Chapter 8
Risk Measures in Multi-Horizon Scenario Trees

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Abstract Production assurance requirements are used to ensure that the operation of natural gas transportation networks is robust with respect to flow and production disruptions. They also affect strategies for optimal infrastructure investments. Motivated by a combined investment and operational optimization model for natural gas transport, we describe how to address such requirements through risk measure formulations such as Average Value-at-Risk. The large number of operational scenarios required for a meaningful analysis of the risk measures creates a computational challenge. A new scenario tree structure, multi-horizon scenario trees, can improve computational tractability. We investigate properties of the risk measures such as time consistency for such scenario trees and illustrate this discussion with a stylized example.

8.1 Introduction

Industries with large capital investments are exposed to both long-term uncertainty and short-term or operational variations and uncertainty. Long-term uncertainty includes trends in demand or price developments, costs for infrastructure elements,

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R.M. Kovacevic et al. (eds.), *Handbook of Risk Management in Energy Production and Trading*, International Series in Operations Research & Management Science 199,
DOI 10.1007/978-1-4614-9035-7_8,

and new resource discoveries. Short-term uncertainty comprises daily variations in demand or prices and unplanned events such as production stoppages and outages.

In optimization models, this uncertainty can be handled by applying stochastic programming methodology, where the uncertain parameters are represented by discrete values in scenario trees for possible future realizations of the parameters. Computational tractability is usually of great concern in such models, because variables and constraints are duplicated for each scenario in the scenario tree. If all daily variations are to be included, the scenario tree and, hence, the optimization model will become intractable. For this reason, operational details are often aggregated for strategic models. By applying a multi-horizon scenario tree structure as described in Sect. 8.2, relevant operational detail can be included into stochastic programming models without sacrificing computational tractability.

We discuss the application of risk measures on stochastic programming models with multi-horizon scenario trees and how risk measures can be applied not only to monetary performance. Our motivation stems from investment problems for the natural gas sector, in particular the project "Regularity and uncertainty analysis and management for the Norwegian gas processing and transportation system" (Ramona, funded by the Norwegian Research Council); see Sect. 8.3. This project developed new methodology and tools for optimizing production assurance and capacity utilization in natural gas production, processing, and transportation systems. During the project period, an optimization model was developed to find infrastructure solutions for processing and transporting natural gas from fields (reservoirs) to the markets. Such an infrastructure must be robust and flexible, allowing reliable and profitable operations under various, also adverse, situations. The main driving force is profitability, and the objective is to maximize net present value.

In addition to the obvious financial benefit of being able to fulfill contract obligations through delivery to the marketplaces, the producers value high production assurance, i.e., the capability of a system with respect to production performance or to meet the demand for deliveries. Hence, production assurance requirements can be imposed at both the production side (fields) and the consumption side (markets) of the network. The ability to deliver with high certainty is seen as a strategic goal, and the producers believe that a good reputation in this respect will translate to higher prices, increased sales, and better contract terms. These benefits are not straightforward to include directly into the model objective. Instead, we have chosen to add requirements on the deliveries using risk measures.

Risk measures (Sect. 8.4) are often applied to monetary losses where undesirable deviations are limited or penalized in the objective. We apply risk measures to the physical flow to ensure that the producer is able to deliver the contracted volumes with a high degree of certainty. The target production assurance is valid both in the short term and in the long term, and we apply the risk measures on the operational scale. We show that our approach implies time consistency of the risk measures for multi-horizon trees.

Moreover, problems found in the literature often consider risk aversion which means that one seeks to *minimize* risk by including risk measures in the objective function. In contrast, our application example *limits* the risk through constraints.

That is, we consider risk acceptable as long as it is below a given threshold—on the other hand, risk minimization (in the objective function) allows one to end up with an optimal level of risk above such a threshold value. Moreover, the objective function in our application example is a function of several decision variables and the risk measure is another function of a subset of these. Most applications consider a much more direct relation between the objective and risk functions, e.g., profits and losses.

Finally, a stylized example in Sect. 8.5 illustrates the application of production assurance requirements by way of Average Value-at-Risk constraints on a model with a multi-horizon tree structure and shows how different modeling choices can affect the optimal solution.

8.2 Treating Uncertainty: Multi-horizon Scenario Trees

Decisions on a long-term or strategic level often define a framework for short-term or operational aspects. Hence, when finding strategic decisions such as investments into equipment, it is important to assess their impact on operations and vice versa, and an optimization model should take into account both decision horizons. This becomes even more relevant when these decisions must be found under uncertainty. In many situations, one can distinguish between uncertainty on a longer-term perspective (trends in the development of consumer prices or demand, volumes available for production in newly discovered natural gas reservoirs, technology development, climate change, etc.) and uncertainty on a shorter time scale (daily price or demand variations, weather variations). Obviously, this suggests that a stochastic model combining both time scales in a common scenario tree should distinguish between scenario tree nodes dedicated to strategic uncertainty and decisions and nodes dealing with the operational uncertainty and decision process.

A straightforward combination of the two kinds of nodes in a common scenario tree structure leads to the tree size quickly growing out of hand: To represent the operational conditions during a strategic time period adequately, one should include quite a number of realizations of the uncertain operational parameters and, hence, branchings at the strategic nodes in the considered period. These nodes representing short-term uncertainty must then be combined with the nodes representing long-term uncertainty in the next strategic time period. In particular, if short-term and long-term uncertainties are independent, the resulting tree may contain many duplicate values.

Figure 8.1 shows an example of combining long- and short-term uncertainty in a traditional scenario structure where ◯ represents strategic and □ operational nodes. The tree spans just three strategic time periods with one operational period each. Strategic uncertainty is represented by three branchings at the first strategic stage and two at the second while there are two possible realizations of the uncertain operational parameters at each operational stage. This small example yields 48 scenarios and, in total, 93 tree nodes (62 operational and 31 strategic).

However, strategic decisions often depend on the *overall* operational performance in the time since the previous strategic decision rather than directly on a specific short-term scenario (and the corresponding operational decisions). For example, decisions about investments in natural gas transport infrastructure rarely depend on the infrastructure's performance on a specific day but rather on how the infrastructure is *expected* to perform under varying daily conditions. In this case, a multi-horizon scenario tree structure is well suited. With such a structure, it is sufficient to branch at the strategic nodes while the operational nodes are embedded as subtrees associated with the respective strategic node. Hence, the operational feasibility and profitability of the decisions made in the strategic nodes can be tested on the corresponding subtrees. Moreover, testing infrastructure reliability typically requires many operational scenarios to ensure robustness under a vast variety of situations. This indicates that a multi-horizon tree structure is particularly well suited for such purposes; see also the discussion in Sect. 8.3.3.

Kaut et al. [9] discuss this approach in more detail and draw comparisons to traditional scenario tree structures, also with respect to the growth in tree sizes.

An example of a multi-horizon scenario tree is given in Fig. 8.1b. The tree has the same number of stages and of realizations of the uncertain strategic parameters as the traditional scenario tree in Fig. 8.1a but twice as many realizations of the uncertain operational parameters (i.e., a finer presentation of the short-term uncertainty). Here, we have 40 operational and 6 strategic scenarios while the tree contains just 50 nodes (whereof 40 operational and 10 strategic).

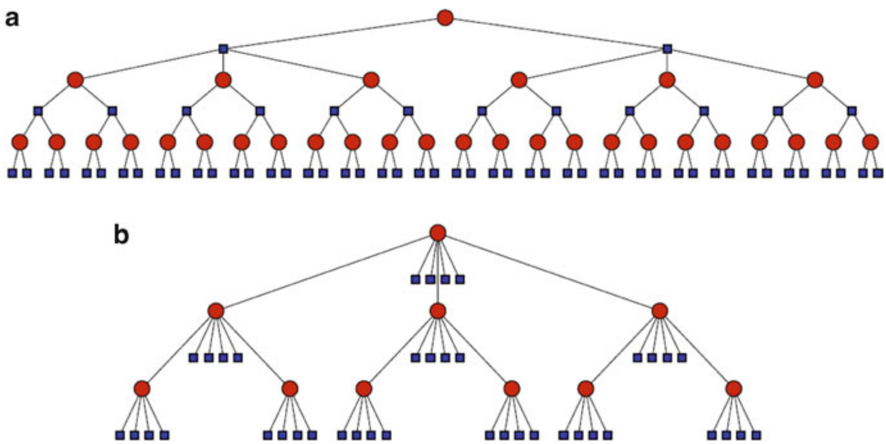

Fig. 8.1 Examples of traditional and multi-horizon scenario trees. (**a**) Traditional scenario tree structure with a combination of strategic and operational uncertainty. (**b**) Multi-horizon tree with the same number of strategic branchings but double number of operational branchings as the traditional tree

The multi-horizon scenario tree structure can be interpreted as a contingent scenario analysis of the operational problem for each strategic node. In general, it is a relaxation of the information structure represented by a traditional scenario tree.

If the strategic uncertainty is independent of the operational uncertainty and the strategic decisions do not depend on particular operational decisions at the previous strategic stage, the information structure is exactly the same as with the traditional approach. This condition is often satisfied quite easily. Moreover, in a multi-horizon tree structure, there is no connection between operational scenarios of two consecutive strategic nodes. Hence, the first operational decision associated with a strategic node should not depend on the last operational decision or state from the previous strategic period. This condition may require a careful definition of the strategic time periods; see also the discussion in Kaut et al. [9].

In the remainder of this chapter, the notation concerning uncertainty is chosen with a multi-horizon scenario tree structure in mind. For a strategic node $i \in \mathscr{N}^{Strat}$, we denote its time period by $\tau(i) \in \mathscr{T}$, relative probability by $\mathbb{P}_i^{Strat}$, and all operational nodes in the associated subtree by $j \in \mathscr{N}_i^{Op}$. The relative probability of an operational node $j \in \mathscr{N}_i^{Op}$ is denoted by $\mathbb{P}_j^{Op}$. Observe that a subtree representing the operational uncertainty is always associated with a certain strategic node. Hence, this strategic node can be interpreted as the root node of the considered subtree and the probabilities of the operational scenarios are considered only within the context of the respective subtree.

For the ease of notation we assume throughout this chapter that there are no bindings between consecutive operational time periods (e.g., due to storage modelling). That is, each operational node can be considered independent of other operational nodes. Hence, each operational node in a given subtree represents a single scenario. Consequently, we have just one operational time period in a subtree and we will, therefore, ignore the operational time index in the subsequent discussion. Moreover, the strategic node i is the parent node Pa_j of all operational nodes $j \in \mathscr{N}_i^{Op}$ in the associated subtree.

8.3 Application Example: The Ramona Optimization Model

The stochastic optimization model developed in the Ramona project combines both infrastructure and operational decisions under uncertainty in a common framework. It reflects, hence, both technological properties of the natural gas transport network and economic and business requirements.

Pressures and flows in one part of the network may influence transportation capacity in other infrastructure parts [14]. Such system effects must be taken into account when deciding about new investment in order to avoid a negative impact on the existing or future infrastructure. Hence, a portfolio perspective is advisable rather than an evaluation of single investment options in isolation and independently of the total system. For example, being able to address gas quality problems from new fields through blending in already existing facilities rather than investing in extra processing capacity can save investment costs. This means that, in addition to economics, operational aspects (physical processes and daily gas routing decisions) must be taken into account.

An increased focus on production assurance and security of supply makes it paramount to evaluate how the infrastructure will perform during daily operations and what the financial effects (costs and revenue) will be. For example, would a new pipeline allow to better satisfy delivery contracts in critical times or to route gas not bound in contracts to the most profitable markets? How would it affect gas flows in other pipelines? How does it affect operational costs and cash flow? Also the timing of investments is important for satisfying production obligations and developing new fields in a good way, e.g., allowing to reuse infrastructure.

Obviously, both investment and routing decisions are subject to various kinds of uncertainty. Discoveries of new reservoirs, gas composition and volumes in undeveloped reservoirs, or long-term changes (trends) in price and demand levels are examples of uncertainty on the strategic level. In contrast, uncertain parameters on the operational level may concern daily nominations in long-term delivery contracts or prices and demands at the markets. Also unplanned events such as network outages represent short-term uncertainty, reducing capacity in the affected infrastructure parts drastically reduced for some, often short, time and, hence, affecting production assurance. This combination of diverse kinds of long- and short-term uncertainty suggests the utilization of multi-horizon scenario trees in order to tackle realistically sized problem instances.

In order to further reduce the scenario tree size, one may consider only a representative selection of operational scenarios. For example, to estimate the profitability of the strategic decisions, it may be sufficient to study a few typical days in a year (spring, summer, winter, and a few variations). To test network flexibility and robustness, some extreme or "critical" scenarios are included. For example, this allows to assess the network's robustness in terms of production assurance, taking into account also unplanned events. Assigning (near-)zero probability to these "critical" scenarios, the feasibility of the strategic decisions also for these scenarios can be tested, but they do not affect profitability evaluations unduly.

The complete model constitutes a unified framework to analyze investment decisions under uncertainty in a portfolio perspective, taking into account physical properties of the network and the dynamics of short-term planning as well as long- and short-term uncertainty. Here, we present only the most important aspects of this multistage stochastic mixed-integer programming problem. Hellemo et al. [7] give an overview of a deterministic version of the model including a discussion of system effects and quality aspects while Hellemo et al. [8] present a comprehensive description of the full stochastic optimization model.

8.3.1 Model Overview

In the strategic nodes $i \in \mathcal{N}^{Strat}$, the (binary) decision variables $X^{Strat} = \{x_i^{Strat}, i \in \mathcal{N}^{Strat}\}$ refer to decisions about investments into network infrastructure elements (production facilities, pipelines, processing facilities, and markets). They establish the framework for the decisions $X^{Op} = \{x_j^{Op}, j \in \mathcal{N}_i^{Op}\}$ in the associated operational nodes $j \in \mathcal{N}_i^{Op}$ which concern the flow f_{nj} through each network infrastructure element n and the pressures at its in- and outlets. These decisions, in turn, determine the cash flow and production assurance achievable with the found network design.

The objective of the model is to maximize the expected net present value (NPV) of investments and operations which is determined by the costs C_i^{Strat} and revenue R_i^{Strat} from decisions at all strategic nodes $i \in \mathcal{N}^{Strat}$ and costs C_j^{Op} and revenue R_j^{Op} from operations at the associated operational nodes $j \in \mathcal{N}_i^{Op}$:

$$NPV(X^{Strat}, X^{Op}) \tag{8.1}$$
$$= \sum_{i \in \mathcal{N}^{Strat}} \mathbb{P}_i^{Strat} \delta_{\tau(i)} \left(R_i^{Strat} - C_i^{Strat} + \sum_{j \in \mathcal{N}_i^{Op}} \mathbb{P}_j^{Op} \gamma_j \left(R_j^{Op} - C_j^{Op} \right) \right).$$

The costs C_i^{Strat} from investment decisions x_i^{Strat} at the node $i \in \mathcal{N}^{Strat}$ are mainly costs for installing new and decommissioning network infrastructure elements. In principle, they may depend on the strategic time period $\tau(i)$ or on the age of the element. The mathematical model uses two kinds of cost profiles to reflect this. However, we do not go into the details of these profiles here. Decommissioned network elements may have a positive salvage value contributing to revenue R_i^{Strat} directly arising from strategic decisions. The factor $\delta_{\tau(i)}$ denotes the discount factor at the time period of the strategic node i.

Operational costs C_j^{Op} are related to operating and maintaining the infrastructure. For each network element n, they are composed of operational expenditure (depending on the element's age), fixed (depending on calendar time), and variable costs (depending on the flow f_{nj}). Again, time-dependent profiles are used to express the different dependencies. As mentioned above, the operational nodes often represent only a selection of scenarios such that a scaling factor γ_j is applied to the values R_j^{Op} and C_j^{Op} which represents the weight of the considered operational node. For example, if all operational time periods have a length of one day and the strategic time period is a year, then the weights of all operational nodes associated with a strategic node must sum up to 365.

For a given market, the daily natural gas sales price may be stochastic, and the revenue R_j^{Op} achieved in an operational node $j \in \mathcal{N}_i^{Op}$ is the sum over sales at all markets.

Investment decisions are subject to constraints on the network elements' start-up (within a time window), shutdown, availability for production, and capacity. Operational decisions concern routing gas through the network and, obviously, the constraints on these decisions primarily model the physics of the network. The most important constraints express flow–pressure relationships and ensure mass balances as well as limits on the flow and pressure in each network element. The latter constraints take also care of capacity variations throughout the lifetime of an infrastructure element. Moreover, they can be used to model unforeseen events affecting network capacity. Reservoir constraints ensure that the amount of gas produced at the single production facilities complies with yearly production plans and limits on the totally available gas in the corresponding reservoir.

8.3.2 Multi-Horizon Scenario Trees in the Ramona Model

A typical realistic model instance comprises about 200 network elements. While investment analysis may span a time horizon of between twenty and fifty years, operational decisions are found with a daily time resolution. Consequently, a three-stage stochastic model with 12 strategic periods, 10 branches per strategic node at each stage, and 10 operational profiles over 365 days will have about 100 million decision variables when using a multi-horizon scenario tree structure. With a traditional scenario tree structure, the model would be practically unsolvable—it would contain about 9 billion variables.

For this application, multi-horizon scenario trees represent an approximation of the information structure represented in a traditional scenario tree (cf. discussion in Sect. 8.2): Obviously, while production plans made at the strategic level give guidelines for the produced volumes to optimally deplete the reservoirs, the actual production depends on the operational scenarios. Hence, specific operational scenarios do indeed affect the decision space for the production plans in subsequent strategic periods to some degree. However, also the total volume in a reservoir available for production is not perfectly known, and the dependency of this volume on operational production decisions may be considered negligible.

8.3.3 Production Assurance Requirements

At the production side, production assurance requirements refer to the flow into the network relative to the production plan. On the other hand, at the consumption side, they consider the deviation of actually delivered volumes from the company's delivery obligations agreed upon in the contract with its customers. Moreover, production assurance may be measured at several levels: separately for each market or field, at a cluster of or at all markets fields, or even across the whole network. For our subsequent discussion, it is not important what exactly the notion refers to as we focus on a discussion of this concept in the context of the time and information structure provided by multi-horizon scenario trees. Therefore, we refrain from a network topology index for the concerned variables and parameters.

As an example, production assurance at a market can be found by analyzing *daily deliverability* over a given strategic time period. Typically, deliverability measures the deviation of the gas volume actually delivered in an operational node $j \in \mathscr{N}_i^{Op}$ (e.g., a day) against the demand, i.e., the nominated or contracted volume. Here, the nomination or contracted flow f_j^{Con} represents a random parameter while the delivered flow f_j^{Del} is a decision variable. Depending on the definition of the production assurance requirement, f_j^{Del} may refer to the flow f_{nj} in a specific market node n but also to the aggregated flow in a cluster of (or all) market nodes. Obviously, a similar understanding holds for the nomination f_j^{Con}.

Often, there is no proper incentive to perform better than required (e.g., gas volume exceeding nominations may rather be sold in a more lucrative market). Hence, we define deliverability with a focus on cases where the delivered volume is insufficient compared to the nomination:

$$Del\left(f_j^{Del}, f_j^{Con}\right) = \min\left\{1, \frac{f_j^{Del}}{f_j^{Con}}\right\}, \quad \forall\, j \in \mathcal{N}_i^{Op},\ i \in \mathcal{N}^{Strat},$$

and ignore the (theoretically possible) option of delivering more than specified.

For the ease of notation, we suppress the reference to the volumes f_j^{Con} and f_j^{Del} when mentioning the deliverability in an operational scenario in the following: $Del_j = Del\ (f_j^{Del}, f_j^{Con})$.

Observe that production assurance requirements apply to underdeliveries compared to nominated gas volumes (or underproduction compared to planned gas volumes) while the profitability evaluations included in the objective function are based on the actually delivered (or produced) volumes and the gas price at the considered operational nodes. In other words, production assurance requirements do not consider the *value* of the gas. Rather, production assurance requirements must be satisfied no matter what the current gas price is. If the risk measure took into account gas prices, thus focusing on the risk of lost profit, a violation of delivery obligations would matter more when the gas price is high and less when it is low.

There is no clear-cut and unified way to specify production assurance requirements, and we discuss some ways to specify such requirements in the following.

One may state a threshold value PA_t below which the daily deliverability should not fall in all strategic nodes i in a time period t (i.e., for all nodes $i \in \mathcal{N}^{Strat}$ with $\tau(i) = t$). For example, one may require a deliverability of at least 99%. Due to the uncertainty about operational parameters—which also affects the daily deliverability—it may not be wise to formulate this requirement as a constraint to be satisfied under all circumstances:

$$Del_j \geq PA_{\tau(i)}, \quad \forall j \in \mathcal{N}_i^{Op},\ i \in \mathcal{N}^{Strat}.$$

This would result in a network infrastructure with a high degree of redundancy and, consequently, unduly high investment costs to ensure that this constraint is satisfied at any time. Instead one may allow a violation of the requirement, but encourage solutions ensuring a high degree of production assurance. For example, a penalty M may be imposed for each day $j \in \mathcal{N}_i^{Op}$ with insufficient deliverability:

$$M \max\{0, PA_{\tau(i)} - Del_j\},$$

which is then summed up in the objective function over the whole optimization horizon, taking into account the probabilities of the strategic scenarios and of the operational nodes within each subtree:

$$\sum_{i \in \mathcal{N}^{Strat}} \mathbb{P}_i^{Strat} \sum_{j \in \mathcal{N}_i^{Op}} \mathbb{P}_j^{Op} M \max\{0, PA_{\tau(i)} - Del_j\}. \tag{8.2}$$

This corresponds to penalizing the expected insufficient deliverability $\max\{0, PA_{\tau(i)} - Del_j\}$ over the whole horizon. Hence, the variability of this random variable cannot be taken into account properly and such a penalty term cannot control how the target values are satisfied over the operational nodes $j \in \mathcal{N}_i^{Op}$ associated with a strategic node $i \in \mathcal{N}^{Strat}$. In other words, a very low deliverability on one day and good performance else are considered comparable to a constant slight underperformance. Moreover, the penalty factor M must be chosen very carefully to achieve the desired results, weighting production assurance against expected net present value of investments and operations. As it is difficult to quantify such quality-oriented aspects, this is a rather daunting task.

The former challenge can be addressed by the following formulation: Given the threshold value PA_t for acceptable deliverability, set a limit on the percentage (or number) of days in the strategic node $i \in \mathcal{N}^{Strat}$ (with $\tau(i) = t$) where this threshold is not reached. This can be formulated by way of *chance constraints*. The threshold $\alpha_{\tau(i)}$ specifies the minimum percentage of operational scenarios (days) in strategic period i with sufficient deliverability:

$$\mathbb{P}\left\{Del_j \geq PA_{\tau(i)}, j \in \mathcal{N}_i^{Op}\right\} \geq \alpha_{\tau(i)}, \quad \forall i \in \mathcal{N}^{Strat}. \tag{8.3}$$

Such a formulation avoids the difficulty of quantifying the company's deliverability record. Since the operational scenarios represent a discretization of the distribution of the uncertain operational parameters, this constraint can be expressed by an LP formulation by introducing auxiliary binary and continuous variables [22, 27].

If, for example, it is required that "deliverability at a market node shall be over 0.99 on, at least, 97% of all days in a year" in all years during the optimization horizon (and, consequently, in all strategic tree nodes), constraint (8.3) would read

$$\mathbb{P}\left\{Del_j \geq 0.99, j \in \mathcal{N}_i^{Op}\right\} \geq 97\%, \quad \forall i \in \mathcal{N}^{Strat}.$$

In other words, this formulation ensures that the percentage of days with insufficient deliverability is not too high—but it also allows to underperform quite drastically in all those days.

The thread of excessive underperformance may be taken care of by setting a *lower limit on the average performance in the worst outcomes*. This way, some really low deliverability is still allowed, but only occasionally. However, it is difficult to define the "worst outcomes": If all days with a deliverability below PA_t are considered unacceptable, the requirement just leads to a lower average deliverability as there is no limit on the number of these days. Consequently, there is no incentive to perform better than PA_t.

Alternatively, one may sort all days $j \in \mathscr{N}_i^{Op}$ associated with a strategic node $i \in \mathscr{N}^{Strat}$ according to their deliverability and consider a given percentage $\tilde{\alpha}_{\tau(i)}$ of these days as the worst outcomes, no matter how "bad" they actually are. Then, the average deliverability on these days may be limited from below by a limit $\widetilde{Del}_{\tau(i)}$:

$$\mathbb{E}\{Del_j | Del_j \leq VaR_{\tilde{\alpha}_{\tau(i)}}(Del), j \in \mathscr{N}_i^{Op}\} \geq \widetilde{Del}_{\tau(i)}, \quad \forall i \in \mathscr{N}^{Strat}. \tag{8.4}$$

For example, one may require that the average deliverability on the 15% days with lowest deliverability in a strategic node i still shall be above 0.9:

$$\mathbb{E}\{Del_j | Del_j \leq VaR_{15\,\%}(Del), j \in \mathscr{N}_i^{Op}\} \geq 90\%, \quad \forall i \in \mathscr{N}^{Strat}.$$

Also this requirement can be reformulated to a set of linear constraints by introducing auxiliary continuous variables [22, 27].

Clearly, the deliverability targets PA_t (and threshold probabilities α_t) should be the same for all strategic nodes in a given time period t: The production assurance requirement should not depend on the gas network configuration or realizations of the uncertain strategic parameters. Moreover, they may also be the same for several or all strategic time periods t. The same holds for the lower limits $\widetilde{Del}_t$ and for the percentages $\tilde{\alpha}_t$ of the days with lowest deliverability within the time period t.

However, in general, there is no direct relationship between the target value PA_t and the limit $\widetilde{Del}_t$ or between the threshold α_t and the percentage $\tilde{\alpha}_t$ (although it might be more natural to assume a relationship between the latter than between the first). Intuitively, one may set $\tilde{\alpha}_t$ somewhat higher than α_t and / or $\widetilde{Del}_t$ somewhat lower than PA_t.

Evidently, production assurance is an operational concept and confined to a certain (strategic) time period, e.g., year or month. It is determined for a given infrastructure configuration (existing network and potential investment decisions) and requires, in order to give meaningful results, many operational scenarios associated with each strategic decision point. Hence, to compare or decide between several investment options under similar operational conditions, a traditional scenario tree structure would require a large degree of duplicate values. Consequently, one can solve only relatively small examples rather than realistic-sized cases.

Observe, however, that formulations (8.3) and (8.4) of production assurance requirements do not span several strategic time periods—they are considered independently for each strategic period. Hence, this aspect is well suited a model with a multi-horizon scenario tree structure. This tree structure allows many more operational scenarios for each strategic decision point than a traditional structure interspersing operational and strategic tree nodes in each scenario.

In the following section, we briefly introduce static and dynamic risk measures before we relate them to the multi-horizon scenario tree structure and the production assurance concepts discussed in Sect. 8.3.3. In particular, we turn our attention to the question of time consistency in a multistage setting.

8.4 Risk Measures

Section 8.3.3 outlined several approaches to model risk aversion when making decisions under uncertainty. On a more general level, risk measures as functionals on random variables have been studied intensely over the past decade and have become popular in particular in finance. The seminal paper by Artzner et al. [1] addresses axioms that are considered natural when quantifying risk by assigning a single number to the random variable representing potential outcomes. Krokhmal et al. [13] provide an overview over risk-modeling concepts in a static (single-period) setting.

The expectation operator $\mathbb{E}(\cdot)$ employed in formulation (8.2) represents the simplest form of a risk measure; it assigns the single number $\mathbb{E}Y$ to the possible outcomes represented by a random variable Y. For the example of production assurance described, the random parameter may be the daily nominations f_j^{Con} (and other uncertainties not discussed closer here), while the flow f_j^{Del} is the considered decision variable such that the random variable Y corresponds to the deliverability Del_j. The respective probabilities are modeled through a probability measure involved when calculating the expectation.

As mentioned above, this measure does not take into account properties of the considered random variable such as its variability, i.e., the distribution of the single values over all outcomes. Risk measures exploiting more properties of the random variable are, for example, chance constraints [exemplified by (8.3)] or the *Average Value-at-Risk* ($\mathbb{A}$V@R) as formulated in (8.4).

Chance constraints (8.3) ensure that the deliverability targets PA_t are satisfied with a certain probability as specified in the contract with the company's customers. However, this formulation treats any underdeliveries equally, no matter how large the shortfall is.

The $\mathbb{A}$V@R illustrated in (8.4) reflects another important risk measure which is also known as expected shortfall or Conditional Value-at-Risk (CVaR). This measure does not only take into account at which probability the demand is satisfied but also the level of demand satisfaction. This constraint is sometimes more conservative than a chance constraint. On the other hand, it is a convex constraint and certainly easier to handle computationally than, say, chance constraints. Our subsequent discussion will focus on this risk measure.

With our application in mind, the $\mathbb{A}$V@R of a random variable Y at the confidence level α can be defined generally as the expectation of all outcomes in the lower α-quantile of the probability distribution of Y:

$$\mathbb{A}\mathrm{V@R}_\alpha(Y) = \mathbb{E}\{Y | Y \leq V@R_\alpha(Y)\}. \tag{8.5}$$

However, in the case that the probability space contains atoms, this formulation has a drawback. In this case, the event $\{Y | Y \leq V@R_\alpha(Y)\}$ may have a probability other than α despite $V@R_\alpha(Y)$ being involved in the definition of this event. Exactly this situation occurs in the Ramona model involving finitely many (operational) scenarios of deliverability values as each has a (strictly) positive probability and, hence, is an atom.

Using the Fenchel–Moreau Theorem (cf. [26]), the $\mathbb{AV@R}$ can be expressed by its dual formula:

$$\mathbb{AV@R}_\alpha(Y) = \inf\left\{\mathbb{E}YZ | 0 \le Z \le \frac{1}{\alpha}, \mathbb{E}Z = 1\right\}. \tag{8.6}$$

This representation outlines that the mapping $Y \mapsto \mathbb{AV@R}_\alpha(Y)$ is concave.

Alternatively, the $\mathbb{AV@R}$ can be expressed as

$$\mathbb{AV@R}_\alpha(Y) = \max_{q\in\mathbb{R}} q - \frac{1}{\alpha}\mathbb{E}(q-Y)_+,$$

where x_+ is the positive part, $x_+ := \max\{0, x\}$. This expression has been introduced by Rockafellar and Uryasev [19] while the general formulation is stated in Pflug [15]. It replaces the infimum in (8.6) by a maximum and has become popular in stochastic optimization (cf., e.g., [4]) and appears to be tailor-made for the model discussed here.

A risk constraint similar to the production assurance requirements presented in Sect. 8.3.3 may require that the Average Value-at-Risk of the random variable Y at the level α is above a given threshold $\tilde{q}$:

$$\mathbb{AV@R}_\alpha(Y) \ge \tilde{q}.$$

8.4.1 Risk Measures in Multistage Optimization Problems

Intuitively, the static concept may be extended easily to a dynamic or multi-stage situation. However, due to relations between the decisions and parameters at the different stages affecting the properties of the risk measures, this is not quite straightforward and has spawned increased research interest in the recent years. For example, Kozmík and Morton [12] consider a stochastic programming problem structure with multiple recourse stages. They consider risk aversion, i.e., minimize risk and study stage-wise independent uncertain parameters and a risk measure as a function of the recourse value at each stage.

Risk measures may be applied separately at each stage of the underlying scenario tree or as a nested measure spanning several or all stages. We focus here on the former, conceptually simpler approach. More formally, we consider $\mathbb{R}$-valued random variables dependent on some previous decisions $\mathbf{x} \in \mathbb{X}$ and a random parameter $\xi \in \Xi$, that is, $Y = Y(\mathbf{x}, \xi) \in \mathbb{R}$ for the objective and (possibly different) random variables $Y_t^c = Y_t^c(\mathbf{x}, \xi) \in \mathbb{R}$ for the constraints, which are observed at the times $t \in \{0, \dots, T\}$. The vector $\mathbf{x} = (x_0, \dots, x_T)$ collects all decisions made at $T+1$ subsequent instants of time $t \in \{0, \dots, T\}$. We use the notation $Y(\mathbf{x})$ also for the random variables $Y(\mathbf{x}) : \xi \mapsto Y(\mathbf{x}, \xi)$ (and $Y_t^c(\mathbf{x})$ for $Y_t^c(\mathbf{x}) : \xi \mapsto Y_t(\mathbf{x}, \xi)$, respectively). Importantly, the decisions up to time t are $x_0, \dots, x_t$, and a random variable Y_t^c observed at that time t is determined by $x_0, \dots, x_t$. Expressed in mathematical terms, it holds that

$$Y_t^c(\mathbf{x}) = Y_t^c(\mathbf{x}') \tag{8.7}$$

whenever $(x_0,\ldots,x_t) = (x'_0,\ldots,x'_t)$, where $\mathbf{x} = (x_0,\ldots,x_T)$ and $\mathbf{x}' = (x'_0,\ldots,x'_T)$.

Similar to the static formulation (8.5), an $\mathbb{AV@R}$ measure with a (potentially different) level α_t can be applied to the random variable Y_t at each stage $t \in \{0,\ldots,T\}$ of the underlying scenario tree. Constraints may require that any $\mathbb{AV@R}$ at a given stage t shall exceed a given threshold value q_t (recall that $\mathbb{AV@R}$ is concave). Observe that, if this constraint is required to hold separately at each tree node at this stage, the $\mathbb{AV@R}$ is considered conditionally on realizations of Y up to this stage. (Obviously, if there is only one $\mathbb{AV@R}$ constraint involving all Y_t for given t, this measure does not depend on previous realizations.) This is made evident through the filtration $\mathscr{F}_t$ of the tree: the increasing sigma algebras $\mathscr{F}_t \subset \mathscr{F}_{t+1}$ represent the information available at time t [17].

Hence, a multistage stochastic optimization problem with risk constraints at each time period t can be formulated as

$$\text{maximize} \quad \mathbb{E}Y(\mathbf{x}) \tag{8.8a}$$

$$\text{subject to} \quad \mathbb{AV@R}_{\alpha_t}\left(Y_t^c(\mathbf{x})|\mathscr{F}_t\right) \geq q_t, \quad \forall t \in \{0,\ldots,T\}, \tag{8.8b}$$

$$\mathbf{x} \in \mathbb{X}_0 \times \cdots \times \mathbb{X}_T, \tag{8.8c}$$

where all $x_t, t \in \{0,\ldots,T\}$ are measurable with respect to the sigma algebra $\mathscr{F}_t$. The latter condition expresses the *nonanticipativity* constraints on the decisions x_t.

Observe that the risk measure applies to the random variables Y_t^c while the random variable Y employed in the objective function may be a different function of the decisions $\mathbf{x}$. For example, the Ramona model comprises risk constraints involving natural gas volumes delivered to the markets while maximizing the expected profit from these deliveries. Only the latter involves current market prices.

A sufficiently large sample size is necessary to get acceptable approximations, particularly as a non-biased estimator for the Average Value-at-Risk does not exist ([10]) in general. More specific, the number of considered realizations of Y should be of order $\frac{1}{\alpha} \approx \frac{1}{P(Y \leq VaR_\alpha(Y))}$. As a consequence, the size of multistage stochastic programming problems relying on a traditional scenario tree structure quickly grows out of hand if risk measures are applied not only to the leaf nodes at the final stage. Also Kozmík and Morton [12] point out the need for many scenarios at each stage and the resulting computational challenges when considering risk in a multistage setting: As only a small number of the realizations of the random parameter at each stage contribute to calculating the risk, a large number of nodes would be required. They suggest SDDP, i.e., sampling during the solution process, relying on stage-independent scenario trees. Note that this requirement of stage-wise independent random parameters excludes time-series models.

Alternatively, the scenario tree size may be reduced drastically without sacrificing model quality by utilizing the properties of the model at hand. For the Ramona model, we can distinguish clearly between operational and strategic decisions, and the risk measures apply only to all operational outcomes associated with a given strategic node. This indicates that the subtrees associated with each strategic node at any stage can be of the required size and, consequently, risk constraints can be applied at "any" strategic stage throughout the optimization horizon.

With a multi-horizon scenario tree structure, one can calculate a risk measure at each strategic node i, spanning all operational outcomes (i.e., all operational nodes j representing days) associated with this node i. For all strategic nodes in a given strategic period t, that is, $\{i \in \mathcal{N}^{Strat} : \tau(i) = t\}$, the requirements are the same, i.e., they are characterized by the same parameters α_t and q_t. Consequently, decisions in any strategic node should be found such that (a) the risk requirement covering all operational nodes associated with this strategic node is satisfied, and (b) they allow to make decisions in all subsequent strategic nodes such that the corresponding risk requirements in these nodes are satisfied. In general, the operational scenarios associated with a strategic node are considered to be independent of the operational scenarios associated with other strategic nodes. However, due to b), operational scenarios associated with later strategic nodes in the same strategic scenario may affect earlier strategic decisions—in particular, if the strategic decision space in these later nodes is quite confined. We will resume these considerations in Sect. 8.4.2 discussing time consistency of dynamic risk measures.

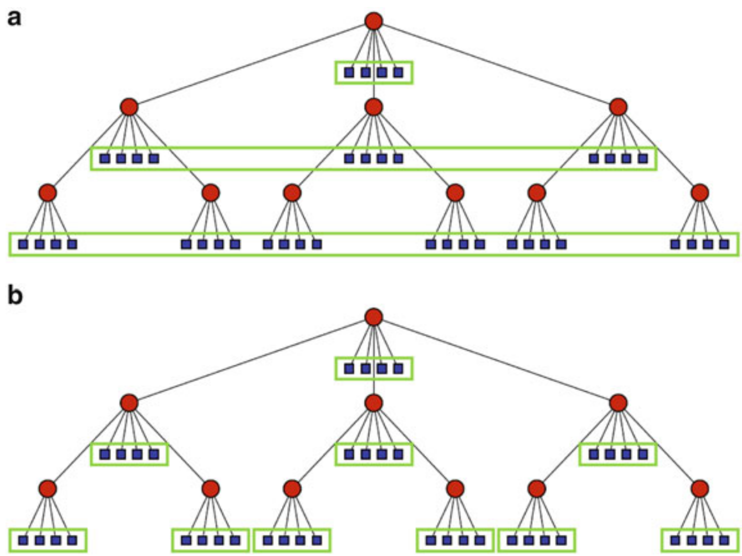

Fig. 8.2 Different scopes of risk measures. (**a**) Risk measure on a traditional scenario tree spanning all nodes at a stage. (**b**) Risk measures on a multi-horizon scenario tree spanning all operational nodes associated with a strategic node

8.4.2 Time Consistency

The principle of time consistency for multistage optimization problems derives from dynamic optimization [5] and has been analyzed from many perspectives in the literature; see, e.g., [2, 11, 24]. Although its meaning is intuitively evident, a common and widely accepted definition does, apparently, not exist yet. A common consensus, however, is that it means that decisions which are made at a certain stage of time should not be withdrawn at a later stage of time. That is, contradictions that may occur during the decision process should be excluded.

Carpentier et al. [3] informally formulate this principle as follows:

The principle of time (or dynamic) consistency. *The sequence of optimization problems is said to be dynamically consistent if the optimal strategies obtained when solving the original problem at time t_0 remain optimal for all subsequent problems. In other words, dynamic consistency means that strategies obtained by solving the problem at the very first stage do not have to be questioned later on.*

When discussing time consistency, one often distinguishes between optimization problems which involve a risk functional in the objective and which are solved at multiple, subsequent stages in time and problems which involve a risk functional in the constraints.

We will follow this distinction here and discuss time consistency for risk measures in the objective and the constraints separately. Then, we address time consistency on multi-horizon scenario trees.

8.4.2.1 Risk Measures in the Objective

Typically, multistage stochastic optimization problems with a risk measure in the objective employ a composition of risk measures at subsequent stages. Several results are known about such compositions of risk measures which often generalize initial properties of one-period risk measures ([6, 20, 21]). Schachermayer and Kupper [23] show that the only risk measure that is closed under time-consistent compositions is the functional

$$Y \mapsto u^{-1}\left(\mathbb{E}\left(u(Y)|\mathscr{F}_t\right)\right).$$

Moreover, Shapiro [25] states that the composition of risk functionals is not necessarily a law-invariant risk measure anymore, except for the expectation

$$\mathbb{A}\mathrm{V@R}_1(\cdot) = \mathbb{E}(\cdot) \tag{8.9}$$

and the min-risk functional

$$\mathbb{A}\mathrm{V@R}_0(\cdot) = \lim_{\alpha \searrow 0} \mathbb{A}\mathrm{V@R}_\alpha(\cdot) = \mathrm{essinf}(\cdot). \tag{8.10}$$

Moreover, the composition lacks a natural interpretation: it is not clear what the Average Value-at-Risk of an Average Value-at-Risk could be. However, a composition of risk measures can be easily applied and it is convenient in computations. Often it is considered a—rather conservative—alternative to a single-period $\mathbb{A}\mathrm{V@R}$ measure in the objective.

A possible way to overcome these challenges is by changing the level of the Average Value-at-Risk according to the representation

$$\mathbb{AV@R}_{\alpha}(Y) = \inf \mathbb{E}\, Z_t \cdot \mathbb{AV@R}_{\alpha \cdot Z_t}(Y|\mathscr{F}_t), \tag{8.11}$$

where the infimum is taken over all random variables Z_t, measurable with respect to $\mathscr{F}_t$, satisfying $\mathbb{E}\, Z_t = 1$ and $0 \le Z_t \le \frac{1}{\alpha}$. The essential difference to (8.6) is that the dual variable Z_t is measurable with respect to $\mathscr{F}_t$ and the level $\alpha \cdot Z_t$ in (8.11) is random itself [16].

This demonstrates that the level of the Average Value-at-Risk has to be changed in order to allow a combination of conditional Average Value-at-Risk measures. Equation (8.11) does not represent a composition but a change of measure instead (change of numéraire, cf. [18]).

Notably, the level of the Average Value-at-Risk represents the risk which the decision maker should accept in order to handle the optimization problem. Hence, one may conclude that the perception of risk may vary in different situations which, indeed, reflects a natural situation: Having observed a comfortable past which makes the initial objective more likely to achieve, a decision maker may be more relaxed in the future. Conversely, having observed a difficult past making the initial goal unlikely to be achieved, a decision maker may impose tougher conditions to ensure that the initial goal can still be achieved.

This is especially important in the case of rolling-horizon solution approaches.

8.4.2.2 Risk Measures in Constraints

The problem formulation (8.8) considers risk measures in the constraints at different levels. Studying time consistency of such risk measures, it appears natural to ask if $\mathbb{AV@R}_{\alpha}(Y|\mathscr{F}_t) \ge q$ means that also $\mathbb{AV@R}_{\alpha}(Y) \ge q$. More generally, if a random variable Y_1 is preferred over a variable Y_2 at a stage t, can it then be concluded that this random variable is preferable at an earlier stage as well; that is,

$$\mathbb{AV@R}_{\alpha}(Y_1|\mathscr{F}_t) \ge \mathbb{AV@R}_{\alpha}(Y_2|\mathscr{F}_t) \Longrightarrow \mathbb{AV@R}_{\alpha}(Y_1) \ge \mathbb{AV@R}_{\alpha}(Y_2).$$

Similar to the case of risk measures in the objective, this holds obviously for $\alpha = 1$ (the expectation) and $\alpha = 0$ (the min-risk functional).

Figure 8.3 illustrates that time consistency of the Average Value-at-Risk measure cannot be guaranteed for values of α other than 0 and 1: Assuming an Average Value-at-Risk at the level $\alpha = \frac{2}{3}$ at both stages, the example demonstrates that Y is acceptable when employing the criterion $\mathbb{AV@R}_{\frac{2}{3}} > 13$ at every subtree. However, applying the same criterion $\mathbb{AV@R}_{\frac{2}{3}} > 13$ to the complete problem, the variable Y is not acceptable. The simple Average Value-at-Risk is, therefore, not a time-consistent risk functional in this specified sense whenever $\alpha \in (0,1)$.

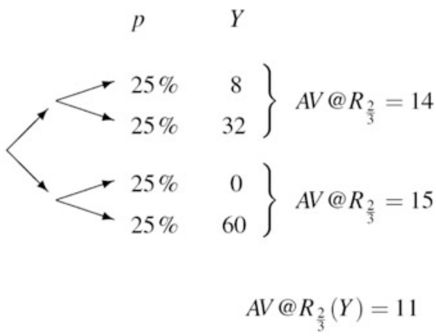

Fig. 8.3 This random variable Y satisfies $\mathbb{AV@R}_{2/3} > 13$ for every partial observation in the subtree (specified by $\mathscr{F}_1$). However, the combined observation does not satisfy $\mathbb{AV@R}_{2/3} > 13$. It is, however, correct, that $\mathbb{AV@R}_\alpha \leq \min \mathbb{AV@R}_\alpha(\cdot|\mathscr{F}_1)$

8.4.3 Consistency of Risk Measures in a Multi-horizon Tree Formulation

This section addresses time consistency of the multi-horizon problem (8.8). Note that its objective is an expectation and, moreover, the constraints in this problem are not compositions of risk measures as discussed in the previous section. The way the problem is formulated ensures its time consistency according to the principle formulated on page 192.

Proposition (Time consistency of the multi-horizon problem). *Let*

$$\mathbf{x}^* := (x_0^*, \ldots, x_T^*)$$

be an optimal solution of the multi-horizon problem (8.8)*. Then* $\mathbf{x}^*$ *solves also the problem with respect to the conditional probability measure* P^i*, where* $i \in \mathscr{N}_t^{Strat}$ *is an arbitrary (strategic) node at stage* t *and* $P^i(\cdot) = P(\cdot|i)$ *is the conditional probability satisfying* $P^i(i) = 1$*.*

Hence, the problem is time consistent in the sense of the principle given on page 192.

Remark. Incorporating the measure P^i—that is, conditioning on the node i—ensures that the tree process will move through the node i with probability one. The latter preposition ensures, therefore, that the problem can be reconsidered at a certain stage, and the initial solution will remain optimal even for the new subproblem which is reconsidered at a later time t ($i \in \mathscr{N}_t^{Strat}$). Hence, the problem is time consistent in the described sense and the initial solution does not have to be changed retrospectively.

Proof. Without loss of generality, we assume $P(i) > 0$.

Let $\mathbf{x}^*$ denote the optimal (minimal) solution, and assume that $\mathbf{x}^*$ is not optimal for the subproblem conditional on P^i. Denote the optimal solution of the subproblem with respect to P^i by $(x_0^*, \dots, x_{t-1}^*, \tilde{x}_t^*, \dots, \tilde{x}_T^*)$, and define the new decision as

$$\tilde{\mathbf{x}} := \begin{cases} (x_0^*, \dots, x_{t-1}^*, \tilde{x}_t^*, \dots, \tilde{x}_T^*) & \text{if } i \text{ is contained in the path,} \\ (x_0^*, \dots, x_{t-1}^*, x_t^*, \dots, x_T^*) & \text{else,} \end{cases}$$

which we apply to the initial problem.

The new strategy $\tilde{\mathbf{x}}$ is a potential solution of the initial problem as $\mathbb{X} = \mathbb{X}_0 \times \mathbb{X}_1 \times \dots \mathbb{X}_T$. Moreover, $\tilde{\mathbf{x}}$ is feasible for the initial problem: Indeed, if $t' \le t$, then, from (8.7), $Y_{t'}(\mathbf{x}) = Y_{t'}(\tilde{\mathbf{x}})$ as $x_{t'} = \tilde{x}_{t'}$ for all $t' \le t$. Further, if $t' > t$, then

$$\mathbb{AV@R}(Y_t(\mathbf{x})|\mathscr{F}_t) \ge q_t$$

in both cases, that is, no matter whether i is in the path or not.

Finally, the objective $\mathbb{E}Y(\tilde{\mathbf{x}})$ of the new strategy $\mathbf{x}$ is superior as

$$\begin{aligned} \mathbb{E}Y(\tilde{\mathbf{x}}) &= \mathbb{E}\mathbb{E}(Y(\tilde{\mathbf{x}})|\mathscr{F}_t) \\ &< \mathbb{E}\mathbb{E}(Y(\mathbf{x})|\mathscr{F}_t) = \mathbb{E}Y(\mathbf{x}) \end{aligned}$$

due to the assumption $P^i(i) > 0$ and since $\tilde{\mathbf{x}}$ is better than $\mathbf{x}$ on the node i.

Summarizing, $\tilde{\mathbf{x}}^*$ is a better strategy than $\mathbf{x}^*$ on the entire tree. This, however, contradicts the assumption that $\mathbf{x}^*$ is optimal. Hence, the strategy $\mathbf{x}^*$ is also optimal for the subproblem conditioned on P^i. This proves the assertion. □

Observe that the argument is valid also for operational nodes $j \in \mathscr{N}_i^{Op}$ associated with any strategic node $i \in \mathscr{N}^{Strat}$.

8.5 Illustrative Example

To illustrate the implementation of $\mathbb{AV@R}$ on multi-horizon trees we use a stylized example. We show in the example that different ways of modeling risk aversion can change the optimal decisions in the optimization model. As a case study, we consider a network that consists of a field connected to a market through a single pipeline. This is illustrated in Fig. 8.4. The production capacity in the field node is assumed to not constrain our solution, but the pipeline that connects the two nodes has a capacity limit of 100 units. Furthermore, we assume that the market price is fixed at 1 million per unit, while the demand is stochastic. The scenario tree that we use in our example consists of two strategic periods and three strategic nodes (i.e., $i \in \{1,2,3\}$; see Fig. 8.5), each with an associated operational subtree.

We represent the demand uncertainty by 100 equiprobable scenarios in each of the operational subtrees. The demand uncertainty in the subtrees in strategic nodes 1 and 2 is identical and uniformly distributed between 50.5 and 100 units (plot to

Fig. 8.4 The simple network used in our example, consisting of a single field that supplies a market through a pipeline

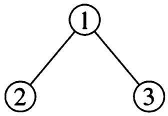

Fig. 8.5 Strategic nodes in the tree

the left in Fig. 8.6), while the demand uncertainty in the subtrees in strategic node 3 is uniformly distributed between 52.5 and 102 units (plot to the right in Fig. 8.6). The probability of strategic nodes 2 and 3 is equal (0.5).

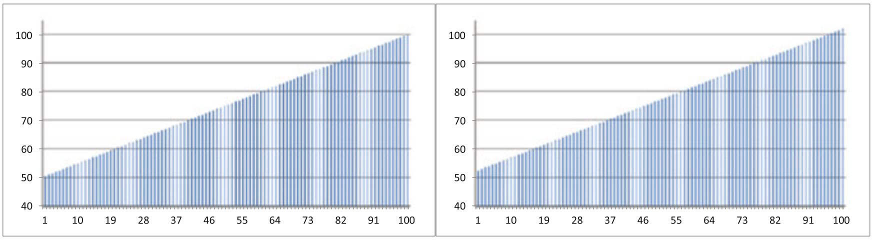

Fig. 8.6 Demand scenarios. The *left plot* shows the demand realizations in the operational subtree linked to the strategic nodes 1 and 2 (uniformly distributed between 50.5 and 100), while the *right plot* shows the slightly higher demand realizations for the subtree in strategic node 3

The company has an investment opportunity that will increase the pipeline capacity from 100 units to 110 units. The cost of this capacity increase is 0.1 million. To simplify our model we assume that the network is operated only in the two strategic periods considered in our example. This means that there are no end-of-horizon effects in our model. We also disregard discount rates and, as there is only a single field, pipeline, and market, we ignore network element indices. This simplified investment model can then be formulated as

$$\max_{\substack{\lambda_i, f_j \\ i \in \mathscr{N}^{Strat}, j \in \mathscr{N}_i^{Op}}} \sum_{i \in \mathscr{N}^{Strat}} \mathbb{P}_i^{Strat} \sum_{j \in \mathscr{N}_i^{Op}} \mathbb{P}_j^{Op} p f_j - \sum_{i \in \mathscr{N}^{Strat}} \mathbb{P}_i^{Strat} \lambda_i I, \tag{8.12}$$

where p is the price in the market, f_j is the volume sold in the market in operational scenario j, λ_i is the (binary) investment decision, and I is the investment cost. The production, flow in the pipeline, and sale in the market are constrained by the pipeline capacity K and the market demand F_j. The company can invest in additional capacity L. The set $\mathscr{N}_{A(i)}^{Strat}$ contains all ancestor nodes for node i as well as the node

i itself (i.e., all the nodes on the path from the root node to node i):

$$f_j \leq K + \sum_{i' \in \mathscr{N}^{Strat}_{A(i)}} \lambda_{i'} L, \quad j \in \mathscr{N}^{Op}_i, i \in \mathscr{N}^{Strat}, \tag{8.13a}$$

$$f_j \leq F_j, \quad j \in \mathscr{N}^{Op}_i, i \in \mathscr{N}^{Strat}. \tag{8.13b}$$

The company can make the investment only once in each scenario:

$$\sum_{i' \in \mathscr{N}^{Strat}_{A(i)}} \lambda_{i'} \leq 1, \quad i \in \mathscr{N}^{Strat}. \tag{8.14}$$

We can then solve the profit-maximizing model (8.12)–(8.14) to find the optimal investment decision. The solution to this problem is trivial since the only node where the capacity extension would influence the revenue is strategic node 3 (the demand associated with strategic nodes 1 and 2 is already covered by the capacity of the pipeline without the investment). The additional expected revenues from having a capacity of 110 units in node 3 are 0.05 million (additional sales in the four scenarios where demand exceeds 100). Since these revenues are smaller than the investment cost, the investment will not be made in any of the strategic nodes.

Let us now consider how risk measures may influence this solution. We assume that the company that operates the field has an obligation to deliver according to the demand level in the market node. The performance is regulated with an $\mathbb{AV}@\mathrm{R}$ constraint enforcing that the expected delivery rate in the worst 5% of the scenarios should be at least 0.995 (meaning that the expectation of the actually delivered volumes divided by the demand in the 5% worst scenarios should be at least 0.995). Figures 8.7 and 8.8 show two different ways of implementing this $\mathbb{AV}@\mathrm{R}$ constraint. Figure 8.7 illustrates the traditional approach where the $\mathbb{AV}@\mathrm{R}$ constraint is based on all operational observations within a given *time period* while Fig. 8.8 illustrates the approach used in the Ramona model. In this case, a separate $\mathbb{AV}@\mathrm{R}$ constraint refers to all operational subtrees associated with a strategic *node*. In the following, we show that these different $\mathbb{AV}@\mathrm{R}$ calculations can indeed influence the investment decision in the model.

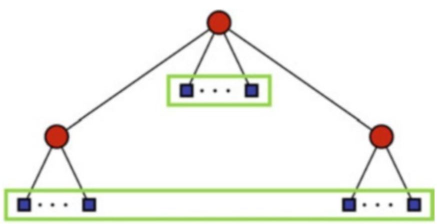

Fig. 8.7 Stage-wise constraints on $\mathbb{AV}@\mathrm{R}$, involving all operational nodes in a strategic period. With this implementation of $\mathbb{AV}@\mathrm{R}$, the optimal investment decision in our model is to not invest in capacity extension in any of the strategic nodes

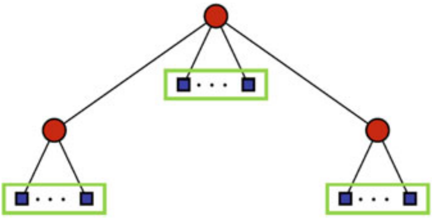

Fig. 8.8 Node-wise constraints, involving all operational nodes associated with a strategic node. With this implementation of the $\mathbb{A}$V@R constraints, the optimal decision in our example is to invest in additional pipeline capacity in strategic node 3

The mathematical formulation of the $\mathbb{A}$V@R constraints that include all operational observations within a given time period can be given as

$$\mathbb{E}\{Del_j | Del_j \leq VaR_{\tilde{\alpha}_t}(Del), j \in \widetilde{\mathscr{N}_t^{Op}}\} \geq \widetilde{Del}_t, \quad \forall t \in \mathscr{T}, \tag{8.15}$$

where the set $\widetilde{\mathscr{N}_t^{Op}}$ includes all operational nodes linked to a strategic time period t. The mathematical formulation used in the Ramona model can be given as

$$\mathbb{E}\{Del_j | Del_j \leq VaR_{\tilde{\alpha}_{\tau(i)}}(Del), j \in \mathscr{N}_i^{Op}\} \geq \widetilde{Del}_{\tau(i)}, \quad \forall i \in \mathscr{N}^{Strat}. \tag{8.16}$$

For more explanation on this $\mathbb{A}$V@R formulation, see the discussion linked to Equation 8.4 on page 187.

In our example, $\widetilde{Del}_t$ and $\widetilde{Del}_{\tau(i)}$ are equal to 0.995 while $\tilde{\alpha}_t$ and $\tilde{\alpha}_{\tau(i)}$ are equal to 5%.

First, let us consider the $\mathbb{A}$V@R constraints that are based on all observations in a time period. We already know that the investment project is not profitable, so to solve the model we only need to check if the $\mathbb{A}$V@R constraint holds. For the subtree linked to the first strategic node, this constraint is clearly satisfied, since the demand is met in all scenarios (the expected delivery rate in the 5% worst scenarios is 1). Considering the operational subtrees linked to the second strategic period, we find that the 5% worst scenarios have an expected delivery rate of 0.995 without any investments (calculated as the expected delivery rate in the 10 scenarios with highest demand). This means that including the $\mathbb{A}$V@R constraint on the model will not alter the optimal decisions.

Now, let us study how $\mathbb{A}$V@R constraints on all operational subtrees linked to a strategic node will influence the optimal solution from the model. Again, we know that the investment option alone is not profitable and we only need to check the $\mathbb{A}$V@R constraints. Obviously, in the operational subtrees linked to strategic nodes 1 and 2, these constraints are satisfied even without the investment (the demand will not exceed the original capacity of the pipeline). In the operational subtree linked to strategic node 3, however, the expected delivery rate of the 5% worst scenarios (the 5 scenarios with highest demand) is 0.990 without the investment. If the pipeline capacity is increased to 110, the $\mathbb{A}$V@R constraint is also satisfied in these operational scenarios (the new expected delivery rate is 1). This means that the inclusion of $\mathbb{A}$V@R constraints in every subtree will force an investment in additional pipeline capacity in strategic node 3.

Obviously, this example is simplified and rather far removed from real investment decisions. The influence of the modeling choice of $\mathbb{A}$V@R constraints on the decision space and, hence, the optimal decisions found by the model is, however, a general result. We chose a simple example to transparently illustrate this effect. We can also note that while the first approach to modeling $\mathbb{A}$V@R is not time consistent, the second approach is. It can be easily seen that the optimal decisions on a rolling horizon will change when using the first approach: it will be necessary to change the original decision of not investing in strategic node 3. The second approach to modeling $\mathbb{A}$V@R is, however, time consistent, and the decisions will not change if we consider a rolling-horizon solution approach.

Acknowledgements The presented work builds on research performed in the Ramona project (The Research Council of Norway, project number 175967) on regularity and security of supply for natural gas transport. We thank our industrial partners Statoil and Gassco for many inspiring discussions and support.
We would also like to thank Michal Kaut at SINTEF for his valuable help while preparing this manuscript.

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