# A Dynamic CAPM with Supply Effect Theory and Empirical Results

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**Abstract**

Breeden (1979) and Grinols (1984) and Cox et al. (1985) have described the importance of supply side for the capital asset pricing. Black (1976) derives a dynamic, multiperiod CAPM, integrating endogenous demand and supply. However, Black's theoretically elegant model has never been empirically tested for its implications in dynamic asset pricing. We first theoretically extend Black's CAPM. Then, we use price, dividend per share, and earnings per share to test the existence of supply effect with US equity data. We find the supply effect is important in US domestic stock markets. This finding holds as we break the companies listed in the S&P 500 into ten portfolios by different level of payout ratio. It also holds consistently if we use individual stock data.

A simultaneous equation system is constructed through a standard structural form of a multiperiod equation to represent the dynamic relationship between supply and demand for capital assets. The equation system is exactly identified under our specification. Then, two hypotheses related to supply effect are tested regarding the parameters in the reduced form system. The equation system is estimated by the seemingly unrelated regression (SUR) method, since SUR allow one to estimate the presented system simultaneously while accounting for the correlated errors.

**Keywords**

CAPM • Asset • Endogenous supply • Simultaneous equations • Reduced form • Seemingly unrelated regression (SUR) • Exactly identified • Cost of capital • Quadratic cost • Partial adjustment

## 93.1 Introduction

Breeden (1979) and Grinols (1984) and Cox et al. (1985) have described the importance of supply side for the capital asset pricing. Cox et al. (1985) study a restricted technology to allow them to explicitly solve their model for reduced form. Grinols (1984) focuses on describing market optimality and supply decisions which guide firms in incomplete markets in the absence of investor unanimity. Black (1976) extends the static CAPM by Sharpe (1964), Litner (1965), and Mossin (1966) explicitly allowing for the endogenous supply effect of risky securities to derive the dynamic asset pricing model.[1] Black modifies the static model by explicitly allowing for the existence of the supply effect of risky securities. In addition, the demand side for the risky securities is derived from a negative exponential function for the investor's utility of wealth. Black finds that the static

[1]This dynamic asset pricing model is different from Merton's (1973) intertemporal asset pricing model in two key aspects. First, Black's model is derived in the form of simultaneous equations. Second, Black's model is derived in terms of price change, and Merton's model is derived in terms of rates of return.

CAPM is unnecessarily restrictive in its neglect of the supply side and proposes that his dynamic generalization of the static CAPM can provide the basis for many empirical tests, particularly with regard to the intertemporal aspects and the role of the endogenous supply side. Assuming that there is a quadratic cost structure of retiring or issuing securities and that the demand for securities may deviate from supply due to anticipated and unanticipated random shocks, Black concludes that if the supply of a risky asset is responsive to its price, large price changes will be spread over time as specified by the dynamic capital asset pricing model. One important implication in Black's model is that the efficient market hypothesis holds only if the supply of securities is fixed and independent of current prices. In short, Black's dynamic generalization model of static wealth-based CAPM adopts an endogenous supply side of risky securities by setting equal quantity demanded and supplied of risky securities. Lee and Gweon (1986) extend Black's framework to allow time-varying dividend payments and then test the existence of supply effect in the situation of market equilibrium. Their results reject the null hypothesis of no supply effect in the US domestic stock market. The rejection seems to imply a violation of efficient market hypothesis in the US stock market.

It is worth noting that some recent studies also relate return on portfolio to trading volume (e.g., Campbell et al. 1993; Lo and Wang 2000). Surveying the relationship between aggregate stock market trading volume and the serial correlation of daily stock returns, (Campbell et al. 1993) suggest that a stock price decline on a high-volume day is more likely than a stock price decline on a low-volume day. They propose an explanation that trading volume occurs when random shifts in the stock demand of non-informational traders are accommodated by the risk-averse market makers. Lo and Wang (2000) also examine the CAPM in the intertemporal setting. They derive an intertemporal CAPM (ICAPM) by defining preference for wealth instead of consumption, by introducing three state variables into the exponential types of investor's preference as we do in this paper. This state-dependent utility function allows one to capture the dynamic nature of the investment problem without explicitly solving a dynamic optimization problem. Thus, the marginal utility of wealth depends not only on the dividend of the portfolio but also on future state variables. This dependence introduces dynamic hedging motives in the investors' portfolio choices. That is, this dependence induces investors to care about future market conditions when choosing their portfolio. In equilibrium, this model also implies that an investor's utility depends not only on his wealth but also on the stock payoffs directly. This "market spirit," in their terminology, affects investor's demand for the stocks. In other words, for even the investor who holds no stocks, his utility fluctuates with the payoffs of the stock index.

Black (1976), Lee and Gweon (1986), and Lo and Wang (2000) develop models by using either outstanding shares or trading volumes as variables to connect the decisions in two different periods, unlike consumption-based CAPM which uses consumption or macroeconomic information. Black (1976) and Lee and Gweon (1986) both derive the dynamic generalization models from the wealth-based CAPM by adopting an endogenous supply schedule of risky securities.[2] Thus, the information of quantities demanded and supplied can now play a role in determining the asset price. This proposes a wealth-based model as an alternative method to investigate intertemporal CAPM.

In this chapter, we first theoretically extend the Black's dynamic, simultaneous CAPM to be able to test the existence of the supply effect in the asset pricing determination process. We use two datasets of price per share and dividend per share to test the existence of supply effect with US equity data. The first dataset consists most companies listing in the S&P 500 of the US stock market. The second dataset is the companies listed in the Dow Jones Index. In this study, we find the supply effect is important in the US stock market. This finding holds as we break the companies listed in the S&P 500 into ten portfolios. It also holds if we use individual stock data. For example, the existence of supply effect holds consistently in most portfolios if we test the hypotheses by using individual stock as many as 30 companies in one group. We also find that one cannot reject the existence of supply effect by using the stocks listed in the Dow Jones Index.

This chapter is structured as follows. In Sect. 93.2, a simultaneous equation system of asset pricing is constructed through a standard structural form of a multiperiod equation to represent the dynamic relationship between supply and demand for capital assets. The hypotheses implied by the model are also presented in this section. Section 93.3 describes the two sets of data used in this paper. The empirical finding for the hypotheses and tests constructed in previous section is then presented. Our summary is presented in Sect. 93.4.

## 93.2 Development of Multiperiod Asset Pricing Model with Supply Effect

Based on the framework of Black (1976), we derive a multiperiod equilibrium asset pricing model in this section. Black modifies the static wealth-based CAPM by explicitly allowing for the endogenous supply effect of risky securities. The demand for securities is based on the well-known model of James Tobin (1958) and Harry Markowitz (1959). However, Black further assumes a quadratic cost function of changing short-term capital structure under long-run optimality condition. He also assumes that the demand for security may deviate from supply due to anticipated and unanticipated random shocks.

Lee and Gweon (1986) modify and extend Black's framework to allow time-varying dividends and then test the existence of supply effect. In Lee and Gweon's model, two major differing assumptions from Black's model are: (1) the model allows for time-varying dividends, unlike Black's assumption constant dividends,

[2]It should be noted that Lo and Wang's model did not explicitly introduce the supply equation in asset pricing determination. Also, one can identify the hedging portfolio using volume data in the Lo and Wang model setting.

and (2) there is only one random, unanticipated shock in the supply side instead of two shocks, anticipated and unanticipated shocks, as in Black's model. We follow the Lee and Gweon set of assumptions. In this section, we develop a simultaneous equation asset pricing model. First, we derive the demand function for capital assets, then we derive the supply function of securities. Next, we derive the multiperiod equilibrium model. Thirdly, the simultaneous equation system is developed for testing the existence of supply effects. Finally, the hypotheses of testing supply effect are developed.

### 93.2.1 The Demand Function for Capital Assets

The demand equation for the assets is derived under the standard assumptions of the CAPM.[3] An investor's objective is to maximize their expected utility function. A negative exponential function for the investor's utility of wealth is assumed:

$$U = a - h \times e^{\{-bW_{t+1}\}}, \tag{93.1}$$

where the terminal wealth $W_{t+1} = W_t(1 + R_t)$; $W_t$ is initial wealth; and $R_t$ is the rate of return on the portfolio. The parameters $a$, $b$, and $h$ are assumed to be constants.

The dollar returns on N marketable risky securities can be represented by

$$X_{j,\ t+1} = P_{j,\ t+1} - P_{j,\ t} + D_{j,\ t+1},\ j = 1, \ldots, N, \tag{93.2}$$

where

$P_{j,\ t+1}$ = (random) price of security j at time t + 1

$P_{j,\ t}$ = price of security j at time t

$D_{j,\ t+1}$ = (random) dividend or coupon on security at time t + 1

These three variables are assumed to be jointly normally distributed. After taking the expected value of Eq. 93.2 at time t, the expected returns for each security, $x_{j,\ t+1}$, can be rewritten as

$$x_{j,t+1} = E_t X_{j,t+1} = E_t P_{j,t+1} - P_{j,t} + E_t D_{j,t+1}, \quad j = 1, \ldots, N, \tag{93.3}$$

where

$$\begin{aligned} E_t P_{j,t+1} &= E\left(P_{j,t+1} | \Omega_t\right) \\ E_t D_{j,t+1} &= E\left(D_{j,t+1} | \Omega_t\right) \\ E_t X_{j,t+1} &= E\left(X_{j,t+1} | \Omega_t\right) \end{aligned}$$

$\Omega_t$ is the given information available at time t.

[3]The basic assumptions are as follows: (1) a single period moving horizon for all investors; (2) no transaction costs or taxes on individuals; (3) the existence of a risk-free asset with rate of return, r*; (4) evaluation of the uncertain returns from investments in terms of expected return and variance of end-of-period wealth; and (5) unlimited short sales or borrowing of the risk-free asset.

Then, a typical investor's expected value of end-of-period wealth is

$$w_{t+1} = E_t W_{t+1} = W_t + r^* \left( W_t - q_{t+1}{}' P_t \right) + q_{t+1}{}' x_{t+1}, \tag{93.4}$$

where
$P_t = (P_{1,\ t}, P_{2,\ t}, P_{3,\ t}, \ldots, P_{\ N,\ t})'$
$x_{t+1} = (x_{1,\ t+1}, x_{2,\ t+1}, x_{3,\ t+1}, \ldots, x_{N,\ t+1})' = E_t P_{t+1} – P_t + E_t D_{t+1}$
$q_{t+1} = (q_{1,\ t+1}, q_{2,\ t+1}, q_{3,\ t+1}, \ldots, q_{N,\ t+1})'$
$q_{j,t+1}$ = number of units of security j after reconstruction of his portfolio
r* = risk-free rate

In Eq. 93.4, the first term on the right hand side is the initial wealth, the second term is the return on the risk-free investment, and the last term is the return on the portfolio of risky securities. The variance of $W_{t+1}$ can be written as

$$V(W_{t+1}) = E(W_{t+1} - w_{t+1})(W_{t+1} - w_{t+1})' = q_{t+1}{}' S q_{,\, t+1}, \tag{93.5}$$

where $S = E(X_{t+1} – x_{t+1})(X_{t+1} – x_{t+1})'$ = the covariance matrix of returns of risky securities.

Maximization of the expected utility of $W_{t+1}$ is equivalent to:

$$\text{Max } w_{t+1} - \frac{b}{2} V(W_{t+1}), \tag{93.6}$$

By substituting Eqs. 93.4 and 93.5 into Eq. 93.6, Eq. 93.6 can be rewritten as:

$$\text{Max}(1 + r^*) W_t + q_{t+1}{}'(x_{t+1} - r^* P_t) - (b/2) q_{t+1}{}' S q_{t+1.} \tag{93.7}$$

Differentiating Eq. 93.7, one can solve the optimal portfolio as:

$$q_{t+1} = b^{-1} S^{-1} (x_{t+1} - r^* P_t). \tag{93.8}$$

Under the assumption of homogeneous expectation, or by assuming that all the investors have the same probability belief about future return, the aggregate demand for risky securities can be summed as:

$$Q_{t+1} = \sum_{k=1}^{m} q_{t+1}^{k} = c S^{-1} [E_t P_{t+1} - (1 + r^*) P_t + E_t D_{t+1}], \tag{93.9}$$

where $c = \Sigma (b^k)^{-1}$.

In the standard CAPM, the supply of securities is fixed, denoted as $Q^*$. Then, Eq. 93.9 can be rearranged as $P_t = (1/r^*)(x_{t+1} – c^{-1}\ S\ Q^*)$, where $c^{-1}$ is the market price of risk. In fact, this equation is similar to the Lintner's (1965) well-known equation in capital asset pricing.

### 93.2.2 Supply Function of Securities

An endogenous supply side to the model is derived in this section, and we present our resulting hypotheses, mainly regarding market imperfections. For example, the existence of taxes causes firms to borrow more since the interest expense is tax-deductible. The penalties for changing contractual payment (i.e., direct and indirect bankruptcy costs) are material in magnitude, so the value of the firm would be reduced if firms increase borrowing. Another imperfection is the prohibition of short sales of some securities.[4] The costs generated by market imperfections reduce the value of a firm, and, thus, a firm has incentives to minimize these costs. Three more related assumptions are made here. First, a firm cannot issue a risk-free security; second, these adjustment costs of capital structure are quadratic; and third, the firm is not seeking to raise new funds from the market.

It is assumed that there exists a solution to the optimal capital structure and that the firm has to determine the optimal level of additional investment. The one-period objective of the firm is to achieve the minimum cost of capital vector with adjustment costs involved in changing the quantity vector, $\mathrm{Q}_{\mathrm{i,\ t+1}}$:

$$\begin{aligned}&\mathrm{Min}\ \ \mathrm{E_t}D_{\mathrm{i,t+1}}\mathrm{Q}_{\mathrm{i,t+1}} + (1/2)\left(\Delta\mathrm{Q}_{\mathrm{i,t+1}}{}'\mathrm{Ai}\Delta\mathrm{Q}_{\mathrm{i,t+1}}\right),\\&\text{subject to}\,P_{\mathrm{i,t}}\Delta\mathrm{Q}_{\mathrm{i,t+1}} = 0,\end{aligned} \tag{93.10}$$

where $\mathrm{A_i}$ is a $\mathrm{n_i} \times \mathrm{n_i}$ positive-definite matrix of coefficients measuring the assumed quadratic costs of adjustment. If the costs are high enough, firms tend to stop seeking raise new funds or retire old securities. The solution to Eq. 93.10 is

$$\Delta\mathrm{Q}_{\mathrm{i,t+1}} = \mathrm{A_i}^{-1}\left(\lambda_{\mathrm{i}}P_{\mathrm{i,t}} - \mathrm{E_t}D_{\mathrm{i,t+1}}\right), \tag{93.11}$$

where $\lambda_i$ is the scalar Lagrangian multiplier.

Aggregating Eq. 93.11 over N firms, the supply function is given by

$$\Delta\mathrm{Q}_{\mathrm{t+1}} = \mathrm{A}^{-1}(\mathrm{B}P_{\mathrm{t}} - \mathrm{E_t}D_{\mathrm{t+1}}), \tag{93.12}$$

where $A^{-1} = \begin{bmatrix} A_1^{-1} & & & \\ & A_2^{-1} & & \\ & & \ddots & \\ & & & A_N^{-1} \end{bmatrix}$, $B = \begin{bmatrix} \lambda_1 I & & & \\ & \lambda_2 I & & \\ & & \ddots & \\ & & & \lambda_N I \end{bmatrix}$, and

$$Q = \begin{bmatrix} Q_I \\ Q_2 \\ \vdots \\ Q_N \end{bmatrix}.$$

[4]Theories as to why taxes and penalties affect capital structure are first proposed by Modigliani and Miller (1958) and then Miller (1977). Another market imperfection, prohibition on short sales of securities, can generate "shadow risk premiums" and, thus, provide further incentives for firms to reduce the cost of capital by diversifying their securities.

Equation 93.12 implies that a lower price for a security will increase the amount retired of that security. In other words, the amount of each security newly issued is positively related to its own price and is negatively related to its required return and the prices of other securities.

### 93.2.3 Multiperiod Equilibrium Model

The aggregate demand for risky securities presented by Eq. 93.9 can be seen as a difference equation. The prices of risky securities are determined in a multiperiod framework. It is also clear that the aggregate supply schedule has similar structure. As a result, the model can be summarized by the following equations for demand and supply, respectively:

$$Q_{t+1} = cS^{-1}(E_t P_{t+1} - (1 + r^*)P_t + E_t D_{t+1}), \tag{93.9}$$

$$\Delta Q_{t+1} = A^{-1}(BP_t - E_t D_{t+1}). \tag{93.12}$$

Differencing Eq. 93.9 for period t and t+1 and equating the result with Eq. 93.12, a new equation relating demand and supply for securities is

$$cS^{-1}[E_t P_{t+1} - E_{t-1}P_t - (1 + r^*)(P_t - P_{t-1}) + E_t D_{t+1} - E_{t-1}D_t] = A^{-1}(BP_t - E_t D_{t+1}) + V_t, \tag{93.13}$$

where $V_t$ is included to take into account the possible discrepancies in the system. Here, $V_t$ is assumed to be random disturbance with zero expected value and no autocorrelation.

Obviously, Eq. 93.13 is a second-order system of stochastic differential equation in $P_t$ and conditional expectations $E_{t-1}P_t$ and $E_{t-1}D_t$. By taking the conditional expectation at time $t-1$ on Eq. 93.13, and because of the properties of $E_{t-1}[E_t P_{t+1}] = E_{t-1}P_{t+1}$ and $E_{t-1}E(V_t) = 0$, Eq. 93.13 becomes

$$\begin{aligned} &cS^{-1}[E_{t-1}P_{t+1} - E_{t-1}P_t - (1 + r^*)(E_{t-1}P_t - P_{t-1}) + E_{t-1}D_{t+1} - E_{t-1}D_t] \\ &= A^{-1}(BE_{t-1}P_t - E_{t-1}D_{t+1}). \end{aligned} \tag{93.13$'$}$$

Subtracting Eq. 93.13′ from Eq. 93.13,

$$\begin{aligned} &\left[(1 + r^*)cS^{-1} + A^{-1}B\right](P_t - E_{t-1}P_t) = cS^{-1}(E_t P_{t+1} - E_{t-1}P_{t+1}) \\ &+\left(cS^{-1} + A^{-1}\right)(E_t D_{t+1} - E_{t-1}D_{t+1}) - V_t. \end{aligned} \tag{93.14}$$

Equation 93.14 shows that prediction errors in prices (the left hand side) due to unexpected disturbance are a function of expectation adjustments in price (first term on the right hand side) and dividends (the second term on the right hand side) two periods ahead. This equation can be seen as a generalized capital asset pricing model.

One important implication of the model is that the supply side effect can be examined by assuming the adjustment costs are large enough to keep the firms from seeking to raise new funds or to retire old securities. In other words, the assumption of high enough adjustment costs would cause the inverse of matrix A in Eq. 93.14 to vanish. The model is, therefore, reduced to the following certain equivalent relationship:

$$P_t - E_{t-1}P_t = (1+r^*)^{-1}(E_tP_{t+1} - E_{t-1}P_{t+1}) + (1+r^*)^{-1}(E_tD_{t+1} - E_{t-1}D_{t+1}) + U_t, \tag{93.15}$$

Where $U_t = -c^{-1}S(1 + r^*)^{-1}V_t$.

Equation 93.15 suggests that current forecast error in price is determined by the sum of the values of the expectation adjustments in its own next-period price and dividend discounted at the rate of $1 + r^*$.

### 93.2.4 Derivation of Simultaneous Equation System

From Eq. 93.15, if price series follow a random walk process, then the price series can be represented as $P_t = P_{t-1} + a_t$, where $a_t$ is white noise. It follows that $E_{t-1}P_t = P_{t-1}$, $E_tP_{t+1} = P_t$ and $E_{t-1}P_{t+1} = P_{t-1}$. According to the results in Appendix 1, the assumption that price follows a random walk process seems to be reasonable for both datasets. As a result, Eq. 93.14 becomes

$$-\left(r^*cS^{-1} + A^{-1}B\right)(P_t - P_{t-1}) + \left(cS^{-1} + A^{-1}\right)(E_tD_{t+1} - E_{t-1}D_{t+1}) = V_t. \tag{93.16}$$

Equation 93.16 can be rewritten as

$$G\,p_t + H\,d_t = V_t, \tag{93.17}$$

where

$G = -\,(r * cS^{-1} + A^{-1}B)$

$H = (cS^{-1} + A^{-1})$

$d_t = E_tD_{t+1} - E_{t-1}D_{t+1}$

$p_t = P_t - P_{t-1}$.

If Eq. 93.17 is exactly identified and matrix G is assumed to be nonsingular, then as shown in Greene (2004), the reduced form of this model may be written as[5]

$$p_t = \Pi d_t + U_t, \tag{93.18}$$

[5]The identification of the simultaneous equation system can be found in Appendix 2.

where Π is a n-by-n matrix of the reduced form coefficients and $U_t$ is a column vector of n reduced form disturbances. Or

$$\Pi = -G^{-1}H, \text{and } U_t = G^{-1}V_t. \tag{93.19}$$

Equations 93.18 and 93.19 are used to test the existence of supply effect in the next section.

### 93.2.5 Test of Supply Effect

Since the simultaneous equation system as in Eq. 93.17 is exactly identified, it can be estimated by the reduced form as Eq. 93.18. A proof of identification problem of Eq. 93.17 is shown in Appendix 2. That is, Eq. 93.18, $p_t = \Pi d_t + U_t$, can be used to test the supply effect. For example, in the case of two portfolios, the coefficient matrix G and H in Eq. 93.17 can be written as[6]

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} -(r^*cs_{11} + a_1b_1) & -r^*cs_{12} \\ -r^*cs_{21} & -(r^*cs_{22} + a_2b_2) \end{bmatrix},$$

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} cs_{11} + a_1 & cs_{12} \\ cs_{21} & cs_{22} + a_2 \end{bmatrix}. \tag{93.20}$$

Since $\Pi = - G^{-\ 1}$ H in Eq. 93.21, Π can be calculated as

$$\begin{aligned} -G^{-1}H &= \begin{bmatrix} r^*cs_{11}+a_1b_1 & r^*cs_{12} \\ r^*cs_{21} & r^*cs_{22}+a_2b_2 \end{bmatrix}^{-1} \begin{bmatrix} cs_{11}+a_1 & cs_{12} \\ cs_{21} & cs_{22}+a_1 \end{bmatrix} \\ &= \frac{1}{|G|} \begin{bmatrix} r^*cs_{22}+a_2b_2 & -r^*cs_{12} \\ -r^*cs_{21} & r^*cs_{11}+a_1b_1 \end{bmatrix} \begin{bmatrix} cs_{11}+a_1 & cs_{12} \\ cs_{21} & cs_{22}+a_1 \end{bmatrix} \\ &= \frac{1}{|G|} \begin{bmatrix} (r^*cs_{22}+a_2b_2)(cs_{11}+a_1) - r^*cs_{12}cs_{21} & (r^*cs_{22}+a_2b_2)cs_{12} - r^*cs_{12}(cs_{22}+a_1) \\ -r^*cs_{21}(cs_{11}+a_1) + (r^*cs_{11}+a_1b_1)cs_{21} & -r^*cs_{21}cs_{12} + (r^*cs_{11}+a_1b_1)(cs_{22}+a_1) \end{bmatrix} \\ &= \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}. \end{aligned} \tag{93.21}$$

From Eq. 93.21, if there is a high enough quadratic cost of adjustment, or if $a_1 = a_2 = 0$, then with $s_{12} = s_{21}$, the matrix would become a scalar matrix in which diagonal elements are equal to $r^*c^2$ $(s_{11}\ s_{22} - s_{12}{}^2)$, and the off-diagonal elements are all zero. In other words, if there is high enough cost of adjustment, firm tends to stop seeking to raise new funds or to retire old securities. Mathematically, this will be represented in a way that all off-diagonal elements are all zero and all diagonal

[6] $s_{ij}$ is the *i*th row and *j*th column of the variance-covariance matrix of return. $a_i$ and $b_i$ are the supply adjustment cost of firm *i* and overall cost of capital of firm *i*, respectively.

elements are equal to each other in matrix Π. In general, this can be extended into the case of more portfolios. For example, in the case of N portfolios, Eq. 93.18 becomes

$$\begin{bmatrix} p_{1t} \\ p_{2t} \\ \vdots \\ p_{Nt} \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{N1} & \pi_{N2} & \cdots & \pi_{NN} \end{bmatrix} \begin{bmatrix} d_{1t} \\ d_{2t} \\ \vdots \\ d_{Nt} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{Nt} \end{bmatrix}. \tag{93.22}$$

Equation 93.22 shows that if an investor expects a change in the prediction of the next dividend due to additional information (e.g., change in earnings) during the current period, then the price of the security changes. Regarding the US equity market, if one believes that how the expectation errors in dividends are built into the current price is the same for all securities, then, the price changes would be only influenced by its own dividend expectation errors. Otherwise, say if the supply of securities is flexible, then the change in price would be influenced by the expectation adjustment in dividends of all other stocks as well as that in its own dividend.

Therefore, two hypotheses related to supply effect to be tested regarding the parameters in the reduced form system shown in Eq. 93.18 are as follows:

Hypothesis 1: All the off-diagonal elements in the coefficient matrix Π are zero if the supply effect does not exist.

Hypothesis 2: All the diagonal elements in the coefficients matrix Π are equal in the magnitude if the supply effect does not exist.

These two hypotheses should be satisfied jointly. That is, if the supply effect does not exist, price changes of a security should be a function of its own dividend expectation adjustments, and the coefficients should all be equal across securities. In the model described in Eq. 93.16, if an investor expects a change in the prediction of the next dividend due to the additional information during the current period, then the price of the security changes.

Under the assumption of the efficiency in the domestic stock market, if the supply of securities is fixed, then the expectation errors in dividends are built in the current price is the same for all securities. This phenomenon implies that the price changes would only be influenced by its own dividend expectation adjustments. If the supply of securities is flexible, then the change in price would be influenced by the expectation adjustment in dividends of all other securities as well as that of its own dividend.

## 93.3 Data and Empirical Results

In this section, we derive the test by analyzing the US domestic stock market. Most details of the model, the methodologies, and the hypotheses for empirical tests are previously discussed in Sect. 93.2. However, before testing the hypotheses, some other details of the related tests that are needed to support the assumptions used in the model are also briefly discussed in this section.

This section examines the hypotheses derived earlier for the US domestic stock market by using the companies listed in S&P 500 and, then, by using the companies listing in Dow Jones Index. If the supply of risky assets is responsive to its price, then large price changes, which are due to the change in expectation of future dividend, will be spread over time. In other words, there exists supply effect in the US domestic stock markets. This implies that the dynamic instead of static CAPM should be used for testing capital assets pricing in the equity markets of the United States.

### 93.3.1 Data and Descriptive Statistics

Three hundred companies are selected from the S&P 500 and grouped into ten portfolios with equal numbers of 30 companies by their payout ratios. The data are obtained from the Compustat North America industrial quarterly data. The data starts from the first quarter of 1981 to the last quarter of 2002. The companies selected satisfy the following two criteria. First, the company appears on the S&P 500 at some time period during 1981 through 2002. Second, the company must have complete data available – including price, dividend, earnings per share, and shares outstanding – during the 88 quarters (22 years). Firms are eliminated from the sample list if one of the following two conditions occurs:

(i) Reported earnings are either trivial or negative.
(ii) Reported dividends are trivial.

Three hundred fourteen firms remain after these adjustments. Finally, excluding those seven companies with highest and lowest average payout ratio, the remaining 300 firms are grouped into ten portfolios by the payout ratio. Each portfolio contains 30 companies. Figure 93.1 shows the comparison of S&P 500 index and the value-weighted index of the 300 firms selected (M). Figure 93.1 shows that the trend is similar to each other before the third quarter of 1999. However, there exist some differences after third quarter of 1999.

To group these 300 firms, the payout ratio for each firm in each year is determined by dividing the sum of four quarters' dividends by the sum of four quarters' earnings; then, the yearly ratios are further averaged over the 22-year period. The first 30 firms with highest payout ratio comprise portfolio one, and so on. Then, the value-weighted average of the price, dividend, and earnings of each portfolio is computed. Characteristics and summary statistics of these ten portfolios are presented in Tables 93.1 and 93.2, respectively. Table 93.1 presents information of return, payout ratio, size, and beta for ten portfolios. From the results of this table, there appears to exist an inverse relationship between return and payout ratio, payout ratio and beta. However, the relationship between payout ratio and beta is not so clear. This finding is similar to that of Fama and French (1992).

Table 93.2 shows the first four moments of quarterly returns of the market portfolio and ten portfolios. The coefficients of skewness, kurtosis, and Jarque-Bera statistics show that one cannot reject the hypothesis that log return of most

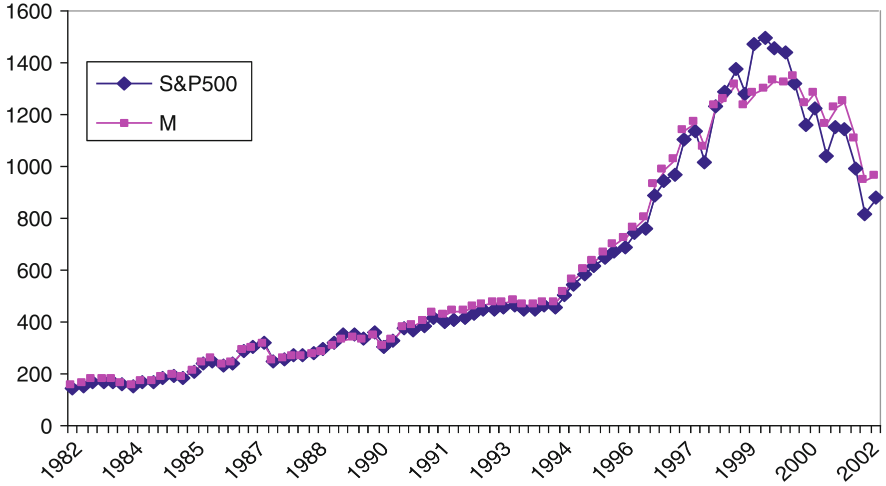


**Fig. 93.1** Comparison of S&P 500 and market portfolio

**Table 93.1** Characteristics of ten portfolios

| Portfolio[a] | Return[b] | Payout[c] | Size (000) | Beta (M) |
|---|---|---|---|---|
| 1 | 0.0351 | 0.7831 | 193,051 | 0.7028 |
| 2 | 0.0316 | 0.7372 | 358,168 | 0.8878 |
| 3 | 0.0381 | 0.5700 | 332,240 | 0.8776 |
| 4 | 0.0343 | 0.5522 | 141,496 | 1.0541 |
| 5 | 0.0410 | 0.5025 | 475,874 | 1.1481 |
| 6 | 0.0362 | 0.4578 | 267,429 | 1.0545 |
| 7 | 0.0431 | 0.3944 | 196,265 | 1.1850 |
| 8 | 0.0336 | 0.3593 | 243,459 | 1.0092 |
| 9 | 0.0382 | 0.2907 | 211,769 | 0.9487 |
| 10 | 0.0454 | 0.1381 | 284,600 | 1.1007 |

[a]The first 30 firms with highest payout ratio comprise portfolio one, and so on
[b]The price, dividend, and earnings of each portfolio are computed by value-weighted of the 30 firms included in the same category
[c]The payout ratio for each firm in each year is found by dividing the sum of four quarters' dividends by the sum of four quarters' earnings; then, the yearly ratios are then computed from the quarterly data over the 22-year period

portfolios is normal. The kurtosis statistics for most sample portfolios are close to three, which indicates that heavy tails are not an issue. Additionally, Jarque-Bera coefficients illustrate that the hypotheses of Gaussian distribution for most portfolios are not rejected. It seems to be unnecessary to consider the problem of heteroskedasticity in estimating domestic stock market if the quarterly data are used.

**Table 93.2** Summary statistics of portfolio quarterly returns[a]

| Country | Mean (quarterly) | Std. dev. (quarterly) | Skewness | Kurtosis | Jarque-Bera |
|---|---|---|---|---|---|
| Market portfolio | 0.0364 | 0.0710 | −0.4604 | 3.9742 | 6.5142* |
| Portfolio 1 | 0.0351 | 0.0683 | −0.5612 | 3.8010 | 6.8925* |
| Portfolio 2 | 0.0316 | 0.0766 | −1.1123 | 5.5480 | 41.470** |
| Portfolio 3 | 0.0381 | 0.0768 | −0.3302 | 2.8459 | 1.6672* |
| Portfolio 4 | 0.0343 | 0.0853 | −0.1320 | 3.3064 | 0.5928 |
| Portfolio 5 | 0.0410 | 0.0876 | −0.4370 | 3.8062 | 5.1251 |
| Portfolio 6 | 0.0362 | 0.0837 | −0.2638 | 3.6861 | 2.7153 |
| Portfolio 7 | 0.0431 | 0.0919 | −0.1902 | 3.3274 | 0.9132 |
| Portfolio 8 | 0.0336 | 0.0906 | 0.2798 | 3.3290 | 1.5276 |
| Portfolio 9 | 0.0382 | 0.0791 | −0.2949 | 3.8571 | 3.9236 |
| Portfolio 10 | 0.0454 | 0.0985 | −0.0154 | 2.8371 | 0.0996 |

[a]Quarterly returns from 1981:Q1to 2002:Q4 are calculated
* and ** denote statistical significance at the 5 % and 1 % level, respectively

### 93.3.2 Dynamic CAPM with Supply Side Effect

If one believes that the stock market is efficient (i.e., if one believes the way in which the expectation errors in dividends are built in the current price is the same for all securities), then price changes would be influenced only by its own dividend expectation errors. Otherwise, if the supply of securities is flexible, then the change in price would be influenced by the expectation adjustment in dividends of other portfolios as well as that in its own dividend. Thus, two hypotheses related to supply effect are to be tested and should be satisfied jointly in order to examine whether there exists a supply effect.

Recalling from the previous section, the structural form equations are exactly identified, and the series of expectation adjustments in dividend, $d_t$, are exogenous variables ($d_t$ can be estimated from earnings per share and dividends per share by using a partial adjustment model as presented in Appendix 3). Now, the reduced form equations can be used to test the supply effect. That is, Eq. 93.22 needs to be examined by the following hypotheses:

Hypothesis 1: All the off-diagonal elements in the coefficient matrix $\Pi$ are zero if the supply effect does not exist.

Hypothesis 2: All the diagonal elements in the coefficients matrix $\Pi$ are equal in the magnitude if the supply effect does not exist.

These two hypotheses should be satisfied jointly. That is, if the supply effect does not exist, price changes of each portfolio would be a function of its own dividend expectation adjustments, and the coefficients should be equal across all portfolios.

The estimated coefficients of the simultaneous equation system for ten portfolios are summarized in Table 93.3.[7] Results of Table 93.3 indicate that the estimated

[7]The results are similar when using either the FIML or SUR approach. We report here the estimates of the SUR method.

**Table 93.3** Coefficients for matrix Π' (ten portfolios)[a]

| | P_P1 | P_P2 | P_P3 | P_P4 | P_P5 | P_P6 | P_P7 | P_P8 | P_P9 | P_P10 |
|---|---|---|---|---|---|---|---|---|---|---|
| P1 | 15.57183 | −12.60844 | 13.15747 | −8.58455 | 8.62495 | −2.486287 | 10.48123 | 1.959701 | −1.274653 | −13.4239 |
| | −23.5688 | −24.513 | −22.2507 | −22.3461 | −25.6377 | −26.1305 | −24.906 | −24.181 | −16.358 | −29.1236 |
| | [0.6607] | [−0.5144] | [0.5913] | [−0.3842] | [0.3364] | [−0.0952] | [0.4208] | [0.0810] | [−0.0779] | [−0.4609] |
| P2 | −16.67868 | −18.7728 | −24.73303 | −12.19542 | −18.61126 | 5.326864 | −16.99283 | −5.675232 | −1.795597 | 13.98581 |
| | −14.2287 | −14.7988 | −13.433 | −13.4906 | −15.4778 | −15.7753 | −15.036 | −14.5984 | −9.8755 | −17.5823 |
| | [−1.1722] | [−1.2685] | [−1.8412] | [−0.9040] | [−1.2025] | [0.3377] | [−1.1301] | [−0.3888] | [−0.1818] | [0.7955] |
| P3 | 140.7762 | 117.8989 | 180.973 | 128.0238 | 161.9093 | 44.47442 | 115.7946 | 103.2686 | 74.30349 | 74.72393 |
| | −73.6813 | −76.6333 | −69.5607 | −69.8588 | −80.1493 | −81.69 | −77.8617 | −75.5953 | −51.1387 | −91.047 |
| | [1.9106] | [1.5385] | [2.6017] | [1.8326] | [2.0201] | [0.5444] | [1.4872] | [1.3661] | [1.4530] | [0.8207] |
| P4 | −79.569 | −82.9826 | −16.2607 | −71.5316 | −38.36708 | −29.88297 | −43.8957 | −20.7400 | −10.4372 | −2.02316 |
| | −64.5317 | −67.1171 | −60.9228 | −61.1839 | −70.1966 | −71.5459 | −68.193 | −66.208 | −44.7884 | −79.741 |
| | [−1.2330] | [−1.2364] | [−0.2669] | [−1.1691] | [−0.5466] | [−0.4177] | [−0.6437] | [−0.3133] | [−0.2330] | [−0.0254] |
| P5 | 25.63953 | 29.0526 | 54.39686 | 6.087413 | 31.12653 | 7.582502 | 30.88937 | 19.3122 | 17.58315 | −0.01716 |
| | −25.521 | −26.5435 | −24.0937 | −24.197 | −27.7613 | −28.2949 | −26.9689 | −26.1839 | −17.7129 | −31.5359 |
| | [1.0047] | [1.0945] | [2.2577] | [0.2516] | [1.1212] | [0.2680] | [1.1454] | [0.7376] | [0.9927] | [−0.0005] |
| P6 | −12.46593 | −8.734942 | −45.85208 | −25.53128 | −17.06422 | −18.11443 | −23.51969 | −1.723033 | −4.492465 | −31.53814 |
| | −12.1881 | −12.6764 | −11.5065 | −11.5558 | −13.2581 | −13.5129 | −12.8796 | −12.5047 | −8.45921 | −15.0607 |
| | [−1.0228] | [−0.6891] | [−3.9849] | [−2.2094] | [−1.2871] | [−1.3405] | [−1.8261] | [−0.1378] | [−0.5311] | [−2.0941] |
| P7 | −84.5262 | −35.03964 | −114.7987 | −19.48548 | −97.9274 | 4.402397 | −57.69584 | −58.88397 | −68.04914 | 3.566607 |
| | −56.1062 | −58.354 | −52.9685 | −53.1955 | −61.0314 | −62.2046 | −59.2894 | −57.5636 | −38.9406 | −69.3296 |
| | [−1.5065] | [−0.6005] | [−2.1673] | [−0.3663] | [−1.6045] | [0.0708] | [−0.9731] | [−1.0229] | [−1.7475] | [0.0514] |
| P8 | −5.497057 | −4.463256 | −31.77293 | 29.38345 | −8.488357 | 0.394223 | −21.59846 | −45.72339 | 19.80597 | −107.4715 |
| | −62.0465 | −64.5323 | −58.5765 | −58.8276 | −67.4932 | −68.7905 | −65.5667 | −63.6582 | −43.0635 | −76.67 |
| | [−0.0886] | [−0.0692] | [−0.5424] | [0.4995] | [−0.1258] | [0.0057] | [−0.3294] | [−0.7183] | [0.4599] | [−1.4017] |

*(continued)*

**Table 93.3** (continued)

| | P_P1 | P_P2 | P_P3 | P_P4 | P_P5 | P_P6 | P_P7 | P_P8 | P_P9 | P_P10 |
|---|---|---|---|---|---|---|---|---|---|---|
| P9 | 20.70817 | 28.77904 | 15.61156 | 23.14069 | 25.93932 | 35.08121 | 23.73591 | 15.46799 | 18.15523 | 25.27915 |
| | −15.5463 | −16.1691 | −14.6768 | −14.7398 | −16.911 | −17.236 | −16.4283 | −15.9501 | −10.7899 | −19.2103 |
| | [1.3320] | [1.7799] | [1.0637] | [1.5700] | [1.5339] | [2.0353] | [1.4448] | [0.9698] | [1.6826] | [1.3159] |
| P10 | −14.64016 | −51.1797 | −49.51991 | −64.67943 | −23.53575 | 67.38674 | 7.053653 | −30.23067 | −15.54273 | 36.60222 |
| | −112.584 | −117.094 | −106.288 | −106.743 | −122.467 | −124.821 | −118.971 | −115.508 | −78.1391 | −139.118 |
| | [−0.1300] | [−0.4371] | [−0.4659] | [−0.6059] | [−0.1922] | [0.5399] | [0.0593] | [−0.2617] | [−0.1989] | [0.2631] |
| $R^2$ | 0.083841 | 0.096546 | 0.283079 | 0.134377 | 0.088212 | 0.075947 | 0.091492 | 0.027763 | 0.065971 | 0.138979 |
| F-st | 0.772786 | 0.902404 | 3.334318 | 1.310894 | 0.816966 | 0.694038 | 0.850408 | 0.241141 | 0.596435 | 1.363029 |

[a]Standard errors in ( ) and t-statistics in [ ]

diagonal elements seem to vary across portfolios and most of the off-diagonal elements are significant from zero. However, simply observing the elements in matrix Π directly cannot justify either accept or reject the null hypotheses derived for testing the supply effect. Two tests should be done separately to check whether these two hypotheses are both satisfied.

For the first hypothesis, the test of supply effect on off-diagonal elements, the following regression in accordance with Eq. 93.22 is run for each portfolio:

$$p_{i,t} = \beta_i d_{i,t} + \Sigma_{j \neq i} \beta_j d_{j,t} + \varepsilon_{i,t}, i, j = 1, \ldots, 10. \tag{93.23}$$

The null hypothesis then can be written as $H_0$: $\beta_j = 0$, $j = 1, \ldots, 10$, $j \neq i$. The results are reported in Table 93.4. Two test statistics are reported. The first test uses an F distribution with 9 and 76 degrees of freedom, and the second test uses a chi-squared distribution with 9 degrees of freedom. The null hypothesis is rejected at 5 % significance level in six out of ten portfolios, and only two portfolios cannot be rejected at 10 % significance level. This result indicates that the null hypothesis can be rejected at conventional levels of significance.

For the second hypothesis of supply effect on all diagonal elements of Eq. 93.22, the following null hypothesis needs to be tested:

$$H_0 : \pi_{i,i} = \pi_{j,j} \quad \text{for all } i, j = 1, \ldots, 10.$$

To do this null hypothesis test, we need to estimate Eq. 93.22 simultaneously, and then, we calculate Wald statistics by imposing nine restrictions on this equation system. Under the above nine restrictions, the Wald test statistic has a chi-square distribution with 9 degrees of freedom. The statistic is 18.858, which is greater than 16.92 at 5 % significance level. Since the statistic corresponds to a $p$-value of 0.0265, one can reject the null hypothesis at 5 %, but it cannot reject $H_0$ at a 1 % significance level. In other words, the diagonal elements are not similar to each other in magnitude. In conclusion, the above empirical results are sufficient to reject two null hypotheses of nonexistence of supply effect in the US stock market.

In order to check whether the individual stocks can hold up to the same testing, we use individual stock data as many as 30 companies in one group. The results are summarized in Table 93.5. From Table 93.5, we find that the above conclusion seems to be sustainable if we use individual stock data. More specifically, the diagonal elements are not equal to each other at any conventional significant level and the off-diagonal elements are significantly from zero in each group composed of 30 individual stocks.

We also find that one cannot reject the existence of supply effect by using the stocks listed in the Dow Jones Index. Again, to test the supply effect on off-diagonal elements, Eq. 93.23 is run as the following for each company:

$$p_{i,t} = \beta_i d_{i,t} + \Sigma_{j \neq i} \beta_j d_{j,t} + \varepsilon_{i,t}, \quad i, j = 1, \ldots, 29. \tag{93.23$'$}$$

**Table 93.4** Test of supply effect on off-diagonal elements of matrix $\Pi^{a,b}$

| | $R^2$ | F- statistic | *p*-value | Chi-square | *p*-value |
|---|---|---|---|---|---|
| Portfolio 1 | 0.1518 | 1.7392 | 0.0872 | 17.392* | 0.0661 |
| Portfolio 2 | 0.1308 | 1.4261 | 0.1852 | 14.261 | 0.1614 |
| Portfolio 3 | 0.4095 | 5.4896 | 0.0000 | 53.896*** | 0.0000 |
| Portfolio 4 | 0.1535 | 1.9240 | 0.0607 | 17.316** | 0.0440 |
| Portfolio 5 | 0.1706 | 1.9511 | 0.0509 | 19.511** | 0.0342 |
| Portfolio 6 | 0.2009 | 1.2094 | 0.2988 | 12.094 | 0.2788 |
| Portfolio 7 | 0.2021 | 1.8161 | 0.0718 | 18.161* | 0.0523 |
| Portfolio 8 | 0.1849 | 1.9599 | 0.0497 | 19.599** | 0.0333 |
| Portfolio 9 | 0.1561 | 1.8730 | 0.0622 | 18.730** | 0.0438 |
| Portfolio 10 | 0.3041 | 3.5331 | 0.0007 | 35.331*** | 0.0001 |

[a] $p_{i,\,t} = \beta_i{}'\, d_{i,\,t} + \Sigma_{j \neq i}\beta_j{}'\, d_{j,\,t} + \varepsilon{}'_{i,\,t}$ i, $j = 1, \ldots, 10$.
Hypothesis: all $\beta_j = 0, j = 1, \ldots, 10, j \neq i$
[b] The first test uses an F distribution with 9 and 76 degrees of freedom, and the second uses a chi-squared distribution with 9 degrees of freedom
*, **, and *** denote statistical significance at the 10 %, 5 %, and 1 % level, respectively

The null hypothesis then can be written as $H_0$: $\beta_j = 0$, j = 1, . . ., 29, j ≠ i. The results are summarized in Table 93.6. The null hypothesis is rejected at 1 % significance level in 26 out of 29 companies. For the second hypothesis of supply effect on all diagonal elements, the following null hypothesis is also tested: $H_0$: $\pi_{i,\,i} = \pi_{j,\,j}$, for all i, j = 1, . . ., 29.

The Wald test statistic has a chi-square distribution with 28 degrees of freedom. The statistic is 86.35. That is, one can reject this null hypothesis at 1 % significance level.

## 93.4 Summary

We examine an asset pricing model that incorporates a firm's decision concerning the supply of risky securities into the CAPM. This model focuses on a firm's financing decision by explicitly introducing the firm's supply of risky securities into the static CAPM and allows the supply of risky securities to be a function of security price. And thus, the expected returns are endogenously determined by both demand and supply decisions within the model. In other words, the supply effect may be one important factor in capital assets pricing decisions.

Our objective is to investigate the existence of supply effect in the US stock markets. We find that supply effect is important in the US stock market. This finding holds as we break the companies listed in the S&P 500 into ten portfolios. It also holds if we use individual stock data. These test results show that two null hypotheses of the nonexistence of supply effect do not seem to be satisfied jointly. In other words, this evidence seems to be sufficient to support the existence of supply effect and, thus, imply a violation of the assumption in the one-period static CAPM, or to imply a dynamic asset pricing model may be a better choice in the US domestic stock markets.

**Table 93.5** Test of supply effect (by individual stock)

| | Test of supply effect on the diagonal elements: $H_0$: $\pi_{ii} = \pi_{jj}$ for all i, $j = 1, 2, \ldots, 30$ | Test supply effect on off-diagonal elements: $p_{i,t} = \beta_i{}' d_{i,t} + \Sigma_{j \neq i}\beta_j{}' d_{j,t} + \varepsilon'_{i,t}$, for i, $j = 1, 2, \ldots, 30$; $H_0$: all $\beta_j = 0$, $j = 1, 2, \ldots, 30$, $j \neq i$ Different significant level 1 % | 5 % | 10 % |
|---|---|---|---|---|
| Group 1 | $\chi^2 = 113.65$, $p$-value = 0.0000 ➔ Reject $H_0$ at 1 % | Reject 23 in 30 equations | Reject 25 in 30 equations | Reject 25 in 30 equations |
| Group 2 | $\chi^2 = 52.08$, $p$-value = 0.0053 ➔ Reject $H_0$ at 1 % | Reject 21 in 30 equations | Reject 24 in 30 equations | Reject 25 in 30 equations |
| Group 3 | $\chi^2 = 86.53$, $p$-value = 0.0000 ➔ Reject $H_0$ at 1 % | Reject 26 in 30 equations | Reject 27 in 30 equations | Reject 28 in 30 equations |
| Group 4 | $\chi^2 = 88.58$, $p$-value = 0.0000 ➔ Reject $H_0$ at 1 % | Reject 21 in 30 equations | Reject 24 in 30 equations | Reject 25 in 30 equations |
| Group 5 | $\chi^2 = 101.14$, $p$-value = 0.0000 ➔ Reject $H_0$ at 1 % | Reject 25 in 30 equations | Reject 26 in 30 equations | Reject 28 in 30 equations |
| Group 6 | $\chi^2 = 69.14$, $p$-value = 0.0000 ➔ Reject $H_0$ at 1 % | Reject 17 in 30 equations | Reject 21 in 30 equations | Reject 22 in 30 equations |
| Group 7 | $\chi^2 = 181.10$, $p$-value = 0.0000 ➔ Reject $H_0$ at 1 % | Reject 29 in 30 equations | Reject 30 in 30 equations | Reject 30 in 30 equations |
| Group 8 | $\chi^2 = 116.97$, $p$-value = 0.0000 ➔ Reject $H_0$ at 1 % | Reject 29 in 30 equations | Reject 29 in 30 equations | Reject 29 in 30 equations |
| Group 9 | $\chi^2 = 117.44$, $p$-value = 0.0000 ➔ Reject $H_0$ at 1 % | Reject 27 in 30 equations | Reject 28 in 30 equations | Reject 29 in 30 equations |
| Group 10 | $\chi^2 = 109.50$, $p$-value = 0.0000 ➔ Reject $H_0$ at 1 % | Reject 25 in 30 equations | Reject 27 in 30 equations | Reject 27 in 30 equations |

For the future research, we will first modify the simultaneous equation asset pricing model defined in Eqs. 93.9 and 93.12 to allow for testing the existence of market disequilibrium in dynamic asset pricing. Then, we will use disequilibrium estimation methods developed by Amemiya (1974), Fair and Jaffe(1972), and Quandt (1988) to test whether there is price adjustment in response to an excess demand in equity market.

## Appendix 1: Modeling the Price Process

In Sect. 93.2.3, Eq. 93.16 is derived from Eq. 93.15 under the assumption that all countries' index series follow a random walk process. Thus, before further discussion, we should test the order of integration of these price series. From Hamilton (1994), we know that two widely used unit root tests are the Dickey-Fuller (DF) and the augmented Dickey-Fuller (ADF) tests. The former can be represented as $P_t = \mu + \gamma P_{t-1} + \varepsilon_t$, and the latter can be written as $\Delta P_t = \mu + \gamma P_{t-1} + \delta_1 \Delta P_{t-1} + \delta_2 \Delta P_{t-2} + \ldots + \delta_p \Delta P_{t-p} + \varepsilon_t$.

**Table 93.6** Test of supply effect (companies listed in the Dow Jones Index)

| GVKEY | Security $i$ | $R^2$ of each equation $i$ | $H_0$: all $\beta_j = 0, j = 1, 2, \ldots, 29, j \neq i$ Chi-square | *p*-value |
|---|---|---|---|---|
| 1300 | Honeywell International Inc | 0.7088 | 137.08 | 0.0000 |
| 1356 | Alcoa Inc | 0.6716 | 120.84 | 0.0000 |
| 1447 | American Express | 0.4799 | 55.47 | 0.0015 |
| 1581 | AT&T Corp | 0.5980 | 56.16 | 0.0012 |
| 2285 | Boeing Co | 0.5291 | 66.75 | 0.0001 |
| 2817 | Caterpillar Inc | 0.5887 | 83.10 | 0.0000 |
| 2968 | JPMorgan Chase & Co | 0.5352 | 68.12 | 0.0000 |
| 3144 | Coca-Cola Co | 0.5927 | 87.04 | 0.0000 |
| 3243 | Citigroup Inc | 0.6082 | 88.63 | 0.0000 |
| 3980 | Disney (Walt) Co | 0.6457 | 104.06 | 0.0000 |
| 4087 | Du Pont (E I) De Nemours | 0.6231 | 98.37 | 0.0000 |
| 4194 | Eastman Kodak Co | 0.3793 | 36.14 | 0.1416 |
| 4503 | Exxon Mobil Corp | 0.5653 | 76.50 | 0.0000 |
| 5047 | General Electric Co | 0.5425 | 61.17 | 0.0003 |
| 5073 | General Motors Corp | 0.5372 | 66.73 | 0.0001 |
| 5606 | Hewlett-Packard Co | 0.4755 | 53.61 | 0.0025 |
| 5680 | Home Depot Inc | 0.6753 | 106.00 | 0.0000 |
| 6008 | Intel Corp | 0.5174 | 60.05 | 0.0004 |
| 6066 | Intl Business Machines Corp | 0.5596 | 75.31 | 0.0000 |
| 6104 | Intl Paper Co | 0.5512 | 72.58 | 0.0000 |
| 6266 | Johnson & Johnson | 0.5211 | 67.59 | 0.0000 |
| 7154 | McDonalds Corp | 0.4416 | 45.53 | 0.0195 |
| 7257 | Merck & Co | 0.4109 | 40.82 | 0.0558 |
| 7435 | 3M CO | 0.6344 | 105.07 | 0.0000 |
| 8543 | Altria Group Inc | 0.5751 | 72.25 | 0.0000 |
| 8762 | Procter & Gamble Co | 0.5816 | 84.19 | 0.0000 |
| 9899 | SBC Communications Inc | 0.5486 | 72.81 | 0.0000 |
| 10983 | United Technologies Corp | 0.6595 | 116.19 | 0.0000 |
| 11259 | Wal-Mart Stores | 0.6488 | 111.85 | 0.0000 |

Test the off-diagonal elements: $p_{i,t} = \beta_i{}' d_{i,t} + \Sigma_{j \neq i} \beta_j{}' d_{j,t} + \varepsilon'_{i,t}$, for $i, j = 1, \ldots, 29$, null hypothesis $H_0$: all $\beta_j = 0, j = 1, 2, \ldots, 29, j \neq i$

Test of supply effect on the diagonal elements; $H_0$: $\pi_{ii} = \pi_{jj}$ for all $i, j = 1, 2, \ldots, 29$ Result: $\chi^2 = 86.35$, $p$-value $= 0.0000 \rightarrow$ Reject $H_0$ at 1 %

Microsoft Corp. is not included since it had paid dividends twice for the whole sample period

Similarly, in the US stock markets, the Phillips-Perron test is used to check whether the value-weighted price of market portfolio follows a random walk process. The results of the tests for each index are summarized in Table 93.7. It seems that one cannot reject the hypothesis that all indices follow a random walk process since, for example, the null hypothesis of unit root in level cannot be rejected for all indices but are all rejected if one assumes there is a unit root in the first-order difference of the price for each portfolio. This result is consistent with most studies that find that the financial price series follow a random walk process.

**Table 93.7** Unit root tests for $P_t$

| | $P_t = \mu + \gamma P_{t-1} + \varepsilon_t$ | | Phillips-Perron test[a] | |
|---|---|---|---|---|
| | Estimated $c_2$ (std. error) | Adj. $R^2$ | Level | 1st difference[b] |
| Market portfolio | 1.0060 (0.0159) | 0.9788 | −0.52 | −8.48** |
| S&P 500 | 0.9864 (0.0164) | 0.9769 | −0.90 | −959** |
| Portfolio 1 | 0.9883 (0.0172) | 0.9746 | −0.56 | −8.67** |
| Portfolio 2 | 0.9877 (0.0146) | 0.9815 | −0.97 | −9.42** |
| Portfolio 3 | 0.9913 (0.0149) | 0.9809 | −0.51 | −13.90** |
| Portfolio 4 | 0.9935 (0.0143) | 0.9825 | −0.61 | −7.66** |
| Portfolio 5 | 0.9933 (0.0158) | 0.9787 | −0.43 | −9.34** |
| Portfolio 6 | 0.9950 (0.0150) | 0.9808 | −0.32 | −8.66** |
| Portfolio 7 | 0.9892 (0.0155) | 0.9793 | −0.64 | −9.08** |
| Portfolio 8 | 0.9879 (0.0166) | 0.9762 | −0.74 | −9.37** |
| Portfolio 9 | 0.9939 (0.0116) | 0.9884 | −0.74 | −7.04** |
| Portfolio 10 | 0.9889 (0.0182) | 0.9716 | −0.69 | −9.07** |

*5 % significant level; ** 1 % significant level
[a]The process assumed to be random walk without drift
[b]The null hypothesis of zero intercept terms, $\mu$, cannot be rejected at 5 %, 1 % level for all portfolios

## Appendix 2: Identification of the Simultaneous Equation System

Note that given G is nonsingular, $\Pi = -G^{-1}H$ in Eq. 93.19 can be written as

$$\text{where } A = [\,G \quad H\,] = \begin{pmatrix} g_{11} & g_{12} & \cdots\cdots & g_{1n} & h_{11} & h_{12} & \cdots\cdots & h_{1n} \\ g_{21} & g_{22} & \cdots\cdots & g_{1n} & h_{21} & h_{22} & \cdots\cdots & h_{2n} \\ . & & & & & & . & \\ . & & & & & & . & \\ g_{n1} & g_{n2} & \cdots\cdots & g_{nn} & h_{n1} & h_{n2} & \cdots\cdots & h_{nn} \end{pmatrix}$$

$$W = [\,\Pi \quad I_n\,]' = \begin{pmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1n} & 1 & 0 & \cdots\cdots & 0 \\ \pi_{21} & \pi_{22} & \cdots & \pi_{1n} & 0 & 1 & \cdots\cdots & 0 \\ . & & & & & & . & \\ . & & & & & & . & \\ \pi_{n1} & \pi_{n2} & \cdots & \pi_{nn} & 0 & 0 & \cdots\cdots & 1 \end{pmatrix} \tag{93.24}$$

That is, A is the matrix of all structure coefficients in the model with dimension of $(n \times 2n)$, and W is a $(2n \times n)$ matrix. The first equation in Eq. (93.24) can be expressed as

$$A_1 W = 0, \tag{93.25}$$

where $A_1$ is the first row of A, i.e., $A_1 = [g_{11}\ g_{12} \ldots.\ g_{1n}\ h_{11}\ h_{12} \ldots..\ h_{1n}]$.

Since the elements of $\Pi$ can be consistently estimated, and $I_n$ is the identity matrix, Eq. 93.25 contains 2n unknowns in terms of $\pi$s. Thus, there should be n restrictions on the parameters to solve Eq. 93.25 uniquely. First, one can try to impose normalization rule by setting $g_{11}$ equal to 1 to reduce one restriction. As a result, there are at least n−1 independent restrictions needed in order to solve Eq. 93.25.

It can be illustrated that the system represented by Eq. 93.17 is exactly identified with three endogenous and three exogenous variables. It is entirely similar to those cases of more variables. For example, if $n = 3$, Eq. 93.17 can be expressed in the form

$$-\begin{pmatrix} r*cs_{11}+a_1b_1 & r*cs_{12} & r*cs_{13} \\ r*cs_{21} & r*cs_{22}+a_2b_2 & r*cs_{23} \\ r*cs_{31} & r*cs_{32} & r*cs_{33}+a_3b_3 \end{pmatrix}\begin{pmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{pmatrix} + \begin{pmatrix} cs_{11}+a_1 & cs_{12} & cs_{13} \\ cs_{21} & cs_{22}+a_2 & cs_{23} \\ cs_{31} & cs_{32} & cs_{33}+a_3 \end{pmatrix}\begin{pmatrix} d_{1t} \\ d_{2t} \\ d_{3t} \end{pmatrix} = \begin{pmatrix} v_{1t} \\ v_{2t} \\ v_{3t} \end{pmatrix} \tag{93.26}$$

where
$r^*$ = scalar of risk-free rate
$s_{ij}$ = elements of variance-covariance matrix of return
$a_i$ = inverse of the supply adjustment cost of firm i
$b_i$ = overall cost of capital of firm i

For example, in the case of $n = 3$, Eq. 93.17 can be written as

$$\begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix}\begin{pmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{pmatrix} + \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}\begin{pmatrix} d_{1t} \\ d_{2t} \\ d_{3t} \end{pmatrix} = \begin{pmatrix} v_{1t} \\ v_{2t} \\ v_{3t} \end{pmatrix}. \tag{93.27}$$

Comparing Eq. 93.26 with Eq. 93.27, the prior restrictions on the first equation take the form $g_{12} = -r^*h_{12}$ and $g_{13} = -r^*h_{13}$ and so on.

Thus, the restriction matrix for the first equation is of the form

$$\Phi = \begin{pmatrix} 0 & 1 & 0 & 0 & r^* & 0 \\ 0 & 0 & 1 & 0 & 0 & r^* \end{pmatrix} \tag{93.28}$$

Then, combining Eq. 93.25 and the parameters of the first equation gives

$$[g_{11} \quad g_{12} \quad g_{13} \quad h_{11} \quad h_{12} \quad h_{13}]\begin{pmatrix} \pi_{11} & \pi_{12} & \pi_{13} & 0 & 0 \\ \pi_{21} & \pi_{22} & \pi_{13} & 1 & 0 \\ \pi_{31} & \pi_{32} & \pi_{33} & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & r* & 0 \\ 0 & 0 & 1 & 0 & r* \end{pmatrix}[0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]. \tag{93.29}$$

That is, extending Eq. 93.29, we have

$$\begin{aligned} &g_{11}\pi_{11} + g_{12}\pi_{21} + g_{13}\pi_{31} + h_{11} = 0,\\ &g_{11}\pi_{12} + g_{12}\pi_{22} + g_{13}\pi_{32} + h_{12} = 0,\\ &g_{11}\pi_{13} + g_{12}\pi_{23} + g_{13}\pi_{33} + h_{13} = 0,\\ &g_{12} + r^{*}h_{12} = 0, \text{and}\\ &g_{13} + r^{*}h_{13} = 0. \end{aligned} \tag{93.30}$$

The last two ($n-1 = 3-1 = 2$) equations in Eq. 93.30 give the value $h_{12}$ and $h_{13}$, and the normalization condition, $g_{11} = 1$, allows us to solve Eq. 93.25 in terms of πs uniquely. That is, in the case $n = 3$, the first equation represented by Eq. 93.25, $A_1W = 0$, can be finally rewritten as Eq. 93.30. Since there are three unknowns, $g_{12}$, $g_{13}$, and $h_{11}$, left for the first three equations in Eq. 93.30, the first equation $A_1$ is exactly identified. Similarly, it can be shown that the second and the third equations are also exactly identified.

## Appendix 3: Derivation of the Formula Used to Estimate $d_t$

To derive the formula for estimating $d_t$, we first define the partial adjustment model as

$$D_t = a_1 + a_2D_{t-1} + a_3E_t + u_t \tag{93.31}$$

where $D_t$ = dividend per share in period t, $D_{t-1}$ = dividend per share in period t−1, $E_t$ = earnings per share in period t are dividends and earnings, and $u_t$ = error term in period t. Similarly,

$$D_{t+1} = a_1 + a_2D_t + a_3E_{t+1} + u_{t+1}. \tag{93.31$'$}$$

And thus,

$$\mathrm{E}_{t-1}[D_t] = a_1 + a_2D_{t-1} + a_3\mathrm{E}_{t-1}[E_t], \tag{93.32}$$

$$\mathrm{E}_t[D_{t+1}] = a_1 + a_2D_t + a_3\mathrm{E}_t[E_{t+1}], \tag{93.33}$$

$$\mathrm{E}_{t-1}[D_{t+1}] = a_1 + a_2\mathrm{E}_{t-1}[D_t] + a_3\mathrm{E}_{t-1}[E_{t+1}]. \tag{93.34}$$

Substituting Eq. 93.32 to Eq. 93.34, we have

$$E_{t-1}[D_{t+1}] = a_1 + a_1a_2 + a_2^2D_{t-1} + a_2a_3\mathrm{E}_{t-1}[E_t] + a_3\mathrm{E}_{t-1}[E_{t+1}]. \tag{93.34$'$}$$

Subtracting Eq. 93.34′ from Eq. 93.33 on both hand sides, we have

$$\begin{aligned} \mathrm{E}_t[D_{t+1}] - \mathrm{E}_{t-1}[D_{t+1}] = &-a_1a_2 + a_2D_t - a_2^2D_{t-1} - a_2a_3\mathrm{E}_{t-1}[E_t]\\ &+ a_3\mathrm{E}_t[E_{t+1}] - a_3\mathrm{E}_{t-1}[E_{t+1}]. \end{aligned} \tag{93.35}$$

Equation Eq. 93.35 can be investigated depending upon whether $E_t$ is following a random walk.

**Case 1** $E_t$ follows an AR(p) process.

If the time series of $E_t$ is assumed to be stationary and follows an AR(p) process, then after taking the seasonal differences, we obtain

$$dE_t = \rho_0 + \rho_1 dE_{t-1} + \rho_2 dE_{t-2} + \rho_3 dE_{t-3} + \rho_4 dE_{t-4} + \varepsilon_t, \tag{93.36}$$

where $dE_t = E_t - E_{t-4}$.

The expectation adjustment in seasonally differenced earnings, or the revision in forecasting future seasonally differenced earnings, can be solved as

$$\mathrm{E}_t[dE_{t+1}] - \mathrm{E}_{t-1}[dE_{t+1}] = \rho_1(-\rho_0 + dE_t - \rho_1 dE_{t-1} - \rho_2 dE_{t-2} - \rho_3 dE_{t-3} - \rho_4 dE_{t-4}). \tag{93.37}$$

Since $\mathrm{E}_t[dE_{t+1}] - \mathrm{E}_{t-1}[dE_{t+1}] = \mathrm{E}_t[E_{t+1}] - \mathrm{E}_{t-1}[E_{t+1}]$, we have

$$\mathrm{E}_t[E_{t+1}] - \mathrm{E}_{t-1}[E_{t+1}] = \rho_1(-\rho_0 + dE_t - \rho_1 dE_{t-1} - \rho_2 dE_{t-2} - \rho_3 dE_{t-3} - \rho_4 dE_{t-4}). \tag{93.38}$$

Furthermore, from Eq. 93.36, we have

$$\mathrm{E}_{t-1}[dE_t] = \rho_0 + \rho_1 dE_{t-1} + \rho_2 dE_{t-2} + \rho_3 dE_{t-3} + \rho_4 dE_{t-4}. \tag{93.39}$$

Similarly, $\mathrm{E}_{t-1}[dE_t] = \mathrm{E}_{t-1}[E_1 - E_{t-4}] = \mathrm{E}_{t-1}[E_t] - E_{t-4}$; thus, $\mathrm{E}_{t-1}[E_t]$ can be found by

$$\mathrm{E}_{t-1}[E_t] = \rho_0 + \rho_1(dE_{t-1}) + \rho_2(dE_{t-2}) + \rho_3(dE_{t-3}) + \rho_4(dE_{t-4}) + E_{t-4}. \tag{93.40}$$

Finally, the expectation adjustment in dividends, $d_t$, can be found by plugging Eqs. 93.38 and 93.40 into Eq. 93.35:

$$\begin{aligned} d_t \equiv \mathrm{E}_t[D_{t+1}] - \mathrm{E}_{t-1}[D_{t+1}] &= -a_1 a_2 + a_2 D_t - a_2^2 D_{t-1} \\ &- a_2 a_3(\rho_0 + \rho_1 dE_{t-1} + \rho_2 dE_{t-2} + \rho_3 dE_{t-3} + \rho_4 dE_{t-4} + E_{t-4}) \\ &+ a_3 \rho_1(-\rho_0 + dE_t - \rho_1 dE_{t-1} - \rho_2 dE_{t-2} - \rho_3 dE_{t-3} - \rho_4 dE_{t-4}) \end{aligned} \tag{93.41}$$

Or

$$\begin{aligned} d_t = C_0 + C_1 D_t + C_2 D_{t-1} + C_3 dE_t + C_4 dE_{t-1} + C_5 dE_{t-2} + C_6 dE_{t-3} \\ + C_7 dE_{t-4} + C_8 E_{t-4} \end{aligned} \tag{93.42}$$

where $C_0$ to $C_8$ are functions of $a_1$ to $a_1$ and $\rho_0$ to $\rho_4$.

That is, the expectation adjustment in dividends, $d_t$, can be found by the coefficients estimated in Eqs. 93.31 and 93.36, i.e., $a_1$ to $a_3$ and $\rho_0$ to $\rho_4$, and the observable data from the time series of $D_t$ and $E_t$.

**Case 2** $E_t$ follows a random walk process.

If the series of earnings, $E_t$, follows a random walk process, i.e., $\mathrm{E}_t[E_{t+1}] = E_t$, $\mathrm{E}_{t-1}[E_t] = E_{t-1}$, and $\mathrm{E}_{t-1}[E_{t+1}] = E_{t-1}$, then Eq. 93.35 can be redefined:

$$d_t \equiv \mathrm{E}_t[D_{t+1}] - \mathrm{E}_{t-1}[D_{t+1}] = C_0 + C_1 D_t + C_2 D_{t-1} + C_3 E_t + C_4 E_{t-1} \quad (93.43)$$

where $C_0 = -a_1 a_2$ $C_1 = a_2$, $C_2 = -\ a_2^2$, $C_3 = a_3$, and $C_4 = -\ a_3(1 + a_2)$.

That is, the expectation adjustment in dividends, $d_t$, can be found by the observable data from the time series of $D_t$ and $E_t$.

In this study, we assumed that $E_t$ follows a random walk process. Therefore, we used Eq. 93.43 instead of Eq. 93.42 to estimate $\mathrm{d}_t$ in Eqs. 93.22 and 93.23 in the text.

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