considered part of physics). For those wishing to obtain scholarships from universities, state or county, there were 'Scholarship Papers' of a more testing kind.

### 2.5.2 Three Government Committees and the Form of State Secondary Education

A government committee (Hadow) had reported in 1926 that some form of post-primary education should be provided for all children and suggested that three types of secondary schools were needed: grammar, 'modern' and technical (Trade) schools. Some local authorities attempted to put this plan into practice, but little was done to make such a scheme universal for the resources needed to put the proposals into practice were non-existent. A later committee (Spens) reported in 1938 on secondary education in grammar and technical schools and its report included some interesting remarks concerning mathematics teaching: 'The content of school mathematics should be reduced', its teaching suffered 'from the tendency to stress secondary rather than primary aims', it concentrated too much on 'tricky problem solving' rather than giving a 'broad view', the type and 'rigour' of the logic it presented had 'not been properly adjusted to the natural growth of young minds', … (pp. 235–242). The virtues of Hadow's 'tripartite' system were upheld by the Norwood Committee when it reported in 1943. As one critic, S. J. Curtis, wrote, according to Norwood the Almighty had benevolently created three types of children in just those proportions which would gratify educational administrators and, moreover, which class a child belonged to was clearly to be observed by the age of 11 (quoted in Howson 1982, p. 277).

[7]The move to comprehensive schools within the state system has been a long one and is by no means complete. London had 11 by 1957 and there were others in rural areas such as Cumberland. However, in 1965 the new Labour Government circulated all LEAs requiring them to draw up plans to convert to comprehensive education. This circular was rescinded by the Conservative government when it took office in 1970, but by then, the plans were so far advanced that, in fact, more comprehensives were created under that government than under any other. Yet some English (but no Welsh) authorities still retain grammar schools, as does Northern Ireland. Since 1988 various new types of schools have arisen and the situation is now extremely complex, but outside the remit of this chapter.

### 2.5.3 The 1944 Education Act

This important act, produced in the closing years of the Second World War, followed the lead given by Hadow and Norwood and established secondary school education for all based on a tripartite system with entry to state grammar schools being dependent on passing an examination taken at 11. The school leaving age was raised to 15 and when possible (in the event 1972) to 16. Entry to state grammar schools was now to be solely on merit. Previously parents could purchase a state grammar school education for their children at a relatively small cost, provided the school would accept them. Now, middle-class parents of children who failed the '11+' and who could not afford to send their children to a non-state school were often dissatisfied by the education provided, and the status enjoyed, by the new secondary modern schools. They were to play a part in replacing the tripartite system, in the 1970s, by local comprehensive schools: as was, more importantly, the growing belief that valid decisions concerning a child's future could not be taken at the age of 11.[7] (Nevertheless, selection had thrown an academic lifeline to able children from poor working class districts.)

The percentages of children who attended a grammar school varied from 10 % to 30 % depending on their local authority, but in 1961, according to official figures, only 22.1 % of pupils in England and Wales were in maintained (state) grammar schools. The technical schools intended to supply pupils with a specialist form of practical education had a mere 3.1 % of pupils, and 10.4 % were in independent or 'direct grant' schools (the latter occupying a middle position between state and independent schools: a position that ceased to exist when, later, such schools had to choose between becoming comprehensive or independent – the vast majority choosing the latter option). The majority of pupils were, then, in the secondary modern schools created following the 1944 Education Act.

In the late 1940s revisions had been made to the school examination system. School and Higher School Certificates were replaced by Ordinary Level and Advanced Level General Certificates of Education (O-level GCE and A-level GCE), usually taken, respectively, at age 16 and 18. In theory, these were qualifications intended for grammar school pupils. However, increasingly secondary modern schools began to enter their more able students (some of whom, had they lived elsewhere, would have qualified for a grammar school education) for some O-level GCEs. This was thought to be distorting the general curriculum of such schools, and so in 1965 a new Certificate of Secondary Education (CSE) was created intended to satisfy their particular needs. Again, diversity was actively encouraged, for schools were offered three options: to be examined externally on a board's syllabus, to be examined externally on the school's own syllabus and to set their own examinations, with external moderation, on their own syllabus (examples of papers set are reprinted in Griffiths and Howson 1974). This did not deter schools from entering 'border-line' pupils for both GCE and CSE examinations (with the result that the latter became, in practice, merely watered-down GCE ones), and in 1986 the difficulties caused by running GCE and CSE courses in parallel in comprehensive schools led to their abandonment and replacement by a General Certificate of Secondary Education intended to serve the needs of all types of student.[8]

### 2.5.4 Curriculum Reform

By the 1960s dissatisfaction was beginning to grow both with the curriculum that had not changed significantly for many years and the way that mathematics was being taught. An MA report, published in 1959, *Mathematics in Secondary Modern Schools*, made many useful suggestions, but was

[8] The problems this caused, for it did nothing to solve the great problem of varying mathematical abilities in pupils, and the subsequent claims of a consequent 'dumbing down' of the GCE have meant many independent and, more recently, a few state schools opting for other international qualifications.

criticised by Cyril Hope, a leading figure in the ATAM and ATM, for the backward-looking nature of the mathematical content (Secondary School Mathematics 1959, pp. 37–38).

However, encouraged by the publication of a radically new text by D. E. Mansfield and D. Thompson, and by what was happening in other countries – particularly the United States, a variety of projects sprang up from 1962, of which the most important were the School Mathematics Project (SMP) (ages 11–18), Mathematics in Education and Industry (MEI) (ages 16–18) and the Midland Mathematics Experiment (MME) (ages 11–16). SMP's early work (including syllabuses and examination papers) is documented in Thwaites (1972) and Howson (1987, 2009), and an account of MEI's history can be found on its web site, http://www.mei.org.uk. Howson (1978) and Watson (1976) provide general surveys of these and other projects, and their social and educational contexts are described in Cooper (1985); a brief account of a variety of textbooks used in English schools (primary and secondary) in the 1950s and 1960s is in Breakell (n.d). MME differed from SMP and MEI in that, from its initiation, it directed its work to secondary modern schools in addition to grammar schools. It failed to make a lasting impact, not on mathematical grounds but because it lacked the money that SMP and MEI were able to attract and because the schools attached to it did not have the prestige and status of those connected with those two projects.

The reforms did not emphasise sets as much as, say, the SMSG in the United States or abstract algebra as in France. Much new material such as co-ordinate geometry, probability and statistics entered the 11–16 curriculum and has stayed there. Other innovations, such as transformation geometry, stay only in an emaciated form, and other topics simply disappeared – sometimes because of the changed nature of schools and examinations. (The SMP's transformation geometry soon lost its initial goals – including a first introduction to matrices and, from ages 16–18, group theory and linear algebra. Providing a sound geometry course has remained a major problem (Royal Society 2001).)

### 2.5.5 Increased Diversity and Attempts to Bring Unity

These curricular innovations were made possible because of the freedom given to schools, or groups of schools, to create their own syllabus, provided that an examination board would agree to set examinations on it. This led to a great number of syllabuses on offer to schools with very different mathematical demands – and schools were free to select the examination board (now with greatly diminished university influence) and syllabus most suited to their own goals. (See, e.g. the examination papers reproduced in Thwaites 1972 and Griffiths and Howson 1974.) In the 1970s it was estimated that about a third of secondary schools were still following a traditional-style syllabus, a third modern ones, and the remaining third hybrids. It was a time of curricular chaos and the first attempt to rectify this came when governmentally established bodies over-seeing examinations drew up lists of 'core' items that had to be present in all curricula. Such restrictions made innovation increasingly difficult. Moreover differences began to grow in what was taught, and how, to more able pupils in the better independent, fee-paying schools and those in state comprehensive ones (see, e.g. the two 1980s SMP series: the New Books 1–5 used almost only in independent schools and the 11–16 series used in the majority of comprehensive ones).

Many questions came to be raised about the mathematical standards of school leavers and this resulted in the establishment of a government enquiry which led in 1982 to the publication of the Cockcroft Report. This set out many ideas for improving the teaching of mathematics at all levels, but, like so many other reports of its type did little to solve any problems. More significant were the projects for low attainers that were established in its wake: the Low Attainers Mathematics Project (LAMP), Raising Achievement in Mathematics Project (RAMP) and the SMP Graduated Achievement Project. Basically, the problems consequent upon the establishment of comprehensive schools and the greater number of students aspiring to 18+ qualifications and university entrance have still to be solved in a satisfactory manner.

It was in an attempt to end such diversity and ensure that all schools had the same goals, which in 1988 an Education Act was passed which established a National Curriculum in Mathematics for students aged 5–16 to be followed in all state-funded schools (but not necessarily in independent ones). The hastily assembled curriculum (see Howson 1991) was designed to fit into a controversial and untried scheme for testing students at various attainment levels at ages 7, 11, 14 and 16. The years since then have seen continuous attempts to solve the problems created by the poorly designed curriculum, the testing proposals (used in incompatible ways reminiscent of 'payment by results', i.e. to assess pupils' progress and to rank schools for accountability) and more general social changes. At the time of writing, yet more changes – and even some diversity – are promised!

## 3 Scotland

**Leo Rogers**
University of Oxford, Oxford, UK

### *3.1 Prelude*

In the past, Scotland suffered from periods of instability, and incursions from England, but developed independent trading and political links with many countries (Mitchison 2002) For more details on the general situation described here, the reader is referred to the works of Knox (1953), Hunter (1968) and Rogers (2012). These connections as well as economic contexts and differences in religion influenced the country's culture and its legal and education system. General histories of both education systems can be found in Gillard (2011) and Scotland (1969).

Scotland followed the typical pattern of European education with the Church organising elementary schools in the main Burghs (local administrative regions) and parishes where writing and basic elements of arithmetic were taught in the vernacular and by the end of the fifteenth century some of these schools admitted girls (Wright 1898, pp. 244–246). A few grammar schools were attached to cathedrals or abbeys, to train boys for the priesthood. In 1496 a Scottish Parliament Act required all sons of landowners to attend grammar schools and the landowners to provide funds for the upkeep of the schools and a salary for the teachers (Wright 1898, pp. 47–48).

The University of St Andrews was founded in 1413, followed by the Universities of Glasgow (1451) and Aberdeen (1495). In 1574 the humanist Andrew Melville (1545–1622) was appointed to Glasgow University and began a liberal arts programme with a 4-year curriculum (for his biography see University of Glasgow n.d.). In 1580 he moved to St Andrews and continued his curriculum there (Wright 1898, pp. 70–81; Wilson 1935, pp. 7–19). In 1582 Edinburgh University was founded and followed the same tradition.

By the end of the sixteenth century, Scotland had free primary and secondary education and a university system with an established humanist curriculum of 4 years and these remain as a significant feature of Scottish Education. Following Henry VIII's Reformation, John Knox (1514–1572) reformed the Church of Scotland on Presbyterian principles. The Scottish Parliament renounced the authority of the Pope and redistributed the wealth of the Church to the ministry, the schools and the poor. John Knox, with five colleagues, wrote *The First Book of Discipline* (1560) that required all citizens to read the Bible, so literacy became important (Laing 1895, vol. 2, pp. 183–260). Free education for the poor was established, so Scotland had a public education system committed to widespread literacy and funded in part, by both Church and State (Wright 1898, pp. 82–92).

Education Acts of 1633 and 1646 extended taxes to municipalities, providing a better foundation for schools, and another Act of 1696 regulating elementary education continued in force until 1872.

Mathematics books written in English appeared in the sixteenth century, and *Abacus*[9] mathematics was taught in the vernacular. Parish and Burgh schools provided elementary and some secondary education. In spite of the emphasis on the 'three Rs' (reading, writing and arithmetic), the erratic attendance of children from an agrarian society meant that numeracy and literacy were poor. By the end of the seventeenth century, practical arithmetic consisted of the four rules in integers, vulgar fractions and the rule of three, but extraction of roots was often left until the university (Yealdham 1936, pp. 75–87). Other aspects of mathematics were not taught in most Scottish schools until the end of the century (Wright 1898). However, state schools were not providing the mathematics people needed for commerce or practical occupations, so private schools for writing and arithmetic and some 'commercial academies' were set up by independent teachers from the early seventeenth century.

By the end of the century, mathematics was becoming well established in the universities, including the teaching of Euclid's Elements, plane and spherical trigonometry, astronomy and navigation and algebra up to quadratic equations (Wilson 1935, pp. 23–26). The contribution of well-known individuals as John Napier and James and David Gregory to the development of mathematics is described in detail elsewhere (see, e.g. The MacTutor History of Mathematics archive is a website at St Andrews University: http://www-history.mcs.st-and.ac.uk/history/).

## 3.2 *The Eighteenth Century*

In 1707 the English and the Scottish Parliaments signed the Treaty of Union by which the two kingdoms became the Kingdom of Great Britain with one Parliament based at Westminster, London. This treaty maintained the religious, legal and educational independence of Scotland.

### 3.2.1 The Universities

The work of a number of well-known mathematicians (particularly Simson, Stirling, Maclaurin and Playfair) who all taught at one or other of the universities began to develop university courses. Only two of these figures had any direct impact on the teaching of mathematics in school.

Robert Simson (1687–1768) became professor of mathematics at Glasgow in 1711. His best-known work, *The elements of Euclid*, was published in 1756, is the translation of Euclid (Books I–VI and XI and XIII). Separate editions in Latin and in English were printed, and the English version appeared in many schools, becoming the model for editions by later authors (Simson 1756).

John Playfair (1748–1819) appointed professor of mathematics at Edinburgh in 1765 was also involved in the foundation of the Royal Society of Edinburgh. In 1795 he published an edition of the *Elements* for use by his students. His work began to standardise the notation for points and sides of figures and introduced his alternative to Euclid's parallel axiom (Playfair 1795, pp. 295–296).

### 3.2.2 Writing Schools and Commercial Academies

Frequently in competition with the state system, writing schools taught arithmetic and practical mathematics, surveying, navigation, basic trigonometry and occasionally algebra. Writing schools, their teachers and some academies were often subsidised by town authorities to encourage business development. More commercial academies appeared that taught practical arithmetic and geometry to those who needed it for business; for surveying, gauging and astronomy. Navigation was a practical art, part learned at sea, and boys were trained from quite young. Typical textbooks included the use of

[9] The term *Abacus mathematics* indicates the traditional European body of mathematics for business and commerce.

the globe and the celestial sphere, with measure in degrees for astronomical calculations. (A similar development appeared in England (Rogers 1993).)

Some academies had a wider curriculum including foreign languages and took pupils from elementary and grammar schools for professional training. This influenced some grammar schools, forcing changes in their mathematics curriculum, but others resisted this attack on their monopoly of classical secondary education.

Founded in 1760, Perth Academy provided an ambitious programme in the higher branches of arithmetic; mathematical, physical and political geography; algebra, including the theory of equations; differential calculus; geometry, consisting of the first six books of Euclid; plane and spherical trigonometry; mensuration of surfaces and solids; navigation and fortification; analytical geometry and conic sections; and natural philosophy, consisting of statics, dynamics, hydrostatics, pneumatics, optics and astronomy. Later, this curriculum included the binomial theorem for a positive power and evolution of algebraic expressions up to the sixth root and some infinite series; arithmetic and geometric progressions, permutations, combinations and compound interest; probability, co-ordinate geometry, graphs and fluxions. Wilson (1935, pp. 75–79), so some academies, provided a range of knowledge that began to rival the universities.

### 3.2.3 Secondary Schools

At this time a confusing mixture of institutions provided some form of secondary education: traditional grammar schools, academies and writing schools, and even some primary schools whose teachers were sufficiently knowledgeable. There were no formal qualifications or age limits for pupils to progress from primary to secondary schools, nor to university. Lucky pupils progressed if they had the knowledge or ability, or if they were able to pay for private tutors as the growing middle class was able to afford to send its children to the better establishments. However, many pupils left education without any academic or technical qualification.

Algebra as 'generalised arithmetic' became accepted in school mathematics; arithmetic and geometric progressions, permutations and combinations, probability, co-ordinate geometry, graphs and elementary differential and integral calculus were commonly taught. Simultaneous linear equations in two variables were usually solved by substitution and quadratic and cubic solutions were limited to finding one positive root. Geometrical constructions and surveying instruments of the period were in common use.

Arithmetic books developed beyond explaining the basic operations to provide more sophisticated mathematics and many of the practical applications for commercial use. Algebra was finding its way into the general curriculum; both John Mair's *Arithmetic, Rational and Practical* (1766) and Hamilton's *Introduction to Merchandise* (1788) included sections on algebra. Cocker's *Arithmetic* (see Hawkins 1667) was produced in Scottish editions from 1748, and other texts from England were used to prepare pupils for commercial life and entry to the universities. Wilson (1935, pp. 89–91) has a list of over 60 mathematics books of the period showing two thirds were published in Scotland. Many of these were used well into the next century. Trigonometry was included in most books on pure and applied geometry, Wilson's *Trigonometry, with an Introduction to the Use of both Globes and Projection of the Sphere* (1714), became the standard reference work for more than a century, and most books had tables of trigonometrical functions and logarithms.

### 3.2.4 Progression from Primary Schools

Towards the end of this period, girls and boys in primary schools received the same teaching in basic arithmetic as in the previous century. Boys who achieved reasonable success in the 3Rs, proceeded to secondary education, while some girls became bookkeepers in family businesses.

Sufficient knowledge, or the ability to pay fees, enabled progress to secondary school at age 8 or 9 and to enter university at about 12, but ages of transfer began to increase as the secondary and university curriculum expanded. Bright pupils from parish schools were lucky if they met a primary teacher who knew enough to get them into university. For the majority who continued in primary school, their mathematical diet was basic, repetitive and arithmetic.

In the closing years of the eighteenth century, Edinburgh was the centre for the *Royal Society of Scotland* and maintained active and enlightened interest in science. The university chairs were occupied by men who had a competent knowledge of mathematics and who conducted courses efficiently; at the same time advances in knowledge were being made and the level of attainment was high; in the country, some schools included conics and elementary calculus, indicating a provision of teachers and an outlook for their pupils that could only be met by the universities.

### *3.3 The Industrial Revolution and Education of the Workers*

In the 1750s Scotland was a poor, agricultural society with a population of about 1.3 million. But Glasgow and Edinburgh grew, and by the 1850s the total population was some 2.6 million. The Treaty of Union had increased access to the British Empire, and Glasgow developed as a centre of trade, while Edinburgh became the administrative and intellectual centre of the Scottish Enlightenment where the views of David Hume, Adam Smith, Thomas Muir and Adam Ferguson influenced the development of modern social and political theory. These people contributed to the intellectual and scientific life of the whole United Kingdom.

With the development of steam power and the change to manufacturing, entrepreneurs, like James Watt and Thomas Telford, developed engineering and infrastructure; James Young set up the first oil refinery; James Brown founded the shipbuilding industry; these and many others led industrial development in Scotland through the late eighteenth and nineteenth centuries (Whyte 1995).

There was enormous social upheaval. The mechanisation of weaving destroyed the cottage-based industries and drove people into factory work; two famines in Ireland, in 1739–1741 and 1845–1852, drove many people to Liverpool and Glasgow; and the Clearances (the forced displacement of the population of the Scottish Highlands during the eighteenth and nineteenth centuries due to agricultural reform) destroyed the life of the Highlands so that thousands moved to urban centres or emigrated to the colonies.

From the latter part of the eighteenth century, liberal-minded people and factory owners became concerned about the state of the poor and established public parks, libraries and new educational institutions. The improvement in technology provided cheaper books and printed material, improving access to information. During the early nineteenth century, the Westminster Parliament passed a series of acts that restricted child labour and laid down a maximum number of hours of work for adults.

Education for working people began when John Anderson and George Birkbeck at Glasgow University started giving free lectures to workers in the 1780s. Anderson died in 1796, leaving money to provide education for the 'un-academic classes', leading to the foundation of Anderson's College and the beginning of the 'Mechanics Institute Movement'. Institutes were founded to provide technical education for working people and to incorporate fundamental scientific thinking and research into engineering. They revolutionised education in science and technology for ordinary working people. Birkbeck later moved to London and became the first president of the London Mechanics Institute in 1824. Many industrialists supported this movement, and by the mid-nineteenth century, there were institutes in towns and cities across the United Kingdom that provided access to scientific and practical knowledge for working people.

## 3.4 *Education in the Nineteenth Century*

By 1800 the education system had failed to cope with the upheaval of the industrial revolution and was showing signs of strain. In 1834 it was estimated that less than 8 % of the school-age populations of Glasgow and Dundee attended educational institutions, and in the Highlands many inhabitants could neither read nor write. In spite of the emphasis on the 3Rs, erratic attendance meant that the standards were poor. Institutions, like the Church and the landowners, found it increasingly difficult to provide funds for educating a growing population swollen by Irish immigrants. The serious Disruption in the Church in 1843[10] and the need to provide for Roman Catholic pupils increased the problems.

### 3.4.1 Schools and Pupils

By now, the role of Scottish schools had significantly changed. The curricula of the Burgh and grammar schools (the original secondary schools) had met the entry requirements for the universities. However, owing to the competition from the parish and writing schools (originally the primary sector) who were taking subjects to a higher level, the Burgh and grammar schools began to admit primary pupils. Thus, both types of schools became providers of general education. The better primary schools had 'upper classes' with pupils aged 11 or 12 who were achieving a standard equal to some secondary schools (see Scottish School Reforms of the 1870s at The MacTutor History of Mathematics archive is a website at St Andrews University: http://www-history.mcs.st-and.ac.uk/history/).

In 1868 the Argyll Commission showed that over 50 % of students attending Scottish universities had come directly from parish schools (Cruickshank 1967). An education (Scotland) act of 1872 legislated for inspections of parish schools, and in 1882 inspections were extended to all Burgh and grammar schools. This applied also to the academies, which had gained independent secondary status or merged with Burgh or grammar schools. Attendance was low, and finding a job was the real priority for children from working-class backgrounds (Knox 1953) so that under 5 % of pupils attended.

### 3.4.2 Nineteenth-Century Secondary School Mathematics

The mathematics taught at this time can be judged by many surviving documents. Madras College was founded in 1832 in St Andrews by Andrew Bell[11] and gained a high reputation. An account of the classes of one of its headmasters, W. O. Lonie, tells us that from 1846 to 1894, 'their programme exhibited an extensive course of teaching in geometry, practical mathematics, algebra, and geography' (see Lonie's biography at the MacTutor History of Mathematics archive). James Walker's *Fair Book* of 1852, from a pupil at the college, (also found on the MacTutor site) contains pages of typical exercises covering a variety of applications of mathematics; the four rules in arithmetic (with quite large numbers and six decimal places), use of six-figure logarithms; common and vulgar fractions, ratio and proportion, conversion of money and mensuration; capacity and gauging; measurement of areas and other geometric and trigonometric exercises using identities; exercises on the circle, parabola and ellipse; numerical solution of elementary quadratic equations; and questions on 'artillery'. Apart from the explicit mention of some trigonometric formulae and Heron's formula for the area of a triangle, it seems that pupils were taught procedures for solving standard problems.

---

[10] The Disruption of 1843 was a schism within the established Church of Scotland where 450 ministers broke away over the issue of the relationship with the State to form the Free Church of Scotland. It had a serious effect not only on the Church but also on Scottish civic life.

[11] Andrew Bell was one of the founders of the 'Pupil-Teacher' or 'Monitorial' system which he introduced in the late eighteenth century. The method became very popular and was based on the abler pupils being used as 'helpers' to the teacher, passing on the information they had learned to other students.

Finding 'artillery' in a school exercise is not unusual; John Davidson's *System of Practical Mathematics* (1832) is a typical compendium of the 'mixed mathematics'. As well as the usual sections on algebra, geometry, plane and spherical trigonometry, it deals with the measurement of heights, distances, surfaces, and solids, specific gravity, conic sections, land and wood measurement, and artificers (craftsmen) works, gauging, gunnery, geographical and astronomical problems and navigation. Books like this provided an important social function; apart from their use in schools, property owners used them for checking craftsmen's work and calculating payment.

During the nineteenth century there were considerable advances in university mathematics, but in most secondary schools, arithmetic and geometric series and quadratic equations were the limits of algebra, Euclidean geometry was quite formal, and enough plane and spherical trigonometry were taught for practical surveying and navigation.

Important and influential changes in teaching in the second half of the century can be exemplified by George Chrystal's textbooks. Chrystal graduated from Aberdeen University in 1871 and became second wrangler at Cambridge, in 1875. He was professor of mathematics at St Andrews and then in 1879 took the chair of mathematics at Edinburgh University. Chrystal was concerned about standards in schools and in 1883 he supported the foundation of the Edinburgh Mathematical Society by two schoolteachers, demanding better trained schoolmasters and higher teaching standards. Chrystal's lasting contribution to education was his concern with the teaching of mathematics, particularly algebra.

He published the first volume of *Algebra: An Elementary Text-Book for the Higher Classes of Secondary Schools and for Colleges* in 1886. In his preface he warns that the book is not for beginners and says,

> [It] becomes necessary if Algebra is to be anything more than a mere bundle of unconnected rules, to lay down generally the fundamental laws of the subject and to proceed deductively—in short to introduce the idea of *Algebraic Form* which is the foundation of all modern developments of Algebra….

Volume 2 Algebra: An elementary text-book for the higher classes of secondary schools and for colleges followed in 1889.

This 'elementary' textbook was for the 'higher classes', who would have already studied algebra, as far as quadratic equations during their first 3 years of secondary education. The expected level for the higher classes was considerable, and the results of the first leaving certificate examinations were very poor.

Probably the most significant publication that would influence school mathematics throughout the United Kingdom for the next half century was Chrystal's *Introduction to Algebra for the Use of Secondary Schools and Technical Colleges* published in 1898. In his Preface (1898, pp. vii–xii), Chrystal is admitting that his earlier work contained too little practical application, and not enough *graphical illustration* for students in technical colleges. His motivation is to reform the 'English text-books in vogue during the latter part of this century [that] have tended to denigrate [algebra] into a mere farrago of rules and artifices, directed to the solution of examination puzzles of a somewhat stereotyped character having little visible relation to one another and still less bearing on practice' (1866, p. viii).

He wanted students to be aware of *algebraic form* and demonstrated the commutative, associative and distributive properties of arithmetic operations applied in a generalised form and introduced graphical methods. The contents of the book display a progressive demonstration of structure and technique in a rational reorganisation of the material found in earlier texts, and his examples and exercises are deliberately chosen to achieve a standard that set a clear example for many subsequent algebra textbooks.

### 3.4.3 The Leaving Certificate

In 1885, Henry Craik, secretary of the Scottish Education Department (SED), appointed a committee to examine the state of education. Following its report, it was decided to conduct an inspection of all schools teaching higher classes (for pupils aged 15–17). Thus began the first move to achieve common standards for secondary education. Chrystal took part in the inspections, and the report showed many school staff were inadequate and underpaid, the curricula inappropriate, and teaching methods ineffective.

It was decided that inspections would monitor school programmes and certificates should be awarded to pupils on the result of an examination. This provided a link to the universities and became an important factor in improving teaching methods and results in secondary schools.

Chrystal examined pupils in the higher classes of 12 secondary schools. He designed questions so that pupils who knew their ‘bookwork (routine procedures and basic facts) could just pass. The mathematics covered was geometry, algebra, trigonometry and arithmetic, and special papers were offered in differential calculus, analytical geometry and geometrical conics (Philip 1992). Mathematics was a compulsory subject in this examination, and all professions required some mathematical competence.

A leaving examination would show that pupils were academically equipped to enter university. However, Chrystal realised that many who ‘failed’ this new examination had reached the level demanded by the General Medical Council and other professional bodies. So it was decided that two levels of certificate (so-called higher and honours higher) would be issued and the first leaving certificate examination was sat in June 1888.

In mathematics, the higher and honours levels consisted of three separate papers, each requiring answers to seven compulsory questions: arithmetic (1½ hrs.) standard questions plus optional questions using logarithms; geometry (2 h) standard questions on Euclidean geometry using memorised proofs and optional questions on solid geometry that might need the use of logarithms and trigonometry; and in algebra (2 h) with standard questions on polynomials, permutations, complex numbers and trigonometry, using the cosine rule, trigonometric expansions and inverse ratios. Since different institutions catered for different professions, questions were written with options to meet their needs. For those who chose honours grade, candidates were encouraged to complete as many questions as possible. Mathematics was the only one of the six leaving certificate subjects to publish notes for guidance from the beginning.

## *3.5 The Scottish Mathematics Education System 1900 to the 1980s*

The first half of the twentieth century saw many changes in educational administration, with increased state intervention as schools were seen as an important agency for social welfare. In 1901 the school leaving age was raised to 14, and in the 1908 Education Act, parents were made responsible for their children’s attendance, medical inspection was introduced, and free meals are given for needy children. After the First World War, the 1918 Education Act created new educational authorities and brought Catholic schools into the state system (Humes and Paterson 1983; Finn 1983). The leaving certificate also underwent many changes in detail, and it played a vital role in the development of secondary education. It raised the standard in secondary schools so that they began to cover work previously studied in the universities.

In the 1920s and 1930s policymakers faced a growing demand for secondary education and divided children into two types: the ‘academic’ and the ‘nonacademic’. To tackle this problem, ‘advanced divisions’ were created in elementary schools to provide post-primary education. In 1936, maintaining the meritocratic system from the nineteenth century, the secondary schools were divided into 3-year junior secondary schools and 5-year senior secondary schools.

By this time, the post-primary education offered consisted of the following:

- Primary schools with advanced division classes, where it was still possible for some pupils to proceed to a junior secondary school or, very exceptionally, to gain a leaving certificate
- Junior secondary schools with a 3-year curriculum that later ran more ‘academic’ classes to enable a few pupils to transfer into the higher class schools, but the majority received no formal qualifications
- Senior secondary (higher class) schools with a 5-year curriculum and the expectation of entry into university or the professions

However, in spite of more reforms in the 1940s, 87 % of young adults in the age group 20–24 in 1951 had left school at age 15 or younger. It was not until the introduction of Comprehensive schools[12] that the inequality between senior and junior secondary schools was finally addressed. In 1947 raising the school leaving age to 15 increased the school population by 40,000. The system required more teachers and serious thought about teaching larger groups of pupils of a wide range of ability for longer than before (Cruickshank 1970).

To describe the problems and development of mathematics education during a relatively prolonged period of time is difficult. Below an attempt is made to do so, relying on figures from certain official reports. For a more detailed analysis of them, see Rogers (2012).

### 3.5.1 The Hamilton Fyfe Report 1947

This report, Secondary education: A report of the advisory council on education in Scotland, chaired by William Hamilton Fyfe (1947) considered compulsory secondary schooling from age 12 to 15 and was an opportunity to reconsider the curriculum and the majority of pupils unable to achieve the leaving certificate because they were judged only suitable for vocational occupations.

The authors thought that different aspects of mathematics should be relevant to the real world and not independent from each other. These views follow the Spens Report where treating aspects of mathematics as if they were different subjects is described as ‘partial, unhistorical and un-philosophical’ (Spens 1938, p. 236).

The report presented recommendations for curriculum reform; re-thinking attitudes, connecting subjects to the culture, teaching methods and administrative organisation. They confronted the system where a junior secondary school pupil had virtually no possibility of transfer to a senior school at age 14 or 15. Proposals were made to develop apprenticeships and semi-professional work and to allow transfer to senior school.

According to the report, mathematics in Scottish schools needed a ‘drastic overhaul’ (paragraphs 441–478); teaching was unsatisfactory and the majority of pupils of average ability suffered from ‘the dullness and futility of much school teaching’. Mathematics was ‘devoid of appeal to ordinary youngsters’ and the nature of the Leaving Certificate in Mathematics made it difficult for all schools to break with the formal academic nature of the subject. Provision for the ‘lower ability’ pupils should be simple everyday arithmetic, easy mensuration and the elements of graph work, with clear practical applications. Diagnostic tests would be more suitable than formal examinations (p. 445).

The report made a number of critical comments and suggestions about teaching specific topics in mathematics and took a practical, visual, tactile approach to the curriculum, encouraging engagement of the senses and making a cultural and aesthetic appeal to the historical development of concepts, art, architecture and the technical and social applications of mathematics.

Finally, a special note was made that girls should have access to a similar course, and since the school leaving age had been raised to 15, the report challenged Scottish teachers to address the problem of suitable practical activities for girls.

### 3.5.2 The Revised Mathematics Syllabuses of 1950 and the Arrival of the Comprehensive System

In 1950 the SED published a series of five new mathematics syllabuses for junior secondary schools, together with some brief teaching notes, and in the same document, syllabuses for Ordinary and Higher courses for Senior Schools. It continued the social stratification; the language of ‘ability’ and

[12] ‘Comprehensive’ in the United Kingdom means that a state school does not select its intake on the basis of academic achievement or aptitude.

division of the sexes was maintained, and pupils would be destined for low-level occupations. Three-year courses had no provisions that would enable transfer to a year 4 or 5, nor ideas that would integrate mathematical topics into a meaningful technical programme.

The SED acknowledged pure mathematics as an abstract subject suited the ablest pupils and recognised that 'for the less well-endowed the academic rigours must be tempered'. The *'need for a greater differentiation of courses has become still more pronounced with the general acceptance of the principle that promotion to the secondary stage should in the main be determined by age rather than by attainment. The raising of the school leaving age has at the same time made it necessary to reconsider existing schemes of work'* (SED 1950 p. 5).

The five syllabuses were intended to suit pupils' abilities, with content progressively modified to relate them to the 'needs of life' as perceived by the Department. The most challenging Syllabus 1 was described as a general course in mathematics for pupils capable of passing the leaving certificate. This contained arithmetic with a commercial bias; algebra with solution of linear and quadratic equations and inverse proportion, including graph work; and geometry with practical knowledge and use of instruments and selected theorems and proofs including some circle geometry, mensuration and elementary trigonometry. Contrarily, Syllabus V was described briefly as a very simple course for pupils of low ability with a limited list of topics concentrating on everyday quantities, units and measurements, probably repeating what they had failed to learn in primary school.

In spite of the clear intentions of the 1947 report (SED 1947), *these syllabuses were all content driven and fundamentally unchanged since the previous century*; the teaching notes emphasised speed and accuracy in calculation and precision with language. *With these attitudes and no change in educational philosophy, schools were unprepared for Comprehensive Education in 1965 when the state grammar and secondary schools were combined and teachers had to deal with teaching the 'academic' and 'less academic' pupils in the same school and often in the same class.* In 1967 the Ruthven Committee report on the Organisation of Secondary Certificate Courses recommended that all pupils should follow a common course (which included mathematics) in their first 2 years of secondary school. This was not intended to be taught at the same rate, use the same methods as 'normal' classes or share a common syllabus. When the school leaving age was raised to 16, a report by inspectors (SED 1972) found that just under half of secondary schools had no clearly defined policy for the first 2 years of secondary education.

### 3.5.3 The Scottish Mathematics Group (SMG) 1965–1975 and Other Important Projects

Facing the challenge of mixed ability classes and the 'new mathematics',[13] the SED established[14] a committee of inspectors, lecturers and teachers in 1964 to prepare a new syllabus and textbooks for an Alternative Ordinary Grade examination for the new Scottish Certificate of Education.

SMG mathematics was, to a large extent, promoted by the work of Geoffrey Sillito, a lecturer at Jordanhill College of Education and a key member of the *Association of Teachers of Mathematics* (ATM), who had arranged conferences where teachers and administrators were challenged to think differently about school mathematics, its content and its teaching. The original Scottish Mathematics Group, who wrote the first texts for *Modern Mathematics for Schools*[15] (SMG 1965a, b) had 16

[13] This term is only very roughly comparable to the 'New Math' of the Americas. The United Kingdom was not driven by government bodies and committees of mathematicians but influenced more by European sources and our own *laissez faire* local and 'grass roots' curriculum development.

[14] This government initiative contrasted with curriculum development in England where individual local authorities, teachers' organisations and publishers were encouraged to develop their own schemes.

[15] This series 'For pupils taking the General Certificate of Education Ordinary Level, or an equivalent examination' was also clearly intended for a wider market and was used by some Secondary schools in England and other countries.

members whose names appeared in the front of each book. The series was published in Glasgow from 1965 onwards.

Some 7,000 pupils took part in piloting in 1964, and the first textbooks appeared in 1965. Since the SED was the only authority responsible for both examinations and curriculum development, all agencies concerned cooperated to train teachers, and the new syllabuses and books were quickly adopted (the Scottish Government does not supply or recommend all books for use in school). The authors claimed that the most appropriate areas of the 'new mathematics' were 'blended with the necessary elements of the traditional'.

The course emphasised the relevance of mathematics in the world today, offering pupils material for developing mathematical ideas, with a balance of exploration and computation. The conflict of the new and old ideologies is clear in the approach towards pupil-centred activities of investigation and discovery, stressing the learning situation and at the same time supplying adequate material 'in a form which could easily be adapted to suit individual schemes and teaching methods'. The textbooks emphasised the structurally integrating areas of algebra, geometry and arithmetic (Trigonometry was introduced in books 5 and 6 and some elementary Calculus in book 7), and in each book the content was organised under these headings, while topics were developed within each section.

Six subthemes are found throughout: the language of sets; number systems; relations, mappings and functions; coordinates and graphs; logic; and deduction. About one third of the algebra and the arithmetic was new, including iterative solution of equations, linear programming, elementary probability and more statistics. The remaining material was approached 'from a new angle or set in a new context'. For teachers brought up on a traditional syllabus, with fixed views about pupils' abilities, this was highly radical. It was not surprising to find strong conservatism among teachers and, over time, much of the original content was modified, and the geometry chapters in particular were largely rewritten (SMG 1967).

Among other attempts to solve the problems of mixed ability classes was the Fife[16] Mathematics Project which started in 1970 and lasted until 1977. The leader of the project Geoffrey Giles described the project as 'an open-ended experiment aimed at developing *teaching methods* for mixed-ability classes in the first 2 years of a comprehensive system'. The educational aims encouraged pupils to take responsibility for their own learning, stimulate understanding through activity and develop self-confidence. A number of evaluations of the project were undertaken including Giles (1977), Crawford (1975) and Morgan (1977). A critique and extended quotations from Crawford and Morgan can be found in Howson et al. (1981) (pp. 217–225).

### 3.5.4 Changes in the Primary Curriculum 1946–1980

Hamilton Fyfe also chaired a report on primary school education, *Primary education: A report of the advisory council on education in Scotland*, published in 1946. It criticised the system that still relied on old traditions and unfounded beliefs; arithmetic was condemned as outdated, and the new proposals for education were radical. Classrooms needed displays and equipment, children should be more active, their senses developed, and subjects no longer taught separately. Teachers should become aware of recent research and educational theory.[17] The report shows concern with transfer from

[16]The County of Fife is a peninsula on the East of Scotland. The county government supported this project. Many other authorities in the United Kingdom supported local curriculum projects at this time.

[17]The Scottish Council for Research in Education (SCRE) was set up by the Scottish Teachers' union (The Educational Institute of Scotland) in 1928. It was the first institution of its kind in the world. It now forms part of the Faculty of Education at the University of Glasgow. (The National Foundation for Educational Research (NFER) in England was founded in 1946.)

primary to secondary education and addresses the means of selection for junior or senior secondary schools, discussing intelligence, attainment and aptitude tests with sensitivity. Numerical test results cannot be ‘final verdicts’ on pupils, and personal and educational development will change possibilities for a child. Unfortunately, teachers were slow to change and primary mathematics remained as training in arithmetic with some simple mensuration (SED 1946).

In 1965, the SED published the ‘Primary Memorandum’ (SED 1965) that set out a philosophy of education starting with the needs and interests of the child, was appropriate to age, aptitude and ability and saw pupils as active in their own learning. This was followed by a progress report (SED 1971) on guidance for teachers offered by education authorities, colleges of education and head teachers. However, progress was still slow. A further report SED (1973) *Primary Education in Scotland: Mathematics* was issued updating the Memorandum, providing national guidelines for the Primary mathematics curriculum. However, a research report (SED 1980) on pupils aged 9 and 12 found only formal aspects of mathematics being taught. Teachers favoured closed responses, and there was very little discovery learning, open-ended questioning and discussion (Hartley 1987).

## 4 Conclusion

Two reports commissioned by the SED published in 1977 were to illustrate again the fundamental conflict in the Scottish education system brought on by raising the school leaving age and comprehensive education (Ewan 1978). The Munn report of the Committee to Review the Secondary Curriculum (SED 1977) recommended some measures in the third and fourth years of secondary school to meet the needs of pupils of all abilities. The Dunning Committee (SED 1977) worked on the aims and purposes of assessment in the higher grade Scottish Certificate of Education and the Certificate of Sixth Year Studies.

However, that the real issues were avoided again and the problem about producing an integrated system that could meaningfully and fairly assess the ‘less able’ remained. In 1983 new National Qualifications were introduced where the Scottish Government (SED 1983) declared its intention to reduce content and place more emphasis on the learning process and skills of a generic nature, and while the content changed as pupils progressed through the secondary school, the three-level certificate remained, leading to Highers and Advanced Highers for university entrance (see Learning and Teaching Scotland (n.d.) for more details).

Since then, we have seen more centralisation of the curriculum and more government intervention in England. Scotland has been able to resist this to some extent, but has been influenced by events over the border, and in 2008 launched a new ‘Curriculum for Excellence’.

## References

Full publication details of governmental reports, projects and other books mentioned in the text but not included in this list of references can be found by consulting the Copac website, http://copac.ac.uk/, which lists all the books contained in the United Kingdom’s leading academic libraries. Descriptions of projects such as LAMP can be found on the Internet and Howson (1982) gives numerous references relating to the history of mathematics education in England.

Board of Education. 1912. *Special reports on educational subjects* (2 vols.). London: HMSO.

Breakell, J. n.d. An analysis of mathematics textbooks and reference books in use in primary and secondary schools in England and Wales in the 1960s. http://faculty.ed.uiuc.edu/westbury/paradigm/breakell.pdf

Brock, W.H. and M.H. Price. 1980. Squared paper in the nineteenth century. *Educational Studies in Mathematics* 11, 365–381

Chrystal, G. 1886. *Algebra: An elementary text-book for the higher classes of secondary schools and for colleges. Part 1*. London/Edinburgh: Adam and Charles Black.
Chrystal, G. 1889. *Algebra: An elementary text-book for the higher classes of secondary schools and for colleges. Parts I & II*. London/Edinburgh: Adam and Charles Black.
Chrystal, G. 1898. *Introduction to algebra for the use of schools and technical colleges*. London: Adam & Charles Black.
Cooper, B. 1985. *Renegotiating secondary school mathematics: A study of curriculum change and stability*. London: Falmer.
Corston, G.B. 1969. Spreading the new ideas. In *Children at school*, ed. A.G. Howson, 157–170. London: Heinemann.
Crawford, D.H. 1975. *The fife project: An experiment in individualised learning*. London: Oxford University Press.
Cruickshank, M. 1967. The Argyll commission report 1865–8: A landmark in Scottish education. *British Journal of Educational Studies* 15(2): 133–147.
Cruickshank, M. 1970. *A history of the training of teachers in Scotland*. London: ULP.
Davidson, John A.M. 1832. *A system of practical mathematics containing geometrical problems, plane trigonometry: The mensuration of heights and distances, surfaces and solids, conic sections, specific gravity, ships, cattle and hay, land and wood and artificers' works, gauging, gunnery, spherical trigonometry, geographical, geodesical and astronomical problems, and navigation*. Edinburgh: Bell and Bradfute.
De Bellaigue, C. 2007. *Educating women: Schooling and identity in England and France 1800–1867*. Oxford: Oxford University Press.
De Morgan, A. 1831. On mathematical instruction. *Quarterly Journal of Education* 2: 264–265.
Ewan, E.A. 1978. Reports in Scottish education: (1) Introducing pack, Munn and Dunning. *Educational Management Administration & Leadership* 7(1): 126.
Finn, M.E. 1983. Social efficiency, progression and secondary education in Scotland. In *Scottish culture and Scottish education 1800–1980*, ed. W.M. Humes and H.M. Paterson, 175–196. Edinburgh: John Donald.
Fyfe, Hamilton W. 1946. *Primary education: A report of the advisory council on education in Scotland*. Edinburgh: HMSO.
Fyfe, Hamilton W. 1947. *Secondary education: A report of the advisory council on education in Scotland*. Edinburgh: HMSO.
Giles, G. 1977. *A comparison of the cognitive effects of individualised learning and conventional teaching*. Stirling: DIME, Stirling University.
Gillard, D. 2011. *Education in England: A brief history.* www.educationengland.org.uk/history
Griffiths, H.B., and A.G. Howson. 1974. *Mathematics: Society and curricula*. Cambridge: Cambridge University Press.
Hamilton, R. 1788. *Introduction to merchandize*. Edinburgh: Charles Elliot.
Hartley, D. 1987. The convergence of learner-centred pedagogy in primary and further education in Scotland 1965–1985. *British Journal of Educational Studies* 35(2): 115–128.
Hawkins, J. (ed.). 1667. *Cocker's arithmetic: Being a plain and familiar method suitable to the meanest capacity for the full understanding of that incomparable art, as it is now taught by the ablest school-masters in city and country*. London: Richard & Henry Causton.
Horn, P. 2010. *The Victorian and Edwardian schoolchild*. Stroud: Amberley.
Howson, A.G. 1974. Mathematics: The fight for recognition. *Mathematics in School* 3(6): 7–8.
Howson, A.G. 1978. Change in mathematics education since the late 1950s: Ideas and realisations: Great Britain. *Educational Studies in Mathematics* 10: 183–223.
Howson, A.G. 1982. *A history of mathematics education in England*. Cambridge: Cambridge University Press.
Howson, A.G. 1984. Euclid: A very English subject. *Sciences et Techniques en Perspective* V: 60–102.
Howson, A.G. (ed.). 1987. *Challenges and responses in mathematics: Essays to celebrate the twenty-fifth anniversary of the School Mathematics Project*. Cambridge: Cambridge University Press.
Howson, A.G. 1991. *National curricula in mathematics*. Leicester: The Mathematical Association.
Howson, A.G. 2009. The origins of mathematics education research in the UK: A tribute to Brian Griffiths. *Research in Mathematics Education* 11(2): 97–114.
Howson, A.G. 2010. Mathematics, curricula and society in nineteenth-century England. *International Journal for the History of Mathematics Education* 5(1): 21–51.
Howson, A.G. 2011. Informal mathematics education in England prior to 1870. *International Journal for the History of Mathematics Education* 6(1): 25–38.
Howson, G., C. Keitel, and J. Kilpatrick. 1981. *Curriculum development in mathematics*, 217–225. Cambridge: C.U.P.
Howson, A.G., T. Harries, and R. Sutherland. 1999. *Primary school mathematics textbooks: An international summary*. London: QCA.
Humes, W.M., and H.M. Paterson (eds.). 1983. *Scottish culture and Scottish education 1800–1980*. Edinburgh: John Donald.
Hunter, S.L. 1968. *The Scottish educational system*, Commonwealth & international library. London: Pergamon.
Knox, H.M. 1953. *Two hundred and fifty years of Scottish education 1696–1946*. London: Oliver & Boyd.

Laing, D. (ed.). 1895. *The works of John Knox*. Edinburgh, James Thin.
Learning and Teaching Scotland. n.d. http://www.ltscotland.org.uk/nationalqualifications. Accessed 14 Nov 2011.
MacTutor History of Mathematics archive. http://www-history.mcs.st-and.ac.uk/history/
Mair, John. 1766. *Arithmetic, rational and practical: Wherein the properties of numbers are clearly pointed out, the theory of the science deduced from first principles, the methods of operation demonstratively explained, and the whole reduced to practice, in a great variety of useful rules*. Edinburgh: Kincaid, Creech & Bell.
Mathematical Association. 1955. *The teaching of mathematics in primary schools*. London: Bell.
Mitchison, R. 2002. *A history of Scotland*, 3rd ed. Abingdon: Taylor & Francis.
Morgan, J. 1977. *Affective consequences for the learning and teaching of mathematics of an individualised learning programme*. Stirling: DIME, Stirling University.
Orme, N. 2006. *Medieval schools: From Roman Britain to Renaissance England*. New Haven: Yale University Press.
Perry, J. 1902. *The teaching of mathematics*. London: Macmillan.
Philip, H.L. 1992. *The higher tradition: A history of public examinations in Scottish schools and how they influenced the development of secondary education*. Dalkeith: Scottish Examination Board.
Playfair, J. 1795. *Elements of geometry: Containing the first six books of Euclid, with a supplement on the quadrature of the circle, and the geometry of solids*. Edinburgh: Bell & Bradfute.
Price, M.H. 1986. The Perry movement in school mathematics. In *The development of the secondary curriculum*, ed. M.H. Price, 103–155. London: Croom Helm.
Price, M.H. 1994. *Mathematics for the multitude? – A history of the Mathematical Association*. Leicester: The Mathematical Association.
Rich, R.W. 1933. *The training of teachers in England and Wales during the nineteenth century*. Cambridge: Cambridge University Press.
Rogers, L. 1993. *The mathematical curriculum and pedagogy in England 1780–1900: Social and cultural origins. Proceedings*. Montpelier: European Summer University.
Rogers, L. 2012. Mathematics education in Scotland: A brief history. *International Journal for the History of Mathematics Education* 6(2): 23–54.
Royal Society. 2001. *Teaching and learning geometry*, 11–19. London: Royal Society.
Scotland, J. 1969. *The history of Scottish education*. London: University of London Press.
Secondary School Mathematics. 1959. *Mathematics Teaching* 11: 37–38.
SED (Scottish Education Department). 1946. *Primary education. A report of the advisory council on education in Scotland*. Edinburgh: HMSO (His Majesty's Stationery Office).
SED. 1947. *Secondary education. A report of the advisory council on education in Scotland*. Edinburgh: HMSO.
SED. 1950. (Reprint 1954). *Mathematics in schools: Syllabuses for 3-year secondary schools, leaving certificate and highers.* Edinburgh: Her Majesty's Stationery Office.
SED. 1965. *Primary education in Scotland*. Edinburgh: HMSO.
SED. 1971. *Primary education: Organisation for development*. Edinburgh: HMSO.
SED. 1972. *The first two years of secondary education: A report of a survey by H.M. Inspectors of the Scottish education department*. Edinburgh: HMSO.
SED. 1977. *The structure of the curriculum in the third and fourth years of the secondary school.* (The Munn Report). Edinburgh: HMSO.
SED. 1983. *16-18s in Scotland: An action plan*. Edinburgh: HMSO.
Simson, R. 1756. *The elements of Euclid viz: The first six books together with the eleventh and twelfth*. Glasgow: Glasgow University.
Scottish Mathematics Group (SMG). 1965a. *Modern mathematics for schools Book 1*. Glasgow: Author.
SMG. 1965b. *Notes on modern mathematics for schools*. Glasgow: Author.
SMG. 1967. *Modern mathematics for schools Book 4*. Glasgow: Author.
Spens Report. 1938. Report of the consultative committee on secondary education. In *Secondary education: With special reference to grammar schools and technical high schools.* London: HMSO.
Taylor, E.G.R. 1954. *The mathematical practitioners of Tudor and Stuart England*. Cambridge: Cambridge University Press.
Taylor, E.G.R. 1966. *The mathematical practitioners of Hanoverian England 1714–1840*. Cambridge: Cambridge University Press.
Thwaites, B. 1972. *The school mathematics project: The first ten years*. Cambridge: Cambridge University Press.
University of Glasgow. n.d. Andrew Melville's biography. http://www.gla.ac.uk/ and http://www.universitystory.gla.ac.uk/biography/?id=WH0004&type=P. Accessed 14 Nov 2011.
Walker, James. 1852. *Fair book*. http://www-history.mcs.st-nd.ac.uk/history/HistTopics/fair_book.html. Accessed 14 Nov 2011.
Wardle, D. 1976. *English popular education 1780–1975*, 2nd ed. Cambridge: Cambridge University Press.
Warwick, A. 2003. *Masters of theory. Cambridge and the tise of mathematical physics*. Chicago: Chicago University Press.
Watson, F. 1908. *The English grammar schools to 1660*. Cambridge: Cambridge University Press.

Watson, F.R. 1976. *Developments in mathematics teaching*. London: Open Books.
Whyte, I.D. 1995. *Scotland before the industrial revolution: And economic and social history, c1050-c1750*. London: Longman.
Wilson, J. 1714. *Trigonometry, with an introduction to the use of both globes and projection of the sphere*. Edinburgh: Kincaid & Creech.
Wilson, D.K. 1935. *The history of mathematical teaching in Scotland to the end of the eighteenth century*. London: University of London Press.
Wolff, G. 1915. *Der mathematische Unterricht der höheren Knabenschulen Englands*. Leipzig: Teubner.
Wright, A. 1898. *The history of education and of the old parish schools of Scotland*. Edinburgh: John Menzies & Co.
Yealdham, Florence A. 1936. *The teaching of arithmetic through four hundred years (1535–1935)*. London: George Harrap.