Phase-Change Phenomena and Two-Phase Flow

<div style="text-align:right">**7**</div>

Phase change usually occurs as a result of heat transfer, but it can also take place in response to a change of pressure. Using heat to produce power relies on the water to steam phase change which occurs by boiling or flashing, and sometimes the phase change of fluids other than water. Boiling takes place in one of two possible modes, nucleate boiling, which involves surface tension, and homogenous nucleation. These are introduced first, followed by flashing, which is the change from liquid to gas resulting from pressure reduction, and then condensation. The nature and possible causes of thermal explosions and hydrothermal eruptions are briefly discussed, since phase-change phenomena are involved. An introduction to two-phase flow then follows, since it is empirically based and requires the introduction of many new variables to the governing equations established in Chap. 4. Finally, the physics of aqueous solutions of gas is explained.

7.1   Background

Geothermal engineering usually involves the extraction of hot water from a resource and flashing it in stages to produce dry steam. Flashing might appear today as a minor industrial activity, but in the early part of the twentieth century, storing saturated water in an "accumulator" and using it to produce extra steam when required was a standard practice in factories—see, for example, Lyle [1947] and books of similar vintage. Accumulators were insulated horizontal cylinders kept partly full of water at saturation conditions so that a large surface area existed from which steam could evaporate when the pressure was reduced in response to demand. Alternatively, steam could be supplied at higher pressure when there was a surplus, so that it condensed. The process of heat storage as a means of fuel economy was well established, and many of the steamfield practices of liquid-dominated geothermal resource use must have originated from this background.

Detailed studies of the boiling process intensified in the mid-twentieth century as the level of heat flux used in equipment increased. A natural heat flux of 800 mW/m^2 is regarded as high over a geothermal resource, but engineering equipment such as boilers, nuclear reactors and chemical processing plant are often designed to operate with heat fluxes of the order of 1 MW/m^2. Details of the flashing process have been less important to normal engineering operations, but very rapid flashing, boiling and condensation have all attracted attention in studies of explosions, both in natural and in engineering equipment.

7.2   Surface Tension

The saturation pressure and temperature form a unique set of conditions at which water and steam (or any liquid and its gas) are in equilibrium. Unfortunately, phase change creates a surface, the pressure on each side of which is only saturation pressure for a pure substance if the surface is flat. This is illustrated in Fig. 7.1—the liquid molecules at the surface are maintained in close contact by the short-range forces which act throughout the liquid including along the surface, giving it a "surface tension".

For water, the surface tension ζ (N/m) varies with saturation pressure and becomes zero at the thermodynamic critical point; the interface between liquid and gas disappears at the critical point, and watching this happen is a classic laboratory demonstration (though not easy to set up). The IAPWS [1994] equation is written in terms of saturation temperature; thus (Fig. 7.2),

$$\zeta = 235.8\Gamma^{1.256}(1 - 0.625\Gamma) \tag{7.1}$$

where $\Gamma = \left(1 - \frac{\Theta}{\Theta_C}\right)$ and the critical temperature $\Theta_C = 647.096K$.

The surface tension influences the shape of the interface between phases and may shift the local conditions away from the saturation pair. Consider a steam bubble of radius r and examine the force balance which must exist to keep its diameter constant, by imagining it cut in half—Fig. 7.3.
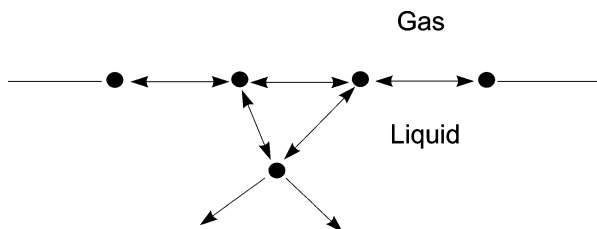

Fig. 7.1 Sketch of water molecules at a surface, to illustrate surface tension

Fig. 7.2 Surface tension (N/m) for water as a function of saturation temperature calculated from IAPWS [1994]
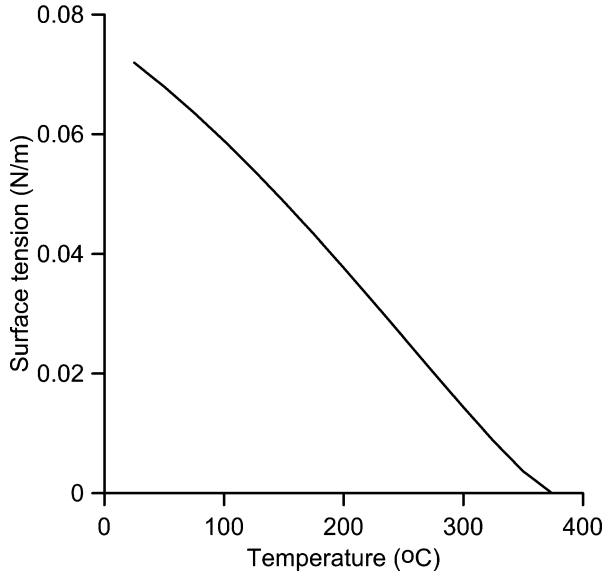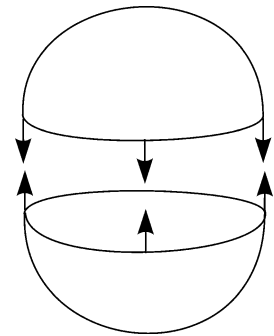

Fig. 7.3 The action of surface tension on a bubble

The surface tension around the perimeter keeps the two halves together but must be balanced by a pressure difference over the wall of the bubble to prevent it collapsing; thus,

$$\zeta.2\pi r = \Delta P.\pi r^2 \tag{7.2}$$

or

$$\Delta P = \frac{2\zeta}{r} \tag{7.3}$$

If the bubble and surrounding liquid are isothermal, then the liquid must be slightly superheated with respect to the steam in the bubble. A similar analysis can be carried out for a water droplet in steam.

7.3 Boiling

Quantifying the rate of heat transfer by boiling is seldom necessary in geothermal engineering, but the physical processes are worth understanding. Boiling studies are often reported in engineering literature as one of two types, pool boiling and flow boiling, depending on whether the liquid being heated is a fixed volume held as a "pool" in a container, or is flowing, as through a heated pipe. But the process is characterised by the way the phase change takes place, and three modes are commonly identified, nucleate boiling, film boiling and homogenous nucleation. The specific volume of steam is several orders of magnitude greater than that of water, so it is evident that steam generated at a surface has the potential to prevent replacement water from reaching it. In nucleate boiling the phase change occurs at nucleation sites on the heated surface, and bubbles are created which leave at regular intervals allowing new water to occupy the sites. If the heat flux at a surface is greater than a critical value, replacement water cannot reach the surface, which becomes blanketed by a layer of steam through which heat must pass in order to cause more evaporation at the steam–water interface. This is referred to as film boiling, in which the rate of heat transfer is very much less than in nucleate boiling. It is a mode that designers work to avoid—it is a failure condition of high heat flux equipment and is barely relevant to geothermal engineering, although it may arise in volcanic circumstances.

7.3.1 Nucleate Boiling

However smooth the surface of a container for boiling water appears, it will have microscopic pits and crevices. If the container can be evacuated first and then filled with water free of dissolved gas, the pits can be entirely filled with water, but normally air remains in the bottom when the water is added—see the sketch of Fig. 7.4a. The air pocket is compressed under hydrostatic head until it takes up a curvature which satisfies Eq. (7.3); it then becomes a pocket of air mixed with a small amount of steam, before the surface is heated. The air cannot condense, so even when the surface is cold, the pocket will not collapse. When the surface is heated, conduction ensures that the water adjacent to the surface and pit (more properly referred to as a nucleation site) is heated, and water molecules transfer across the interface from the liquid side. A bubble grows in the manner depicted by
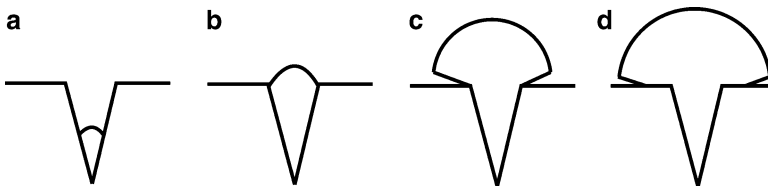

Fig. 7.4 Stages in the growth of a bubble at a nucleation site, from *left* to *right*

the remaining sketches of Fig. 7.4; it grows hemispherically, eventually leaving behind a thin layer of water on the heated surface as an annulus, with a dry patch around the nucleation site. Thermal conduction is sufficient to evaporate the thin layer (micro-layer). Relatively little evaporation takes place from around the dome of the bubble.

The surface and adjacent liquid are at a slightly higher temperature than the steam, providing heat transfer into the bubble. The growing bubble eventually becomes large enough to develop a buoyancy force sufficient to lift it off the heated surface, and it forms a neck which eventually breaks, leaving the bubble free to rise. The nucleation site then returns to its starting point, but the original air that filled it before heating started has been diluted and the new interface forms over mainly steam. Evaporation again takes place and the bubble growth and departure is repeated.

As the surface temperature is increased, the heat flux transferred increases, but eventually a stage is reached at which the rising bubbles obstruct the inflow of replacement water and the surface then becomes covered entirely by steam. In nuclear reactor fuel elements heated by fission and also in fossil-fuelled boiler tubes heated by thermal radiation from flames, the physical boundary condition of the surface is one of constant heat flux, not constant temperature. Given a heat flux entirely independent of temperature, it is possible for the surface to reach temperatures at which it will fail if the heat transfer rate is insufficient, and the term "burnout" is sometimes used to describe the critical heat flux condition.

There is a temperature level for any surface above which water sprayed onto it will not touch it but remains separated by a film of steam; the Leidenfrost drop is the name usually associated with this phenomenon.

Nucleate boiling also occurs in heated pipes through which liquid is flowing—called flow boiling; bubble growth can be very rapid, and the only difference from pool boiling is that the bubble is subjected to fluid drag as it gets bigger, so it is swept off the nucleation site instead of leaving it under its buoyancy force. The boiling crisis is likewise an issue with flow boiling. Some engineering equipment has nucleation sites deliberately created on the heated surfaces. The bumping in round-bottomed flasks in chemistry laboratories is caused by a lack of nucleation sites—it is traditionally cured by adding broken pieces of pottery, in other words, by adding nucleation sites. The pores of a permeable formation provide nucleation sites, as do particles suspended in a fluid. In the absence of nucleation sites, boiling does not occur with the few degrees of superheat that they permit, but instead the violent process known as homogenous nucleation takes place.

7.3.2   Homogenous Nucleation

Homogenous nucleation takes place within the body of the fluid when there are no nucleation sites. It is an explosive process, i.e. it is very rapid and the specific volume increase from water to steam produces high acceleration rates and hence pressure waves. The physical idea is that as the level of superheat increases, the

amplitude of vibration of the liquid molecules increases—they are bound by short-range forces so movement is restricted but becomes greater with the increased kinetic energy associated with increasing temperature. A minimum linear dimension can be defined beyond which adjacent molecules will break free, and as superheat increases, spaces within the liquid will open up randomly and allow the formation of a bubble. In nucleate boiling the superheat provided at the surface is only a degree or two, but 50 °C or more may occur in the absence of nucleation sites. The rate of evaporation is correspondingly high. Homogenous nucleation is a rare phenomenon because nucleate boiling usually begins first at a container wall, but it can occur under very high rates of heating. Skripov [1974] set out the theory which was expanded upon by Blander [1979]; Hasan et al. [2011] have recently provided a literature survey. This topic is very relevant to the search for the causes of thermal explosions in industrial equipment and of hydrothermal eruptions and explosive volcanism.

7.4   Flashing

Flashing is the phase change from liquid to gas produced by reducing the pressure without adding heat, which distinguishes it from boiling. It is a common process both in natural geothermal flows and in wells and steamfield equipment. Hot fluids convect upwards in permeable ground and spring conduits and undergo falling pressure as a result, and they flow upwards in discharging wells.

If a mass of water at saturation conditions suffers a pressure reduction, the fraction of the water which will evaporate is determined by the absolute pressure level and the pressure change. How the evaporation takes place depends on the rate of fall of pressure and the surface area of the water from which steam can leave. Suppose the mass of water is contained in a cylinder by means of a piston and in the initial condition the entire volume is taken up by water. The pressure is then reduced by withdrawing the piston slowly. The only surfaces over which evaporation can take place are those in the bottom of the nucleation sites, as already discussed, which are water–air surfaces or water–air + steam surfaces. Bubbles would appear on the vessel walls. Hahne and Barthau [2000], who reviewed previous studies, carried out experiments of this type in which the starting point was a vessel part filled with liquid and the remainder filled with its gas (vapour), both at rest and in thermal equilibrium. Slow pressure reductions resulted in nucleation around the liquid line in the cylinder, while rapid reductions caused an evaporation wave to traverse down through the liquid, forming a mixture of droplets and gas behind the wave.

If a pressurised liquid is to be flashed to a lower pressure, then spraying it into the available space will help the process by maximising the liquid surface area, and this is the usual practice. Given the high specific volume of steam, it is possible that spraying water into a vessel would generate vapour faster than it could escape, so that the pressure in the vessel could increase and restrict the flashing process—in

other words, the vessel must be large enough to accept the steam generated at the required pressure.

In discharging wells the liquid becomes superheated as it flows upwards because of the effective hydrostatic pressure gradient. The heat loss to the casing can be neglected if the discharge has been taking place for some time and is steady, and the flow will be approximately adiabatic. The distribution of the two phases will depend on the particular parameters of the flow, and this is the main issue in the study of flow in Sect. 7.7.

The flashing process is used in the desalination of seawater and chemical processes for concentrating solutes and recovering solvents. "Multistage flash" in geothermal terms means 2 or 3, but 25–30 stage flash evaporators are used to produce drinking water from seawater in the Middle East countries where there is ample fuel supply but little fresh water. There is a difference in focus between the two industries: in geothermal the aim is to produce steam from the available energy in the well discharge, and in desalination it is to produce the maximum amount of pure water from the seawater after heating it only once, at the inlet, and then conserving the heat by recovering the h_{fg} specific enthalpy change by means of a condenser in every stage. There is no economic advantage in increasing the number of flash stages in a geothermal plant because it would make the turbine too complicated, as will become clear in Chap. 11, although this is an opinion unsupported by any analysis. The literature on multistage flash evaporation is rather impenetrable, but papers by Lior [1986] and Khademi et al. [2009] provide an inroad in the vernacular of this book.
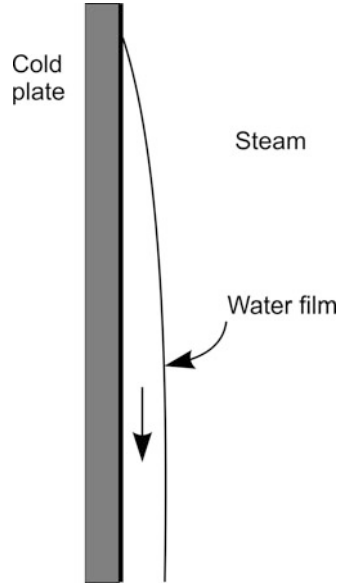
7.5 Condensation

Both steady-state condensation and very rapid transient condensation are relevant to geothermal engineering. The physical process is the reverse of phase change by boiling. It can be achieved by either decreasing the temperature of the gas or increasing its pressure, and it can take place homogenously or heterogeneously (via nucleation sites).

Homogenous condensation arises when the gas is supersaturated enough to cause molecules to come into close enough proximity to allow short-range forces to form a liquid droplet. Once a liquid surface is established, then condensation will occur via the capture of individual molecules, and the distribution of the condensed liquid controls the rate of further condensation via the surface area exposed. Nusselt (1916) developed the theory associated with the idea of a continuous film condensing on a vertical cooled surface. The film starts at the top and grows in thickness as it flows down the wall. If the surface is held at constant temperature by removing the heat given up in the condensation process, the rate of condensation decreased lower down the plate because of the increasing temperature difference required to drive the heat flow through the film to the wall—Fig. 7.5.

The patterns of condensation described so far occur when the gaseous form of a single pure substance condenses on a clean surface. The rate of heat transfer can be

Fig. 7.5 Illustrating the process
of film condensation

increased by the presence of other substances, just as it can be decreased by the presence of non-condensable gases. The surface can be treated with organic compounds which cause the condensation to form into drops (hence the name drop-wise condensation) which run off the surface quickly and leave it free for further condensation. If present, non-condensable gas (ncg) molecules mixed with the condensable gas molecules are carried to the cool surface but remain there after the liquid has formed. If the concentration of ncg molecules increases, they blanket the surface and prevent condensable molecules reaching it, thus reducing the condensation rate.

Transient condensation of very small bubbles has received a great deal of attention because it is the cause of the phenomenon known as cavitation, which occurs in high-speed flows. Many small bubbles form on the underwater surfaces of high-speed marine vessels and in the films of lubricant in high-speed rotating machines because even though the fluid is not at high temperature, local pressures may be transiently below saturation pressure under some flow circumstances—Bernoulli's equation applies and local pressure may fall considerably. The collapse of the bubbles causes metal fatigue and pitting of the surfaces. If the bubble is spherical, the condensation process proceeds fast enough for the liquid to accelerate towards the bubble centre symmetrically, resulting in a very high inertial force. Jones and Edwards [1960] reported peak pressures of more than 10,000 atm; Wang and Chen [2007] provide a review and the results of recent experiments.

A related phenomenon occurs in engineering plant when a steam cavity condenses. The maximum pressure achieved by the collapse is less than in a cavitation bubble because the geometry is usually not spherical, but the effect can in some circumstances be amplified by the local pressure increase travelling along a

pipe until it rebounds and returns—the phenomenon known as water hammer. Chou and Griffith [1990] carried out experiments with the aim of developing protocols for the safe injection of cold water into pipework containing steam, in connection with water-cooled nuclear reactor safety studies. The avoidance of steam cavity collapse is a consideration in the design of geothermal pipelines, as will be discussed in Chap. 13.

7.6   Thermal Explosions and Hydrothermal Eruptions

Material used in engineered systems at ever higher temperatures and heat fluxes has given rise to explosive phenomena, the mechanisms of which remain to be established. Explosions also occur naturally in geothermal resources and volcanoes, some small and some very large. There are only a few fundamental physical processes which could contribute to these, yet they have so far defied explanation.

As regards hydrothermal eruptions, McKibben [2007] reviewed his previous work on events following the explosion itself but without identifying its origins, considering that the outflow of material may be the result of a failure of the containment of saturated water or other high-pressure fluids. Vessels filled with water at saturation temperature are known to produce considerable reactive forces when they burst. Ohba et al. [2007] examined the circumstances of a small isolated eruption on the flanks of a volcano, involving mud but not magma. Browne and Lawless [2001] presented evidence from the craters formed by eruptions from hundreds to thousands of years ago and addressed the terminology, which is poorly defined because the physics of the explosion have not been clearly identified. Phreato-magmatic and hydrothermal are two names often used, the first implying an explosion in which magma played a part and the second where no magma was present.

In engineering equipment the explosions are called steam explosions or vapour explosions, and they have been the subject of a great deal of laboratory experimentation. They occur, for example, when molten metal is dropped into water. Their history is reviewed by Hasan et al. [2011], and a recent numerical modelling study reported by Ursic et al. [2012] presents a picture very similar to the idea of a detonation wave. In the process called detonation, some combustible material is distributed throughout a volume of air which is traversed by a compression wave. Normally a wave of locally high-pressure fluid (air) will attenuate as it travels, but if it is fuelled by initiating combustion at or just behind the wave front, the energy release accentuates and accelerates the wave instead. The numerical model mentioned includes the molten metal being fragmented into very small drops which release their heat rapidly, in milliseconds, the result being an explosive expansion. Explosions of this type are of concern in foundries and also in the nuclear industry as the extreme outcome of fuel melting; the process also occurs in some types of volcanic eruption.

A connection between geothermal-related explosions and steam explosions is offered in a report by Stanmore and Desai [1993] of a total of 17 explosions which have occurred in fossil-fuelled boilers when dropping hot ash from the base of the boiler into a purpose-built pool of water beneath. Coal ash is primarily silica and

alumina in particles of diameter less than 30 μm. A method of avoiding the explosions was recommended but the cause was not identified.

Watson [2002] identified and compared four physical processes which give rise to an explosion, namely, the failure of a vessel containing high-pressure fluid, a chemical explosion (TNT, gelignite, etc.), homogenous nucleation and the collapse of a vapour bubble. There are similarities in the craters caused by buried explosives and natural hydrothermal eruption craters and breccia pipes, but the natural events are so rare and so unpredictable that real progress in understanding will not be made without laboratory and small-scale field experimentation.

7.7 Two-Phase Flow

Two-phase flows occur in geothermal wells, pipelines, condensers and heat exchangers and also in the flows induced in a formation by the discharge from wells. They occur throughout the petroleum and chemical processing industries, often with more than one chemical species present. Only single-component (chemical species) two-phase flows are discussed here.

7.7.1 The Approach to Analysing a Two-Phase Flow

Some of the flow patterns that a steady mass flow rate of water goes through in changing into steam on its way through a vertical heated tube are illustrated in Fig. 7.6. An entirely liquid water flow enters at the bottom and gradually collects bubbles from the wall. Partway up the pipe, there is enough steam for it to collect as larger volumes separated by bodies of mainly water; still further up, the flow may become less organised before adopting an annular pattern, with most of the water flowing up the wall in an annular layer. The annular flow eventually dries out as the water evaporates. At any point in the flow, no matter what the frame of reference, the conditions vary randomly. There is no resemblance at all to the smooth variation of velocity with position in the single-phase flows described in Chap. 4.

The problems facing the designer of a two-phase flow pipe network are the same as for single-phase flow, usually calculating the pressure drop for a given pipe diameter and flow rate and optimising pumping power or energy loss. The fundamental approach of creating equations from the statements of conservation of mass, momentum and energy flow rates is still adopted, but a mainly new set of variables is required to express them. The mass flow rate through a pipe may be constant, but the fluid can change phase.

The flow rates of each phase are related—they add up to the constant total flow rate—and this introduces simple though clumsy algebraic relations into the equations. As in the treatment of turbulent single-phase flows, no solutions are possible without introducing experimentally determined relationships. In the case of two-phase flows, a set of experimental data must be provided for each of the flow patterns characterising the flow in Fig. 7.4 (there are more recognised patterns than
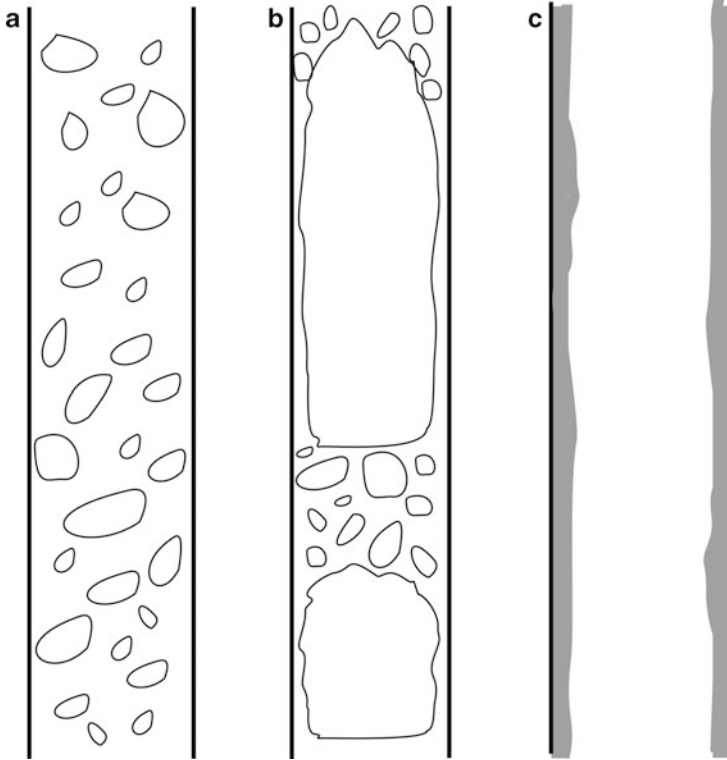
Fig. 7.6 Regimes of steady two-phase flow in a vertical, heated pipe; (**a**) bubbly, (**b**) slug and (**c**) annular

are shown here). The patterns have a measure of subjectiveness, which adds uncertainty to the outcome of solving the equations. The experimental data is provided as a number of correlations written in terms of the new variables already mentioned.

Quite separate from the flow patterns, there are three well-known methods of considering the flow in setting up the governing equations. Wallis [1969] described them as models, and they remain in use today—see Ghiaasiaan [2008].

They are:

- Homogenous flow, in which the two phases are well mixed and travel together at the same velocity. The flow can be represented by a single momentum equation because the phases are in thermodynamic equilibrium.
- Separated flow, in which it is accepted that phase change from liquid to gas induces a velocity change because of the change in specific volume, and hence a momentum equation for each phase is introduced.
- The drift flux model in which the phases travel as in the separated flow model, but the equations are expressed in terms of the relative velocity between the phases.

Only homogenous flow will be dealt with here as it is sufficient to illustrate the approach.

7.7.2   The New Variables

7.7.2.1 Void Fraction, α

Consider a pipe of cross-sectional area A and imagine the flow passing through a plane at some particular value of z, the direction of the pipe axis. At any instant, part of the cross-sectional area will be occupied by steam and part by water, and because of random shape and size of the "lumps" of water and steam, these areas will be constantly changing. The void fraction is the average fraction of A occupied by the steam phase:

$$\alpha = A_g/A \tag{7.4}$$

For dealing with the simple, steady-state homogeneous flow, the concept of a simple average is sufficient, but in general the phases can travel at a different velocity and the method of averaging requires careful consideration. By the definition of mean density, the void fraction is related to the phase densities as follows:

$$\overline{\rho} = \alpha \rho_f + (1 - \alpha)\rho_g \tag{7.5}$$

7.7.2.2 Mass Quality, x_m

If a 1 m length of the pipe being considered could be instantaneously removed, taking with it the flow that happened to be there at the time, and if the fluid content was weighed and the mass of steam measured, then the dryness fraction could be found. On the other hand, if the average flow rate of each phase could be measured at a cross section of the pipe, then the proportion of the flow that was steam could be calculated—this is the mass quality, which is a characteristic of a flow rather than a given mass.

$$x_m = \frac{\dot{m}_s}{\dot{m}} \tag{7.6}$$

The mean density can be written in terms of mass quality:

$$\frac{1}{\overline{\rho}} = \frac{x_m}{\rho_g} + \frac{(1 - x_m)}{\rho_f} \tag{7.7}$$

7.7.2.3 Mass Velocity G

The mass velocity is the mass flow rate per unit area, in other words a mass flux (kg/sm^2),

$$G = \frac{\dot{m}}{A} \tag{7.8}$$

It can be written for each phase.

7.7.2.4 Volumetric Flux or Superficial Velocity j

This is similar to the mass velocity, but is the volumetric flow rate per unit area instead of the mass flow rate. The volumetric flux is better known as the superficial velocity because it has the units of a velocity (m^3/s per m^2), and it may be defined for the total flow as well as each phase:

$$j = \frac{Q}{A} \tag{7.9}$$

7.7.3 The Governing Equations for Homogenous Flow

The flow is vertically upwards, steady state and one dimensional, varying only with z (positive upwards). It is also isothermal and finely mixed, whatever the proportions of steam and water. The phases travel at the same speed, and this has an influence on the averaging process and hence on the value of mean density. It is a single-component flow (H_2O) through a pipe of diameter d.

The continuity equation, (4.7), is most useful when written as a simple statement that the mass flow rate is constant:

$$\dot{m} = \overline{\rho} w \frac{\pi d^2}{4} \tag{7.10}$$

The momentum equations reduce to one only, which may be recognised as being similar to Eqs. (4.10) and (4.18) but with more basic terms since the flow is one dimensional:

$$\overline{\rho} w \left(\frac{\pi d^2}{4}\right) \frac{dw}{dz} = -\left(\frac{\pi d^2}{4}\right) \overline{\rho} g - \left(\frac{\pi d^2}{4}\right) \frac{dP}{dz} - (\pi d)\tau_w \tag{7.11}$$

On the right-hand side, the terms are gravitational, pressure gradient and wall shear stress acting on the pipe perimeter. This equation reduces to

$$\overline{\rho} w \frac{dw}{dz} = -\overline{\rho} g - \frac{dP}{dz} - \frac{4}{d}\tau_w \tag{7.12}$$

Since the flow is steady state, the energy equation is a version of the Steady Flow Energy Equation (3.6). However, in geothermal engineering, the flows are mainly adiabatic—at least, they are not heated and the heat loss is small, either through a well casing or through an insulated pipeline—so the energy equation provides no input to the set.

If Eq. (7.12) is rearranged as an expression for pressure gradient,

$$\frac{dP}{dz} = -\overline{\rho} g - \overline{\rho} w \frac{dw}{dz} - \frac{4}{d}\tau_w \tag{7.13}$$

it can be interpreted as a statement that the pressure gradient is the sum of components from three separate effects:

$$\frac{dP}{dz} = \left(\frac{dP}{dz}\right)_{grav} + \left(\frac{dP}{dz}\right)_{accel} + \left(\frac{dP}{dz}\right)_{fric} \tag{7.14}$$

To calculate the pressure gradient, each of the component parts must be provided: gravity, acceleration and friction. The gravitational contribution depends only on the mean density, which is a function of the flow regime and the details of mass flow rate, water and steam properties, etc., and must be found from experimental data. The acceleration component is the result of the volumetric flow rate changing as a result of heating or flashing and likewise depends on the mean density. The frictional component requires experimental data, very similar to the friction factor for single-phase turbulent flow.

7.7.4   Correlations for Obtaining Flow Parameters

Examining Eq. (7.12), the opportunities for introducing experimental data to help with the solution are only in the average density $\bar{\rho}$ and the wall shear stress τ_w.

With the flow to be analysed fixed in terms of a pipe diameter, mass flow rate, local quality, pressure and temperature, the mean density can be found from the definition, Eq. (7.10). The gravitational and acceleration pressure gradient components can then be calculated.

The wall shear stress can be considered to be proportional to the kinetic head of the flow, with a friction factor as the constant of proportionality, exactly as for single-phase fluids:

$$\tau_w = \frac{1}{2}\bar{\rho}w^2 f \tag{7.15}$$

This can be made more convenient by replacing the kinetic head by more practical parameters:

$$\frac{1}{2}\bar{\rho}w^2 = \frac{1}{2\bar{\rho}}\left(\frac{\dot{m}}{A}\right)^2 \tag{7.16}$$

But this simply defers the real issue of providing an appropriate friction factor. ESDU [2008] cites experimental data correlations by four different authors who adopted a common approach. The friction factor was assumed to be related to the Reynolds number in exactly the same way as for single-phase pipe flow, being represented by the experimental correlation for fully developed turbulent flow

$$f = 0.046\,\mathrm{Re}^{-0.2} \quad \text{for Re} > 20,000 \tag{7.17}$$

and the theoretical solution for fully developed laminar flow (Re < 2,000)

$$f = \frac{16}{\text{Re}} \tag{7.18}$$

But instead of using the fluid viscosity μ to calculate Reynolds number, each author provided a new definition of viscosity, mostly based on mass quality, and provided a formula for calculating it using the viscosities of water and steam at the appropriate pressure and temperature. Each formula was based on a separate set of experiments.

Adopting the homogenous flow model defines some of the flow parameters very tightly, leaving few options for experimenters to correlate the measured variables, but it is simple. The separated flow and drift flux models are less restrictive, providing a better chance of good predictions.

7.7.5   Two-Phase Flow in Permeable Formations

In Sect. 7.1 reference was made to factory accumulators, large vessels half filled with saturated water overlain by saturated steam. If the steam demand suddenly increased, a resulting sudden drop in pressure in the supply lines was avoided by the saturated water flashing; the process is very rapid. Each phase has its own compressibility, as defined earlier by Eq. (4.80):

$$c = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial P} \right)_T \tag{4.80}$$

But when dealing with the response of a fixed volume of a mixture at saturation conditions to a pressure change, the volumes of each phase change because of not only their individual compressibilities but also the exchange of mass between the phases. The exchange of mass can be considered as an effective compressibility. Transient pressure testing of formation properties relies on measuring the pressure change induced by discharging a well or pumping water into it, and if the formation being tested contains two-phase fluid, then this effective compressibility is important. Work on this topic culminated in an important contribution by Grant and Sorey [1979] in the form of an expression for the effective compressibility:

$$c_{tp} = \frac{\left[(1 - \phi)\rho_R Cp_R + \phi S_f \rho_f Cp_f\right]\left(\rho_f - \rho_g\right)}{\phi h_{fg} \left(\frac{dP}{dT}\right)_S \rho_f \rho_g} \tag{7.19}$$

In this expression the compressibilities of the individual phases have been neglected because they are small compared to the effective compressibility due to the phase change. The denominator contains the Clausius–Clapeyron equation given earlier as Eq. (3.16).

The saturation S appears in this equation, being defined as the volume fraction of a porous medium occupied by either of the phases; it is equivalent to the dryness fraction for mass fractions. The "rules" in respect of saturation are

$$S_f + S_g = 1 \qquad (7.20)$$

and, for example, mean density is given by the following, with other properties similarly derived:

$$\overline{\rho} = \rho_f S_f + \rho_g S_g \qquad (7.21)$$

7.8   Geothermal Liquids with Dissolved Gases

Geothermal liquids usually contain dissolved gases, and as they rise, the pressure falls and the gases come out of solution. The presence of gas in solution changes the pressure at which the water–steam phase change takes place for a given solution temperature, in other words the pressure at which bubbles form. The saturation line for pure water is now redundant, replaced by a new relationship involving the concentration of gas in solution. The presence of the gas changes the mechanics of the flow, both directly by forming bubbles and indirectly by altering the parameters at which phase change takes place; the bubbles contain both gas and steam. Fortunately most of the gas dissolved in geothermal water is CO_2 and it can be considered as the only solute so far as flow effects are concerned, for the present purposes.

An ideal gas is one in which the molecules move independently of each other, except for elastic collisions. The pressure (force per unit area) on the wall of the container is created by the change of momentum of the molecules hitting it. If two ideal gases are present as a mixture, red molecules and blue molecules, they still act independently, there is a pressure contribution from each colour, and the total pressure is the sum of the two. This was established by Dalton (1802) and is known as Dalton's law of partial pressures. Each pressure contribution is called the partial pressure of that species.

For some pairs of chemical species, the equilibrium vapour pressures formed throughout the entire range of mixture strengths, from a dilute solution of the red species in the blue as solvent, to the other way around, the total pressure above a liquid mixture is the sum of the partial pressures, and the partial pressure of each species is proportional to its concentration. This is an ideal gas-like behaviour and is referred to as Raoult's law, but it is not the general case. More often, the linear relationship of partial pressure to concentration occurs only for low-solute concentrations, when it is referred to as Henry's law

$$P_{pp} = K_H . C_M \qquad (7.22)$$

in which

P_{pp} is the partial pressure of the gas in bars

C_M is the molar fraction of the gas in solution

K_H is Henry's constant in bars/molar fraction.

Not all chemical species behave as ideal gases, and a parameter known as fugacity is introduced as a measure of the departure from ideal. Fugacity is not needed here as Henry's law is an accurate enough representation for typical geothermal calculations.

Lu [2004] reviewed the experimental measurements of Henry's constant for dilute aqueous solutions of CO_2 and their correlation by Alkan et al. [1995]:

$$K_H = 406.41 + 47.088T + 7.6975x10^{-2}T^2 - 7.4695x10^{-4}T^3 \qquad (7.23)$$

$$\text{for } 25 < T < 172 \ ^\circ C$$

Sutton [1976] described a method of calculating the equilibrium mass fraction of CO_2 in an aqueous solution over a range of pressures and temperatures, from which the new "boiling point" curve can be calculated for a given solute concentration. This will be referred to again in Chap. 8.

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