Chapter 6
Medium-Term Operational Planning for Hydrothermal Systems

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Abstract The planning of operations of hydrothermal systems is, in general, divided into coordinated steps which focus on distinct modeling details of the system for different planning horizons. The medium-term operation planning (MTOP) problem, one of the operation planning steps and the focus of this chapter, aims at defining weekly generation for each power plant with the minimum expected operational cost over a specific planning horizon, with regard especially to the uncertainties related to reservoir inflows. Consequently, it is modeled as a stochastic problem and solving it requires the use of multistage stochastic optimization algorithms. In this sense, the objective of this chapter is to discuss the problem features, its particularities, and its importance in the overall operational planning. The stochastic methods usually used to solve this problem and some applications are also presented.

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6.1 Introduction

The operation of electric power systems[1] covers a broad spectrum of activities or studies, among which the planning/scheduling of operations stands out (Test, 1797) [1, 2]. In general, this problem [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21] is divided into several steps (long term, medium term, and short term), which have different planning horizons and which, consequently, each prioritize distinct details of the problem modeling. Briefly, the global problem involves the analysis of the important operational aspects of the system to define the optimal level of energy production to meet demand in an economical and reliable manner.

The medium-term operation planning (MTOP) problem, the focus of this chapter, aims to set weekly generation for each power plant with the minimum expected operation cost over a specific planning horizon, which can vary from months to a year or two, especially taking into account the uncertainties associated with some problem data (inflows and demand, among others). In addition, results from the MTOP can be used to set the spot energy price, depending on the regulatory framework.

Considering the importance of the MTOP, the credibility of the results is essential for the System Operator (SO), which is responsible for settling generation targets, and for the Energy Market (EM) agents, given the economic impacts of the transactions in the market energy environment. In this sense, constant improvements in the MTOP optimization model are required in order to satisfy or update the system and EM participants' requirements. This is the reason why a lot of work has focused on this step of the operation planning problem [6, 15, 22, 23, 24, 25, 26].

As the MTOP is part of a scheduling chain, it can be tightly linked with other steps in this chain in order to obtain the global solution of the operation planning problem, as occurs, for instance, in Brazil. The purpose is to maintain a temporal connection between scheduling chain steps. Briefly, the idea is to exchange information concerning the operational policies, which aim to propitiate coherent global decisions, as illustrated in Fig. 6.1, avoiding, for example, the inefficient use of the generation resources.

Given some problem features, especially those related to data uncertainties, the MTOP is quite complex to solve. As a consequence, solutions obtained by models that do not recognize the uncertainties can produce unsatisfactory results. In other words, the MTOP problem is essentially a stochastic optimization problem [27, 28]. In general, these uncertainties are associated with future inflows into the reservoirs for hydro or hydrothermal systems. Demand and future fuel or energy spot prices can also be modeled as random data, according to the predominance of generation resources of the system or the main objective of the problem.

Like most practical stochastic optimization problems, solving the MTOP problem requires a very significant computational effort, given that the size of the problem increases substantially with the representation of the uncertainties, with the number of stages and with the level of detail in system modeling. Therefore, it

[1] It is important to remark that the systems can be composed of hydro, thermal, or both power plants.

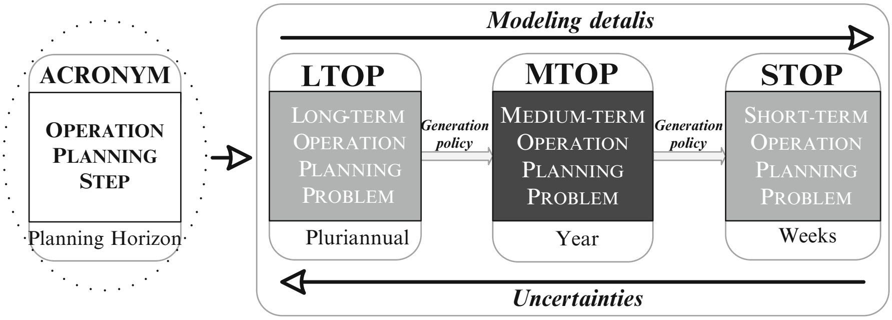

Fig. 6.1 Scheduling chain

is important to balance the level of detail used in modeling uncertainties and system operations by introducing some simplifications to allow the resolution of the problem in reasonable CPU times. For instance, hydro [16, 29] and thermal production and cost functions, which are typically nonlinear and non-convex [1], are usually modeled as linear or a piecewise linear functions to overcome their nonlinearity features. In this way, the resulting optimization problem can be approximated by a linear model.

One therefore ends up normally with a large multistage linear stochastic (MLS) problem [30], for which the use of stochastic decomposition algorithms [31] is essential to reduce the computational burden (in fact, attacking this problem directly keeping its standard structure, known as the deterministic equivalent problem, DEP, is often computationally infeasible). In this context, one must highlight the algorithms based on the Benders Decomposition (BD) principle [32], which display an excellent computational performance when dealing with problem instances with a representative number of scenarios, as shown in [33].

BD-based algorithms, such as the nested decomposition algorithm (NDA) [27] or stochastic dual dynamic programming (SDDP) [4], are broadly used to solve large-scale operation planning problems. The idea underlying NDA is to decompose the DEP into smaller subproblems with a restricted set of variables and constraints, which are easier to solve. These subproblems are solved individually and the coordination among them is performed by means of optimality constraints, which are built and updated iteratively. Although this is an efficient method, some disadvantages are often highlighted for this class of methods [27], as the difficulty to improve the quality of the solution when the iterative process is close to the optimal value. A similar idea can be found behind SDDP, which, however, is more suited to large-scale problems with many stages and scenarios. In this algorithm, scenario sampling, based on the original probability distributions of the random variables [2, 34, 35, 36, 37], is used to reduce the size of the problem, as discussed in another chapter of this book.

Algorithms with different characteristics in relation to the BD-based approach, such as the augmented Lagrangian (AL)-based algorithm [30, 38], have been successfully used to solve MLS problems. In this sense, it is possible to highlight the progressive hedging algorithm (PHA), which has been applied in several fields:

financial market [39], network flow problems [40] and, more recently, operation planning problems [22, 23, 26].

Like other AL-based algorithms, the PHA proceeds by relaxing the nonanticipativity constraints[2] [27, 41] providing an independent set of quadratic subproblems, which are linked via Lagrangian multipliers and penalized by a positive scalar parameter. In this class of methods, the aforementioned disadvantage of BD-based methods can be mitigated because the resulting subproblems are quadratic and they are still sensitive to the use of warm start techniques [25]. Nevertheless, its main disadvantages are associated with the penalty parameter adjustment, which is crucial to algorithm success. Otherwise, it can take a long time to converge.

Based on the aspects mentioned before, the remainder of this chapter aims to describe in detail the main features of the MTOP problem, the mathematical structure of the methods usually used to solve this kind of problem, some application, and important remarks concerning the problem resolution. More precisely, in Sect. 6.2, the stochastic optimization aspects are pointed out, emphasizing the challenges of the MTOP problem resolution. The general problem formulation is presented in Sect. 6.3. In Sect. 6.4, a brief idea of the algorithms and some application results are discussed. Finally, conclusions are presented in Sect. 6.5.

6.2 Stochastic Optimization Aspects

Unlike deterministic problems in which there are no uncertainties with respect to the future data, in stochastic programming problems it is necessary to optimize taking into account the data unpredictability. In order to make the problem resolution computationally viable, it is essential to ensure that the problem horizon has a finite number of stages and, additionally, knowing beforehand the probability distribution of the random variables.

In this context, some important aspects are often addressed in the literature [42], such as the modeling of random variables, the quality solution analysis based on the set of random variables, the problem resolution, solution algorithms, among other aspects.

Once the MTOP problem is essentially a stochastic problem, this section focuses on presenting a brief review associated with the stochastic programming, highlighting some important features which can make the MTOP formulation and challenges more understandable.

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[2] The algorithm details and the stochastic theoretical aspects will be discussed in the next sections.

6.2.1 Problem Description

Assuming that the discrete probability distribution of the random variable is known, the classical two-stage linear stochastic programming problem or two-stage equivalent stochastic problem can be defined as follows:

$$\begin{aligned} &\min c_1^{\mathrm{T}} x_1 + \sum_{\omega \in \Omega_2} p_2^{\omega} c_2^{\mathrm{T}} x_2^{\omega} \\ &\text{s.t}: \quad A_1 x_1 = b_1, \\ &A_2^{\omega} x_2^{\omega} + B_2^{\omega} x_1 = b_2^{\omega}, \\ &x_1 \geq 0, x_2^{\omega} \geq 0. \end{aligned} \tag{6.1}$$

where:

T	Total of stages
t	Index of stage, so that $t = 1,T$
Ω_t	Set of realizations (nodes) on stage t
ω_t	Index associated with a specific realization (node) in the stage t, so that $\omega \in \Omega_t$
c_t	Cost vector related to stage t;
x_t	Vector decision of stage t, $x \in \Re^n$
p_t^{ω}	Probability associated to the each node ω, such that $\sum_{\omega \in \Omega_t} p_t^{\omega} = 1$
A_t^{ω}	Coefficient matrix in stage t (m_t x n_t)
B_t^{ω}	The technology matrix at stage t (m_t x n_t)
b_t^{ω}	Right-hand side for a specific realization ω at stage t

Based on this formulation, as called Deterministic Equivalent (ED) [43] of the stochastic problem, notice that uncertainties can be related to A or B matrices, as well as the vector b. Nevertheless, in the MTOP problem, the randomness is usually associated with the vector b, given that the inflows or the demand are the most future unpredictable data.

Solving this stochastic problem requires the use of methods that exploit the matrices structure of the problem, such as simplex method and interior point method [44].

Notice that the uncertainties ω are addressed to the second stage. The objective of problem (6.1) aims to minimize the cost over two stages, being composed by the costs associated with the decisions x_1 plus the expected future value of the second stage decisions. The remaining equations of (6.1) correspond to sets of constraints related to the first and second stages, interconnected by the technology matrix, B_2, beyond the variable bounds.

As it will be discussed later, this is the same idea of the MTOP problem formulation, i.e., an objective function composed of the total operational cost over the planning horizon and a set of constraints that are associated with the operational features or particularities of the system.

6.2.2 Scenario Tree

Considering the beforehand aspects with respect to the discrete probability distribution, the problem uncertainties are represented by means of a scenario tree. Thus, for a finite number of second stage realization Ω_2, the scenario tree can be represented as illustrated in Fig. 6.2.

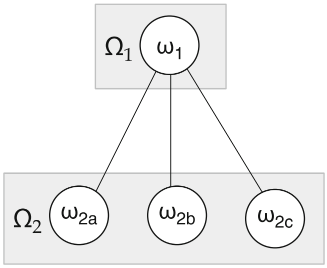

Fig. 6.2 Scenario tree

It is possible to say that the uncertainties representation by means of graphs aims to show the set of the problem realizations in each stage and the link (transition) among the each node decision. Thus, each node of the scenarios tree is associated with a specific realization or a set of random variables. In this context, a specific scenario can be defined as a path from the initial stage to the last stage, regarding a single realization at each stage or, simply, a set of realization from stage 1 onwards. In Fig. 6.2, for instance, a scenario 1 is composed of the realization ω_1 and ω_{2a}, the scenario 2 composed of the realization ω_1 and ω_{2b}, and the scenario 3 composed of ω_1 and ω_{2c}.

According to scenario assumptions, all scenarios share the same first stage realization/decision regardless of the second stage realization (the set of random variables of the first stage is identical in all of them). In this sense, it is possible to define an important concept in stochastic optimization: the nonanticipativity condition [41]. It means that the decisions are not made regarding future expectations, but based on past and current realizations of the random variables. In other words, if two different scenarios have identical path up to stage t, they must have the same decisions until this stage t regardless the next realizations.

Therefore, the scenario tree shown in Fig. 6.2 can also be represented as it is illustrated in Fig. 6.3.

It is possible to say that the nonanticipativity condition is modeled in an implicit way in (6.2) [43], i.e., the first stage decision, equal in all scenarios, is only represented by a unique vector x_1. On the other hand, in Fig. 6.3, the nonanticipativity condition is represented in an explicit way, given that there are nodes associates

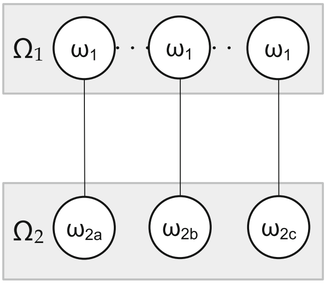

Fig. 6.3 Scenario tree representation—another possibility

with each stage for all scenarios. Therefore, the problem formulation (6.2) can be rewritten as follows:

$$
\begin{aligned}
& f = \min \sum_{s=1}^{S} p^s (c_1^s x_1^s + c_2^s x_2^s) \\
& \text{s.t.}: A_1^s x_1^s = b_1^s, \\
& A_2^s x_2^s + B_2^s x_1^s = b_2^s, \\
& x_1^s - x_1^{\bar{s}} = 0, \ \forall \bar{s} \in \Psi_1^s, \\
& x_1^s \geq 0, x_2^s \geq 0,
\end{aligned}
\tag{6.2}
$$

where:

S	Total number of scenarios
s	Index of scenario, so that, s=1,...,S
Ψ_t^s	Set of all scenarios related to scenario s at stage t by the nonanticipativity condition, including itself
$\bar{s}$	Index associated with Ψ_t^s
p^s	Probability associated with the scenario s

Observe that, in this case, the nonanticipativity condition becomes a set of additional constraints explicit written which aims to ensure the same decision at stage 1. Thus, in short, there are two ways to model a stochastic optimization problem, depending on the nonanticipativity constraints management.

6.2.3 Data Structure

Based on the aspects aforementioned, the nonanticipativity constraints impacts into the problem formulation are pointed out. Initially, observe the matrix structure of problem (6.2) highlighted in Fig. 6.4.

c_1	$p_2^{\omega_{2a}} c_2$	$p_2^{\omega_{2b}} c_2$	$p_2^{\omega_{2c}} c_2$
A_1			
$B_2^{\omega_{2a}}$	$A_2^{\omega_{2a}}$		
$B_2^{\omega_{2b}}$		$A_2^{\omega_{2b}}$	
$B_2^{\omega_{2c}}$			$A_2^{\omega_{2c}}$

Fig. 6.4 Structure of the DE implicit

According to Fig. 6.4, it is easy to see that the structure of stochastic programming problems is substantially sparse. Thus, solving a stochastic problem considering this formulation, the ED implicit of stochastic problem, can require a high computational effort, especially in a multistage case.

For this reason, decomposition algorithms [45] are most often used to overcome the computational burden. By this structure, the decomposition idea is to solve each node subproblem individually, maintaining the link among them by means of some mathematical strategies. It is possible since for a feasible decision x_1, the remaining node subproblems can be solved recursively with a specific set of constraints and variables. For instance, methods based on BD principle "attack" this ED representation.

In turn, problem (6.2) matrix structure is shown in Fig. 6.5.

$p^1 c_1^1$	$p^1 c_2^1$	$p^2 c_1^2$	$p^2 c_2^2$	$p^3 c_1^3$	$p^3 c_2^3$
A_1^1					
B_2^1	A_2^1				
W_1^1		W_1^2		W_1^3	
		A_1^2			
		B_2^2	A_2^2		
W_2^1		W_2^2		W_1^3	
				A_1^3	
				B_2^3	A_2^3
W_3^1		W_3^2		W_3^3	

Fig. 6.5 Data structure of the DE explicit

Notice that the number of variables and constraints of the problem increases when compared to Fig. 6.4. Thus, the computation burden tends to be higher than the ED implicit modeling and, thereby, the ED explicit is basically used by some kind of specific decomposition algorithm. It means that, in case of solving the ED of the stochastic problem, the implicit structure is more indicated.

By this approach, only the nonanticipativity constraints, W^s, connect the scenarios decisions. Thus, the ED explicit decomposition algorithm aims to decompose the problem into scenario subproblems in order to obtain smaller subproblems with variables and constraints that belong to a particular scenario s. In general, algorithms based on AL-based methods, such as PHA, use this formulation to decompose the original stochastic problem.

Finally, it is important to remark that there are algorithms that reduce the problem size by means of exploring the sparse structure of the problem without decomposing it [46, 47].

6.3 MTOP Problem

The MTOP problem presents many particularities, which make it a complex problem to be solved. It is possible to highlight the uncertainties related to some data, such as the inflows, besides other operating characteristics intrinsic to each system. In this sense, the purpose of this section is to discuss some operational aspects related to the majority of MTOP problems with predominance of hydro resources and its consequence into the problem modeling. Additionally, an idea of the problem formulation is presented.

6.3.1 Problem Features

As aforementioned, the MTOP problem usually takes part of a scheduling chain which aims to define the optimal dispatch of all system power plants and, depending on the system regulatory framework, it can also be responsible for giving an economic sign of the energy price. For this reason, besides the stochastic aspects, the modeling of the operational characteristic can be crucial to provide a satisfactory operation of the system, making the MTOP model an important tool for all system agents (operator, generator, regulator, among others).

6.3.1.1 Stochastic Aspects

Although there are other uncertainties in the MTOP problem with hydro power plant predominance, the inflows into the reservoirs modeling has received special attention. In recent years, the studies focusing on the representation of the stochastic issues into the MTOP problem are, in general, only associated with this random variable.

The modeling of these uncertainties is, in general, based in the discrete probability distribution of a historical data. Therefore, one of the most important challenges concerning the MTOP problem is to find a way to represent rightly this original information. In other words, it is necessary to study the best strategy to represent the original infinite scenario tree by means of a representative discrete scenario tree [48], as illustrated in Sect. 6.2.

For that, some issues are extremely important, such as the number of realization in each stage, the precise representation of statistic aspects of the original probability distribution, and the quality solution of the related scenario tree [42].

As introduced before, the demand, the future fuel thermal price and the inflows into the reservoirs can be modeled as a random data in the operation planning problem context. Depending on the MTOP problem features or its main purpose, some of them can be concerned stochastic or simply represented as a deterministic data. For instance, in problems with the hydro power plant predominance, the inflow uncertainties modeling have received special attention in recent years.

6.3.1.2 General Aspects

In addition to the stochastic factors, the MTOP problem has other operational important features which must be modeled in order to represent the physics characteristics and the dynamic of the system. Among them, it is possible to highlight the streamflow balance for each reservoir, the hydro production function [16, 49], the future cost-go function, the load levels, the bounds of the exchange power flow among subsystems or zones, and water travel time among hydro plant located on the same cascade, among others.

Given that the general idea of MTOP problem formulation is detailed in the next subsection, some of these model aspects should be emphasized to make understanding of the modeling easier. For example, the hydro production function and the thermal cost function must be linearized, given that they are essentially nonlinear functions and the MTOP problem must be modeled as a linear programming problem in order to propitiate a computational resolution feasible. It is possible to say that it is not a trivial task, especially in systems where there are many power plants, such as in Brazil, Canada, Colombia, and Norway [17, 18].

Concerning the hydro power plant, the challenge is to obtain a linear function with the same operative characteristics when compared to the original nonlinear function. It can require a high effort, considering that there are hydro plants with production functions neither concave nor convex [50].

Another aspect that adds complexities into the problem formulation is representation of load levels. These levels emulate the load variation over a specific stage. In other words, the purpose consists in divide the demand into the different stage levels, in order to represent the peaks and valleys of the demand. For this reason, the number of variables increases substantially.

Obviously that the modeling depends on the system characteristics and thereby the idea in this section is to show some general features. For instance, in Brazil, the future cost-go function used in the MTOP problem, it is a result from LTOP, which is not currently used in Canada. Moreover, aspects as evaporation and operational particularities of power plants should be represented to propitiate real-life results.

6.3.2 MTOP Problem Formulation

According to the assumptions presented in the previous subsection, it is possible to present the general idea of the problem formulation highlighting the most important constraints and variables based on the Brazilian and Canadian systems. For that, the ED implicit modeling is used and thereby can be written as follows.

6.3.2.1 Objective Function

The objective function aims to minimize the expected operational cost over a specific horizon. It is composed of the fuel thermal cost over the total MTOP horizon in case of hydrothermal system, the penalty associated with the slack of energy (deficit level cost), and the expected future cost, which depends on the reservoir level at the end of the MTOP horizon, T:

$$\text{Min} \quad F = \sum_{t=1}^{T} \sum_{\omega \in \Omega_t} p_t^{\omega_t} \sum_{u \in U} \sum_{e \in E} \left(\sum_{i \in I_e} ct_{iu} gt_{iut}^{\omega_t} + \sum_{\delta=1}^{\Delta} cd_{\delta eu} d_{\delta uet}^{\omega_t} \right) + \alpha_T, \tag{6.3}$$

where:

F	Objective function ($)
E	Energy subsystems or operative zones
e	Index of subsystems or operative zones, so that, e=1,. . . ,E
U	Total number of load levels
u	Index of load levels, u=1,. . . ,U
Δ	Total number of deficit levels
δ	Index of deficit level, δ=1,. . . , Δ
I	Total number of thermal plants
i	Index related to thermal plants, i=1,. . . ,I
ct_{iu}	Thermal incremental cost of the ith thermal plant ($/MWh)
gt_{iut}	Generation of thermal plant i, load level u, and stage t (MWh)
$cd_{\delta eu}$	Deficit incremental cost in δth deficit level, uth load level, and subsystem e ($/MWh)
$d_{\delta eut}$	Deficit in the δth deficit level, uth load level, subsystem e, and stage t (MWh)
α_T	Expected value of operation cost from state T+1 onwards, i.e., a future cost-go function which can be a result from LTOP ($)

6.3.2.2 Supply the Demand

These constraints aim to ensure that the sum of all generation resources is equal to the system demand for all stages, subsystems, and load levels, respecting the operational power plant bounds:

$$\sum_{i=1}^{I} gt_{iut}^{\omega_t} + \sum_{r=1}^{R} gh_{rut}^{\omega_t} + \sum_{l \in \Gamma_e} Int_{leut}^{\omega_t} + \sum_{\delta=1}^{\Delta} d_{\delta uet}^{\omega_t} = L_{uet}, \tag{6.4}$$

where:

R	Total number of hydro plants
r	Index of hydro plants, r=1,...,R
gh_{rut}	Generation of hydro plant r, load level u, and stage t (MWh)
Int	Power interchange from subsystem l to subsystem e, load level u, and stage t (MWh)
L_{uet}	System demand of the uth load level, eth subsystem, and stage t (MWh)
Γ_e	Set of subsystems linked to the subsystem e

6.3.2.3 Stream-Flow Balance

The stream-flow balance constraint ensures that the final volume at the end of a specific stage must be equal to the initial volume plus inflows and minus the total released outflow, regardless casual losses or evaporations:

$$v_{rt}^{\omega_t} - v_{r,t-1}^{\omega_{t-1}} + C\sum_{u}^{U}\left[q_{rut}^{\omega_t} + sp_{rut}^{\omega_t} - \sum_{m \in M_i}\left(q_{mu,t-\tau_{mr}}^{\omega_t} + sp_{mu,t-\tau_{mr}}^{\omega_t}\right)\right] = Cy_{rt}^{\omega_t}, \tag{6.5}$$

where

$v_{rt}^{\omega_t}$	Volume of rth hydro plant reservoir at the end of stage t (hm^3) considering a specific node ω
$q_{rut}^{\omega_t}$	Discharge outflow of hydro plant r, load level u, and stage t (m^3/s)
$sp_{rut}^{\omega_t}$	Spillage of hydro plant r, load level u, and stage t (m^3/s)
$y_{rt}^{\omega_t}$	Incremental inflow of rth hydro plant reservoir and stage t (m^3/s)
C	Conversion factor of water discharge unit (m^3/s) in volume units (hm^3)
M_r	Set of upstream reservoirs from hydro plant r
m	Index of upstream reservoirs, m=1,...,M
τ_{mr}	Number of stages that the total outflow of a hydro plant m takes to reach the downstream hydro plant r

6.3.2.4 Hydro Piecewise Linear Function

$$gh_{rut}^{\omega_t} = \Theta\left(v_{rt}^{\omega_t}, q_{rut}^{\omega_t}, sp_{rut}^{\omega_t}\right). \tag{6.6}$$

The hydro production function is a nonlinear function which related the net head, the generator units efficiency, and the release of the power plant [51]. Nevertheless, in the MTOP problem context, the hydro production is, in general, modeled as piecewise linear function which can depend on the released outflow, the volume, and the spillage, as detailed in [50], or simply depend on the release outflow and the net head. In literature, alternatives approaches are also broadly used to represent the hydro production function, as discussed in [16, 52], because it is definitely not a trivial task to obtain a good hydro production function linearization.

6.3.2.5 Future Cost-Go Function

This function can be given by the LTOP problem, if there is a scheduling chain models or built taking into the future level of the reservoirs. Thus, it estimates the expected future cost. In short, it is a piecewise linear function depending on the volume of water in the reservoirs at the end of the planning horizon, T. In other words, it represents the expected future cost from $T+1$:

$$\alpha_T - \sum_{\omega \in \Omega_T} \sum_{r \in R} \gamma_{rj} v_{rt}^{\omega_T} \geq \alpha_r^0, \tag{6.7}$$

where:

J	Number of linear constraints used in the piecewise future cost-go function
j	Index related to the piecewise future cost function, with j=1,...,J
γ_{rj}	Slope associated with jth linear segment of the future cost-go function associated with hydro plant r (MW/hm3)

6.3.2.6 Bounds

The individual variable limits must be also considered which aims to determine the physic operational of the system and its generator resources, such as the exchange bounds, released outflow, and reservoir volumes:

$$\begin{aligned}
Int_{leut}^{\min} &\leq Int_{leut}^{\omega_t} \leq Int_{leut}^{\max}, \\
v_r^{\min} &\leq v_{rt}^{\omega_t} \leq v_r^{\max} \\
0 &\leq q_{rut}^{\omega_t} \leq q_r^{\max}, \\
0 &\leq sp_{rut}^{\omega_t} \leq sp_r^{\max}, \\
0 &\leq gt_{iut}^{\omega_t} \leq gt_i^{\max}, \\
0 &\leq gh_{rut}^{\omega_t} \leq gh_r^{\max}.
\end{aligned} \tag{6.8}$$

Thus, the (6.3)–(6.8) formulation summarizes the MTOP problem formulation. Obviously that in case of a pure hydro systems or other system peculiarities, some variable considered above must be disregarded or adapted to the system reality.

6.4 Decomposition Algorithms

Based on the features and challenges associated with the MTOP problem resolution presented so far, the study of the stochastic decomposition algorithms becomes mandatory [53, 54]. Depending on the planning horizon and the operational features which are directly related to the problem size, some decomposition algorithms are more appropriated. It is possible to highlight the algorithm based on BD [14, 27, 55, 56, 57, 58], AL [23, 24, 38, 40, 41, 59, 60, 61, 62], and its particularities [63], among others [64].

Thus, in this section, a simplified MTOP problem example is used to show the main features of the two algorithms broadly used to solve this kind of problem. The idea is to discuss the main differences between the algorithms and its properties.

For that, consider the simple hydrothermal system and the inflow scenario tree of the problem illustrated in Fig. 6.6.

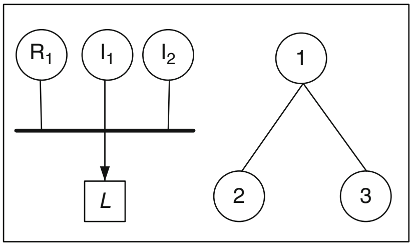

Fig. 6.6 Hydrothermal system and scenario tree

Notice that the hydrothermal system has only one hydro plant, R, and two thermal plants, I_1 and I_2. In this example, the hydro production function is represented by a simple linear function depending on the reservoir released outflow as it is shown in (6.9). With these generator system resources, the objective is to supply a constant demand L with the minimum expected cost over two-stage planning horizon, considering only the inflow uncertainties:

$$gh = q. \tag{6.9}$$

The inflow scenario tree has two possible inflow realizations in the second stage. As a consequence, there are three nodes and two scenarios, given that a scenario can be defined as a complete path from node 1 in the first stage to a specific node in the last (second) stage. The inflows scenario tree data and additional information concerning the hydrothermal system are presented in Tables 6.1 and 6.2, respectively.

Table 6.1 Data of scenario tree?

Stage (t)	Node (ω)	Probability (p)	Inflow [y (hm^3)]	Demand [L (MW)]
1	1	1	300	450
2	2	0.5	100	450
2	3	0.5	400	450

Table 6.2 Additional information of the hydrothermal system

Power plant	Incremental cost [c (\$/MW)]	Maximum generation [gt or gh (MW)]	Initial volume [v_0 (hm^3)]
I_1	50	60	–
I_2	200	450	–
H_1	–	450	150

According to the data shown, it is possible to write the general stochastic optimization problem formulation by means of the implicit ED modeling, (6.10)–(6.13), where, for sake of simplification, the variable bounds are omitted:

$$\begin{aligned} \min F = \ & 50gt_{11}^1 + 200gt_{21}^1 + 0.5(50gt_{12}^2 + 200gt_{22}^2) \\ & +0.5(50gt_{12}^3 + 200gt_{22}^3) \end{aligned} \tag{6.10}$$

$$\text{s.t.:} \quad \begin{aligned} & gt_{11}^1 + gt_{21}^1 + q_{11}^1 = 450 && (node1) \\ & v_{11}^1 + q_{11}^1 = 150 + 300 && (node1) \end{aligned} \tag{6.11}$$

$$\begin{aligned} & gt_{12}^2 + gt_{22}^2 + q_{12}^2 = 450 && (node2) \\ & v_{12}^2 + q_{12}^2 - v_{11}^1 = 100 && (node2) \end{aligned} \tag{6.12}$$

$$\begin{aligned} & gt_{12}^3 + gt_{22}^3 + q_{12}^3 = 450 && (node3) \\ & v_{12}^3 + q_{12}^3 - v_{11}^1 = 400 && (node3) \end{aligned} \tag{6.13}$$

The objective function (6.10) aims to minimize the expected value of thermal production over the horizon taking into account the node probabilities. Additionally, there are set of constraints and variables associated with each node, given by (6.11)–(6.13). The constraints represent the demand supply and the stream-flow balance of the reservoir R_1.

Notice that, in the MTOP problem context, the link between different stages is performed by the reservoir volume, i.e., the initial volume for all nodes in the second stage corresponds to the final volume in the first node. For this reason, it is possible to conclude that the reservoir storage is a state variable of the problem. It is an important MTOP characteristic given that the optimal decision is related to the reservoir storage levels. In other words, once defined the volume storage in each stage, the other variable are consequently determined.

As already mentioned, the (6.10)–(6.13) formulation is the simplest way to represent a stochastic optimization problem. According to Sect. 6.2, it is also possible to write the variables and constraints with respect to scenarios, the explicit ED, as follows:

$$\begin{aligned}\min F &= 0.5(50gt_{11}^{1} + 200gt_{21}^{1} + 50gt_{12}^{1} + 200gt_{22}^{1}) \\ &+0.5(50gt_{11}^{2} + 200gt_{21}^{2} + 50gt_{12}^{2} + 200gt_{22}^{2})\end{aligned} \tag{6.14}$$

$$\text{s.t.}:$$

$$\begin{aligned}&\text{scenario1}: \\ &gt_{11}^{1} + gt_{21}^{1} + q_{11}^{1} = 450, \\ &v_{11}^{1} + q_{11}^{1} = 150 + 300, \\ &gt_{12}^{1} + gt_{22}^{1} + q_{12}^{1} = 450, \\ &v_{12}^{1} + q_{12}^{1} - v_{11}^{1} = 100,\end{aligned} \tag{6.15}$$

$$\begin{aligned}&\text{scenario2}: \\ &gt_{11}^{2} + gt_{21}^{2} + q_{11}^{2} = 450, \\ &v_{11}^{2} + q_{11}^{2} = 150 + 300, \\ &gt_{12}^{2} + gt_{22}^{2} + q_{12}^{2} = 450, \\ &v_{12}^{2} + q_{12}^{2} - v_{11}^{2} = 400,\end{aligned} \tag{6.16}$$

$$v_{111} - v_{112} = 0. \tag{6.17}$$

As discussed in the formulation (6.2), the nonanticipativity constraints are added to the problem formulation in order to guarantee the same decision in all scenarios that share the same nodes until stage T-1. In this case, the purpose is to ensure the unique decision for both scenarios that share the same realizations in stage 1. Due to the reservoir volume is a unique state variable of the MTOP problem, using it to represent the nonanticipativity constraints can be an interesting approach to reduce the problem size, as discussed in [22].

It is important to remark that the differences between the formulations are essential for better understanding of the decomposition algorithm strategies presented later in this chapter. In summary, the BD-based algorithm uses the (6.10)–(6.13) representation in order to get node subproblems and, in turn, the AL-based algorithm explores (6.14)–(6.17) structure to obtain scenario subproblems linked by nonanticipativity constraints.

Before discussing the decomposition algorithms, it is convenient to mention that once the MTOP is modeled as a convex optimization problem [65], both formulations of the MTOP problem present the same optimal objective function value, as it is shown in Table 6.3.

Table 6.3 ED solution

Stages	Node	gt_1	gt_2	gh	v
1	1	60	0	390	60
2	2	60	230	160	0
	3	0	0	450	10
Objective function($)27,500					

6.4.1 Benders Decomposition Algorithm

In this subsection, the idea is to present some basic notion of the BD-based algorithms. Given that the test problem illustrated in Fig. 6.6 is a two-stage stochastic problem, the L-shaped algorithm [27, 66] is detailed. Nevertheless, the theoretical concept can be extended to the multistage algorithm, called NDA, which is, for instance, currently used to solve the Brazilian MTOP problem.

In summary, this algorithm solves the first stage subproblem and manages the remaining stages as other subproblems, solving them recursively. Thus, it moves down and up the scenario tree, also called forward and backward recursions, by means of solving each node subproblem passing forward information to immediate successors to form the right-hand side and passing backward to its ancestors in the form of feasibility cuts (cutting planes) [56].

Thus, the L-shaped resulting subproblems can be written to each node as follows:

$$\begin{aligned} \min f_1 &= 50gt_{11}^1 + 200gt_{21}^1 + \alpha \\ \text{s.t.}: &\quad gt_{11}^1 + gt_{21}^1 + q_{11}^1 = 450 \\ &\quad v_{11}^1 + q_{11}^1 = 150 + 300. \end{aligned} \tag{6.18}$$

$$\begin{aligned} \min f_2 &= 50gt_{12}^2 + 200gt_{22}^2 \\ \text{s.t.}: &\quad gt_{12}^2 + gt_{22}^2 + q_{12}^2 = 450 \\ &\quad v_{12}^2 + q_{12}^2 = v_{11}^2 + 100. \end{aligned} \tag{6.19}$$

$$\begin{aligned} \min f_3 &= 50gt_{12}^3 + 200gt_{22}^3 \\ &\text{s.t.}: gt_{12}^3 + gt_{22}^3 + q_{12}^3 = 450 \\ &v_{12}^3 + q_{12}^3 = v_{11}^1 + 400. \end{aligned} \tag{6.20}$$

Notice that the objective function of subproblem (6.18) has a new variable α, which aims to represent the expected value of the second stage according to the first stage decisions. Consequently, it is updated in each algorithms iteration, as detailed below.

Then, once writing the node subproblems, the L-shaped decisions are sequential. It means that the first stage subproblem (6.18) must be solved to obtain the value of the reservoir volume and, therefore, it uses this information to solve the second stage subproblems: (6.19) and (6.20). This step is called forward and it is briefly illustrated in Fig. 6.7, where the Lagrangian multiplier π associated with the stream-flow balance constraint is also highlighted.

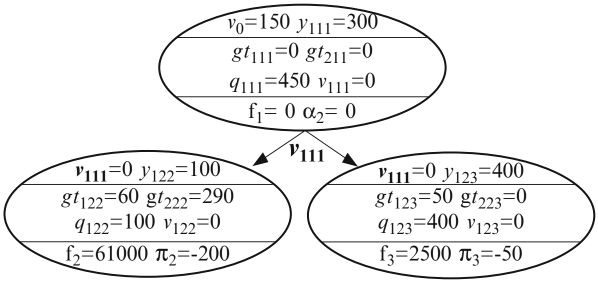

Fig. 6.7 First forward step

After finishing the forward step, by the L-shaped algorithm, it is necessary to compare the total cost of the first stage, the Lower Cost (LC), with the sum of individual cost of each stage f disregarding the future cost, the Upper Cost (UC), as it is detailed in (6.21). If the future cost α accurately represents the second stage cost taking into account a stopping criterion υ, the algorithm is stopped. Based on the results presented in Fig. 6.7, it is easy to notice that the LC, equal to first node cost f_1, is different when compared to the UC showed in (6.21) (in the first iteration, there is no second stage approximation; i.e., α equal to zero):

$$\begin{aligned} &\text{UC} = p_1(f_1 - \alpha) + p_2 f_2 + p_3 f_3, \\ &\therefore \text{UC} = 31{,}750. \end{aligned} \tag{6.21}$$

Thus, the algorithms' next step, called backward, aims to build the expected cost of the second stage. For this, the expected value of the Lagrange multipliers related to the stream-flow balance constraints[3] are used to obtain a Benders cut, which represents the lower bound approximation of the second stage expected cost. In this context, it is possible to write the feasibility cut as follows:

$$\begin{aligned} \alpha - \alpha^* &\geq \bar{\pi}(v_{111} - v_{111}^*), \\ \alpha - (p_2 f_2 + p_3 f_3) &\geq (p_2 pi_2 + p_3 pi_3)(v_{111} - 0), \\ \therefore \alpha + 125 v_{111} &\geq 31{,}750. \end{aligned} \tag{6.22}$$

This constraint is thereby added in the first stage problem (6.18) and, afterwards, a new forward recursion must be initialized. The backward and forward steps should

[3] More precisely, it represents the derivative of dual cost function in relation to volume variable v111, in (R$/hm3).

be continued until the stopping criterion is reached. It is an important remark that the size of the first stage subproblem is increased iteratively, which can cause eventually some impact to the algorithm efficiency.

The convergence process evolution is shown in Fig. 6.8 and the second stage expected cost function α, also called the future cost-go function, is illustrated in Fig. 6.9.

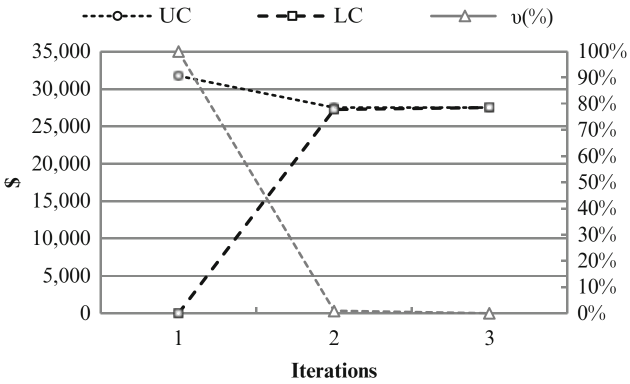

Fig. 6.8 Iterative process

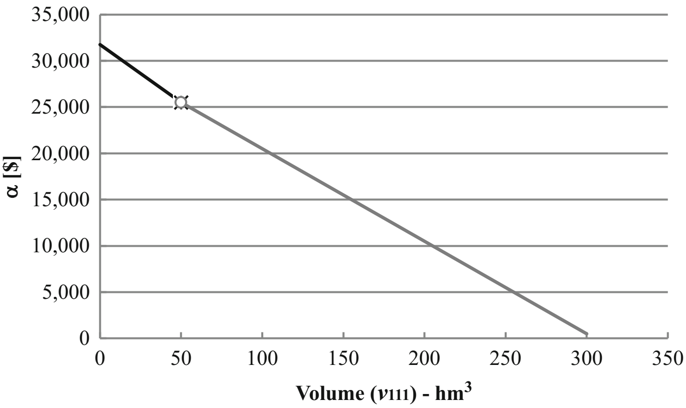

Fig. 6.9 First stage future cost-go function with two linear approximations resulting from two backward steps

Considering that the results are equal to those presented in Table 6.3, two L-shaped aspects should be highlighted: (i) the convergence was attained when the UC and LC were equal (it is an academic example and thereby the stopping criterion υ is equal to zero); (ii) given that three iterations were necessary to reach the stopping criterion (three forward steps), two future cost-go function approximations were built (two backward steps).

6.4.2 Augmented Lagrangian-Based Algorithm

Unlike BD-based algorithms, the AL-based algorithms applied to solve the multistage stochastic problem, such as the PHA, aim to explore the structure of the explicit ED (6.14)–(6.17) in order to decompose it into scenario subproblems. The purpose is to relax the nonanticipativity constraint such that each subproblem presents only variables that belong to a particular scenario.

Thus, following the PHA idea recently applied to solve the Brazilian MTOP problem [22, 23, 24], to obtain the independent scenario subproblems, the first algorithm step is to make the nonanticipativity decisions equal to a constant value corresponding to the expected value of nonanticipativity variables which must be updated iteratively. Thus, regarding the nonanticipativity constraint (6.17), in the PHA, it becomes

$$v_{111} - \bar{v} = 0, \ \ v_{112} - \bar{v} = 0. \tag{6.23}$$

Consequently, problems (6.14)–(6.17) can be rewritten, replacing (6.17) by (6.23), as follows:

$$\begin{aligned}
\min F \ = \ & 0.5(50gt_{111} + 200gt_{211} + 50gt_{121} + 200gt_{221}) \\
& +0.5(50gt_{112} + 200gt_{212} + 50gt_{122} + 200gt_{222}) && (6.24)\\
\text{s.t.}: \ \ & \text{scenario1}: \\
& gt_{111} + gt_{211} + q_{111} = 450, \\
& v_{111} + q_{111} = 150 + 300, && (6.25)\\
& gt_{121} + gt_{221} + q_{121} = 450, \\
& v_{121} + q_{121} - v_{111} = 100, \\
& \text{scenario2}: \\
& gt_{112} + gt_{212} + q_{112} = 450, \\
& v_{112} + q_{112} = 150 + 300, \\
& gt_{122} + gt_{222} + q_{122} = 450, && (6.26)\\
& v_{122} + q_{122} - v_{112} = 400, \\
& v_{111} - \bar{v} = 0, \\
& v_{112} - \bar{v} = 0. && (6.27)
\end{aligned}$$

By the PHA, the next step is to relax (6.28) taking into account the AL concept in order to obtain the following separable problem:

$$\begin{aligned}
\Theta \ = \ & \min 0.5(50gt_{111} + 200gt_{211} + 50gt_{121} + 200gt_{221}) \\
& +0.5(50gt_{112} + 200gt_{212} + 50gt_{122} + 200gt_{222}) \\
& +\pi_1(v_{111} - \bar{v}) + \pi_1(v_{112} - \bar{v}) && (6.28)\\
& +\frac{\mu}{2}\|v_{111} - \bar{v}\|^2 + \frac{\mu}{2}\|v_{112} - \bar{v}\|^2 \\
\text{s.t.}: \ \ & gt_{111} + gt_{211} + q_{111} = 450,
\end{aligned}$$

$$v_{111} + q_{111} = 150 + 300, \tag{6.29}$$
$$gt_{121} + gt_{221} + q_{121} = 450,$$
$$v_{121} + q_{121} - v_{111} = 100,$$

$$\begin{aligned} &gt_{112} + gt_{212} + q_{112} = 450, \\ &v_{112} + q_{112} = 150 + 300, \\ &gt_{122} + gt_{222} + q_{122} = 450, \\ &v_{122} + q_{122} - v_{112} = 400. \end{aligned} \tag{6.30}$$

Therefore, considering fixed values for $\bar{v}$, π_1, π_2, and μ, it is possible to solve problems (6.28)–(6.30) by means of the resolution of each scenario subproblem individually:

$$\Theta = \theta_1 + \theta_2. \tag{6.31}$$

where each scenario subproblem θ can be written as follows:

$$\begin{aligned} \theta_1 = &\min 50gt_{111} + 200gt_{211} + 50gt_{121} + 200gt_{221} \\ &+ \pi_1(v_{111} - \bar{v}) + \frac{\mu}{2}\|v_{111} - \bar{v}\| \end{aligned} \tag{6.32}$$

$$\begin{aligned} \text{s.t.}: \quad &gt_{111} + gt_{211} + q_{111} = 450, \\ &v_{111} + q_{111} = 150 + 300, \\ &gt_{121} + gt_{221} + q_{121} = 450, \\ &v_{121} + q_{121} - v_{111} = 100. \end{aligned} \tag{6.33}$$

$$\begin{aligned} \theta_2 = &\min 50gt_{112} + 200gt_{212} + 50gt_{122} + 200gt_{222} \\ &+ \pi_2(v_{112} - \bar{v}) + \frac{\mu}{2}\|v_{112} - \bar{v}\|^2 \end{aligned} \tag{6.34}$$

$$\begin{aligned} \text{s.t.}: \quad &gt_{112} + gt_{212} + q_{112} = 450, \\ &v_{112} + q_{112} = 150 + 300, \\ &gt_{122} + gt_{222} + q_{122} = 450, \\ &v_{122} + q_{122} - v_{112} = 400. \end{aligned} \tag{6.35}$$

The first iteration solution, considering μ equal to 1, the initial target $\bar{v}$ equal to 0 and the Lagrangian multipliers also equal to 0, is presented in Table 6.4.

Table 6.4 Scenario subproblems solution—first iteration

Stages	Scenario	gt_1	gt_2	gh	v
1	1	60	0	390	60
	2	60	230	160	0
2	1	0	0	450	0
	2	50	0	400	0
Objective function(R$) 27,250					

After solving both scenario subproblems, the next algorithm step consists in calculating the new value of the nonanticipativity condition (6.23), the volume average, and update the Lagrangian multipliers using, for instance, the gradient method [5], as described by (6.36):

$$\pi_s^{iter+1} = \pi_s^{iter} + \mu(v_{11s} - \bar{v}). \tag{6.36}$$

Therefore, the new values of the Lagrangian multipliers are shown in Table 6.5.

Table 6.5 Lagrangian multipliers—first iteration

Scenario	π
1	30
2	-30

Finally, the stopping criterion must be assessed by D_{iter}, as proposed by [22]. It means that while D_{iter} is bigger than error ε, the algorithm process should be continued:

$$D_{iter} = \mathrm{E}\left[\sum_{s=1}^{S=2}\left(\left\|v_{r1s} - \bar{v}\right\|^2 + \frac{1}{\mu^2}\left\|\pi_{s,iter+1} - \pi_{s,iter}\right\|^2\right)\right] < \varepsilon, \tag{6.37}$$

where $E[\cdot]$ represents the expected value.

Figure 6.10 illustrates the "D_{iter} track" iteration after iteration until the stopping criterion is satisfied (in this example problem, D_{iter} is less than 0.1). As the L-shaped algorithm, PHA presented the same optimal results described in Table 6.3 after seven iterations.

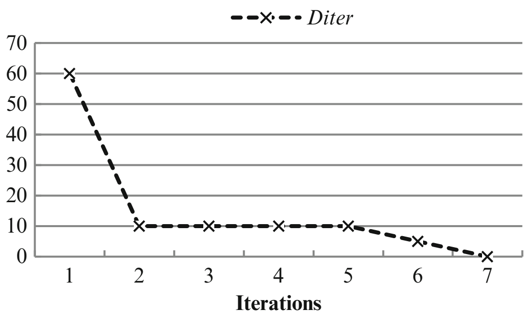

Fig. 6.10 Over the iterative process

Although not mentioned so far, one aspect is essential for the AL-based algorithm success: the choice of the suitable penalty parameter μ. Therefore, instead of using a fixed penalty parameter value during the optimization process as done in this test example, the following update iteratively strategy (6.38) [24] seems more interesting for large-scale cases:

$$\mu_{iter+1} = \mu_{iter} \left\{ \mathrm{E} \left[\sum_{s}^{S} \sum_{t=1}^{T-1} \left(\frac{\|v_{rts} - \bar{v}\|^2}{\left(v_{rt,iter}^{\max} - v_{rt,iter}^{\min} + 1 \right)} \right) \right] \sigma + 1 \right\}. \tag{6.38}$$

According to the algorithm features described above, it is possible to notice the differences between both algorithms presented here . Both algorithms present its particularities as well as advantages and disadvantages. With respect to the PHA, it can mitigate some BD-based algorithms or cutting planes method disadvantages [27] (initial iteration is often inefficient and iterations may become degenerate at the end of the process), given that it provides quadratic subproblems and it is still sensitive to the use of warm start techniques [25], though some other heuristics are required, such as the choice of the penalty parameter.

Finally, it is possible to say that the scenario decomposition algorithms are easily implemented using parallel processing, once it has a weak link between scenario subproblems.

6.5 Conclusions

This chapter focused on the MTOP problem of hydrothermal systems. In summary, this problem aims to define the dispatch of the power plants and the spot energy price depending on system framework, taking into account the uncertainties associates with some problems data.

Given that it is a stochastic problem, important aspects related to the stochastic programming were addressed in this chapter, such as the random variable representation, the solution algorithms, and other theoretical aspects especially associated with the MTOP problem.

With respect to the random variables, the impact of the scenario tree representation into the problem data structure was emphasized, which makes the solution algorithms steps presentation easier. In addition, the stochastic problem formulation related to each scenario tree structure was discussed.

The MTOP problem formulation was also stressed, highlighting some operational features and its particularities. In this context, the current and future challenges taking into account the MTOP modeling were also discussed.

Finally, an academic example was used to help the description of two different solution algorithms with distinct characteristics: L-shaped and an augmented Lagrangian-based algorithm. Both are broadly used in literature to solve these kinds of problems.

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