Chapter 14
Investment in Stochastic Electricity-Production Facilities

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Abstract This chapter considers a profit-oriented private investor interested in building stochastic electricity-production facilities, such as solar and wind power plants. This investor sells its production in a competitive pool-based electricity market and faces uncertainties related to demand growth, its production level, and its investment cost. Adopting a multistage approach, a stochastic complementarity model is formulated to determine the optimal capacity to be built by the investor to maximize its expected profit while minimizing its profit volatility. An example considering a wind power investor is presented to illustrate the working of the proposed model.

14.1 Investment in Stochastic Electricity-Production Facilities

14.1.1 Generation Capacity Investment

Generation capacity investment constitutes a relevant problem in electricity markets. To tackle this problem two different approaches are generally considered: a centralized framework [1] and a market framework [2].

The first of these approaches, i.e., a centralized framework [1], usually determines the generation capacity expansion plan based on a worst scenario case and considering the whole electric energy system. This approach, although relevant for the efficient operation of the system as a whole, is not of interest for a particular investor aiming at maximizing its own expected profit.

The market approach represents the operation of the market in which producers and consumers participate. This approach allows representing the perspective

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of a profit-oriented investor. Within this framework, there are several approaches for the generation capacity expansion (or investment) problem, e.g., static [3] or dynamic [4]; strategic [2] or competitive [5]; and with [6] or without network representation [7].

In this chapter, a market framework and a multistage approach are considered to address a particular but important case of the generation capacity investment problem: the investment in stochastic electricity-production facilities. Stochastic units are those units whose production is variable and uncertain, e.g., solar and wind power units. Note that the production of stochastic units depends on the availability of a natural resource such as solar intensity or wind speed. Thus, conventional models have to be modified to incorporate the uncertain production characteristics of this type of generation units [8, 9].

14.1.2 Uncertainty

There are several parameters that influence investment decisions, e.g., the future demand growth, equipment outages, investment costs, generation costs, and the production of stochastic units. Most of these parameters are subject to uncertainty and thus, an adequate modeling of such uncertainty is a must to achieve effective investment decisions.

Among the parameters subject to uncertainty, those with the greatest influence in the investment in stochastic electricity-production facilities are the demand growth, the production of stochastic units, and the investment cost. This is so because:

1. Under a market framework, the demand in the system and its growth directly influence the market clearing prices that in turn influence the investment decisions.
2. The uncertainty in the production of stochastic units influences the required capacity to be installed.
3. As the technology related to stochastic units matures, their future investment costs are expected to decrease. However, the decrease rate is uncertain, which has a significant impact on investment decisions.

The above sources of uncertainty are represented through a set of scenarios within a stochastic programming model [10, 11], an appropriate model to address this type of investment problems. Section 14.2 of this chapter provides the methodologies used for modeling of different sources of uncertainty.

14.1.3 Planning Horizon

A multistage approach is adopted by considering a planning horizon comprising a specific number of time periods. Each time period spans a known number of years.

This multistage approach allows making investment decisions at different points in time and provides flexibility to adapt to changes in the conditions that influence the investment decisions.

In order to characterize the different sources of uncertainty, each time period is represented by a single year within the corresponding period (e.g., the last year), which is considered the reference year of the whole time period.

The investment decisions concerning the capacities of the stochastic production units to be built are made at the beginning of each time period.

14.1.4 Risk Management

The key objective of a private investor is maximizing its expected profit. However, maximizing expected profit may lead to extreme cases in which the investor achieves a very high profit in some scenarios but incurs very high losses in others. The investor may not be able to assume such losses and might prefer to reduce its profit volatility despite having to reduce its expected profit as well. Thus, it is important to introduce a metric to control the risk of profit volatility. To do so, we use the conditional value-at-risk (CVaR) metric [12, 13] that can be easily implemented through linear constraints.

In a profit maximization problem, the CVaR is defined as the expected value of the profit smaller than the $(1-\alpha)$-quantile of the profit distribution, being α a given confidence level.

There are two manners of implementing the CVaR in a multistage model such as the one considered in this chapter. The first one seeks to reduce the risk of profit volatility per period. The second one seeks to reduce this risk throughout the whole planning horizon. This second approach is used in this chapter.

14.1.5 Complementarity Model

The considered investor aims at making investment decisions to maximize its expected profit from selling its production in the market while minimizing its profit volatility. However, these investment decisions are related to the market in which the investor sells its production once the newly built capacity is ready to operate. Note that the production of the newly built units influences the clearing of the market and that the outputs of the market influence in turn the investment decisions. Thus, the investment model must incorporate the clearing of the market as an additional constraint.

However, note that the market clearing is itself an optimization problem that seeks to maximize social welfare or to minimize generation cost. Thus, the investment model becomes an optimization problem constrained by other optimization problem, i.e., a complementarity model.

Figure 14.1 illustrates the structure of this complementarity model which comprises an upper-level problem and a collection of lower-level problems, i.e., a bi-level model. On one hand, the upper-level problem is an optimization problem that aims at maximizing the investor expected profit while minimizing its profit volatility. On the other hand, the lower-level problems represent the market clearing under different operating conditions, scenarios, and time periods. Note that investment decisions influence the market clearing (through the production of the newly built units), and that the investor obtains the clearing prices from the market clearing problems, which in turn influence its investment decisions.

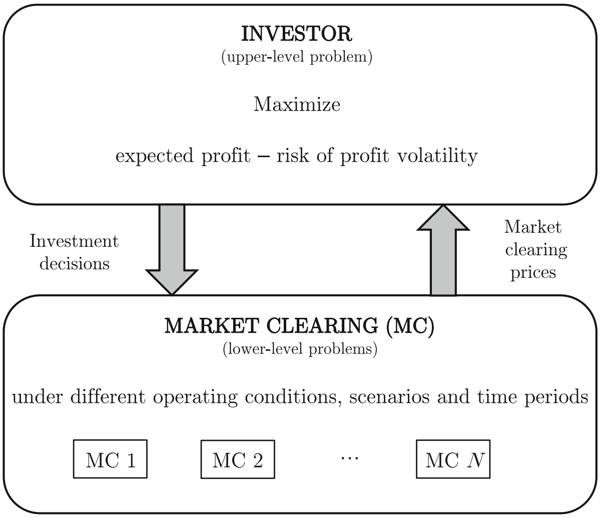

Fig. 14.1 Complementarity model structure

14.1.6 Chapter Organization

The remaining of this chapter is organized as follows. Section 14.2 describes the modeling of the sources of uncertainty. Section 14.3 provides a risk-constrained multistage bi-level model to decide the optimal investment in stochastic electricity-production facilities, as well as a procedure to transform such bi-level model into a mathematical program with equilibrium constraints (MPEC) and then into a mixed-integer linear programming (MILP) model. Both Sects. 14.2 and 14.3 include clarifying examples to illustrate the uncertainty modeling and the bi-level model, respectively. Finally, Sect. 14.4 summarizes the chapter and provides some relevant remarks and conclusions.

14.2 Uncertainty Modeling

This section describes the modeling of the sources of uncertainty that affect the investment decisions in stochastic electricity-production facilities. The explanations below are given for the particular case of investment in wind power facilities. However, extending the analysis to consider alternative stochastic-production units is straightforward.

14.2.1 Sources of Uncertainty

As explained in Sect. 14.1.2 of this chapter, there are three main sources of uncertainty that influence wind power investment decisions, namely, the future demand growth, the wind power production, and the future investment cost.

For the sake of simplicity and due to their high impact on investment decisions, only these three sources of uncertainty are considered below. However, additional sources of uncertainty (e.g., generation costs and equipment outages) can be considered in a similar manner.

14.2.2 Demand and Wind Power Production Uncertainty

The demand and the wind power production are usually anticorrelated since low demands (during the night) generally correspond to high wind power productions and high demands (during the day) generally correspond to low wind power productions. Thus, the uncertain character of both parameters has to be addressed jointly in order to account for this negative correlation.

The aim of the wind power investor is to decide the wind power units to be built throughout an existing electric energy system at the beginning of each of the time periods comprising the planning horizon. As explained in Sect. 14.1.3, each of these time periods is modeled using a representative year. The demand and wind power production uncertainties throughout the representative years are modeled as described below.

We consider hourly historical values (throughout one or several past years) of demand and wind speed in different locations of the electric energy system under study. First, wind speed values are transformed into wind power capacity factors (defined as the wind power production per MW) through appropriate wind speed/wind power production curves. Second, demand values in each location are divided by the peak demand in each particular location, rendering a set of values of demand factors. We obtain hourly values of the demand factor and of the wind power capacity factor in each location of the electric energy system under study which represent different operating conditions.

This set of values represents the historical demand and wind power production profile in the electric energy system under study. This profile also accounts for the correlation among demand and wind power capacity factors. For the sake of simplicity, this demand and wind power production profile is considered fixed throughout the whole planning horizon. Thus, these historical demand and wind power capacity factors are considered to represent the demand and wind power production uncertainty in the representative year of each time period of the planning horizon. It is important to emphasize that wind power capacity factors do not change throughout the years but the demand grows. However, for simplicity we assume that all demands grow in the same proportion, i.e., the *geometry* remains unaltered.

Considering all historical operating conditions as input data of the investment model may result in intractability, particularly for realistic electric energy systems. Thus, the K-means clustering method [14, 15] is used to transform the historical data into a reduced data set that maintains the information of and the correlation among the demand and the wind power capacity factors of the historical data.

The working of the K-means technique to reduce the historical operating condition is summarized below.

We define a cluster as a group of observations (e.g., demand and wind power capacity factors in different locations) that are similar among them but different from the observations in other clusters. Additionally, we define the centroid of each cluster as the mean value of the observations allocated to the cluster. Given these definitions, the K-means follows the iterative algorithm below:

1. Select an appropriate number of clusters.
2. Define initial clusters and the initial centroid of each cluster, e.g., randomly allocating the observations to different clusters.
3. Compute the distances (e.g., quadratic distances) between each historical observation and each cluster centroid.
4. Allocate each historical observation to the closest cluster according to the calculated distances.
5. Recalculate the centroid of each cluster.

Steps 3–5 above are repeated iteratively until there is no change in the compositions of the clusters in two consecutive iterations.

Note that the output of the K-means technique is a set of clusters, each one defined by its centroid and the number of historical observations allocated to it. The centroids comprise the values of the demand and the wind power capacity factors in each location of the system under study, which represent the system operating condition. On the other hand, the number of historical observations allocated to each cluster gives the weight of the cluster. As we represent each period using a reference year, we define this weight as the number of hours in the representative year of each period that are represented by each cluster, i.e., the weight of each operating condition in each representative year. Additional details of the K-means technique can be found in [14].

Demand and wind power capacity factors as per unit values are considered fixed throughout the planning horizon. However, this is not the case for the demand and the wind power production which generally increase. Such variations have to be properly represented. On one hand, the wind power production depends on the installed wind power capacity which is a decision variable of the problem and, thus, its increase is determined by the solution of the problem. On the other hand, the uncertainty in the demand growth throughout the planning horizon is represented using a set of scenarios as depicted in the scenario tree of Fig. 14.2. In this particular example of two time periods, the scenario tree comprises two demand growth scenario realizations in the first time period and two demand growth scenario realizations in the second period for each scenario realization in the first one, which results in four demand growth scenarios (D1, D2, D3, and D4) for the whole planning horizon. In this example, there is only one possible investment decision at the beginning of the planning horizon (i.e., at the beginning of the first period) and two alternative investment decisions at the beginning of the second period depending on the scenario realization in the first one: one investment decision for scenarios D1 and D2; and one investment decision for scenarios D3 and D4.

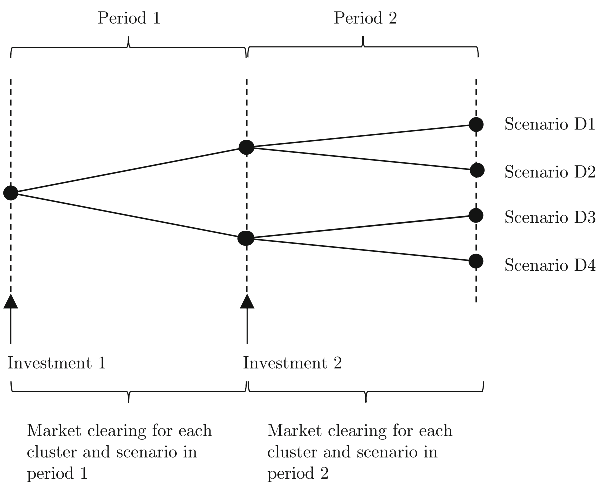

Fig. 14.2 Scenario tree considering uncertainty in demand growth

As explained above, demand and wind power capacity factors are considered fixed throughout the planning horizon. This assumption implies that all demands grow in the same proportion. Note that this may not be the case, specially if the system under study has (or may have in the future) a smart grid technology enabling the implementation of demand response programs that may play a significant role in changing the consumer behavior and thus, the *geometry* of the demand. However, note that if this is the case, different demand and wind power production profiles may be considered through additional scenarios.

14.2.3 Investment Cost Uncertainty

As the wind power technology matures, the future wind power investment cost is expected to decrease. However, this potential decrease is uncertain and thus, this uncertainty has to be properly modeled. To do so, we use a set of scenarios to represent the investment cost uncertainty.

Figure 14.3 depicts an example of a scenario tree representing the investment cost uncertainty. In this example, there are three possible investment cost scenario realizations for the second period (IC1, IC2, and IC3). In this case, there is a single investment decision at the beginning of the first period and three alternatives investment decisions at the beginning of the second one, depending on the investment cost scenario realization.

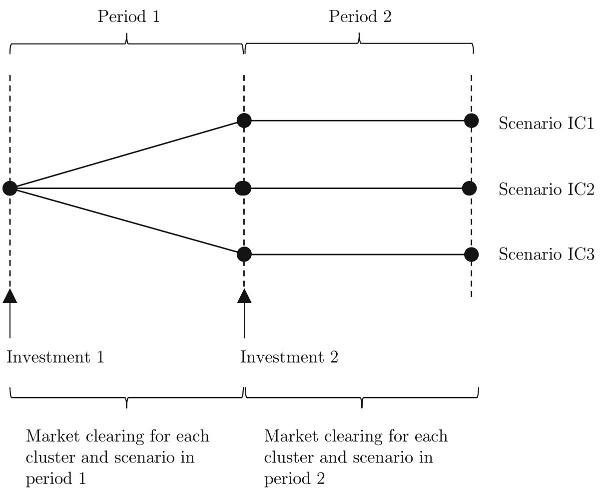

Fig. 14.3 Scenario tree considering uncertainty in investment cost

The main difference in the uncertainty modeling of the investment cost and in the demand growth is that in the first case, the wind power investor knows the actual investment cost at the time it makes its investment decisions. The investment cost in the first period is known at the point in time that the investment decisions for this period are made. However, at this point in time, the investor does not know the investment cost in the second period. Nevertheles, this investment cost is known at the time the investor makes its investment decisions. On the other hand, the demand growth in the first period is not known at the time the wind power investor makes its investment decisions for this period. Thus, the uncertainty in demand growth usually entails a higher risk than the uncertainty associated with investment costs.

The demand growth and the investment cost scenarios are generated using appropriate forecasting tools [16, 17].

14.2.4 Decision Sequence: Scenario Tree

The sources of uncertainty described in Sects. 14.2.2 and 14.2.3 above are independent and thus, the final scenario tree includes all possible scenario combinations. For the considered examples in Figs. 14.2 and 14.3, the final scenario tree comprises twelve scenario realizations (four demand growth scenarios and three investment cost scenarios) as depicted in Fig. 14.4. In this case, there is a single investment decision at the beginning of the planning horizon which does not depend on the future scenario realizations and six possible investment decisions at the beginning of the second period depending on the demand growth and the investment cost scenario realization in the first period (six alternatives).

It is important to note that each of these scenarios comprises a specific number of clusters (those obtained using the K-means technique and representing different system operating conditions) for each time period, which account for the demand and wind power capacity factors variability throughout the reference year of each period.

Given this framework, the decision sequence is as follows:

1. At the beginning of the planning horizon, i.e., at the beginning of the first time period, the wind power investor decides the wind power capacity to be built at this point in time. These investment decisions are *here-and-now* decisions since they do not depend on any scenario realization.
2. Once the investment decisions for the first period are made, the market is cleared for each cluster within each scenario in the first period. From the market clearing we obtain power productions, power flows, market clearing prices, etc.
3. The first period concludes and the wind power investor knows the actual scenario realization in this period (i.e., it knows the demand growth in the first period and the investment cost for the second period). Depending on the scenario realization, the investor makes its investment decisions for the second period. These are *wait-and-see* decisions with respect to the first period since they do depend on the scenario realization in this period and *here-and-now* decisions with respect to the second period (and the following ones if there are more than two) since they do not depend on the scenario realization in the future periods.
4. Once the investment decisions for the second period are made, the market is cleared for each cluster within each scenario in the second period.

Finally, steps 3 and 4 are repeated for each time period of the planning horizon if there were more than two.

14.2.5 Illustrative Example

In order to illustrate the modeling of the different sources of uncertainty, we consider the three-node electric energy system depicted in Fig. 14.5. This system comprises three nodes, three demands, three generation units, and three transmission lines.

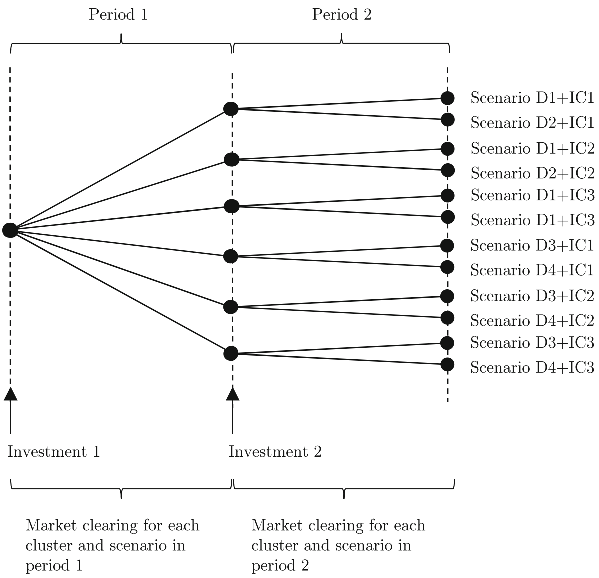

Fig. 14.4 Scenario tree considering uncertainty in both demand growth and investment cost

There are two wind zones: North wind zone (comprising nodes 1 and 2) and south wind zone (node 3).

Data of demand is obtained from aggregated historical data throughout year 2008 in the Iberian Peninsula [18]. The demand values are then divided by the peak demand to obtain demand factors. On the other hand, historical wind speed data of the Spanish towns of Tortosa (Northeast Spain) and Tarifa (Southwest Spain) are considered to characterize the wind speed in the north and south zones, respectively. These wind speed data are obtained using the databases developed by the University of Cantabria [19, 20]. To obtain the corresponding wind power capacity factors, we consider the wind speed/wind power production curve of a Nordex N80/2500 turbine [21]. Finally, we consider a two-period planning horizon, each one comprising 6 years.

14.2.5.1 Clusters

Historical data of demand and wind power capacity factors comprise 8,760 sets of values (demand and wind power capacity factors in each zone and for each hour of year 2008). Each set represents a system operating condition. If we use these historical data as input data of an investment model, we may face intractability. Thus, we apply the K-means technique explained in Sect. 14.2.2 to reduce these historical data into a set of clusters.

Table 14.1 provides the demand and wind power capacity factor data of the ten clusters in which we allocate the historical data. The second column of this table gives the demand factors, which for the sake of simplicity are assumed to be the same at all the nodes of the system. The third and fourth columns provide the wind power capacity factors in the north and south wind zones, respectively. Both the demand and wind power capacity factors within each cluster represent a system operating condition. Finally, the fifth column gives the number of historical observations that are allocated to each cluster, which represent the weight of each cluster in the reference year of each period.

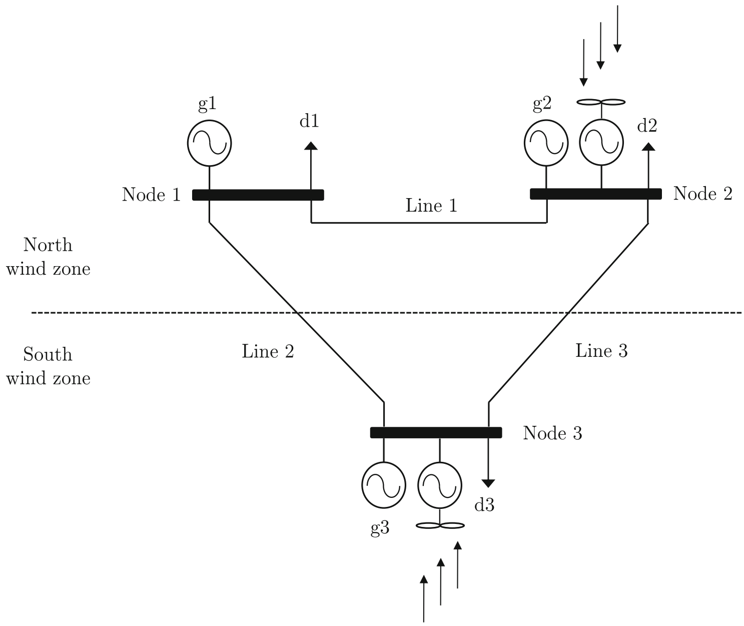

Fig. 14.5 Three-node electric energy system

This set of ten clusters covers the information of the historical observations as well as the correlation among demand and wind power capacity factors in different zones.

14.2.5.2 Demand Growth Scenarios

First, we assume that the future wind power investment cost is not subject to uncertainty. For this purpose, we consider the scenario depicted in Fig. 14.2 comprising two demand growth scenario realizations in the first period and two demand growth scenario realizations in the second period depending on the scenario realization in the first one.

The probability that the demand in the first period is 15 % higher than the demand prior to the beginning of the planning horizon is 0.7, while the probability of being

Table 14.1 Cluster results: demand and wind power capacity factor data

Cluster	Demand factors [p.u.]	Capacity factors [p.u.] Wind zone North	Capacity factors [p.u.] Wind zone South	Number of hours [h]
1	0.8091	0.0331	0.1211	1712
2	0.7362	0.9191	0.1230	715
3	0.7158	0.7389	0.8100	216
4	0.6674	0.6152	0.0962	477
5	0.6885	0.5308	0.5028	300
6	0.7358	0.0598	0.8771	1312
7	0.7138	0.0406	0.4798	1365
8	0.6214	0.0268	0.0773	1810
9	0.8353	0.2746	0.1266	586
10	0.8286	0.4517	0.0933	267

10 % lower than this demand is 0.3. On the other hand, in the second period, the demand may be 10 % higher and 5 % lower than the demand in the first period with probabilities of 0.6 and 0.4, respectively. Table 14.2 summarizes the demand growth scenario data. The total weight of each scenario is computed as the product of the probability of occurrence of the demand variations in the first and second periods.

Table 14.2 Uncertainty in demand growth: scenario data

Scenario	Period 1 Demand growth (%)	Period 1 Weight	Period 2 Demand growth (%)	Period 2 Weight	Total weight
D1	+15	0.7	+10	0.6	0.42
D2			−5	0.4	0.28
D3	−10	0.3	+10	0.6	0.18
D4			−5	0.4	0.12

14.2.5.3 Investment Cost Scenarios

In this case we assume that the uncertainty does not affect the demand growth and we consider the scenario tree depicted in Fig. 14.3, which considers three different investment costs in the second period. The probability that the investment cost in the second period is 15 % higher than that in the first one is 0.2; the probability of being equal to the investment cost in the first period is 0.4, while the probability that the investment cost in the second period is 25 % lower than in the first one is 0.4. Table 14.3 summarizes the investment cost scenario data.

Table 14.3 Uncertainty in investment cost: scenario data

Scenario	Investment cost variation in period 2 (%)	Weight
IC1	+15	0.2
IC2	0	0.4
IC3	−25	0.4

14.2.5.4 Demand Growth and Investment Cost Scenarios

Finally, we consider that the uncertainty affects both the demand growth and the future investment cost. We consider all the scenario combinations of Figs. 14.2 and 14.3 which result in the final scenario tree depicted in Fig. 14.4. Table 14.4 summarizes the scenario data for this case.

There are 12 scenarios for the whole planning horizon as a result of four demand growth scenarios and three investment cost scenarios. The weight of each scenario is computed as the weight of the demand growth scenario times the weight of the investment cost scenario. For example, the weight of scenario D1+IC1 (0.084) is obtained as the weight of scenario D1 (0.42) times the weight of scenario IC1 (0.2).

14.3 Risk-Constrained Multistage Wind Power Investment

In this section, a risk-constrained multistage model is formulated to determine the wind power capacity to be built by a wind power investor to maximize its expected profit while minimizing its profit volatility. This model is based on a bi-level model which can be recast as a MILP problem.

14.3.1 Notation

The notation of the proposed model is provided below for quick reference.

Table 14.4 Uncertainty in both demand growth and investment cost: scenario data

Scenario	Period 1 Demand growth (%)	Period 2 Investment cost variation (%)	Period 2 Demand growth (%)	Total weight
D1+IC1	+15	+15	+10	0.084
D2+IC1			−5	0.056
D1+IC2		0	+10	0.168
D2+IC2			−5	0.112
D1+IC3		−25	+10	0.168
D2+IC3			−5	0.112
D3+IC1	−10	+15	+10	0.036
D4+IC1			−5	0.024
D3+IC2		0	+10	0.072
D4+IC2			−5	0.048
D3+IC3		−25	+10	0.072
D4+IC3			−5	0.048

14.3.1.1 Indices

c	Index for clusters
d	Index for demands
g	Index for generation units (other than candidate wind power units)
l	Index for transmission lines
n	Index for nodes
$r(l)$	Index of the receiving-end node of line l
$s(l)$	Index of the sending-end node of line l
t	Index for time periods
ω	Index for scenarios

14.3.1.2 Constants

a_t	Amortization factor in the tth time period
c_g^{G}	Marginal cost of the gth generation unit
$c_t^{\mathrm{inv}}(\omega)$	Wind power investment cost in the tth time period and scenario ω
$c_t^{\max}$	Investment budget in the tth time period
$k_{d,c}^{\mathrm{D}}$	Demand factor of the dth demand in the cth cluster
$k_{n,c}^{\mathrm{W}}$	Wind power capacity factor at node n in the cth cluster
N_c^{H}	Number of hours comprising the cth cluster
$P_{d,t}^{\mathrm{D}}(\omega)$	Peak load of the dth demand in the tth time period and scenario ω
$P_g^{\mathrm{G,max}}$	Capacity of the gth generation unit

$P_l^{\mathrm{L,max}}$	Transmission capacity of the lth transmission line
x_l	Reactance of line l
$X_n^{\mathrm{W,max}}$	Maximum wind power capacity that can be built at node n throughout the planning horizon
α	Confidence level used to compute the CVaR
β	Weighting parameter used to model the trade-off between expected profit and CVaR
$\gamma(\omega)$	Weight of scenario ω

14.3.1.3 Variables

$P_{g,c,t}^{\mathrm{G}}(\omega)$	Power produced by the gth generation unit in the cth cluster, the tth time period, and scenario ω
$P_{l,c,t}^{\mathrm{L}}(\omega)$	Power flow through the lth transmission line in the cth cluster, the tth time period, and scenario ω
$P_{n,c,t}^{\mathrm{W}}(\omega)$	Wind power production at node n in the cth cluster, the tth time period, and scenario ω
$X_{n,t}^{\mathrm{W}}(\omega)$	Wind power capacity to be built at node n at the beginning of the tth time period, and scenario ω
$\delta_{n,c,t}(\omega)$	Voltage angle at node n in the cth cluster, the tth time period, and scenario ω
$\vartheta_{n,c,t}(\omega)$	LMP at node n in the cth cluster, the tth time period, and scenario ω
$\eta(\omega), \zeta$	Auxiliary variables used to compute the CVaR

14.3.1.4 Dual Variables

The dual variables below are associated with the following constraints:

$\chi_{n,c,t}(\omega)$	Zero voltage angle at the reference node in the cth cluster, the tth time period, and scenario ω
$\lambda_{n,c,t}(\omega)$	Generation-demand balance at node n in the cth cluster, the tth time period, and scenario ω
$\phi_{l,c,t}(\omega)$	Power flow through transmission line l in the cth cluster, the tth time period, and scenario ω
$\phi_{l,c,t}^{\max}(\omega)$	Capacity of transmission line l in direction $s(l)$-$r(l)$ in the cth cluster, the tth time period, and scenario ω
$\phi_{l,c,t}^{\min}(\omega)$	Capacity of transmission line l in direction $r(l)$-$s(l)$ in the cth cluster, the tth time period, and scenario ω
$\varphi_{g,c,t}^{\max}(\omega)$	Capacity of generation unit g in the cth cluster, the tth time period, and scenario ω
$\varphi_{g,c,t}^{\min}(\omega)$	Nonnegativity of the production of generation unit g in the cth cluster, the tth time period, and scenario ω
$\xi_{n,c,t}^{\max}(\omega)$	Upper limit of voltage angle at node n in the cth cluster, the tth time period, and scenario ω
$\xi_{n,c,t}^{\min}(\omega)$	Lower limit of voltage angle at node n in the cth cluster, the tth time period, and scenario ω

14.3.1.5 Sets

Δ^{C}	Set of indices of clusters
Δ^{G}	Set of indices of generation units (other than candidate wind power units)
Δ^{N}	Set of indices of nodes
Δ^{T}	Set of indices of time periods
Δ^{ω}	Set of indices of scenarios
Λ_n^{D}	Set of indices of demands located at node n
Λ_n^{G}	Set of indices of generation units (other than candidate wind power units) located at node n
$\Psi_t(\omega)$	Set of parameters defining scenario ω in the tth time period

14.3.2 Bi-level Formulation

The considered risk-constrained multistage wind power investment model is formulated below [9]:

$$\underset{\Omega^{\mathrm{U}} \cup \Omega^{\mathrm{L}}_{c,t}(\omega)}{\text{Maximize}}$$

$$\sum_{\omega \in \Delta^{\omega}} \gamma(\omega) \left\{ \sum_{t \in \Delta^{\mathrm{T}}} \left[\sum_{c \in \Delta^{\mathrm{C}}} N_c^{\mathrm{H}} \sum_{n \in \Delta^{\mathrm{N}}} \vartheta_{n,c,t}(\omega) P^{\mathrm{W}}_{n,c,t}(\omega) \; - a_t \sum_{n \in \Delta^{\mathrm{N}}} c_t^{\mathrm{inv}}(\omega) X^{\mathrm{W}}_{n,t}(\omega) \right] \right\}$$
$$+ \beta \left(\zeta - \frac{1}{1-\alpha} \sum_{\omega \in \Delta^{\omega}} \gamma(\omega) \eta(\omega) \right) \tag{14.1a}$$

subject to

$$0 \le P^{\mathrm{W}}_{n,c,t}(\omega) \le k^{\mathrm{W}}_{n,c} \sum_{b \le t} X^{\mathrm{W}}_{n,t}(\omega), \quad \forall n, \forall c, \forall t, \forall \omega \tag{14.1b}$$

$$\sum_{n \in \Delta^{\mathrm{N}}} c_t^{\mathrm{inv}}(\omega) X^{\mathrm{W}}_{n,t}(\omega) \le c_t^{\max}, \quad \forall t, \forall \omega \tag{14.1c}$$

$$0 \le \sum_{t \in \Delta^{\mathrm{T}}} X^{\mathrm{W}}_{n,t}(\omega) \le X_n^{\mathrm{W,max}}, \quad \forall n, \forall \omega \tag{14.1d}$$

$$\vartheta_{n,c,t}(\omega) = \lambda_{n,c,t}(\omega), \quad \forall n, \forall c, \forall t, \forall \omega \tag{14.1e}$$

$$X^{\mathrm{W}}_{n,t}(\omega_i) = X^{\mathrm{W}}_{n,t}(\omega_{\tilde{i}}), \quad \forall n, \forall t, \forall \omega_i, \forall \omega_{\tilde{i}} : \Psi_b(\omega_i) = \Psi_b(\omega_{\tilde{i}}), \forall b < t \tag{14.1f}$$

$$\zeta - \sum_{t \in \Delta^{\mathrm{T}}} \left[\sum_{c \in \Delta^{\mathrm{C}}} N_c^{\mathrm{H}} \sum_{n \in \Delta^{\mathrm{N}}} \vartheta_{n,c,t}(\omega) P^{\mathrm{W}}_{n,c,t}(\omega) \right.$$
$$\left. - a_t \sum_{n \in \Delta^{\mathrm{N}}} c_t^{\mathrm{inv}}(\omega) X^{\mathrm{W}}_{n,t}(\omega) \right] \le \eta(\omega), \quad \forall \omega \tag{14.1g}$$

$$\eta(\omega) \ge 0, \quad \forall \omega, \tag{14.1h}$$

where $\lambda_{n,c,t}(\omega), \forall n, \in \arg\left\{ \underset{\Omega^{\mathrm{L}}_{c,t}(\omega)}{\text{Minimize}} \right.$

$$\sum_{g \in \Delta^{\mathrm{G}}} c^{\mathrm{G}}_{g} P^{\mathrm{G}}_{g,c,t}(\omega) \tag{14.2a}$$

subject to

$$\sum_{g \in \Lambda^{\mathrm{G}}_{n}} P^{\mathrm{G}}_{g,c,t}(\omega) + P^{\mathrm{W}}_{n,c,t}(\omega) - \sum_{l|s(l)=n} P^{\mathrm{L}}_{l,c,t}(\omega) + \sum_{l|r(l)=n} P^{\mathrm{L}}_{l,c,t}(\omega)$$
$$= \sum_{d \in \Lambda^{\mathrm{D}}_{n}} P^{\mathrm{D}}_{d,t}(\omega) k^{\mathrm{D}}_{d,c} : \lambda_{n,c,t}(\omega), \quad \forall n \tag{14.2b}$$

$$P^{\mathrm{L}}_{l,c,t}(\omega) = \frac{1}{x_l}\left(\delta_{s(l),c,t}(\omega) - \delta_{r(l),c,t}(\omega)\right) : \phi_{l,c,t}(\omega), \quad \forall l \tag{14.2c}$$

$$-P^{\mathrm{L,max}}_{l} \le P^{\mathrm{L}}_{l,c,t}(\omega) \le P^{\mathrm{L,max}}_{l} : \phi^{\min}_{l,c,t}(\omega), \phi^{\max}_{l,c,t}(\omega), \quad \forall l \tag{14.2d}$$

$$0 \le P^{\mathrm{G}}_{g,c,t}(\omega) \le P^{\mathrm{G,max}}_{g} : \varphi^{\min}_{g,c,t}(\omega), \varphi^{\max}_{g,c,t}(\omega), \quad \forall g \tag{14.2e}$$

$$-\pi \le \delta_{n,c,t}(\omega) \le \pi : \xi^{\min}_{n,c,t}(\omega), \xi^{\max}_{n,c,t}(\omega), \quad \forall n \setminus n\text{: ref.} \tag{14.2f}$$

$$\delta_{n,c,t}(\omega) = 0 : \chi_{n,c,t}(\omega), \quad n\text{: ref.} \tag{14.2g}$$

$\left.\right\}, \forall c, \forall t, \forall \omega,$

where

$$\Omega^{\mathrm{U}} = \left\{ X^{\mathrm{W}}_{n,t}(\omega), \forall n, \forall t, \forall \omega; P^{\mathrm{W}}_{n,c,t}(\omega), \vartheta_{n,c,t}(\omega), \forall n, \forall c, \forall t, \forall \omega; \eta(\omega), \forall \omega; \zeta \right\}, \tag{14.3}$$

and

$$\Omega^{\mathrm{L}}_{c,t}(\omega) = \left\{ P^{\mathrm{G}}_{g,c,t}(\omega), \forall g; P^{\mathrm{L}}_{l,c,t}(\omega), \forall l; \delta_{n,c,t}(\omega), \forall n; \lambda_{n,c,t}(\omega), \forall n; \phi_{l,c,t}(\omega), \right.$$
$$\phi^{\min}_{l,c,t}(\omega), \phi^{\max}_{l,c,t}(\omega), \forall l; \varphi^{\min}_{g,c,t}(\omega), \varphi^{\max}_{g,c,t}(\omega), \forall g; \xi^{\min}_{n,c,t}(\omega), \xi^{\max}_{n,c,t}(\omega),$$
$$\left. \forall n \setminus n\text{: ref.}; \chi_{n,c,t}(\omega), n\text{: ref.} \right\}, \forall c, \forall t, \forall \omega. \tag{14.4}$$

The bi-level model (14.1) and (14.2) comprises an upper-level problem (14.1) and a set of lower-level problems (14.2), one for each cluster c, time period t, and scenario ω. The dual variable associated with each constraint of the lower-level problems is indicated after a colon.

The optimization variables of the lower-level problems are the variables in sets $\Omega^{\mathrm{L}}_{c,t}(\omega)$, $\forall c$, $\forall t$, $\forall \omega$. The lower-level problems constraint the upper-level one and thus, the optimization variables of the upper-level problem include variables in sets $\Omega^{\mathrm{L}}_{c,t}(\omega)$, $\forall c$, $\forall t$, $\forall \omega$ and additional variables in set Ω^{U}.

The upper-level problem (14.1) aims at maximizing the expected profit achieved by the wind power investor plus a coefficient times the CVaR. The CVaR is the metric used to control the risk of profit volatility [12, 13].

The objective function (14.1a) of the upper-level problem comprises three terms:

1. Each term $\vartheta_{n,c,t}(\omega) P^{\mathrm{W}}_{n,c,t}(\omega)$ is the revenue achieved by the wind power investor for selling its wind power production in the pool per cluster, time period, and scenario. The wind power investor is paid the locational marginal price (LMP) of the node at which wind power is produced times the wind power production. Since each time period is represented by a reference year, these revenues are multiplied by the number of hours within each cluster to obtain annual revenues in each time period and for each scenario.
2. Each term $c^{\mathrm{inv}}_t(\omega) X^{\mathrm{W}}_{n,t}(\omega)$ is the investment cost incurred by the wind power investor per time period and scenario for building new wind power capacity. These terms are multiplied in each period by an amortization rate a_t to make investment costs and revenues comparable across time periods.

   Terms in items 1 and 2 are multiplied in each scenario by the weight of the corresponding scenario to obtain expected profits.
3. Term $\beta \left(\zeta - \frac{1}{1-\alpha} \sum_{\omega \in \Delta^{\omega}} \gamma(\omega) \eta(\omega) \right)$ is a coefficient β times the CVaR. Coefficient β is a weighting factor that models the trade-off between expected profit and CVaR.

For the sake of simplicity, we consider that all the monetary values are referred to a single point in time and thus, it is not necessary to multiply them by discount rates.

Constraints (14.1b)–(14.1e) represent the wind power operation and investment limits and conditions for all clusters, periods, and scenarios. Constraints (14.1b) limit the wind power production to the wind power availability at each node and for each cluster, time period, and scenario. Note that the wind power capacity built at the beginning of any time period is available during this and the remaining periods of the planning horizon. Constraints (14.1c) impose a cap on investment budget for each time period and scenario. Equations (14.1d) enforce the nonnegativity of the wind power capacity to be built and limit it to a maximum throughout the planning horizon. Finally, constraints (14.1e) state that the price paid to the wind power investor for its production is the LMP of the node at which wind power is produced. These LMPs are obtained as the dual variables associated with the balance constraints in the lower-level problems for each node, cluster, time period, and scenario.

Constraints (14.1f) are non-anticipativity constraints that avoid anticipating information. They impose that the wind power capacity to be built at the beginning of a time period depends on the scenario realization of the previous periods, but it is unique for all the possible scenario realizations in the current and future periods [22].

Finally, constraints (14.1g) and (14.1h) allow computing the CVaR metric through linear expressions. Further information on the CVaR is available in [12, 13].

Additionally, the upper-level problem is also constrained by a set of lower-level problems (14.2) which represent the clearing of the pool for each cluster, time period, and scenario.

The objective function (14.2a) to be maximized is the social welfare, which is equivalent to minimizing generation costs since demands are considered fixed within each cluster, time period, and scenario; wind power investor offers its production at zero price; and producers other than the wind power investor offer their productions at their marginal costs.

Constraints of these lower-level problems include equalities (14.2b) that represent the generation-demand balance at each node of the system; equalities (14.2c) that define the power flows through transmission lines, limited to the transmission capacity of the corresponding lines by constraints (14.2d); constraints (14.2e) that impose upper and lower bounds on the power produced by generation units other than the wind power units; and finally constraints (14.2f) and (14.2g) that limit the voltage angles and fix equal to zero the voltage angle at the reference node, respectively. Note that the network topology is explicitly modeled by (14.2b)–(14.2d) using a direct current (dc) representation and disregarding losses [23].

14.3.3 MPEC Formulation

Bi-level model (14.1) and (14.2) is transformed into an MPEC following the procedure explained below.

Lower-level problems (14.2) represent the market clearing under different clusters, time periods, and scenarios and constraint the upper-level problem (14.1). Since each of these lower-level problems is linear and thus convex, the Karush–Kuhn–Tucker (KKT) conditions are necessary and sufficient optimality conditions [24]. Thus, each lower-level problem can be replaced by its KKT optimality conditions, which are in turn included as additional constraints of the upper-level problem rendering an MPEC, whose formulation is provided below:

$$\underset{\Omega^{\mathrm{U}} \cup \Omega^{\mathrm{L}}_{c,t}(\omega)}{\text{Maximize}} \quad (14.1\text{a}) \tag{14.5a}$$

subject to

$$\text{Constraints (14.1b)–(14.1h)} \tag{14.5b}$$

$$\left\{ \text{Constraints (14.2b)–(14.2g)} \right. \tag{14.5c}$$

$$c^{\mathrm{G}}_{g,t} - \lambda_{n(g),c,t}(\omega) + \varphi^{\max}_{g,c,t}(\omega) - \varphi^{\min}_{g,c,t}(\omega) = 0, \quad \forall g \tag{14.5d}$$

$$\lambda_{s(l),c,t}(\omega) - \lambda_{r(l),c,t}(\omega) - \phi_{l,c,t}(\omega) + \phi^{\max}_{l,c,t}(\omega) - \phi^{\min}_{l,c,t}(\omega) = 0, \quad \forall l \tag{14.5e}$$

$$\sum_{l|s(l)=n} \frac{1}{x_l}\phi_{l,c,t}(\omega) - \sum_{l|r(l)=n} \frac{1}{x_l}\phi_{l,c,t}(\omega) + \xi^{\max}_{n,c,t}(\omega) - \xi^{\min}_{n,c,t}(\omega) = 0, \quad \forall n \setminus n\text{: ref.} \tag{14.5f}$$

$$\sum_{l|s(l)=n} \frac{1}{x_l} \phi_{l,c,t}(\omega) - \sum_{l|r(l)=n} \frac{1}{x_l} \phi_{l,c,t}(\omega) - \chi_{n,c,t}(\omega) = 0, \quad n\text{: ref.} \tag{14.5g}$$

$$0 \le \phi^{\max}_{l,c,t}(\omega) \perp P^{\mathrm{L,max}}_l - f_{l,c,t}(\omega) \ge 0, \quad \forall l \tag{14.5h}$$

$$0 \le \phi^{\min}_{l,c,t}(\omega) \perp f_{l,c,t}(\omega) + P^{\mathrm{L,max}}_l \ge 0, \quad \forall l \tag{14.5i}$$

$$0 \le \varphi^{\max}_{g,c,t}(\omega) \perp P^{\mathrm{G,max}}_g - P^{\mathrm{G}}_{g,c,t}(\omega) \ge 0, \quad \forall g \tag{14.5j}$$

$$0 \le \varphi^{\min}_{g,c,t}(\omega) \perp P^{\mathrm{G}}_{g,c,t}(\omega) \ge 0, \quad \forall g \tag{14.5k}$$

$$0 \le \xi^{\max}_{n,c,t}(\omega) \perp \pi - \delta_{n,c,t}(\omega) \ge 0, \quad \forall n \setminus n\text{: ref.} \tag{14.5l}$$

$$0 \le \xi^{\min}_{n,c,t}(\omega) \perp \delta_{n,c,t}(\omega) + \pi \ge 0, \quad \forall n \setminus n\text{: ref.} \tag{14.5m}$$

$$\Big\}, \forall c, \forall t, \forall \omega.$$

MPEC (14.5) above is a single-level problem but includes different sources of nonlinearities, namely, the objective function (14.5a), constraints (14.1g), and the complementarity constraints (14.5h)–(14.5m). The following subsection explains how to transform this MPEC into an MILP problem that can be solved using traditional branch-and-cut techniques.

14.3.4 MILP Formulation

The MPEC (14.5) above has the following nonlinearities:

1. Each term $\sum_{n\in\Omega^{\mathrm{N}}} \vartheta_{n,c,t}(\omega) P^{\mathrm{W}}_{n,c,t}(\omega)$ in the objective function (14.5a) and in constraints (14.1g).
2. The complementarity constraints of the KKT conditions (14.5h)–(14.5m).

Each term $\sum_{n\in\Omega^{\mathrm{N}}} \vartheta_{n,c,t}(\omega) P^{\mathrm{W}}_{n,c,t}(\omega)$ can be replaced by an exact equivalent linear expression using the strong duality equality (SDE) as explained below [25]. The SDE states that if a problem is convex (as it is the case of each of the lower-level problems), the objective functions of the primal and dual problems have the same value at the optimum:

$$\sum_{g\in\Omega^{\mathrm{G}}} c^{\mathrm{G}}_{g,t} P^{\mathrm{G}}_{g,c,t}(\omega) = \sum_{n\in\Omega^{\mathrm{N}}} \lambda_{n,c,t}(\omega) \left[\sum_{d\in\Lambda^{\mathrm{D}}_n} P^{\mathrm{D}}_{d,t}(\omega) k^{\mathrm{D}}_{d,c} - P^{\mathrm{W}}_{n,c,t}(\omega) \right]$$
$$- \sum_{l\in\Omega^{\mathrm{L}}} \left[\phi^{\max}_{l,c,t}(\omega) + \phi^{\min}_{l,c,t}(\omega) \right] P^{\mathrm{L,max}}_l - \sum_{g\in\Omega^{\mathrm{G}}} \varphi^{\max}_{g,c,t} P^{\mathrm{G,max}}_g$$
$$- \sum_{n\in\Omega^{\mathrm{N}}\setminus n\text{: ref.}} \left[\xi^{\max}_{n,c,t}(\omega) + \xi^{\min}_{n,c,t}(\omega) \right] \pi, \quad \forall c, \forall t, \forall \omega. \tag{14.6}$$

Since $\vartheta_{n,c,t}(\omega) = \lambda_{n,c,t}(\omega), \forall n, \forall c, \forall t, \forall \omega$, as stated in constraints (14.1e), (14.6) allows reformulating each term $\sum_{n \in \Omega^{\mathrm{N}}} \vartheta_{n,c,t}(\omega) P^{\mathrm{W}}_{n,c,t}(\omega)$ as a function of exclusively linear terms:

$$\sum_{n \in \Omega^{\mathrm{N}}} \vartheta_{n,c,t}(\omega) P^{\mathrm{W}}_{n,c,t}(\omega) = \sum_{n \in \Omega^{\mathrm{N}}} \lambda_{n,c,t}(\omega) \sum_{d \in \Lambda^{\mathrm{D}}_{n}} P^{\mathrm{D}}_{d,t}(\omega) k^{\mathrm{D}}_{d,c}$$
$$- \sum_{l \in \Omega^{\mathrm{L}}} \left[\phi^{\max}_{l,c,t}(\omega) + \phi^{\min}_{l,c,t}(\omega) \right] P^{\mathrm{L,max}}_{l} - \sum_{g \in \Omega^{\mathrm{G}}} \varphi^{\max}_{g,c,t} P^{\mathrm{G,max}}_{g}$$
$$- \sum_{n \in \Omega^{\mathrm{N}} \backslash n:\, \mathrm{ref.}} \left[\xi^{\max}_{n,c,t}(\omega) + \xi^{\min}_{n,c,t}(\omega) \right] \pi - \sum_{g \in \Omega^{\mathrm{G}}} c^{\mathrm{G}}_{g,t} P^{\mathrm{G}}_{g,c,t}(\omega), \quad \forall c, \forall t, \forall \omega. \tag{14.7}$$

On the other hand, the complementarity constraints of the KKT optimality conditions (14.5h)–(14.5m) have the form $0 \leq e \perp h \geq 0$. These terms can be replaced by the following exact equivalent linear expressions based on the Fortuny-Amat transformation [26]:

$$e \leq Mu \tag{14.8a}$$
$$h \leq M(1-u) \tag{14.8b}$$
$$e, h \geq 0 \tag{14.8c}$$
$$u \in \{0,1\}, \tag{14.8d}$$

where M is a sufficiently large positive constant [26].

Using (14.7) and (14.8), the risk-constrained multistage wind power investment problem can be finally formulated as the MILP problem below:

$$\underset{\Omega^{\mathrm{U}} \cup \Omega^{\mathrm{L}}_{c,t}(\omega) \cup \Omega^{\mathrm{A}}_{c,t}(\omega)}{\text{Maximize}}$$

$$\sum_{\omega \in \Delta^{\omega}} \gamma(\omega) \left\{ \sum_{t \in \Delta^{\mathrm{T}}} \left[\sum_{c \in \Delta^{\mathrm{C}}} N^{\mathrm{H}}_{c} \left(\sum_{n \in \Omega^{\mathrm{N}}} \lambda_{n,c,t}(\omega) \sum_{d \in \Lambda^{\mathrm{D}}_{n}} P^{\mathrm{D}}_{d,t}(\omega) k^{\mathrm{D}}_{d,c} \right.\right.\right.$$
$$- \sum_{l \in \Omega^{\mathrm{L}}} \left(\phi^{\max}_{l,c,t}(\omega) + \phi^{\min}_{l,c,t}(\omega) \right) P^{\mathrm{L,max}}_{l} - \sum_{g \in \Omega^{\mathrm{G}}} \varphi^{\max}_{g,c,t} P^{\mathrm{G,max}}_{g}$$
$$\left. - \sum_{n \in \Omega^{\mathrm{N}} \backslash n:\, \mathrm{ref.}} \left(\xi^{\max}_{n,c,t}(\omega) + \xi^{\min}_{n,c,t}(\omega) \right) \pi - \sum_{g \in \Omega^{\mathrm{G}}} c^{\mathrm{G}}_{g,t} P^{\mathrm{G}}_{g,c,t}(\omega) \right)$$
$$\left.\left. - a_{t} \sum_{n \in \Delta^{\mathrm{N}}} c^{\mathrm{inv}}_{t}(\omega) X^{\mathrm{W}}_{n,t}(\omega) \right] \right\} + \beta \left(\zeta - \frac{1}{1-\alpha} \sum_{\omega \in \Delta^{\omega}} \gamma(\omega) \eta(\omega) \right) \tag{14.9a}$$

subject to

$$\text{Constraints (14.1b)–(14.1f) and (14.1h)} \tag{14.9b}$$

$$\zeta - \sum_{t\in\Delta^{\mathrm{T}}}\left[\sum_{c\in\Delta^{\mathrm{C}}} N_c^{\mathrm{H}}\left(\sum_{n\in\Omega^{\mathrm{N}}}\lambda_{n,c,t}(\omega)\sum_{d\in\Lambda_n^{\mathrm{D}}}P_{d,t}^{\mathrm{D}}(\omega)\,k_{d,c}^{\mathrm{D}}\right.\right.$$
$$-\sum_{l\in\Omega^{\mathrm{L}}}\left(\phi_{l,c,t}^{\max}(\omega)+\phi_{l,c,t}^{\min}(\omega)\right)P_l^{\mathrm{L,max}} - \sum_{g\in\Omega^{\mathrm{G}}}\varphi_{g,c,t}^{\max}P_g^{\mathrm{G,max}}$$
$$\left.-\sum_{n\in\Omega^{\mathrm{N}}\setminus n:\,\mathrm{ref.}}\left(\xi_{n,c,t}^{\max}(\omega)+\xi_{n,c,t}^{\min}(\omega)\right)\pi - \sum_{g\in\Omega^{\mathrm{G}}}c_{g,t}^{\mathrm{G}}P_{g,c,t}^{\mathrm{G}}(\omega)\right)$$
$$\left.-a_t\sum_{n\in\Delta^{\mathrm{N}}}c_t^{\mathrm{inv}}(\omega)\,X_{n,t}^{\mathrm{W}}(\omega)\right]\le\eta(\omega),\quad\forall\omega \tag{14.9c}$$

$$\Big\{\ \text{Constraints (14.2b)–(14.2g) and (14.5d)–(14.5g)} \tag{14.9d}$$

$$\phi_{l,c,t}^{\max}(\omega)\le M^{\phi,\max}u_{l,c,t}^{\phi,\max}(\omega),\quad\forall l \tag{14.9e}$$

$$P_l^{\mathrm{L,max}}-f_{l,c,t}(\omega)\le M^{\phi,\max}\left(1-u_{l,c,t}^{\phi,\max}(\omega)\right),\quad\forall l \tag{14.9f}$$

$$\phi_{l,c,t}^{\min}(\omega)\le M^{\phi,\min}u_{l,c,t}^{\phi,\min}(\omega),\quad\forall l \tag{14.9g}$$

$$f_{l,c,t}(\omega)+P_l^{\mathrm{L,max}}\le M^{\phi,\min}\left(1-u_{l,c,t}^{\phi,\min}(\omega)\right),\quad\forall l \tag{14.9h}$$

$$\varphi_{g,c,t}^{\max}(\omega)\le M^{\varphi,\max}u_{g,c,t}^{\varphi,\max}(\omega),\quad\forall g \tag{14.9i}$$

$$P_g^{\mathrm{G,max}}-P_{g,c,t}^{\mathrm{G}}(\omega)\le M^{\varphi,\max}\left(1-u_{g,c,t}^{\varphi,\max}(\omega)\right),\quad\forall g \tag{14.9j}$$

$$\varphi_{g,c,t}^{\min}(\omega)\le M^{\varphi,\max}u_{g,c,t}^{\varphi,\min}(\omega),\quad\forall g \tag{14.9k}$$

$$P_{g,c,t}^{\mathrm{G}}(\omega)\le M^{\varphi,\min}\left(1-u_{g,c,t}^{\varphi,\min}(\omega)\right),\quad\forall g \tag{14.9l}$$

$$\xi_{n,c,t}^{\max}(\omega)\le M^{\xi,\max}u_{n,c,t}^{\xi,\max}(\omega),\quad\forall n\setminus n:\,\mathrm{ref.} \tag{14.9m}$$

$$\pi-\delta_{n,c,t}(\omega)\le M^{\xi,\max}\left(1-u_{n,c,t}^{\xi,\max}(\omega)\right),\quad\forall n\setminus n:\,\mathrm{ref.} \tag{14.9n}$$

$$\xi_{n,c,t}^{\min}(\omega)\le M^{\xi,\min}u_{n,c,t}^{\xi,\min}(\omega),\quad\forall n\setminus n:\,\mathrm{ref.} \tag{14.9o}$$

$$\delta_{n,c,t}(\omega)+\pi\le M^{\xi,\min}\left(1-u_{n,c,t}^{\xi,\min}(\omega)\right),\quad\forall n\setminus n:\,\mathrm{ref.} \tag{14.9p}$$

$$\phi_{l,c,t}^{\max}(\omega),\phi_{l,c,t}^{\min}(\omega)\ge 0,\quad\forall l \tag{14.9q}$$

$$\varphi_{g,c,t}^{\max}(\omega),\varphi_{g,c,t}^{\min}(\omega)\ge 0,\quad\forall g \tag{14.9r}$$

$$\xi_{n,c,t}^{\max}(\omega),\xi_{n,c,t}^{\min}(\omega)\ge 0,\quad\forall n\setminus n:\,\mathrm{ref.} \tag{14.9s}$$

$$u_{l,c,t}^{\phi,\max}(\omega),u_{l,c,t}^{\phi,\min}(\omega)\in\{0,1\},\quad\forall l \tag{14.9t}$$

$$u_{g,c,t}^{\varphi,\max}(\omega), u_{g,c,t}^{\varphi,\min}(\omega) \in \{0,1\}, \quad \forall g \tag{14.9u}$$

$$u_{n,c,t}^{\xi,\max}(\omega), u_{n,c,t}^{\xi,\min}(\omega) \in \{0,1\}, \quad \forall n \setminus n\text{: ref.} \tag{14.9v}$$

$$\Big\}, \forall c, \forall t, \forall \omega,$$

where $M^{\phi,\max}$, $M^{\phi,\min}$, $M^{\varphi,\max}$, $M^{\varphi,\min}$, $M^{\xi,\max}$, and $M^{\xi,\min}$ are large enough constants [26], and

$$\Omega_{c,t}^{\mathrm{A}}(\omega_s) = \Big\{ u_{l,c,t}^{\phi,\max}(\omega), u_{l,c,t}^{\phi,\min}(\omega), \forall l; u_{g,c,t}^{\varphi,\max}(\omega), u_{g,c,t}^{\varphi,\min}(\omega), \forall g; u_{n,c,t}^{\xi,\max}(\omega), u_{n,c,t}^{\xi,\min}(\omega), \forall n \setminus n\text{: ref.} \Big\}, \forall c, \forall t, \forall \omega, \tag{14.10}$$

are sets of auxiliary binary variables.

14.3.5 Illustrative Example

14.3.5.1 Data

MILP problem (14.9) is applied to the three-node system depicted in Fig. 14.5. This system comprises three nodes, one generation unit per node, one demand per node, and three transmission lines. Node 1 is the reference node.

Table 14.5 provides the generation unit and demand data. The first column gives the nodes at which generation units and demands are located. The second and third columns provide the capacity of each generation unit and the corresponding marginal cost, respectively. For the sake of simplicity, both the capacities and marginal costs are considered fixed throughout the planning horizon. Finally, the fourth column provides the peak demand at each node of the system prior to the beginning of the planning horizon. These peak demands multiplied by the corresponding demand factors give the demands for each operating condition, time period, and scenario.

Table 14.5 Generation unit and demand data

Node	Generation units		Peak demand [MW]
	$P_g^{\mathrm{G,max}}$ [MW]	c_g^{G} [\$/MWh]	
1	150	76	150
2	150	58	120
3	120	65	120

All transmission lines are considered to have identical parameters with a reactance equal to 0.2 p.u. and a transmission capacity of 100 MW.

Wind power capacity can be built at nodes 2 and 3 up to 300 MW at each node throughout the planning horizon. Investment costs at the beginning of the planning horizon are $1,000,000 per MW. The investment budget is considered unlimited.

The planning horizon comprises two time periods of 6 years each. The demand and wind power capacity factors throughout the reference year of each period are represented using the ten clusters whose data are provided in Table 14.1. The amortization rates are considered equal to 0.26 and 0.13 in the first and second periods, respectively.

Regarding the uncertainty in the demand growth and investment costs, we consider the three cases below:

1. Uncertainty only affects demand growth. This case corresponds to the scenario data provided in Fig. 14.2 and Table 14.2.
2. Uncertainty only affects investment cost. This case corresponds to the scenario data provided in Fig. 14.3 and Table 14.3.
3. Uncertainty affects both demand growth and investment cost. This case corresponds to the scenario data provided in Fig. 14.4 and Table 14.4.

The subsections below provide the results for the three above cases. MILP problem (14.9) is solved for two different values of parameter β that realizes the trade-off between expected profit and risk of profit volatility:

1. $\beta = 0$: this case corresponds to a risk-neutral investor. This investor aims at maximizing its expected profit regardless of its profit volatility.
2. $\beta = 10$: this case corresponds to a risk-averse investor. This investor prefers to reduce its profit volatility despite the subsequent decrease in its expected profit.

In all cases a confidence level $\alpha = 0.95$ is considered.

Problem (14.9) is solved using CPLEX 12.2.0.1 [27] under GAMS [28] on a Linux-based server with four processors clocking at 2.9 GHz and 250 GB of RAM.

14.3.5.2 Results: Uncertainty in Demand Growth

Results corresponding to the scenario data of Fig. 14.2 and Table 14.2 (uncertainty in demands growth) and for risk-neutral and risk-averse investors are provided in Table 14.6. The first column provides the value of weighting parameter β. The second column indicates the scenarios. The third/fourth and fifth/sixth columns give the wind power capacity to be built at the beginning of the first/second period at nodes 2 and 3, respectively.

Note that in this case there are two demand growth scenario realizations in the first period and two scenario realizations in the second period depending on the scenario realization in the first one. This results in a single investment decision at the beginning of the planning horizon, which does not depend on any scenario realization (i.e., it is a *here-and-know* investment decision) and two alternative investment

Table 14.6 Results: uncertainty in demand growth

Case	Scenario	Wind power capacity to be installed [MW]			
		Node 2		Node 3	
		Period 1	Period 2	Period 1	Period 2
Risk-neutral ($\beta = 0$)	D1	0	0	300	0
	D2				
	D3		0		0
	D4				
Risk-averse ($\beta = 10$)	D1	0	0	279.7	20.3
	D2				
	D3		0		0
	D4				

decisions for the second period, one for scenarios D1 and D2 and other one for scenarios D3 and D4 (i.e., they are *wait-and-see* investment decisions with respect to the first period and *here-and-know* with respect to the second one).

Node 3 located in the south zone has better wind power conditions than node 2, located in the north zone. Thus, the wind power investor prefers to build wind power capacity at node 3 and does not build any wind power capacity at node 2 in any scenario and time period.

Regarding the differences between the investment decisions of a risk-neutral and a risk-averse investor, note that the first one builds 300 MW at the beginning of the planning horizon (which is the maximum wind power capacity that can be installed at each node for the whole planning horizon) while the risk-averse investor prefers to build a smaller wind power capacity at the beginning of the first period (279.7 MW) and wait until the beginning of the second period to decide on further investment: if the demand in the first period has increased (i.e., scenarios D1 and D2), it builds 20.3 MW that complete the 300 MW capacity; however, if the demand in the first period has decreased (i.e., scenarios D3 and D4), the wind power investor builds no additional capacity. This way, the wind power investor reduces its profit volatility.

14.3.5.3 Results: Uncertainty in Investment Cost

Results corresponding to the scenario data of Fig. 14.3 and Table 14.3 (uncertainty in investment cost) and for risk-neutral and risk-averse investors are provided in Table 14.7. The first column provides the value of weighting parameter β. The second column indicates the different scenarios. The third/fourth and fifth/sixth columns give the wind power capacity to be built at the beginning of the first/second period at nodes 2 and 3, respectively.

Table 14.7 Results: uncertainty in investment cost

Case	Scenario	Wind power capacity to be installed [MW]			
		Node 2		Node 3	
		Period 1	Period 2	Period 1	Period 2
Risk-neutral ($\beta = 0$)	IC1	0	0	267.6	0
	IC2		0		0
	IC3		109		32.4
Risk-averse ($\beta = 10$)	IC1	0	0	267.6	0
	IC2		0		0
	IC3		109		32.4

In this case, there is a single investment decision at the beginning of the first period and three different investment decisions at the beginning of the second period depending on the investment cost in the second period with respect to that in the first one: higher than (scenario IC1), equal to (scenario IC2), or lower than (scenario IC3).

The optimal solution consists of installing 267.6 MW at node 3 at the beginning of the planning horizon, and then, if the investment cost in the second period has decreased (i.e., scenario IC3), the wind power investor decides to install 109 and 32.4 MW of additional wind power capacity at nodes 2 and 3, respectively. As in the case of uncertainty in demand growth, the wind power investor prefers to install wind power capacity at node 3, which has better wind power conditions than node 2.

In this case, there are no differences between the optimal solutions of the risk-neutral and the risk-averse investors. This is so because at the time the wind power investor makes its investment decisions, it knows the actual investment costs.

14.3.5.4 Results: Uncertainty in Both Demand Growth and Investment Cost

Results corresponding to the scenario data of Fig. 14.4 and Table 14.4 (uncertainty in both demand growth and investment cost) and for risk-neutral and risk-averse investors are provided in Table 14.8. The first column provides the value of the weighting parameter β. The second column indicates the different scenarios. The third/fourth and fifth/sixth columns give the wind power capacity to be built at the beginning of the first/second period at nodes 2 and 3, respectively.

This case is a combination of the two previous cases since it considers all possible scenario combinations. The results show that, for the risk-neutral case, the wind power investor decides to build the whole 300 MW at node 3 at the beginning of the planning horizon and to build 132.9 and 63.7 MW at node 2 at the beginning of the second period in scenarios D1+IC3/D2+IC3 and D3+IC3/D4+IC3, respectively. That is, it only builds wind power capacity at node 2 if the investment cost in the

Table 14.8 Results: uncertainty in both demand growth and investment cost

<table>
<tr><th rowspan="3">Case</th><th rowspan="3">Scenario</th><th colspan="4">Wind power capacity to be installed [MW]</th></tr>
<tr><th colspan="2">Node 2</th><th colspan="2">Node 3</th></tr>
<tr><th>Period 1</th><th>Period 2</th><th>Period 1</th><th>Period 2</th></tr>
<tr><td rowspan="12">Risk-neutral
($\beta = 0$)</td><td>D1+IC1</td><td rowspan="12">0</td><td rowspan="2">0</td><td rowspan="12">300</td><td rowspan="2">0</td></tr>
<tr><td>D2+IC1</td></tr>
<tr><td>D1+IC2</td><td rowspan="2">0</td><td rowspan="2">0</td></tr>
<tr><td>D2+IC2</td></tr>
<tr><td>D1+IC3</td><td rowspan="2">132.9</td><td rowspan="2">0</td></tr>
<tr><td>D2+IC3</td></tr>
<tr><td>D3+IC1</td><td rowspan="2">0</td><td rowspan="2">0</td></tr>
<tr><td>D4+IC1</td></tr>
<tr><td>D3+IC2</td><td rowspan="2">0</td><td rowspan="2">0</td></tr>
<tr><td>D4+IC2</td></tr>
<tr><td>D3+IC3</td><td rowspan="2">63.7</td><td rowspan="2">0</td></tr>
<tr><td>D4+IC3</td></tr>
<tr><td rowspan="12">Risk-averse
($\beta = 10$)</td><td>D1+IC1</td><td rowspan="12">0</td><td rowspan="2">0</td><td rowspan="12">279.7</td><td rowspan="2">20.3</td></tr>
<tr><td>D2+IC1</td></tr>
<tr><td>D1+IC2</td><td rowspan="2">0</td><td rowspan="2">20.3</td></tr>
<tr><td>D2+IC2</td></tr>
<tr><td>D1+IC3</td><td rowspan="2">132.9</td><td rowspan="2">20.3</td></tr>
<tr><td>D2+IC3</td></tr>
<tr><td>D3+IC1</td><td rowspan="2">0</td><td rowspan="2">0</td></tr>
<tr><td>D4+IC1</td></tr>
<tr><td>D3+IC2</td><td rowspan="2">0</td><td rowspan="2">0</td></tr>
<tr><td>D4+IC2</td></tr>
<tr><td>D3+IC3</td><td rowspan="2">63.7</td><td rowspan="2">20.3</td></tr>
<tr><td>D4+IC3</td></tr>
</table>

second period has decreased with respect to that in period 1. Moreover, the investor builds higher wind power capacity if the demand in the first period has increased (i.e., scenarios D1 and D2) than if the demand in the first period has decreased (i.e., scenarios D3 and D4).

Regarding risk-averse results, the wind power investor prefers in this case to build lower wind power capacity at node 3 at the beginning of the planning horizon than in the risk-neutral case and to wait until it knows the scenario realization in period 1.

Depending on this scenario realization it decides whether or not to build additional capacity.

Finally, Fig. 14.6 depicts the efficient frontier in this case. The efficient frontier shows how the expected profit decreases as the CVaR increases, as a consequence of changes in the weighting parameter β. Note that as parameter β increases, the wind power investor reduces its profit. However, it also reduces its profit volatility, i.e., it increases its CVaR.

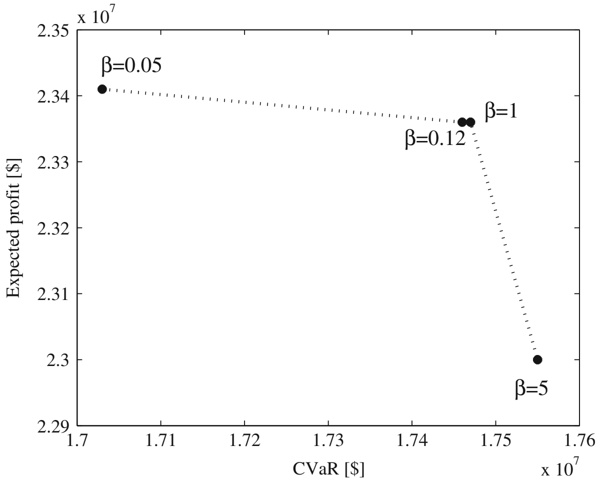

Fig. 14.6 Efficient frontier for the case of uncertainty in both demand growth and investment cost

14.3.5.5 Computational Issues

The computational time required for solving MILP problem (14.9) for the case studies analyzed in the previous subsections is less than 5 s in all cases. However, if problem (14.9) is considered for systems of realistic size and a large number of scenarios are considered as well, it is expected that the computational time drastically increases due mainly to the number of binary variables required to linearize the complementarity constraints.

Nevertheless, note that if investment decisions $X_{n,t}(\omega)$, $\forall n$, $\forall t$, $\forall \omega$, and auxiliary variable ζ are fixed to given values, MILP problem (14.9) decomposes by scenario ω, and Benders' decomposition can be applied [29] to reduce the computational burden.

14.4 Summary and Conclusions

In this chapter we provide a risk-constrained multistage decision-making model to determine the optimal generation investment in stochastic electricity-production facilities. This model is a stochastic bi-level problem that incorporates the clearing of a pool-based electricity market in which the investor sells the production of the newly built capacity. This investor aims at maximizing its expected profit while minimizing its profit volatility.

The multistage approach allows making investment decisions in different points in time to adapt to eventual changes in market and investment cost conditions. In this sense, we consider a planning horizon comprising a number of time periods, each one spanning a specific number of years. Investment decisions, involving stochastic capacity to be built in different locations, can be made at the beginning of each of these time periods.

The risk-constrained approach allows controlling the risk of profit volatility. To do so, the CVaR metric is used to quantify the risk. A weighting parameter in the objective function of the problem allows modeling the trade-off between expected profit and profit volatility and allows obtaining different investment strategies for different risk levels.

The stochastic approach allows incorporating in the model different sources of uncertainty though appropriate scenario trees that represent different realizations of the uncertain parameters.

A study pertaining to wind power investment is presented. In this study, three sources of uncertainty affect the investment decisions: the demand growth, the wind power production, and the investment cost.

Finally, from the theoretical modeling and the study carried out, the conclusions below are in order:

1. A risk-constrained framework is an appropriate approach for investment decision-making since it allows controlling the risk of profit volatility and generally prevents the investor to incur losses.
2. A multistage approach is necessary since demand growth and investment cost are subject to variations in the future and having the possibility of investing in different points in time is advantageous.
3. The resulting bi-level model can be formulated as an MILP problem, which can be solved using available branch-and-cut techniques.
4. The model is computationally tractable provided that the size of the system under study and the number of scenarios are moderate. For large systems and a large number of scenarios, decomposition techniques can be applied to reduce the computational burden.

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