Chapter 1
Introduction

1.1 Classical Local Theory

One of the underlying assumptions in the classical theory is its locality. The classical continuum theory assumes that a material point only interacts with its immediate neighbors; hence, it is a local theory. The interaction of material points is governed by the various balance laws. Therefore, in a local model a material point only exchanges mass, momentum, and energy with its closest neighbors. As a result, in classical mechanics the stress state at a point depends on the deformation at that point only. The validity of this assumption becomes questionable across different length scales. In general, at the macroscale this assumption is acceptable. However, the existence of long-range forces is evident from the atomic theory and as such the supposition of local interactions breaks down as the geometric length scale becomes smaller and approaches the atomic scale. Even at the macroscale there are situations when the validity of locality is questionable, for instance when small features and microstructures influence the entire macrostructure.

Despite the development of many important concepts to predict crack initiation and its growth in materials, it is still a major challenge within the framework of classical continuum mechanics. The main difficulty lies in the mathematical formulation, which assumes that a body remains continuous as it deforms. Therefore, the basic mathematical structure of the formulation breaks down whenever a discontinuity appears in a body. Mathematically, the classical theory is formulated using spatial partial differential equations, and these spatial derivatives are undefined at discontinuities. This introduces an inherent limitation to the classical theory, as the spatial derivatives in the governing equations, by definition, lose their meaning due to the presence of a discontinuity, such as a crack.

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1.1.1 Shortcomings in Failure Prediction

The solutions within the realm of classical continuum mechanics result in infinite (singular) stresses at the crack tips, as derived in the pioneering study by Griffith (1921) that led to the concept of Linear Elastic Fracture Mechanics (LEFM). Within the realm of LEFM, a pre-existing crack in the material is necessary, and the stresses at the crack tip are mathematically singular. Therefore, the onset of crack initiation and crack growth are treated separately by introducing external criteria, such as critical energy release rate, that are not part of the governing equations of classical continuum mechanics. Furthermore, crack nucleation within LEFM still remains an unresolved issue.

Due to the presence of singular stresses, accurate calculation of the stress intensity factor or energy release rate can be highly challenging as these quantities are dependent on loading, geometry, and the numerical solution method. In addition to the requirement of an external criterion for the onset of crack growth, a criterion is also necessary for the direction of crack propagation. Understanding and prediction of the material failure process is rather complex due to the presence of a variety of mechanisms associated with grain boundaries, dislocations, microcracks, anisotropy, etc., each of which plays an important role at a specific length scale.

Many experiments indicated that materials with smaller cracks exhibit higher fracture resistance than those with larger cracks, and yet the solutions utilizing the classical continuum theory are independent of the crack size (Eringen et al. 1977). Furthermore, the classical continuum theory predicts no dispersion while experiments show otherwise for propagation of elastic plane waves with short wavelengths in elastic solids (Eringen 1972a). Within the realm of the classical (local) continuum theory, a material point in a continuum is influenced only by the other material points that are located within its immediate vicinity. Hence, there is no internal length parameter distinguishing different length scales.

Although the classical continuum theory is incapable of distinguishing among different scales, it can capture certain failure processes, and can be applied to a wide range of engineering problems, especially by employing the Finite Element Method (FEM). The FEM is robust in particular for determining stress fields, and it is also exceptionally suitable for modeling structures possessing complex geometries and different materials under general loading conditions. However, its governing equations are derived based on the classical continuum mechanics, and it also suffers from the presence of undefined spatial derivatives of displacements at crack tips or along crack surfaces.

When LEFM is adopted into the FEM, special elements are commonly needed in order to capture the correct singular behavior (mathematical artifact) at the crack tip. With traditional finite elements, the discontinuity in the displacement field that transpires as the crack propagates is remedied by redefining the body, i.e., defining the crack as a boundary.

The field of fracture mechanics is primarily concerned with the evolution of pre-existing cracks within a body, rather than the nucleation of new cracks. Even

when addressing crack growth, the FEM with traditional elements suffers from the inherent limitation that it requires remeshing after each incremental crack growth. In addition to the need to remesh, existing methods for fracture modeling also suffer from the need to supply a kinetic relation for crack growth, a mathematical statement that prescribes how a crack evolves a priori based on local conditions. It guides the analysis as to when a crack should initiate; how fast it should grow and in what direction; whether it should turn, branch, oscillate, arrest, etc. Considering the difficulty in obtaining and generalizing experimental fracture data, providing such a kinetic relation for crack growth clearly presents a major obstacle to fracture modeling using conventional methods. Considering the presence of crack tip singularity, the need for external criteria, the inability to address crack initiation, and the requirement for redefining the body, it is clear that it is nearly impossible to solve a problem with multiple interacting cracks that propagate in a complex manner using traditional finite elements.

1.1.2 Remedies

Numerous studies were performed to improve the shortcomings of the FEM with traditional elements within the realm of LEFM. In particular, the cohesive zone concept introduced by Dugdale (1960) and Barenblatt (1962) has become prevalent among many other fracture criteria. However, the major breakthrough in computational fracture mechanics came with the introduction of Cohesive Zone Elements (CZE) by Hillerborg et al. (1976) for the Mode-I fracture mode and Xu and Needleman (1994) for a mixed-mode fracture. Materials and material interfaces are modeled through a traction-separation law for which the tractions are zero when the opening displacement (separation) reaches a critical value. Cohesive zone elements are usually surface elements that are placed along the element boundaries; hence, crack growth occurs only between traditional (regular) elements. Therefore, the material response exhibits characteristics of both regular and cohesive zone elements; the cohesive elements are only introduced to produce fracture behavior. The number of cohesive elements increases with decreasing mesh size, yet the size of the continuum region remains the same. Hence, softening of material properties can be observed with decreasing mesh size. Furthermore, mesh texture produces anisotropy, and it leads to mesh dependence. Crack paths are highly sensitive to mesh texture and alignment (Klein et al. 2001), and remeshing is required when crack paths are unknown a priori.

In an effort to resolve these difficulties, the concept of the eXtended Finite Element Method (XFEM) was introduced as a technique to model cracks and crack growth within the realm of finite elements without remeshing (Belytschko and Black 1999; Moes et al. 1999). It permits the cracks to propagate on any surface within an element, rather than only along element boundaries. Thus, it removes the limitation of CZE on the admissible direction of new fracture surfaces. XFEM is based on the partition of unity property of finite elements (Melenk and

Babuska 1996). Local enrichment functions, with additional degrees of freedom, are included in the standard finite element approximation. These functions are in the form of discontinuous displacement enrichment, in order to capture displacement discontinuity across a crack, and near-crack tip asymptotic displacement enrichment. Also, the additional number of degrees of freedom is minimized since the enrichment only includes the nodes that belong to the elements cut by cracks (Zi et al. 2007). According to Zi et al. (2007), the elements adjacent to the element in which the crack tip is positioned are partially enriched, and the partition of unity does not hold for them. Hence, the solution becomes inaccurate in the blending region. This prevents such methods from being applicable to problems in which multiple cracks grow and interact in complex patterns. The XFEM has been successfully employed to solve a number of fracture problems; however, it does require external criteria for injection of discontinuous displacement enrichment.

The difficulties encountered in the methods utilizing the classical continuum mechanics can be overcome by performing Molecular Dynamics Simulations (MDS) or atomistic lattice models. The atomistic simulation is certainly the most detailed and realistic one for predicting material fracture (Schlangen and van Mier 1992). The crack initiation and propagation can be simulated using the inter-atomic forces. However, atomistic studies focus on providing a fundamental understanding of the underlying basic physical processes of dynamic fracture, instead of being predictive (Cox et al. 2005). The reason for the limited focus stems from the availability of computational resources. In recent years, large-scale molecular dynamics simulations certainly have become possible with advancements in computer architectures. For instance, Kadau et al. (2006) performed simulations using 320 billion atoms, corresponding to a cubic piece of solid copper with an edge length of 1.56 μm. However, these length scales are still very small for real-life engineering structures. Furthermore, atomistic simulations suffer from the strict limitation on the total time since the time step is very small. Hence, most simulations are performed under very high loading rates, and it is not very clear if the fracture processes at artificially high rates resulting in high stresses are representative of events happening at lower rates.

Inspired from atomistic lattice models, lattice spring models eliminate the inadequacy of the atomistic simulations for large-scale structures by representing materials with discrete units interacting through springs, or, more generally, rheological elements (Ostoja-Starzewski 2002). The interactions among the lattice points can be short range by including nearest neighbors or long range (nonlocal) by containing neighbors beyond the nearest ones. Furthermore, lattice sites can be periodic or disordered and there are many different periodic lattices: triangular, square, honeycomb, etc. However, periodic lattices exhibit directional dependence on the elastic properties. Furthermore, the interaction force for one lattice type cannot easily be utilized for another type, and it is also not clear which lattice type is best suited for a specific problem.

Hence, it is clear that the atomistic simulations are insufficient to model fracture processes in real-life structures. Moreover, the experiments of physicists have revealed that cohesive forces reach finite distances among atoms, yet the classical

continuum theory lacks an internal length parameter permitting modeling at different length scales because it is valid only for very long wavelengths (Eringen 1972a). Therefore, Eringen and Edelen (1972), Kroner (1967), and Kunin (1982) introduced the nonlocal continuum theory in an effort to account for the long-range effects.

1.2 Nonlocal Theories

The nonlocal theory of continuous media establishes the connection between classical continuum mechanics and molecular dynamics. In the case of the local (classical) continuum model, the state of a material point is influenced by the material points located in its immediate vicinity. In the case of the nonlocal continuum model, the state of a material point is influenced by material points located within a region of finite radius. As the radius becomes infinitely large, the nonlocal theory becomes the continuous version of the molecular dynamics model. Therefore, the nonlocal theory of continuous media establishes a connection between the classical (local) continuum mechanics and molecular dynamics models. The relationship between the local and nonlocal continuum models and the molecular dynamics model is illustrated in Fig. 1.1.

Any point $\mathbf{x}$ interacts with other material points within a distance δ. The material points within a distance δ of $\mathbf{x}$ are called the family of $\mathbf{x}$, $H_{\mathbf{x}}$. The number of material points in a family of $\mathbf{x}$, within the realm of classical continuum mechanics, is 3, 5, and 7 (including itself) for one-, two-, and three-dimensional analysis, respectively.

Various nonlocal theories, involving higher-order displacement gradients and spatial integrals, were introduced in the past. Early work by Eringen and Edelen (1972) and Eringen (1972a, b) resulted in a nonlocal continuum theory that accounted for nonlocality in the balance laws and thermodynamic statements. However, the resulting equations were rather complicated, and later work by these researchers simplified the theory by accounting for nonlocality in the

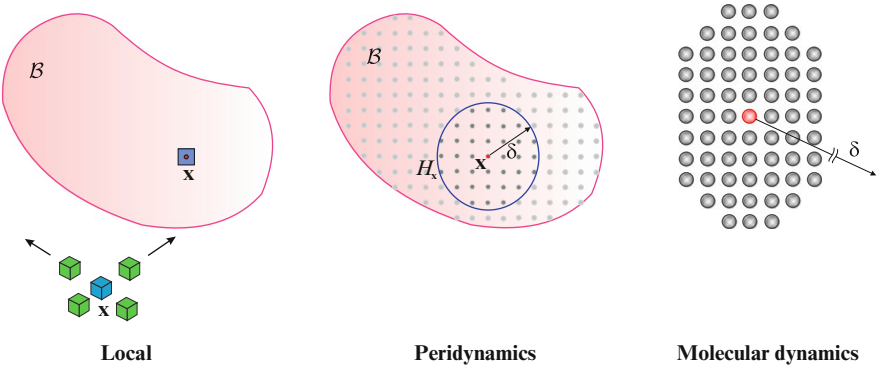

Fig. 1.1 Relationship between local and nonlocal continuum models

constitutive relation while keeping the equilibrium and kinematic equations in the local form (Eringen et al. 1977). Currently, most nonlocal theories account for nonlocality through the constitutive relation. Generally, integral-type nonlocal material models in continuum mechanics have a constitutive law that relates the forces (stresses) at a material point to some weighted average of deformation (strains) of other points that are some finite distance away. On the other hand, gradient-type nonlocal models include higher order derivatives to account for the field in the immediate vicinity of the point, such as the first derivative of the strain to the local constitutive law. Both types of nonlocal models have an associated characteristic length, which can be related to physical lengths such as grain size, fracture process zone size, or pore size.

The nonlocal continuum theory was noted for its ability to not only capture macroscale effects, but also the effects of molecular and atomic scales. Eringen (1972b) showed that a nonlocal model is capable of predicting a wide range of wavelengths. The nonlocal theory still assumes the media as a continuum, however it is computationally less demanding than the molecular dynamics while taking into account the long-range effects. Since the classical theory is the longwave limit of the atomic theory, they showed the ability of the nonlocal theory to capture deformation from the classical longwave limit to the atomic scale. According to Bazant and Jirasek (2002), there are many occasions when it is necessary to adopt a nonlocal approach in continuum mechanics. Such instances include capturing the effects of microstructure heterogeneity on small-scale continuum models. Nonlocality is also required in order to capture size effects—the dependence of nominal strength on structure size, observed in experiments and discrete modeling but not captured by local models. Nonlocality is also exhibited in the phenomena of microcracking. Distributed microcracking has been experimentally observed; however, it is challenging if not impossible to numerically simulate with local models because microcrack growth is not decided by local deformation or local stress. Evidence points to the fact that microcracking is not only dependent on the local deformation at the center of the microcrack, but is also dependent on the deformation that occurs within some neighborhood of the microcrack (Bazant 1991).

The nonlocal theory was also extended to address crack growth prediction. Eringen and Kim (1974a, b) showed that because of the nonlocal nature of the theory, the stress field ahead of the crack tip is bounded as the crack tip is approached asymptotically, rather than unbounded as predicted by the classical continuum theory. Also, Eringen and Kim (1974a) suggested a natural fracture criterion by equating maximum stress to the cohesive stress that holds atomic bonds together. This criterion can be applied everywhere in continuous media without distinguishing discontinuities. Although their nonlocal continuum theory leads to finite stresses at the crack tips, the derivatives of the displacement field are retained in the formulation.

Later, Eringen and his coworkers (1977) applied their nonlocal theory to the modeling of a Griffith crack. The advantage of the nonlocal continuum theory over the local theory in the area of fracture is made obvious by the fact that the nonlocal model predicts a physically meaningful finite stress field at the crack tip. This is

opposed to the local theory that predicts infinite stresses at the crack tip, which is nonphysical because no real material can support infinite stresses. Also, Ari and Eringen (1983) showed that the analysis results of a Griffith crack using nonlocal elasticity are in agreement with the lattice model given by Elliott (1947). In spite of this, the governing equations of their model still lose meaning at the crack, as they are formulated in terms of spatial derivatives. In fact, most nonlocal models still break down in the presence of a discontinuity, such as a crack, because, similar to the classical local theory, spatial derivatives are included in their formulation. Typically, nonlocal models include nonlocality in the stress–strain relation through strain averaging (Eringen et al. 1977; Ozbolt and Bazant 1996) or through adding strain derivatives to the standard constitutive relation, hence retaining the spatial derivative.

Another type of nonlocal theory, introduced by Kunin (1982, 1983) and Rogula (1982), circumvents this difficulty because it uses displacement fields rather than their derivatives. However, it is only given for a one-dimensional medium by Kunin (1982) and Rogula (1982). Kunin (1983) derived a three-dimensional nonlocal model by approximating a continuous medium as a discrete lattice structure. More recently, Silling (2000) proposed a nonlocal theory that does not require spatial derivatives—the peridynamic (PD) theory. Compared to the previous nonlocal theory by Kunin (1982) and Rogula (1982), the PD theory is more general because it considers two- and three-dimensional media in addition to the one-dimensional medium. In contrast to the nonlocal theory by Kunin (1983), the PD theory provides nonlinear material response with respect to displacements. Furthermore, the material response includes damage in the PD theory.

1.2.1 Basics of Peridynamic Theory

In light of the inadequacies of local and nonlocal theories, the peridynamic theory, which is nonlocal, was introduced by Silling (2000) and Silling et al. (2007) in an attempt to deal with the discontinuities. Similar to the nonlocal theory formulated by Kunin (1982), the peridynamic theory employs displacements rather than displacement derivatives in its formulation. Basically, the peridynamic theory is a reformulation of the equation of motion in solid mechanics that is better suited for modeling bodies with discontinuities, such as cracks. The theory uses spatial integral equations that can be applied to a discontinuity. This stands in contrast to the partial differential equations used in the classical formulation, which are not defined at discontinuities. The peridynamic governing equations are defined at fracture surfaces; additionally, material damage is part of the peridynamic constitutive laws. These attributes permit fracture initiation and propagation to be modeled, with arbitrary paths, without the need for special crack growth treatment. Furthermore, interfaces between dissimilar materials have their own properties.

In the peridynamic theory, material points interact with each other directly through the prescribed response function, which contains all of the constitutive

information associated with the material. The response function includes a length parameter called internal length (horizon), δ. The locality of interactions depends on the horizon, and interactions become more local with a decreasing horizon. Hence, the classical theory of elasticity can be considered as a limiting case of the peridynamic theory as the internal length approaches zero. For instance, it has been shown that the peridynamic theory reduces to the linear theory of elasticity with the proper choice of response function (Silling et al. 2003; Weckner and Abeyaratne 2005). In another limiting case where the internal length approaches the inter-atomic distance, it was shown by Silling and Bobaru (2005) that van der Waals forces can be used as part of the response function to model nanoscale structures. Therefore, the peridynamic theory is capable of bridging the nano to macro length scales. With the PD theory, damage in the material is simulated in a much more realistic manner compared to the classical continuum-based methods. As the interactions between material points cease, cracks may initiate and align themselves along surfaces that form cracks, yet the integral equations continue to remain valid.

1.2.2 Attributes and Its Present State

The main difference between the peridynamic theory and classical continuum mechanics is that the former is formulated using integral equations as opposed to derivatives of the displacement components. This feature allows damage initiation and propagation at multiple sites with arbitrary paths inside the material without resorting to special crack growth criteria. In the peridynamic theory, internal forces are expressed through nonlocal interactions between pairs of material points within a continuous body, and damage is part of the constitutive model. Interfaces between dissimilar materials have their own properties and damage can propagate when and where it is energetically favorable for it to do so. The PD theory provides the ability to link different length scales, and it can be viewed as the continuum version of MDS. It provides the ability to address multiphysics and multiscale failure prediction in a common framework.

The ability of the peridynamic theory to represent physical phenomena was demonstrated by Silling (2000). Silling investigated the propagation of linear stress waves and wave dispersion along with the shape of the crack tip within the realm of the peridynamic theory. The peridynamic linear elastic waves with long wavelengths were in agreement with those from the classical theory. At small scales, the peridynamic theory predicts nonlinear dispersion curves, which are found in real materials, unlike the curves predicted by the classical elasticity. In the crack tip study, the peridynamic theory predicts a cusp-like crack tip as opposed to the parabolic crack tip from LEFM. The parabolic crack tip in LEFM is associated with the unphysical unbounded stress at the crack tip.

The original peridynamic formulation by Silling (2000), later coined the "bond-based peridynamic theory," is based on the assumption of pairwise interactions of

the same magnitude, thus resulting in a constraint on material properties, such as requiring the Poisson's ratio to be one-fourth for isotropic materials. Also, it does not distinguish between volumetric and distortional deformations; thus it is not suitable to capture the plastic incompressibility condition, or to utilize the existing material models.

In order to relax the constraint on material properties, Gerstle et al. (2007) introduced a "micropolar peridynamic model" by considering pairwise moments as well as forces in the "bond-based" peridynamics. Although this formulation overcomes the constraint limitation for isotropic materials, it is not clear if it can also capture the incompressibility condition. Therefore, Silling et al. (2007) introduced a more general formulation, coined the "state-based" peridynamic theory, which eliminates the limitations of the "bond-based" peridynamics. The "state-based" PD theory is based on the concept of peridynamic states that are infinite dimensional arrays containing information about peridynamic interactions. Silling (2010) also extended the "state-based" PD theory to account for the effects of indirect interactions between material points on other material points by introducing the "double state" concept. Recently, Lehoucq and Sears (2011) derived the energy and momentum conservation laws of the peridynamic theory by using the principles of classical statistical mechanics. They showed that the nonlocal interaction is intrinsic to continuum conservation laws. Recently, Silling (2011) also extended the use of PD theory for bridging different length scales by introducing a "coarse-graining method." According to this approach, the structural properties at a lower scale are reflected to its upper scale through a mathematically consistent technique.

The peridynamic theory does not concern the concept of stress and strain; however, it is possible to define a stress tensor within the PD framework. Lehoucq and Silling (2008) derived a PD stress tensor from nonlocal PD interactions. The stress tensor is obtained from the PD forces that pass through a material point volume. For sufficiently smooth motion, a constitutive model, and any existing nonhomogeneities, Silling and Lehoucq (2008) showed that the PD stress tensor converges to a Piola-Kirchhoff stress tensor in the limiting case where the horizon size converges to zero.

The integro-differential equation of peridynamic theory is difficult to solve analytically. However, a few analytical solutions exist in the literature. For instance, Silling et al. (2003) investigated the deformation of an infinite bar subjected to a self-equilibrated load distribution. The solution was achieved in the form of a linear Fredholm integral equation and solved by Fourier transformation. This solution revealed interesting results that cannot be captured by the classical theory, including decaying oscillations in the displacement field and progressively weakening discontinuities propagating outside of the loading region. Weckner et al. (2009) also used Laplace and Fourier transforms, and obtained an integral representation for the three-dimensional PD solution by utilizing Green's functions. This approach was independently pursued by Mikata (2012) to investigate peristatic and peridynamic solution of a one-dimensional infinite bar, and it was found that peridynamics can represent negative group velocities for certain wavenumbers,

which can be used for modeling certain types of dispersive media with irregular dispersion.

Peridynamics permits not only linear elastic material behavior, but also nonlinear elastic (Silling and Bobaru 2005), plastic (Silling et al. 2007; Mitchell 2011a), viscoelastic (Kilic 2008; Taylor 2008; Mitchell 2011b), and viscoplastic (Taylor 2008; Foster et al. 2010) material behaviors. Dayal and Bhattacharya (2006) studied the kinetics of phase transformations in solids by using peridynamics. They derived a nucleation criterion by examining nucleation as a dynamic instability.

The solution of PD equations requires numerical integration both in time and space, for which explicit and Gaussian quadrature techniques can be adopted because of their simplicity. The description of these techniques and their application to peridynamics are presented by Silling and Askari (2005). They also provided the stability criterion for convergence of time integration, and discussed the order of accuracy for uniform discretization (grid) for spatial integration. Later, Emmrich and Weckner (2007) presented different spatial discretization schemes and tested them by considering a linear microelastic material of infinite length in one dimension. Recently, Bobaru et al. (2009) and Bobaru and Ha (2011) considered a nonuniform grid and nonuniform horizon sizes for spatial integration. In order to improve the accuracy and efficiency of numerical time integration, Polleschi (2010) proposed a mixed explicit-implicit time integration scheme; the integration through the time steps is explicit with an implicit cycle at every time step. In a similar way, Yu et al. (2011) proposed an adaptive trapezoidal integration scheme with a combined relative-absolute error control. Also, Mitchell (2011a, b) utilized an implicit time integration method.

Although the PD equation of motion includes the effects of inertia, it is possible to use it for quasi-static problems by appropriately allowing the inertia term to vanish through schemes, as demonstrated by Kilic and Madenci (2010a). Alternatively, Wang and Tian (2012) introduced a fast Galerkin method with efficient matrix assembly and storage.

The degree of nonlocality is defined by a PD parameter, referred to as the horizon; therefore, it is crucial to choose an appropriate size for it to obtain accurate results and represent the actual physical reality. In a recent study, Bobaru and Hu (2012) discuss the meaning, selection, and use of horizon in the PD theory and explain under what conditions the crack propagation speed depends on the horizon size and the role of incident waves on this speed. Influence function is another important parameter in PD theory, which determines the strength of interactions between material points. Seleson and Parks (2011) studied the effect of influence function by investigating wave propagation in simple one-dimensional models and brittle fracture in three-dimensional models.

The spatial integration of the PD equation is very suitable for parallel computing. However, the load distribution is a key issue to obtain the most efficient computational environment. An efficient load distribution scheme is described by Kilic (2008). Also, Liu and Hong (2012a) demonstrated the use of Graphics Processing Unit (GPU) architecture towards the same goal.

The PD theory permits crack initiation and growth. Silling et al. (2010) established a condition for the emergence of a discontinuity (crack nucleation) in an elastic body. For crack growth, it requires a critical material failure parameter. The original parameter for a brittle material is referred to as the "critical stretch," and it can be related to the critical energy release rate of the material, as explained in Silling and Askari (2005). Warren et al. (2009) demonstrated the capability of the nonordinary state-based PD theory for capturing failure based on either the critical equivalent strain (measure of shearing strain) or the averaged value of the volumetric strain (dilatation). Recently, Foster et al. (2011) proposed critical energy density as an alternative critical parameter and also related it to the critical energy release rate. As shown by Silling and Lehoucq (2010) and Hu et al. (2012b), the PD theory also permits the calculation of the J-integral value, which is an important parameter of fracture mechanics.

Silling (2003) considered a Kalthoff-Winkler experiment in which a plate having two parallel notches was hit by an impactor and peridynamic simulations successfully captured the angle of crack growth that was observed in the experiments. Silling and Askari (2004) also presented impact damage simulations including the Charpy V-notch test. Ha and Bobaru (2011) successfully captured various characteristics of dynamic fracture observed in experiments, including crack branching, crack-path instability, etc. Furthermore, Agwai et al. (2011) compared their PD analysis results against extended Finite Element Method (XFEM) and Cohesive Zone Model (CZM) predictions. Crack speeds computed from all approaches were found to be on the same order; however, the PD prediction of fracture paths are closer to the experimental observations, including both branching and microbranching behaviors.

The PD theory captures the interaction of local failure such as a crack growth with global failure due to structural stability. Kilic and Madenci (2009a) investigated the buckling characteristics of a rectangular column with a groove (crack initiation site) under compression and a constrained rectangular plate under uniform temperature load. They triggered lateral displacements using geometrical imperfection.

The PD theory also permits multiple load paths such as compression after impact. Demmie and Silling (2007) considered the extreme loadings on reinforced concrete structures by impacts from massive objects and explosive loading of concrete structures. This study was recently extended by Oterkus et al. (2012a) to predict the residual strength of impact damaged concrete structures.

Composite damage has also been modeled with the peridynamic theory. Within the PD framework, the simplest approach to model a composite layer with directional properties is achieved by assigning different material properties in the fiber and other (remaining) directions. The interactions between neighboring layers are defined by using inter-layer bonds. Askari et al. (2006) and Colavito et al. (2007a, b) predicted damage in laminated composites subjected to low-velocity impact and damage in woven composites subjected to static indentation. In addition, Xu et al. (2007) considered notched laminated composites under biaxial loads. Also,

Oterkus et al. (2010) demonstrated that PD analysis is capable of capturing bearing and shear-out failure modes in bolted composite lap joints.

Xu et al. (2008) analyzed the delamination and matrix damage process in composite laminates due to only low-velocity impact. Recently, Askari et al. (2011) considered the effect of both high- and low-energy hail impacts against a toughened-epoxy, intermediate-modulus, carbon-fiber composite. Also, Hu et al. (2012a) predicted the basic failure modes of fiber, matrix, and delamination in laminates with a pre-existing central crack under tension. The analytical derivation of the PD material parameters, including thermal loading conditions, was recently given by Oterkus and Madenci (2012). They also demonstrated the constraints on material constants due to the pairwise interaction assumption. The other approach to model composites was introduced by Kilic et al. (2009) by distinguishing fiber and matrix materials based on the volume fraction. Although this approach may have some advantages by taking into account the inhomogeneous structure, it is computationally more expensive than the homogenized technique. The other approach for modeling composites is the linking of micro- and macroscales as described by Alali and Lipton (2012). The method depends on a two-scale evolution equation. While the microscopic part of this equation governs the dynamics at the length scale of heterogeneities, the macroscopic part tracks the homogenized dynamics.

Since the numerical solution of peridynamic equations of motion is computationally more expensive than the local solutions, such as FEM, it may be advantageous to combine PD theory and local solutions. In a recent study, Seleson et al. (2013) proposed a force-based blended model that coupled PD theory and classical elasticity by using nonlocal weights composed of integrals of blending functions. They also generalized this approach to couple peridynamics and higher-order gradient models of any order. In another study, Lubineau et al. (2012) performed coupling of local and nonlocal solutions through a transition (morphing) that affects only constitutive parameters. The definition of the morphing functions in their approach relies on energy equivalence. In addition to these techniques, Kilic and Madenci (2010b) and Liu and Hong (2012b) coupled FEM and peridynamics. A more straightforward coupling procedure is given in Macek and Silling (2007), where the PD interactions are represented by truss elements. If only some part of the region is desired to be modeled by using peridynamics, then the other sections can be modeled by traditional finite elements. Another simple approach, demonstrated by Oterkus et al. (2012b) and Agwai et al. (2012), was first to solve the problem by using finite element analysis and obtain the displacement field. Then, by using the available information, the displacements can be applied as a boundary condition to the peridynamic model of a critical region.

The peridynamic theory is also suitable for thermal loading conditions; Kilic and Madenci (2010c) included the thermal term in the response function of peridynamic interactions. By utilizing this approach, Kilic and Madenci (2009b) predicted thermally driven crack propagation patterns in quenched glass plates containing single or multiple pre-existing cracks, and Kilic and Madenci (2010c) also

predicted damage initiation and propagation in regions having dissimilar materials due to thermal loading.

Furthermore, the PD theory was extended to consider heat diffusion. Gerstle et al. (2008) developed a peridynamic model for electromigration that accounts for heat conduction in a one-dimensional body. Additionally, Bobaru and Duangpanya (2010, 2012) introduced a multidimensional peridynamic heat conduction equation, and considered domains with discontinuities such as insulated cracks. Both studies adopted the bond-based peridynamic approach. Later, Agwai (2011) derived the state-based peridynamic heat conduction equation. She also further extended it for fully coupled thermomechanics (Agwai 2011).

The peridynamic theory has been utilized successfully for damage prediction of many problems at different length scales from macro to nano. In order to take into account the effect of van der Waals interactions, Silling and Bobaru (2005) and Bobaru (2007) included an additional term to the peridynamic response function to represent van der Waals forces. This new formulation was used to investigate the mechanical behavior, strength, and toughness properties of three-dimensional nanofiber networks under imposed stretch deformation. It was found that the inclusion of van der Waals forces significantly changes the overall deformation behavior of the nanofiber network structure. In a recent study, Seleson et al. (2009) demonstrated that peridynamics can play the role of an upscale version of molecular dynamics and pointed out the extent where the molecular dynamics solutions can be recovered by peridynamics. Celik et al. (2011) utilized peridynamics to extract mechanical properties of nickel nanowires subjected to bending loads in a customized atomic force microscope (AFM) and scanning electron microscope (SEM). SEM images of fractured nanowires are also compared against peridynamic simulation results.

Even though numerous journal articles and conference papers exist in the literature on the evolution and application of the peridynamic theory, it is still new to the scientific community. Because it is based on concepts not commonly used in the past, the purpose of this book is to explain the peridynamic theory in a single framework. It presents not only the theoretical basis but also its numerical implementation.

It starts with an overview of the peridynamic theory and derivation of its governing equations. The relationship between peridynamics and classical continuum mechanics is established, and this leads to the ordinary state-based peridynamics formulations for both isotropic and composite materials. Numerical treatments of the peridynamic equations are presented in detail along with solutions to many benchmark and demonstration problems. In order to take advantage of salient features of the peridynamics and finite element methods, a coupling technique is also presented in detail. Finally, an extension of the peridynamic theory for thermal diffusion and fully coupled thermomechanics is presented with applications. [FORTRAN algorithms providing solutions to many of these benchmark problems can be found at http://extras.springer.com.]

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