Chapter 15
Pricing of Energy Contracts: From Replication Pricing to Swing Options

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Abstract The principle of replication or superhedging is widely used for valuating financial contracts, in particular, derivatives. In the special situation of energy markets, this principle is not quite appropriate and might lead to unrealistic high prices, when complete hedging is not possible, or to unrealistic low prices, when own production is involved. Therefore we compare it to further valuation strategies: acceptability pricing weakens the requirement of almost sure replication and indifference pricing accounts for the opportunity costs of producing for a considered contract. Finally, we describe a game-theoretic approach for valuating flexible contracts (swing options), which is based on bi-level optimization.

15.1 Introduction

This chapter deals with energy delivery contracts and their fair prices both from the seller's and the buyer's points of view. A contract between two parties determines the respective obligations of the two contracting sides to deliver or receive energy and to pay or receive money. Typically a contract is valid for a certain period of time and both the energy deliveries and the financial compensations are made at several moments in time. Some energy-related contracts even do not imply delivery of energy, but only financial transfers, which are however related to observable prices in the energy markets.

Suppose for simplicity that a contract states that payments and energy deliveries are due at times $t = 1, 2, \ldots, T$. The payments (cash flows) are denoted by C_t (in currency units). If both parties transfer money to the other one at the same

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time, only the net amount is recorded. By convention, a positive value C_t indicates a money inflow to the contract seller. One contract may involve several forms of energy $j \in \mathscr{J} = \{0, \ldots, J\}$, such as electricity, oil, gas, and coal. To avoid unit conversions as much as possible amounts of energy sources are measured by their energy content (in MWh). Energy of type j delivered in period t (in the time span $(t, t+1]$) is denoted by $D_{t,j}$. We agree that positive amounts $D_{t,j}$ refer to energy inflows, while negative amounts refer to outflows. If the contract involves only one type of energy we will just write D_t.

Both quantities C_t and D_t may be unknown at the time of contracting and may depend on information which is only available at the respective time of settlement (e.g., actual market prices). However, the amounts must be *determinable* by this information. A clause like "The buyer pays 1,000, if there is no extraterrestrial life" is void, because the validity of the condition it is not determinable. Conditions which are determinable but not known at the time of contracting are modeled as random variables.

Pricing principles determine a reasonable price to be offered to the buyer. The basic pricing principle is known as *replication pricing*: the buyer will not accept the price for the contract, if the market offers an alternative possibility, for which the upfront payment is lower and the cash flows or commodity flows are not smaller than the ones contracted. Thus the maximal offered price can be determined by an optimization problem, which is called the replication problem.

The alternative trading/hedging/production strategy belonging to the replication price is called *replication strategy*, if it produces exactly the same cash or commodity flows as the contract under consideration. If it produces larger flows, it is called a *superreplication strategy*. These types of strategies are riskless for the seller: following this strategy, the seller can under no circumstances make losses since the (super)replication must hold with probability 1. This is however a very strong requirement. Quite often, replication strategies do not exist and the superreplication leads to unrealistically high prices, which no buyer would accept. If there are contracts for which replication strategies do not exist in the market, the market is called *incomplete*. Electricity markets are typically incomplete, since replication strategies must use contracts offered on the wholesale market and these few types of contracts are quite simple compared to the possible variety of demand patterns.

As an example, consider a contract for energy delivery of amounts given by Fig. 15.1, upper graph. The lower graph shows possible hedging contracts. Any nonnegative linear combination of them qualifies as a replication strategy, but none of them replicates the demand shown in the upper graph.

In incomplete markets, replication pricing is not appropriate: if the seller wants to conclude a contract, he/she has to accept a certain risk. The *acceptance pricing rule* accounts for that the acceptance price is the minimal upfront payment, which makes the risk of this contract acceptable (but typically not riskless) for the seller. To quantify the notion of acceptable risk, measures of risk are introduced. Again this rule leads to an optimization problem: the minimal upfront payment has to be found under the constraint that the risk value lies below a certain prespecified value.

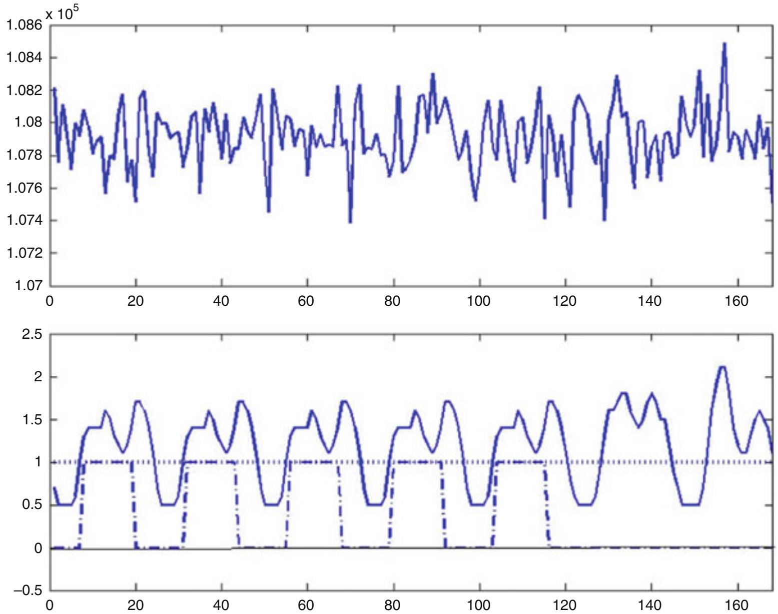

Fig. 15.1 *Above*: a demand profile of an energy buyer for the 168 h of a week. *Below*: the exchange market does only offer specific profiles, the base profile (*dotted*), the peak profile (*dash-dotted*), and the Vattenfall GH0 profile (*solid*)

The acceptance price takes the risk aversion of the seller into account, however does not depend on his/her actual risk exposure. A fine-tailored pricing instrument would take the risk portfolio of the seller into account and would make the contract acceptable only if the total risk exposure of the existing portfolio of contracts augmented by the new contract is acceptable. Notice that this pricing principle depends on the full knowledge of the existing portfolio of resources and contracts, which is not always available. Notice that the same contract may be acceptable for seller A but not acceptable for seller B. Consider for instance the situation when seller A has a lot of baseline energy available, but his contract portfolio is much biased versus peak demand. He would accept a contract which requires delivery in the night hours. On the other hand, if seller B has mostly solar energy to offer, then a contract which delivers at night time risks to require expensive purchases from the spot market and is not advantageous for seller B. The *indifference pricing rule* compares the risk of the existing resource and contract portfolio with the portfolio augmented with the new contract. The indifference price is the lowest price such that the risk of the augmented portfolio is not larger than the risk of the actual portfolio.

The three pricing principles (replication, acceptance, indifference) are applicable for *rigid contracts*, for which all conditions are fixed at contracting time 0. While amounts and prices may depend on future parameters and are considered as random variables at time 0, their way of calculation cannot be changed later by the contract parties. In contrast, *flexible contracts* allow the specification of demands by the buyer at later times. Whenever a contract party has the right to exercise the contract in his discretion, this fact has to be taken into account in the price modeling process. While the ask price of rigid contracts are determined by a regular optimization problem (as described above), the pricing of flexible contracts requires the solution of a bi-level problem. A bi-level problem consists of two coupled optimization problems: an upper-level and a lower-level problem. The lower level describes the optimal reaction of the contract buyer to the price asked by the seller. The seller has to anticipate the reaction of the buyer when calculating the best ask price. Given this reaction, the seller may find the price according to one of the three principles: replication, acceptance, and indifference.

We summarize the mentioned approaches in the following overview:

(1) *(Super)replication* is based on the nonexistence of a better investment strategy for all scenarios.
(2) *Acceptance pricing* is based on the nonexistence of a better investment strategy with an acceptable risk. Superreplication is the special case if only zero risk is acceptable.
(3) *Indifference pricing* considers the actual risk exposure of the seller and accepts only if the additional contract does not increase the risk exposure. It requires to consider and model the full portfolio of all existing contracts and goes far beyond case (2) as there only the contract under consideration has to be considered.
(4) For the pricing of *flexible contracts* the anticipated behavior of the counterparty is taken into account when the price is calculated.

In principle, also the pricing for flexible contracts may be based on (1) replication, (2) acceptance, or (3) indifference. The superreplication principle is quite unrealistic since it is practically impossible to hedge the risk away simultaneously for all reactions of the buyer. We will concentrate on the acceptance principle and the related model structure for flexible contracts in this chapter. However, indifference pricing for flexible contracts can be easily introduced along the lines of the general indifference pricing approach, analyzed in Sect. 15.4.3.

The chapter is organized as follows: in Sect. 15.2 the concept of replication for financial contracts is presented. This concept is adapted to energy contracts in Sect. 15.3. Section 15.4 deals with the more general notion of acceptance pricing and the next Sect. 15.4.3 with the even more powerful notion of indifference pricing. Finally, bi-level acceptance pricing for electricity swing options is presented in Sect. 15.5.

15.2 Replication Pricing of Financial Contracts

As a starting point, we consider financial contracts, which generate for the contract holder a discrete-time sequence of random cash flows $(c_1, \ldots, c_T)$. Unlike for energy contracts, these cash flows are payments from the contract seller to the contract holder (think of the purchase of a share, which requires the initial payment by the buyer but gives him later the benefits of cash flows c_t as dividend payments).

Later, in the context of energy contracts, we set $C_t = -c_t$. We assume that the cash flows are—if necessary—already discounted to the present day by an appropriate discounting scheme [e.g., using a deterministic or stochastic interest rate process (R_t)]. We further assume that the cash-flow process (c_t) is adapted to a filtration $\mathfrak{F} = (\mathscr{F}_0, \mathscr{F}_1, \ldots, \mathscr{F}_{T-1})$, which models the information available at the respective time t.

At time zero money flows only from the buyer to the seller, as the price of the contract is payable at the beginning. From the contract buyer's side, the cash-flow structure is the same, but the signs are opposite. The main questions in contract pricing are what is the maximal price, which is acceptable for the buyer, and what is the minimal price which is acceptable for the seller and are these prices the same?

To answer these questions, alternative investments have to be taken into account: suppose that $m+1$ investment possibilities are given by a stochastic column price vector $S_t = (S_{t,0}, \ldots, S_{t,m})^\top$ (where $S_{t,0}$ relates to the riskless investment), adapted to the filtration $\mathfrak{F}$. Typically $\mathfrak{F}$ will be modeled as the filtration generated by the price process S_t.

Within our setup a hedging strategy is a nonanticipative row vector process $x = (x_0, \ldots, x_{T-1})$ on $\mathbb{R}^{m+1}$, where $x_t = (x_{t,0}, \ldots, x_{t,m})$ denotes the holdings of the $m+1$ investment possibilities during the time interval $[t, t+1]$. Nonanticipativity means that also the decision process x is adapted to the filtration $\mathfrak{F}$. While we prefer intervals with length 1 for notational simplicity, it is easily possible to extend the notation to include periods with different lengths.

With initial capital w and a trading strategy x, let $Y_t^{w,x}$ be the wealth at time t resulting from this strategy. To be more precise, let $Y_{t-}^{w,x}$ be the wealth just before time t. At time t the portfolio may be restructured and $Y_t^{w,x}$ denotes the wealth just after these transactions are made.

15.2.1 The Upper Price

Given investment opportunities as above, a market price π for a contract is acceptable for the buyer only if there is no better investment for the same or a lower price, i.e., there is no initial payment w and trading strategy x, such that $w < \pi$ and $Y_{t-}^{w,x} - Y_t^{w,x} \geq c_t$ (the cash flows can be paid) for all t and $Y_T^{w,x} \geq 0$ (the terminal wealth is nonnegative).

The upper price π_u is the highest price a potential buyer is willing to pay for the contract with the given cash-flow structure c_t. It is given as the minimal value of the following optimization problem:

$$\left\| \begin{array}{ll} \text{Minimize (in } x \text{ and } w\text{): } w & \\ \text{subject to} & \\ Y_0^{w,x} = w, & \\ Y_{t-}^{w,x} - Y_t^{w,x} \geq c_t & t = 1, \ldots, T, \\ Y_T^{w,x} \geq 0, & \\ x_t \text{ is nonanticipative.} & \end{array} \right. \tag{15.1}$$

Notice that in case that the minimal value in (15.1) is attained by a trading strategy x, then the contract seller may execute this strategy to completely hedge the risk away. He/she would take the price w to invest in such a way that the cash-flows c_t are covered by the earnings of the invested portfolio and the final financial position is nonnegative, i.e., at the end the seller is free of debts out of this contract. The optimal $x = (x_0, \ldots, x_{T-1})$ in (15.1) describes the superreplication strategy.

15.2.2 The Lower Price

The seller receives an initial amount w and has to pay the cash flows c_t at later times. The price π of the contract is acceptable for the seller only if there is no alternative strategy, which receives more at the beginning and has lower liabilities later, i.e., there is no initial liability w and a strategy x, such that $w > \pi$ and $Y_{t-}^{w,x} - Y_t^{w,x} \leq c_t$ a.s., where $Y_t^{w,x}$ denotes now the liability process. At the end of the trading period, the liabilities $Y_T^{w,x}$ must be nonpositive.[1] That is, the lower price π_ℓ of this contract is the maximal value of the following optimization problem for liabilities $Y_t^{w,x}$:

$$
\left\|
\begin{array}{ll}
\text{Maximize (in } x \text{ and } w\text{): } w & \\
\text{subject to} & \\
Y_0^{w,x} \geq w, & \\
Y_{t-}^{w,x} - Y_t^{w,x} \leq c_t & t = 1, \ldots, T-1, \\
Y_T^{w,x} \leq 0, & \\
x_t \text{ is nonanticipative.} &
\end{array}
\right.
\tag{15.2}
$$

We call a strategy x which is feasible for this problem a *subreplication strategy*. All prices greater than π_ℓ are in principle acceptable for the seller, although he/she would prefer to get the upper price π_u.

15.2.3 The Linear Setup

In the simplest case with proportional transaction costs and volume-independent prices, the determination of the upper and lower prices amounts to solving a linear (stochastic) program.

If the transaction costs are neglected, the upper price π_u can be calculated by the following linear program[2]:

[1] Negative liabilities are profits.

[2] We denote by $x \cdot S$ the inner product of the vectors x and S.

$$\left\| \begin{array}{l} \text{Minimize (in } x \text{ and } w\text{): } w \\ \text{subject to} \\ x_0 S_0 - w \le 0, \\ x_{t-1} S_t \ge x_t S_t + c_t \qquad t = 1, \ldots, T, \\ x_{T-1} S_T \ge 0, \\ x_t \text{ is nonanticipative.} \end{array} \right. \tag{15.3}$$

In similar manner it is possible to formulate the lower price problem as a linear program:

$$\left\| \begin{array}{l} \text{Maximize (in } x \text{ and } w\text{): } w \\ \text{subject to} \\ x_0 S_0 - w \ge 0, \\ x_{t-1} S_t \le x_t S_t + c_t \qquad t = 1, \ldots, T, \\ x_T S_T \le 0, \\ x_t \text{ is nonanticipative.} \end{array} \right. \tag{15.4}$$

Dualization of these linear programs then entails the following well-known result: let $\tilde{c}_t = c_t / S_{t,0}$ and $\tilde{S}_t = S_t / S_{t,0}$, where $S_{t,0}$ denotes the price of the riskless investment. Then

$$\pi_u = \max\{\sum_{t=1}^{T} \mathbb{E}_Q(\tilde{c}_t) : \ (\tilde{S}_t) \text{ is a martingale under } Q \}, \tag{15.5}$$

$$\pi_\ell = \min\{\sum_{t=1}^{T} \mathbb{E}_Q(\tilde{c}_t) : \ (\tilde{S}_t) \text{ is a martingale under } Q \}. \tag{15.6}$$

Here $\mathbb{E}_Q$ is the expectation w.r.t. the probability measure Q. The upper and the lower price are equal if there is a unique martingale measure Q. In this (rather exceptional) case Q is called *the risk neutral measure* and $\pi_u = \pi_\ell$ for all contracts. If there are several martingale measures, then the model is called an *incomplete market model*. In incomplete markets, typically $\pi_\ell < \pi_u$ and we speak of an *ask-bid interval*. If the price is within the interval $[\pi_\ell, \pi_u]$, then it is acceptable for both parties and none of them may have an arbitrage opportunity.[3] However, even in incomplete markets, there may be contracts for which the inequalities in (15.1) and (15.2) are satisfied as equalities, meaning that the cash-flow c_t can be completely replicated and the optimal superreplication and subreplication strategies coincide. In this case $\pi_\ell = \pi_u$ even in incomplete markets.

On the other hand, for unbounded processes S_t, it may happen that $\pi_u = \infty$ [infeasibility in (15.1)] and/or that $\pi_\ell = -\infty$ [infeasibility in (15.2)]. It is unrealistic to assume that the buyer will pay any arbitrary price if $\pi_u = \infty$. To the contrary, he/she will not conclude the contract at all. That is why we consider acceptance pricing as a realistic alternative.

[3] The ask-bid interval has to be clearly distinguished to the bid-ask spread (bid-price $<$ ask-price) appearing in stock exchanges, when no deal can be made.

Transaction costs lead to an increased ask-bid interval. Furthermore, nonproportional transaction costs result in nonlinear and/or integer programs to be solved for pricing. For a comprehensive overview of convex models see, e.g., [16]. If the hedging process is nonconvex, a duality gap may occur and the notion of martingale measure makes no sense.

15.3 Replication for Energy Contracts

So far we have considered the valuation of pure financial contracts, which leads to classical results of derivative pricing. In the context of energy risk management we have to extend the analysis to deal with energy-related commodities in the following. There is a fundamental difference between pricing in financial markets and pricing in energy markets: while in financial markets the set of feasible trading strategies is considered to be the same for the seller and for the buyer, it is different in energy markets. The seller has a much larger spectrum of possible actions; he/she may use his/her own energy production, buy energy futures, and trade on the wholesale markets; and the buyer typically has no access to these possibilities, maybe for the exception of access to the spot markets. For this reason, the seller may determine the upper price by including all his/her assets in a replication model and may offer this price to the buyer.

15.3.1 Scope and Basic Model Setup

A key difference between financial and commodity derivatives results from the critical role of physical quantities and physical restrictions for the latter. The number of basic financial securities on which derivatives are written (i.e., the market capitalization, number of shares, etc.) is fixed. It is not necessary to produce securities and the market participants can hold arbitrary amounts without physical restrictions at negligible costs. It is also possible to go short in securities to a huge extent. Furthermore constraints on traded amounts and on the speed of trading are almost not existent.

Some energy-related contracts (usually futures) are settled financially and can therefore be viewed as financial contracts. However the picture is completely different for the physical commodities: they are produced and transported, and storage is costly and restricted. In particular negative storage is not possible. Commodities like electricity cannot even be stored. Therefore the usual relation between direct and future financial contracts is in general not valid for commodities and valuation concepts from finance cannot be applied to over-the-counter (OTC) contracts, which are settled physically.

Nevertheless, also for energy contracts we may seek for the smallest amount of cash necessary, at time zero to finance all feasible actions, in particular both physical and financial contractual cash flows. However, but the model has to be extended to capture all peculiarities of energy markets.

In the extended model it is possible to invest in different forms of energies: let $\mathscr{J} = \{0,\dots,J\}$ denote the set of available energy commodities, e.g., gas, heating oil, and electricity, measured by their energy content (MWh). The related spot prices are $S^e_{t,j} \in \mathbb{R}^{J+1}$, and $x^e_{t,j}$ denotes the stored amount of the jth commodities at time t. Throughout this paper the index $j = 0$ is reserved for (physical) electricity delivered at time t.

With minimum storage level zero and maximum levels $\bar{x^e} = (\bar{x}^e_0,\dots,\bar{x}^e_J) \geq 0$ we have to consider constraints

$$0 \leq x^e_t \leq \bar{x}^e. \tag{15.7}$$

For electricity no storage is available and we have $\bar{x}^e_{t,0} = 0$. We assume proportional storage costs ζ^e_j for each storage j.

Moreover, $y^e_t = (y^e_{t,0},\dots,y_{t,J})$ denotes the amounts of energy bought ($y^e_{t,j} \geq 0$) or sold ($y^e_{t,j} \leq 0$) at prices $S^e_{t,j}$ at time t, and matrices Z^e_t with elements $z^e_{t,ij}$ model the amount of energy i used to produce energy j during period $(t,t+1]$. Energy conversion leads to variable operating costs, which we assume to be proportional to input energy. The cost factors can be time dependent and are denoted by $\gamma_{t,ij}$ (currency unit per MWh).

Related to the conversion from energy i to energy j are efficiencies η_{ij}. In this way it is, e.g., possible to model electricity production from different fuels. Conversion between different forms of energy is restricted by lower and upper bounds $\underline{z}_t, \bar{z}_t$, i.e.,

$$0 \leq \underline{z}_{t,ij} \leq z^e_{t,ij} \leq \bar{z}_{t,ij}, \tag{15.8}$$

which reflects physical constraints on production. On the other hand, trading of energy is not restricted in the basic superhedging setup.

For certain energy sources (e.g., stored water in a certain reservoir) one may also consider random inflows, denoted by $d_{t,j} \geq 0$. Further inflows can result from intermittent electricity production by renewable sources like wind or solar power.

Note that in this framework it is also possible to model energy-related contracts with physical delivery, if the lower and upper bounds for conversion and the related conversion costs are modeled time dependent or depending on other variables of the system. As an example consider an electricity future j: conversion between electricity deliverable by the future contract and actual electricity happens during the delivery period. The related conversion factors are described by $\eta_{j0} = 1$. For the seller of the delivery contract we have during delivery $\bar{z}_t = \underline{z}_t = -x_{t,j}/n_t$, where n_t is the number of remaining exercise dates. Outside the delivery period we have $\bar{z}_t = \underline{z}_t = 0$. Finally, the delivery price is modeled by using the operating cost factors $\gamma_{t,j0}$.

In addition to forms of energy and physically settled contracts we use a cash position $x_{t,0}$ with an interest yield of $r_f > 0$ and include financial assets and contracts $\mathscr{I} = \{1,\dots,I\}$ with prices $S^f_{t,i}$, paying cash flows $C^f_{t,i}$ at time t. While typical financial assets are not in the focus for pure energy-related valuation problems, energy derivatives with financial settlement can be modeled in this way. As an example, an electricity future contract i for fuel j with strike price K_i pays $S^e_{t,j} - K_i$ during delivery. Holdings of financial contracts are denoted by $x^f_{t,i}$ and are not restricted; in particular there are no shortselling constraints.

In the extended model we use physical and financial contracts to hedge an OTC contract that is defined by cash flows C_t and physical flows of energy B_t, with $B_{t,j}$ denoting the flow of energy j at time t in MWh. Inflows are reflected by positive and outflows by negative values. Both C_t and D_t can be random and can depend on other variables of the system.

For a simple delivery contract, D_t (delivery of one commodity) and C_t are constant during the whole time of delivery. A swap contract for different forms of energy i, j can be modeled by setting $D_{t,i} \geq 0$ and $D_{t,j} \leq 0$.

Table 15.1 gives an overview of the possible conversions within the proposed framework.

Table 15.1 Possible conversions, including cash

	1.	2.	3.	4.	5.	6.	7.
1. Cash		x	x		x	x	x
2. Energy commodities	x	x			x		
3. Phys. en. contracts	x	x					
4. (Stored water)					x		
5. Electricity	x			x			
6. Phys. el. contracts	x				x		
7. Financial contracts	x						

15.3.2 Formulation of the Extended Valuation Model

Using the outlined notation we can now formally describe the financial sellers' problem to energy markets. We do not consider transaction costs on energy related and financial markets. Hence it is possible to give an LP formulation.

Stored energy starts with an initial storage x^0. In the subsequent periods storages are changed by buying and selling energy, by conversion between energy forms and contractual deliveries of the physical contracts in the portfolio and of the OTC contract under consideration. For all energy contracts except electricity ($j \neq 0$) we formulate this as

$$x^e_{0,j} \leq x^0 + y^e_{0,j} + d_{0,j} \tag{15.9}$$

and

$$x^e_{t,j} \leq x^e_{t-1,j} + y^e_{t,j} + \sum_{i=0}^{J} \eta_{ij} z^e_{t-1,ij} - \sum_{i=1}^{J} z_{t-1,ji} + d_{t,j} + D_{t,j}. \tag{15.10}$$

Note that the index in the first sum begins at $i = 0$. This allows for pumping, if energy j refers to water, stored in a reservoir.

Optimization will lead to boundary solutions in (15.9) and (15.10). For electricity (which is not storable) we require

$$0 = y^e_{0,0} + d_{0,0} \tag{15.11}$$

$$0 = y^e_{t,0} + \sum_{i=1}^{J} \eta_{i0} z^e_{t-1,i0} + d_{t,0} + D_{t,0}. \tag{15.12}$$

Note that (15.9)–(15.12) refer to the point in time immediately before the next production period $[t, t+1)$ begins and recall that storage is also constrained by (15.8), which models physical restrictions as well as contractual limits for physical contracts.

In a model with discrete time, only energy stored at the beginning of a period can be used for conversion during the period. Therefore we introduce the following constraints:

$$\sum_{j=1}^{J} z^e_{t,ij} \leq x^e_{t,i}. \tag{15.13}$$

The cash account is $x^f_{t,0}$. Cash is considered after buying and selling energy and settling all types of contract but before actually converting between energies. Hence with an initial amount w of cash just before the transactions at time zero to be effectuated, $x^f_{0,0}$ must fulfill

$$x^f_{0,0} \leq w - \sum_{j=0}^{J} S^e_{0,j} y^e_{0,j} - \sum_{i=0}^{I} S^f_{0,i} x^f_{0,i}. \tag{15.14}$$

In the subsequent periods $t > 0$ the cash position accumulates gains and subtracts costs from buying and selling energy and financial contracts and the cash flows from the OTC contract under consideration. Furthermore, it has to account for interest on cash, costs for energy conversion, storage costs, and all cash flows from financial contracts. This results in

$$\begin{aligned} x^f_{t,0} \leq\ & (1 + r_f) x^f_{t-1,0} \\ & - \sum_{j=0}^{J} S^e_{t,j} y^e_{t,j} - \sum_{i=1}^{I} S^f_{t,i} (x^f_{t,i} - x^f_{t-1,i}) + \sum_{i=1}^{I} C^f_{t,i} + C_t \\ & - \sum_{i=0}^{J} \sum_{j=0}^{J} \gamma_{t,ij} z_{t,ij} \\ & - \sum_{j=1}^{J} \zeta_j \frac{(x^e_{t,j} + x^e_{t-1,j})}{2}. \end{aligned} \tag{15.15}$$

Note that we do not use a nonnegativity constraint on the cash position, that is, borrowing money is allowed.

Finally the inequality

$$x^f_{T,0}+\sum_{j=1}^{J} S^e_{T,j}x^e_{t,j}+\sum_{i=1}^{I} S^f_{T,i}x^f_{T,i}\geq 0 \tag{15.16}$$

ensures that we search for the smallest initial payment w and a related hedging strategy, such that the asset value—consisting of the final cash position $x^f_{T,0}$ and all physical and financial contracts—is nonnegative after handling the OTC contract under consideration.

Based on the previous considerations and using the hedging approach, the valuation problem can be formulated as the following optimization problem:

$$\left\| \begin{array}{l} \text{Minimize (in } x^e,\, x^f,\, y,\, z \text{ and } w\text{): } w \\ \text{subject to} \\ \text{constraints (15.7)–(15.16),} \\ x^e_t, x^f_t, y_t, z_t \text{ are nonanticipative.} \end{array} \right. \tag{15.17}$$

In principle, both the seller's and the buyer's hedging problem have the same form (15.17). However, the typical situation in energy markets, the buyer's set of possibilities in producing, trading, or hedging is usually restricted. Thus a natural asymmetry between the contracting partners occurs.

Another key difference lies in the fact that the streams C_t and D_t have different signs for the two participants, e.g., for a simple delivery contract for one commodity, the physical flow D_t is negative for the seller and positive for the buyer, while the opposite holds for the cash flow C_t. Hence even if the buyer would have access to all types of financial and commodity contracts, there is still another source of asymmetry: it is impossible to just change the signs of the flows of commodities to get the picture of the other contractor: usually efficiencies are not symmetric, i.e., $\eta_{ij}\neq\eta_{ji}$. Consider e.g., the production of electricity: for pumped turbines it is possible to use electricity for storing it in higher reservoirs, but with low efficiency, compared to the efficiency of producing electricity from stored water. As an extreme example of asymmetry it is not possible to produce fuel from electricity delivered by a contract, while fuel clearly can be used to produce electricity with some positive efficiency. Moreover, by the existence of bounds in production the problem does not scalarize.

15.4 Acceptability Replaces Non-replicability

Clearly, the principle of (super)replication is one of the cornerstones of modern finance. On the other hand it might be too strong under some circumstances. As was already said, it may lead to very large (even infinite) upper prices in incomplete markets.

This is especially true in the case of electricity markets: generating companies have the equipment to buy, store, and use fuel to generate electricity in order to satisfy even very complicated contractual terms. Physical constraints are present but only mildly affect the ability to hedge the flows related to the contract. On the other hand other market participants do not own the same equipment. Pure traders have access to electricity exchanges or pools, and hence to the full spectrum of financially settled contracts, but are not able to produce electricity. End consumers do not even have access to exchange markets. While contractual energy flows can be very complicated for OTC contracts, electricity is not storable and there are only very few instruments (i.e., contracts such as base and peak futures) available to partially hedge a specific contract. So even for a trader with access to an electricity exchange, hedging is difficult and will in general work only approximately—there will always remain residual electricity flows that have to be settled by buying spot electricity without protection from future contracts.

For these reasons we analyze the pricing problem by the notion of acceptability: it is wanted that the difference between the optimal hedge and the cash-flow process is acceptable for the seller, which—in the most basic formulation—means that inequality (15.16) is replaced by

$$\mathscr{A}(C_T + \sum_{j=1}^{J} S^e_{T,j} x^e_{t,j} + \sum_{i=1}^{I} S^f_{T,i} x^f_{T,i}) \geq 0, \tag{16'}$$

where $\mathscr{A}$ is an acceptability functional (see below). In this way, unfavorable scenarios are not avoided completely at the end. Instead, the loss distribution is restricted by the acceptability functional, such that only unfavorable outcomes with small probability are acceptable.

The resulting optimization problem for acceptability pricing is a modification of (15.17) and can be written as

$$\left\| \begin{array}{l} \text{Minimize (in } x^e, x^f, y, z \text{ and } w\text{): } w \\ \text{subject to} \\ \text{constraints (15.7)–(15.15),} \\ \mathscr{A}(C_T + \sum_{j=1}^{J} S^e_{T,j} x^e_{t,j} + \sum_{i=1}^{I} S^f_{T,i} x^f_{T,i}) \geq 0, \\ x^e_t, x^f_t, y_t, z_t \text{ are nonanticipative.} \end{array} \right. \tag{18}$$

15.4.1 Acceptability Functionals

A *probability functional* is an extended real-valued function defined on some random space or on a suitable subset of a random space. Examples are well-known functionals like the expectation, the median, value-at-risk, average (or "conditional") value-at-risk, and variance. If the value of a probability functional depends only on the distribution of the random variable under consideration, it is called *version independent*. If a functional is interpreted in the sense that higher values are preferable to lower values, we call it an *acceptability-type functional*.

Acceptability functionals are probability functionals $\mathscr{A}$, defined on a linear $\mathscr{Y}$ space of random variables on $(\Omega, \mathscr{F}, \mathbb{P})$, such that the following properties are true for all $X, Y \in \mathscr{Y}$:

(A1) Concavity. $\mathscr{A}(\lambda \cdot X + (1-\lambda) \cdot Y) \geq \lambda \cdot \mathscr{A}(X) + (1-\lambda) \cdot \mathscr{A}(Y)$ holds for $\lambda \in [0,1]$.
(A2) Monotonicity. $X \leq Y \; a.s. \Rightarrow \mathscr{A}(X) \leq \mathscr{A}(Y)$.

Often (see, e.g., [17]), acceptability functionals are defined by including the translation equivariance property:

(A3) Translation equivariance. $\mathscr{A}(Y+c) = \mathscr{A}(Y) + c$ holds for all constants c.

An acceptability functional is called *positively homogeneous*, if it satisfies the condition $\mathscr{A}(\lambda Y) = \lambda \cdot \mathscr{A}(Y)$ for all $\lambda \geq 0$. It is called strict, if $\mathscr{A}(Y) \leq \mathbb{E}(Y)$ holds. Recall that for an acceptability functional $\mathscr{A}$ and a random loss Y the valuation $-\mathscr{A}(-Y)$ is a coherent risk measure in the sense of [1].

Throughout this paper we will consider only acceptability functionals with $\mathscr{A}(0) = 0$. If necessary, this can be achieved easily by relocating the functional. Under rather mild conditions (upper semicontinuity) an acceptability functional $\mathscr{A}_t$ has a dual representation

$$\mathscr{A}(Y) = \inf\{\mathbb{E}(YZ) - \mathscr{A}^+(Z) : Z \in \mathscr{Y}^*\},$$

where Z is an element of the dual space $\mathscr{Y}^*$ and $\mathscr{A}^+$ is the concave conjugate (in the sense of Fenchel–Moreau–Rockafellar, see [20]) of the functional $\mathscr{A}$. If the functional is positively homogeneous, then the dual representation simplifies to

$$\mathscr{A}(Y) = \inf\{\mathbb{E}(YZ) : Z \in \mathscr{Z}\}, \tag{19}$$

where $\mathscr{Z}$ is a convex subset of $\mathscr{Y}^*$.

An important, but simple, example for a positively homogeneous acceptability functional is the *average value-at-risk*. For a random variable Y with distribution function G_Y it is defined by $\mathbb{A}\mathrm{V@R}_\alpha(Y) = \frac{1}{\alpha}\int_0^\alpha G_Y^{-1}(u)du$ and is also known as conditional value-at-risk or tail value-at-risk. Its conjugate representation is given as follows [see [17], Theorem 2.34 (ii)]:

$$\mathbb{A}\mathrm{V@R}_\alpha(Y) = \inf\left\{\mathbb{E}(Y \cdot Z) : \mathbb{E}(Z) = 1, 0 \leq Z \leq \frac{1}{\alpha}\right\}. \tag{20}$$

In our examples we will use the average *value-at-risk*, because it is closely related to the notion of value-at-risk, the most important risk measure in practice. The value-at-risk of a random variable X at confidence level $0 \leq \alpha \leq 1$ is basically defined as the value-at-risk of the related distribution (see, e.g., [14]), which is given by $\mathbb{V}\mathrm{@R}_\alpha(Y) = \inf\{v : P\{Y \leq v\} \geq \alpha\}$.

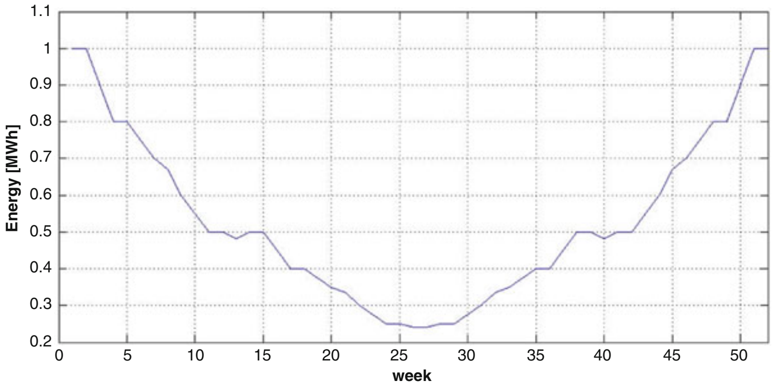

Fig. 15.2 Acceptability pricing: delivery pattern D_t over the 52 weeks of a year

Using the value-at-risk in (16′) is called quantile hedging/pricing; see, e.g., [8]. In this case $\mathbb{V}@\mathrm{R}_\alpha(Y) \geq q$ is equivalent to $P\{Y < q\} \leq \alpha$. $\mathbb{V}@\mathrm{R}$ is monotone, but unfortunately the mapping $Y \mapsto \mathbb{V}@\mathrm{R}_\alpha(Y)$ is not convex and nonsmooth. This makes the usage of constraint (16′) in an optimization problem very difficult.

On the other hand $\mathbb{A}\mathrm{V}@\mathrm{R}$ is an acceptability functional and is a concave minorant of the $\mathbb{V}@\mathrm{R}$. As an alternative to using quantiles or acceptability functionals, one may also consider utility functions U and accept a contract, if $\mathbb{E}[U(Y)] \geq q$. However, the price will then depend on the choice of the entire utility function while in quantile pricing only two parameters, the threshold q and the confidence level $1 - \alpha$, have to be set by the management.

The following example illustrates acceptability pricing, using the average value-at-risk.

Example 1. We consider a planning horizon of 1 year (52 weeks). Electricity spot prices are modeled by geometric Brownian motion with jumps (GBMJ), estimated from EEX Phelix hourly electricity prices (hourly, 09/2008–12/2011, Bloomberg). The pricing model was reformulated and solved on a stochastic tree, generated from the GBMJ model.

The hedging opportunities are represented by four future contracts, related to the quarters of the year, i.e., each of the futures delivers a constant amount of electric energy during one of the quarters. The delivery pattern of the contract to be valued is shown in Fig. 15.2.

Using problem (18) the acceptability price is calculated for a pure trader meaning that only wholesale base quarter future contracts can be used for hedging for different values of the $\mathbb{A}\mathrm{V}@\mathrm{R}$-parameter α, shown in Fig. 15.3. The related optimal hedging strategies can be seen in Fig. 15.4. Finally, Fig. 15.5 shows the density of the optimized distribution of profits and the value-at-risk at level $\alpha = 0.1$.

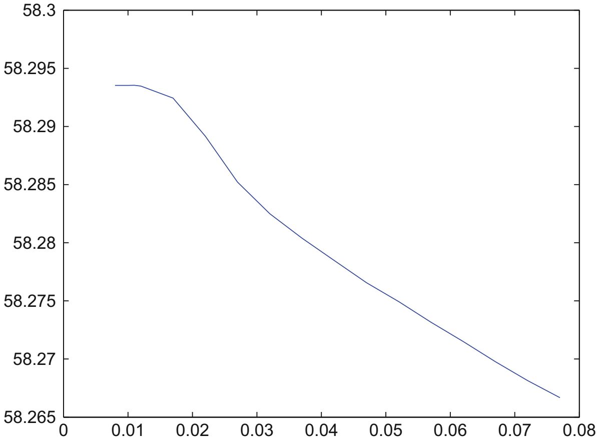

Fig. 15.3 Acceptability pricing: the price of 1 MWh as a function of the acceptance level α

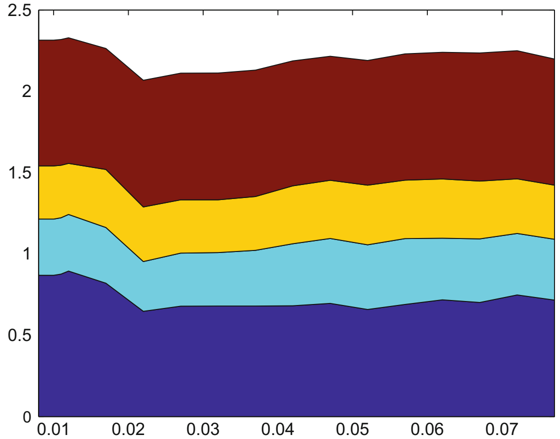

Fig. 15.4 Acceptability pricing: optimal hedges as a function of the acceptance level α

15.4.2 Acceptability Pricing for Financial Contracts

For purely financial contracts, the acceptability upper pricing problem can be investigated in more detail: it is the following variant of the replication problem (15.1). The upper price π_u is the minimal value of

$$\left\| \begin{array}{l} \text{Minimize (in } x \text{ and } w\text{): } w \\ \text{subject to} \\ Y_0^{w,x} = w, \\ Y_{t-}^{w,x} - Y_t^{w,x} \geq c_t \qquad t = 1, \ldots, T, \\ \mathscr{A}(Y_T^{w,x}) \geq 0, \\ x_t \text{ is nonanticipative,} \end{array} \right. \tag{21}$$

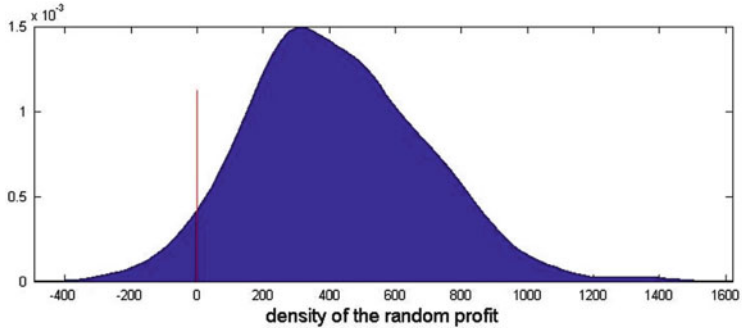

Fig. 15.5 Acceptability pricing: density of the profit variable

which takes the following form in the linear setup:

$$\left\|\begin{array}{l} \text{Minimize (in } x \text{ and } w\text{): } w \\ \text{subject to} \\ x_0 S_0 \leq 0, \\ x_{t-1} S_t - x_t S_t - c_t \geq 0; t = 1, \ldots, T, \\ \mathscr{A}_T(x_T S_T) \geq 0. \end{array}\right. \tag{22}$$

If the functional $\mathscr{A}$ is given by representation (19), then problem (22) has a dual given by

$$\left\|\begin{array}{l} \text{Maximize (in } Z_t\text{) } \sum_{t=1}^{T} \mathbb{E}(c_t Z_t) \\ \text{subject to} \\ \mathbb{E}(S_{t+1} Z_{t+1} | \mathscr{F}_t) = Z_t S_t, \\ Z_t \geq 0; \quad t = 1, \ldots, T-1, \\ Z_T \in \mathscr{Z}. \end{array}\right. \tag{23}$$

The latter problem can be reformulated in an analogous way as in (15.5): Let $\tilde{c}_t = c_t / S_{t,0}$ and $\tilde{S}_t = S_t / S_{t,0}$, where $S_{t,0}$ is the riskless investment. Then the acceptability upper price π_u is given by

$$\max\{\sum_{t=1}^{T} \mathbb{E}_Q(\tilde{c}_t) : (\tilde{S}_t) \text{ is an (equivalent) martingale under } Q \text{ s. t. } \tfrac{dQ}{dP} \in \mathscr{Z}\}. \tag{24}$$

Similarly, the lower price π_ℓ is

$$\min\{\sum_{t=1}^{T} \mathbb{E}_Q(\tilde{c}_t) : (\tilde{S}_t) \text{ is an (equivalent) martingale under } Q \text{ s. t. } \tfrac{dQ}{dP} \in \mathscr{Z}\}. \tag{25}$$

Denote by $\pi_u(\mathscr{Z})$ the upper price in dependency of the considered acceptability functional $\mathscr{A}$ with dual set $\mathscr{Z}$. Notice that

$$\mathscr{Z}_1 \subseteq \mathscr{Z}_2 \quad \text{implies that} \quad \pi_u(\mathscr{Z}_1) \leq \pi_u(\mathscr{Z}_2).$$

The largest price is gotten when full (super)replication is required, meaning that $\mathscr{Z}$ must be equal to all nonnegative random variables, compare (15.5). A smaller and more realistic price is obtained, if the acceptability functional is, e.g., the average value-at-risk $\mathbb{AV@R}_\alpha$, with $\mathscr{Z} = \{Z : 0 \leq Z \leq \frac{1}{\alpha}\}$. The smallest price is given by the choice $\mathscr{Z} = \{1\}$, which corresponds to the acceptability requirement $\mathbb{E}(Y_T^{w,x}) \geq 0$. This simple pricing rule is related to the concept of expected net present value (ENPV) of a contract and can be seen as the absolute minimum price for avoiding bankruptcy. However no seller will be willing to contract on this basis.

15.4.3 From Acceptability Pricing to Indifference Pricing

Acceptability pricing allows a meaningful valuation of contracts, even if full replication of the related flows is not possible or too expensive. However there is another difficulty remaining: the equipment of, e.g., a producer of electricity is never dedicated just to the production of the OTC contract under consideration. From the standpoint of production, delivering new contractual cash and energy flows is always an addendum to previously planned decisions. This means that the value of a contract should be valuated relative to the optimal management of all the other contracts which are already in the portfolio of the seller.

This idea leads to the notion of indifference pricing: the indifference principle states that the seller of a product compares his optimal decisions with and without the contract and then requests a price such that he is at least not worse off when closing the contract. This idea goes back to insurance mathematics (see [4]) but has been used for pricing a wide diversity of financial contracts in recent years, e.g., [5] for an overview.

In order to model the indifference price approach, assume that the total energy deliveries of the actual portfolio are D_t^{old} and the total cash flows out of this portfolio are C_t^{old}. These cash flows must include also the upfront payments at time 0. The additional contract, for which a price is not yet determined, is given by D_t respectively C_t. Indifference pricing happens in two steps:

- Determination of the acceptability of the actual portfolio. To this end, the following problem is solved:

$$\left\| \begin{array}{l} \text{Maximize (in } x,y,z \text{ and } w\text{): } \mathscr{A}(C_T^{old} + \sum_{j=1}^{J} S_{T,j}^{e} x_{t,j}^{e} + \sum_{i=1}^{I} S_{T,i}^{f} x_{T,i}^{f}) \\ \text{subject to} \\ \text{constraints (15.7)–(15.16),} \\ x_t, y_t, z_t \text{ are nonanticipative.} \end{array} \right. \tag{26}$$

  Here the equations are based on D_t^{old} respectively C_t^{old}. The optimal value of this optimization problem, that is, the acceptability level of the actual (old) portfolio, is denoted by a_0.

- Determination of the indifference price of the additional contract. Let the new total deliveries be $D_t^{new} = D_t^{old} + D_t$ and the new cash flows (without the upfront payment for the additional contract) be $C_t^{new} = C_t^{old} + C_t$. The price of the additional contract is denoted by $x_{0,0}$. It is determined by the following problem:

$$\left\| \begin{array}{l} \text{Minimize (in } x,y,z \text{ and } w\text{): } w \\ \text{subject to} \\ \mathscr{A}(C_T^{new} + \sum_{j=1}^{J} S_{T,j}^e x_{t,j}^e + \sum_{i=1}^{I} S_{T,i}^f x_{T,i}^f) \geq a_0 \\ \text{and constraints } (15.7)\text{–}(15.15), \\ x_t, y_t, z_t \text{ are nonanticipative.} \end{array} \right. \tag{27}$$

Of course, here the equations are based on D_t^{new} respectively C_t^{new}.

The following example compares indifference pricing with acceptability pricing for a simple setup with one thermal generation unit and a fixed delivery contract to be valuated.

Example 2. We consider an electricity producer, who has available a single combined cycle plant that is able to use both oil and gas. The machine has maximum power production of 410 MW and efficiencies of 0.575 (gas) and 0.57 (oil). Both fuels can be stored up to some amount ($1.5 \cdot 10^6$ MWh) at storage costs 0.2 Euro/MWh/h. We do not consider future contracts in this setup; hence hedging is possible only by buying fuel at appropriate points in time. Again we use electricity prices and weekly decision periods as described in Example 1. Gas prices are estimated (following [12]) by GBMJ from GPL spot prices (hourly, 04/2007–12/2011, Bloomberg) and oil spot prices for Brent Crude prices (daily, 05/2003–12/2011, Bloomberg).

We valuate a simple delivery contract, which binds the producer to supply a fixed amount of energy, the contract size in MWh, during each stage of the planning problem. The producer is free to buy and store fuel and to produce electric energy for the contract and also for selling it at the spot market. The value of the contract per MWh contains variable operating costs. From this we calculate a contract value per MWh that also includes an amount of coverage for fixed cost, which is proportional to the mean workload of the production unit during the planning horizon.

Within this setup we compare the superhedging approach to indifference pricing. Superhedging is possible for a producer, if the contract size does not exceed the capacity of the combined cycle turbine. For indifference pricing, we use the average value-at-risk at level $\alpha = 0.05$ as the related acceptability functional.

See Fig. 15.6 for the main results. As described above, superhedging leads to contractual deliveries, but does not account for alternative usages of the machine, whereas indifference pricing does. This is the reason why the superhedging price might be considered as too low in this case. Superhedging and indifference pricing also show different amounts of fixed cost, because the related strategies use non-contractual electricity production at different levels.

15.5 Flexible Contracts: Swing Option Pricing

Electricity *swing options* give their buyers the right to obtain electricity at a fixed price K per MWh during some delivery period. The price is set by the seller at contract formation, while the actual purchase quantities can be chosen (within some contractual range) by the buyer during the whole delivery period. Swing options are also known as *flexible nomination contracts*, *take-or-pay contracts*, or *virtual power plants*. See, e.g., [2, 11, 18, 19].

So far, we considered delivery contracts with delivered quantities that were either fixed in advance or depending on some observable (possibly stochastic) variables. Swing options are different, because the delivered quantities are decisions of the option buyer, and the seller has to account for this fact, when making the pricing decision. Hence, two questions are important, when considering swing options:

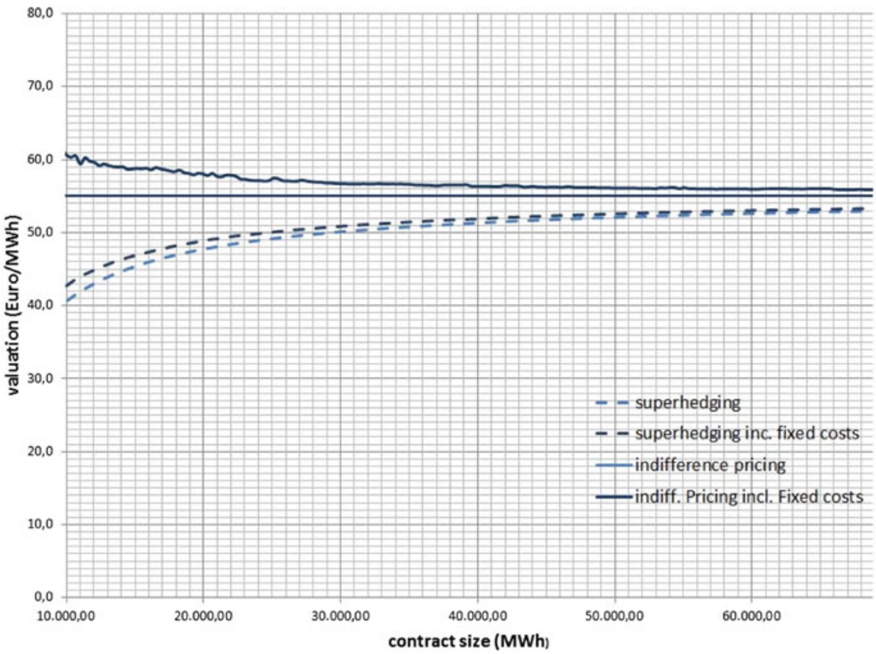

Fig. 15.6 Superhedging and indifference pricing

- The buyer's view: given the exercise price K, what is the optimal demand strategy of the buyer, and what is the resulting value of a swing option?
- The seller's view: what exercise price K should be offered by the seller, in view of the related optimal decision of the buyer?

Clearly the second question goes beyond the pure valuation issues raised and analyzed in the previous sections. While different approaches have been used to answer the first question, we remain in the framework of stochastic optimization and base our elaboration of the buyer's problem on the method proposed first in [9], which was developed further in [10]. For the second question we build on these results and follow the approach in [3], which was extended in several directions by [13].

15.5.1 The Buyer's View

Again, we consider points in time $t \in \{0,1,2,\ldots,T\}$. The delivery price K is fixed at $t = 0$ and delivery (for a single commodity) is possible during some periods $t \in \{t_D,\ldots,T\}$. The buyer specifies the actual consumption from the contract, D_t, one period before delivery. The quantities bought for the tth period are denoted by y_t, where $t \in \mathscr{T} = \{0,1,\ldots,T-1\}$.

The exact strategy of the buyer clearly depends on his own liabilities (e.g., a producer of aluminum will behave differently from a pure trader) and his access to electricity spot markets and other parts of energy markets. We follow [3, 10] and [13], and model a trader with access to the electricity spot market, which is in some sense the worst case from the standpoint of a swing option seller, because the trader is not restricted by own liabilities: the basic problem lies in the fact that the buyer buys swing electricity at the delivery price if he thinks that spot prices will be high and does not buy if he thinks that spot prices will be low. If the buyer is right, this means for the seller that he will have to deliver when prices are high, which clearly is inconvenient.

In this framework, the trader solves the following optimization problem to find an optimal strategy—a consumption pattern $D = (D_0,\cdots,D_{T-1})$ such that D_t is deliverable during period $(t,t+1]$—for a swing option contract with given exercise price K. It is assumed that both the consumption in each period (base line schedule) and the accumulated consumption over the whole delivery period are restricted by lower and upper bounds and that the trader sells any consumption from the contract at the electricity spot market. By $S^e_{t,0}$ we denote the electricity spot prices; $\underline{e}_t$ and $\overline{e}_t$ represent lower and upper bounds for consumption in each period and $\underline{E}$, $\overline{E}$ refer to lower and upper bounds for the cumulated consumption.

$$\left\| \begin{array}{l} \max_D \sum_{t=0}^{T-1} \mathbb{E}\left[D_t\left(S^e_{t+1,0} - K\right)\right] \\ \text{ subject to } \underline{e}_t \le D_t \le \overline{e}_t,\ \forall t \in \{0,\ldots,T-1\}, \\ \underline{E} \le \sum_{t=0}^{T-1} D_t \le \overline{E}, \\ D_t \ge 0,\ \ \forall t \in \mathscr{T}, \\ \sum_{t=0}^{T-1} \mathbb{E}\left[D_t\left(S^e_{t+1,0} - K\right)\right] \ge 0, \\ D_t \text{ is nonanticipative.} \end{array} \right. \tag{28}$$

As pointed out in [10] the optimal value of this problem can be seen as the value of the swing option from the buyer's perspective, as long as it is not negative. If the optimal value is negative, the contract will not be concluded, which justifies the second to last constraint.

The formulation (28) is used in [3, 13] and can be extended in various directions. In particular, ramping constraints with *ratchets* ρ_t can be modeled by

$$-\rho_t \cdot \Delta \le y_t - y_{t-1} \le \rho_t \cdot \Delta, \tag{29}$$

where Δ is the length of the time periods.

15.5.2 The Seller's View

While the buyer's decision problem (28) does not make use of the hedging or acceptability concepts discussed before, the sellers decision is again modeled by acceptability pricing: the seller searches for the minimal delivery price and related hedging and production decisions such that the resulting profit and loss distribution remains acceptable.

The seller's decision problem is similar to (18): the decision variables are augmented by the strike price K and the contractual cashflows C_t are redefined by $C_t = D_t \cdot K$. In particular this means that (15.15) is replaced by

$$\begin{aligned} x^f_{t,0} \le\ & (1+r_f)x^f_{t-1,0} \\ & - \sum_{j=0}^{J} S^e_{t,j} y^e_{t,j} - \sum_{i=1}^{I} S^f_{t,i}(x^f_{t,i} - x^f_{t-1,i}) + \sum_{i=1}^{I} C^f_{t,i} + D_t \cdot K \\ & - \sum_{i=0}^{J}\sum_{j=0}^{J} \gamma_{t,ij} z_{t,ij} \\ & - \sum_{j=1}^{J} \zeta_j \frac{(x^e_{t,j} + x^e_{t-1,j})}{2}. \end{aligned} \tag{30}$$

Unfortunately, acceptability pricing by an extended version of (15.15) cannot be used directly, because the formulation would include the buyer's decisions D_t which are not decision variables of the seller. Instead, the problem has to be reformulated in the framework of bi-level optimization. The papers [3, 13] propose this approach but use simplified versions of this problem, both simplifying production decisions: the first does not model production, while the second uses an internal price for accounting between a production and a trading department as a proxy. The latter paper also gives a broad survey of related models and methods.

The basic problem in bi-level optimization lies in the fact that given a strike price K the buyer's problem (28) may have non-unique optimal decisions. Using the optimistic approach of bi-level optimization and assuming that the lower level chooses among its optimal decisions the best one from the seller's point of view (see, e.g., [7], also for the alternative—the pessimistic approach) the decision problem can be formulated as

$$\left\| \begin{array}{l} \text{Minimize (in } K, x^e, x^f, y, z, w \text{ and D): } w \\ \text{subject to} \\ \text{constraints (15.7)–(15.14) and (30),} \\ \mathscr{A}(C_T + \sum_{j=1}^{J} S^e_{T,j} x^e_{t,j} + \sum_{i=1}^{I} S^f_{T,i} x^f_{T,i}) \geq 0, \\ D \in D^*_K, \\ x^e_t, x^f_t, y_t, z_t \text{ are nonanticipative,} \end{array} \right. \tag{31}$$

where D^*_K denotes the argmin-set (i.e., the set of optimal solutions), given the strike price K, of the buyer's problem (28).

Even in the simplest case bi-level problems like (31) are nonconvex, and strongly NP-hard. See, e.g., [6, 15] for necessary optimality conditions. A specific intricacy of bi-level problems lies in the fact that their feasible set can be disconnected.

Typical standard approaches for stochastic bi-level optimization are stochastic quasi-(sub)gradient methods and the MPEC approach. The first one is applicable only if the argmin-set of the buyer's problem is guaranteed to be a singleton for all relevant prices K. The latter approach consists in formulating the KKT conditions for the buyer's problem and to include them into the upper-level problem in order to code the argmin-set of the buyer's problem. This is very common but hard to use for multistage problems like (28), because complementarity conditions for each node have to be included, which makes the resulting formulation hard to solve. Kovacevic and Pflug [13] give an overview and propose some new algorithms, building on the fact that the optimistic bi-level problem can be approximated by an LP if polyhedral acceptability functionals like the average value-at-risk are used for $\mathscr{A}$ and that all decisions of both the buyer and the seller are dominated by the seller's decision regarding the strike price K.

The following example illustrates the bargaining situations between buyer and seller.

Example 3. We set $\overline{e}_t \equiv 5$, $\overline{E} \equiv 50$. The $\mathbb{A}$V@R-parameter is $\alpha = 0.15$ and the minimum $\mathbb{A}$V@R$_\alpha$-requirement $q = -20$. The spot price process S_t is modeled by a stochastic tree with 6 (monthly) stages. The generating unit and the fuels are as in the previous examples. In addition, we use three futures products (with exercise periods of length 2) for hedging. Three scenarios for lower bounds are considered: $\underline{e} = 0$, $\underline{e} = 0.4\overline{e}$, and $\underline{e} = 0.8\overline{e}$. Furthermore we use $\underline{E} = 0$ and $\overline{E} = 5\overline{e}$.

Figure 15.7 shows effects of these scenarios. In particular increasing lower bounds reduces the value of the contract for the buyer, while it increases the value for the seller. However, the drawback of increasing lower bounds is that also the largest feasible strike price decreases. Note that the feasible prices in this example are comparably small, because they refer to summer months, and the $\mathbb{A}$V@R-requirements are relatively mild.

15.6 Conclusions

We have shown that fair pricing of energy contracts is a difficult task, which can be accomplished by solving various multistage optimization problems. While (super) replication requires the solution of a deterministic program, acceptance and indifference pricing are based on stochastic programs. These programs can be quite complex, especially if the full available portfolio of the contract seller is modeled. For flexible contracts, it is even necessary to solve a stochastic bi-level program, since in this case the optimal pricing must be embedded into a game-theoretic model of the leader-follower type.

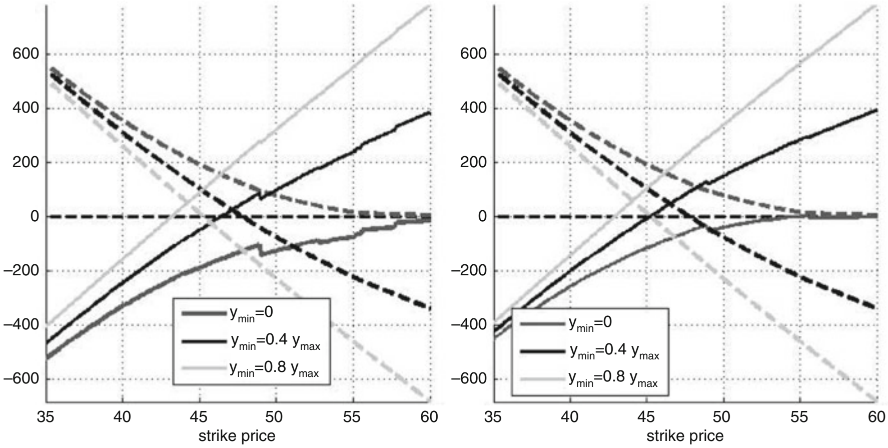

Fig. 15.7 Swing option pricing—the bargaining situation: the *left part* shows the buyer's expectation (*dashed*) versus the seller's average value-at-risk (*solid*). The *right part* shows the buyer's expectation (*dashed*) versus the seller's expectation

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