

Supply Chain Coordination with Energy Price Uncertainty, Carbon Emission Cost, and Product Return

S. Paul, M. I. M. Wahab and X. F. Cao

Abstract An EOQ model for a coordinated two-level supply chain is developed under energy (gasoline) price uncertainty and defective items in transshipment. It is assumed that the transportation cost not only depends on the lot size, but also depends on the gasoline price uncertainty. The purpose is to determine the optimal production–shipment policy for the proposed model by taking into account the percentage of defective items, transportation cost, setup cost, screening cost, holding cost, and carbon emission cost. The objective is to determine the optimal number of shipments and the optimal order quantity that minimize the expected total cost per unit time. Expressions for the optimal order quantity, the optimal number of shipments, the optimal number of buyer’s cycle during which the defective items are stored at the buyer’s warehouse before shipping them to the vendor are derived by minimizing the expected total cost per unit time. In order to illustrate the proposed model, a numerical example is presented and results are discussed. It is found that as the gasoline price uncertainty increases, both the total cost and shipment size increase. This shows that the gasoline price influences the supply chain coordination. Moreover, when the fixed gasoline consumption depending on the vehicle size, type, or age increases, shipment size increases, the number of shipments decreases, and the total cost increases. This implies that when the truck size or type used for shipping changes, the supply chain coordination decision will also change. The variable gasoline consumption increases the total cost of the supply chain. Finally, a similar behavior is observed with respect to fixed and variable carbon emissions costs for the buyer.

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1 Introduction

Rising energy price (e.g., gasoline price) forces companies to rethink about supply chain coordination strategies. It has been reported that \$10 per barrel increase in the oil price results in 4 ¢ per mile increase in transportation cost The Wall Street Journal (2008). In the year 2000, the cost of shipping a 40-foot container from Shanghai, China, to the USA was \$3000 but now it is \$8000 The New York Times (2008). Hence, in order to reduce the transportation cost, companies are trying to reduce the distance between the distribution centers and retailers. For example, Sharp Corporation has moved its flat screen television manufacturing section to Mexico from Asia to be closer to North and South American customers The Wall Street Journal (2008). Johnson & Son Inc. has saved \$1.6 million and reduced fuel use by 168,000 gallons by manufacturing a larger lot size and improving truckload utilization The Wall Street Journal (2008). These are some of the examples of how companies change their supply chain coordination strategies when energy (gasoline) price is very volatile.

In this regard, the coordination between a vendor and a buyer is very important to reduce the total cost. Aderohunmu et al. (1995) showed that a significant cost reduction can be achieved by a joint economic lot-size model as information is shared between the vendor and buyer. A joint economic lot-size model for a single vendor and a single buyer was proposed by Goyal (1976). Then, Benerjee (1986) introduced a joint economic lot-size-inventory policy incorporating a finite production rate for the vendor to satisfy a buyer's demand on a lot-for-lot basis. Again, Goyal (1988) proposed an equal size shipment policy by relaxing the assumption of lot-for-lot policy for the vendor; and it was illustrated that this model can provide a lower total cost than that of the previous models. Later, Goyal (1995) addressed an unequal shipment size policy, where the shipment size increases by a constant factor. Ha and Kim (1997) also developed an integrated single vendor and a single buyer inventory policy under just-in-time and showed that both vendor and buyer could be benefited by adopting that policy. Hoque and Goyal (2000) developed a model for a vendor-buyer production-inventory system with equal and unequal shipment sizes under the shipping capacity constraint. Goyal and Nebebe (2000) proposed a policy that has a smaller shipment size first and then subsequently an equal shipment size. Then, Hill and Omar (2006) developed a different approach of a single-vendor single-buyer integrated production inventory system by decreasing the holding cost as the stock goes down. Moreover, a comprehensive review on vendor-buyer coordination can be found in Ben-Daya et al. (2008).

The above studies have focused on the production-shipment policy between the vendor and buyer ignoring the relationship between shipment size and quality, because these supply chain coordination models have assumed that the items produced are perfect in quality. However, in most industrial applications, because of the damage in transit and/or imperfect production process of the vendor, some defective items may result in a shipment. These defective items will affect the supply chain coordination policy. There are a number of studies that investigate

the effect of defective items on the economic order quantity (EOQ). Cheng (1991) proposed an EOQ model considering imperfect items in the production processes. Along this line, Ben-Daya and Hariga (2000) incorporated the effect of defective items on the economic lot sizing policy. Salameh and Jaber (2000) also developed an EOQ model considering a random proportion of defective items in a lot. Papachristos and Konstantaras (2006) derived the sufficient condition to ensure no shortage due to defective items in Salameh and Jaber (2000). Recently, Wahab and Jaber (2010) modified Salameh and Jaber's model considering different holding costs for good and defective items.

In more recent years, there is a great awareness that carbon emissions will lead to major climate changes in the earth. Governments enforce carbon emission costs to encourage individuals, businesses, industry, and others to reduce their carbon emissions. Consequently, companies undertake initiatives to change their supply chain to "green". In other words, companies do not only concern about reducing the total cost but also thrive for establishing a supply chain to reduce carbon footprint. Hence, logistics and supply chain managers take steps to balance the cost and innovate new environmentally friendly processes Pagell et al. (2004). For example, some reputed companies such as General Electric, DuPont, Alcoa, Procter & Gamble have already started reducing both fuel consumption and carbon emission The New York Times (2008). In 2005, Walmart also declared to reduce greenhouse gases by 20 % and energy use at stores by 30 % in 7 years (MSNBC and Reuters 2005).

In this research, we investigate how the energy price uncertainty and carbon emission costs play a role in a coordinated supply chain, which has some defective items in each shipment because of the damage in transit. It is assumed that the defective items are shipped back to the vendor. As a result, in this particular supply chain, in order to reduce both energy cost and carbon emission cost, the total cost of the supply chain is minimized with respect to the number of shipments from the vendor to the buyer and its shipment size, and the number of shipments from the buyer to the vendor and its shipment size. The results show that both energy price uncertainty and carbon emission cost significantly influence the supply chain coordination decisions.

This chapter is organized as follows: Sect. 2 introduces the mathematical model for coordinated inventory policy. Section 3 presents a numerical example and Sect. 4 concludes the chapter.

2 The Model

We first list the parameters and variables that are used in this chapter.

Nomenclature:

- Q Shipment size from the vendor to the buyer
- n The number of shipments

S_b	Buyer's setup cost per order
D	Buyer's annual demand
ϑ_b	Buyer's fixed carbon emission cost per shipment
Γ_b	Buyer's variable carbon emission cost per unit during the shipment
T	Buyer's cycle time
γ	The percentage of defective items in each shipment
χ	Buyer's screening cost per unit of item
h_b	Buyer's holding per unit per unit time
f_b	Buyer's fixed gasoline consumption per shipment
λ_b	Buyer's variable gasoline consumption per unit during the shipment
η_t	Gasoline price at time t , and $\ln(\eta_t) = \theta_t$
β	Mean reversion coefficient of the gasoline price process
μ	The long-term mean of the gasoline price process
σ	Volatility of the gasoline price process
P	Vendor's production rate
h_v	Vendor's holding cost per unit per unit time during the production
h_d	Vendor's holding cost per defective item per unit time at the buyer's warehouse
N	The number of buyer's cycle during which the defective items are stored at the buyer's warehouse before returning back to the vendor
f_v	Vendor's fixed gasoline consumption per shipment of defective items
λ_v	Vendor's variable gasoline consumption per defective item during the shipment
S_v	Vendor's setup cost per production run
K_v	Vendor's carbon emission cost per unit of item produced
ϑ_b	Vendor's fixed carbon emission cost per shipment of defective items
Γ_b	Vendor's variable carbon emission cost per defective unit during the shipment

We consider a supply chain coordination model that consists of a single vendor and a single buyer. The buyer's demand rate D is assumed to be constant and deterministic. When the buyer places an order for nQ items, it incurs the buyer a setup cost of S_b . The vendor produces nQ items with a production rate of P , where $P > D$ to avoid shortages. The items are shipped to the buyer in equal size of Q ; and there are n number of shipments. Production incurs the vendor a setup cost of S_v per production run and a carbon emission cost of K_v . For each shipment, the buyer has to pay a transportation cost of U_b and a carbon emission cost of E_b . We assumed that the transportation cost not only depends on the shipment size but also depends on the energy (gasoline) price. It is assumed that, due to the damage in transit, each shipment has a percentage of defective items, γ , which is a random variable. That means, in each shipment, there are on average, $QE[\gamma]$ number of defective items and the rest are good items. As soon as the shipment is received, the buyer screens the lot completely at the rate of $\pi(>D)$ during the time t (where

$t = Q/\pi$) with a screening cost of χ per item. The segregated defective items are stored at the buyer's warehouse to be returned to the vendor; and the holding cost for the defective items is paid by the vendor. The holding cost of the good and defective items at buyer's warehouse are h_b and h_d , respectively, where $h_b > h_d$. Once the inventory level of the defective items in the buyer's warehouse reaches to $NQE[\gamma]$, they are shipped back to the vendor. In other words, the defective items are returned to the vendor in every N cycles of the buyer. For this product return, the transportation cost of U_v and the carbon emission cost of E_v incur to the vendor.

2.1 The Buyer's Total Expected Cost

There are five major costs for the buyer: transportation cost, setup cost, screening cost, holding cost, and carbon emission cost.

2.1.1 Transportation Cost

The price of commodities, such as oil, copper, sugar, gas, and electricity, shows a mean-reverting behavior (e.g., Clewlow and Strickland 2000; Hahn and Dyer 2008; Hull 2012; Pinto et al. 2007; Schwartz and Smith 2000). We use the following stochastic differential equation to represent the mean reverting process of the gasoline price.

$$d\theta_t = \beta(\mu - \theta_t)dt + \sigma dw_t. \quad (1)$$

In the above equation, θ_t is the logarithm of gasoline price, η_t , i.e., $\ln(\eta_t) = \theta_t$; β is the mean reversion coefficient; μ is the long-term mean price; σ is the volatility of the process; and dw_t is the standard Wiener process. The price shows a downward trend when the logarithm price, θ_t , is greater than μ and an upward trend when the logarithm price, θ_t , is less than μ . These upward and downward movements are greatly influenced by the mean reversion coefficient, β . Given the logarithm of gasoline price at time $t = 0$, θ_0 , the logarithm of gasoline price at time t , θ_t , is normally distributed with mean

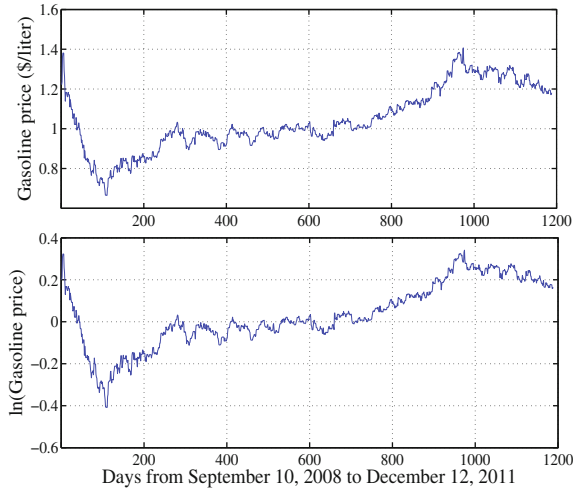
$$\mathbf{E}[\theta_t|\theta_0] = e^{-\beta t}\theta_0 + \mu(1 - e^{-\beta t}), \quad (2)$$

and variance

$$\mathbf{Var}(\theta_t|\theta_0) = (1 - e^{-2\beta t})\frac{\sigma^2}{2\beta}. \quad (3)$$

As a result, the gasoline price, η_t , is log—normally distributed with mean

Fig. 1 Gasoline price in Toronto, Canada, from September 10, 2008 to December 12, 2011



$$\mathbf{E}(\eta_t|\theta_0) = \exp\left\{e^{-\beta t}\theta_0 + \mu(1 - e^{-\beta t}) + (1 - e^{-2\beta t})\frac{\sigma^2}{4\beta}\right\}. \tag{4}$$

Since we are dealing with the long-term average cost in this supply chain model, as the time $t \rightarrow \infty$, the terms $e^{-\beta t}$ and $e^{-2\beta t}$ approach to zero. Hence, $\mathbf{E}(\eta_\infty|\theta_0) = \exp(\mu + \frac{\sigma^2}{4\beta})$. The gasoline price in Toronto, Canada, from September 10, 2008 to December 12, 2011 is presented in Fig. 1, which shows that the gasoline price (per liter) varies from 0.69 CAD to \$1.39 CAD. The required parameters of the mean reverting process are estimated using the procedure described in Dixit and Pindyck (1994). The estimated values of μ , β , and σ are 0.0240, 1.4787, and 0.2552, respectively.

Having defined the process for the gasoline price, we are going to derive the transportation cost per shipment under energy (gasoline) price uncertainty. Some studies such as Darwishi (2008), Ertogral et al. (2007), and Lee (1986) considered transportation cost in a joint vendor–buyer inventory model, and they assumed that the transportation cost depends only on the size of the shipment. However, in reality, it may also depend on energy (gasoline) price. In this chapter, we assume that the transportation cost not only depends on shipment size, but also depends on energy (gasoline) price. In order to incorporate the gasoline price uncertainty in the supply chain model, first we have to determine the gasoline consumption during the shipment. The gasoline consumption mainly depends on factors such as distance between the vendor and the buyer, actual weight of the vehicle including the weight of the shipment, age of the vehicle, condition of the vehicle, type of the vehicle, and traveling speed between the vendor and the buyer (Fitch 1994). However, for a given distance between the vendor and the buyer, a constant speed of traveling between the vendor and the buyer, and for a particular type and age of

the vehicle, the fuel consumption is linearly proportional to the weight of the shipment (Nylund and Erkkilä 2005). Moreover, as one understands, the weight of the shipment is proportional to the size of the shipment. Therefore, we consider a fixed gasoline consumption depending on all the factors except the size of the shipment; and a variable gasoline consumed per unit of the shipment. Let, f_b and λ_b be the fixed and variable gasoline consumptions in liters, respectively. Since, Q is the shipment size from the vendor to the buyer, $(f_b + \lambda_b Q)$ is the total volume of gasoline that will be burnt per shipment. Now, given that the average price of the gasoline is $\exp(\mu + \frac{\sigma^2}{4\beta})$, the buyer's transportation cost per shipment is given by

$$U_b = (f_b + \lambda_b Q)e^{(\mu + \frac{\sigma^2}{4\beta})}. \quad (5)$$

2.1.2 Setup Cost and Screening Cost

The buyer has to pay a fixed setup cost of S_b per order. As each shipment has a certain percentage of defective items, the shipment is screened at unit cost of χ to separate the good items from the defective items. Hence, the total screening cost is $Q\chi$ per buyer's cycle.

2.1.3 Holding Cost

The buyer has to pay the holding cost only for good items. The total inventory level of the good items per cycle is as follows:

$$\frac{\mathbf{E}[\gamma]Q^2}{2\pi} + \frac{Q\mathbf{E}(1 - \gamma)\mathbf{E}[T]}{2}. \quad (6)$$

Since only good items will satisfy the demand during a cycle, the cycle time is $T = (Q/D)(1 - \gamma)$ and the expected cycle time is $\mathbf{E}[T] = (Q/D)(1 - \mathbf{E}[\gamma])$. Now, substituting $\mathbf{E}[T] = (Q/D)(1 - \mathbf{E}[\gamma])$ in Eq. (6), the expected holding cost per unit time can be written as

$$\frac{h_b}{\mathbf{E}[T]} \left[\frac{\mathbf{E}[\gamma]Q^2}{2\pi} + \frac{Q^2\mathbf{E}[(1 - \gamma)^2]}{2D} \right]. \quad (7)$$

2.1.4 Carbon Emission Cost

During the shipment, though gases such as N_2O , CH_4 , and NH_3 have a small contribution to the greenhouse gases emission, the main source of greenhouse gases emission is CO_2 . The amount of the carbon emission depends on several

factors such as distance traveled, fuel consumption, quality of fuel, vehicle type, vehicle age, average speed, actual weight of the vehicle, road type, and climate (Sathaye et al. 2010). For given locations of the vendor and the buyer and a particular type of vehicle with an average speed between the vendor and the buyer, the amount of carbon emission per shipment can be considered to be fixed. On the other hand, the amount of carbon emission varies depending on the actual weight of the shipment. Consequently, we consider a fixed, ϑ_b , and a variable, Γ_b , carbon emission costs for the shipment from the vendor to the buyer. The fixed cost, ϑ_b , depends on the distance between the vendor and the buyer, vehicle type, speed, road type, and climate. The variable cost, Γ_b depends on the actual size of the shipment. Since the shipment size is Q , the total carbon emission cost per buyer's cycle can be expressed as

$$E_b = \vartheta_b + \Gamma_b Q. \quad (8)$$

Having defined the relevant costs of the buyer, we can express the total expected cost of per unit time as follows:

$$\begin{aligned} \mathbf{E}[TC(Q, n)]_b &= \frac{1}{\mathbf{E}[T]} \left(\frac{S_b}{n} + Q\lambda + \vartheta_b + \Gamma_b Q \right) + \frac{1}{\mathbf{E}[T]} \left[(f_b + \lambda_b Q) e^{(\mu + \frac{\sigma^2}{4\beta})} \right] \\ &+ \frac{h_b}{\mathbf{E}[T]} \left(\frac{\mathbf{E}[\gamma]Q^2}{2\pi} + \frac{Q^2 \mathbf{E}[(1-\gamma)^2]}{2D} \right). \end{aligned} \quad (9)$$

2.2 The Vendor's Total Expected Cost Per Unit Time

The vendor's total expected cost per unit time is the summation of holding cost for the average accumulation of items during the production run, holding cost for defective items that are stored at the buyer's warehouse, setup cost, and transportation cost, and carbon emission cost for bringing back the defective items from the buyer.

2.2.1 Holding Cost

During the production run, the cumulative inventory of the vendor is expressed as

$$I_v = \left\{ nQ \left(\frac{Q}{P} + (n-1)\mathbf{E}[T] \right) - \frac{n^2 Q^2}{2P} \right\} - \mathbf{E}[T][Q + 2Q + \dots + (n-1)Q]. \quad (10)$$

The vendor has to pay holding cost of h_v per unit per unit time for accumulated inventory. Hence, the holding cost per unit time is

$$H_{v_1} = \frac{h_v Q^2}{\mathbf{E}[T]} \left[\frac{1}{P} \left(1 - \frac{n}{2} \right) + \frac{1}{2D} (n-1)(1 - \mathbf{E}[\gamma]) \right]. \quad (11)$$

The holding cost for the defective items stored in the buyer's warehouse is paid by the vendor at a rate of h_d per unit per unit time. The expected number of defective items per buyer's cycle is $Q\mathbf{E}[\gamma]$ and this amount of segregated defective items is stored in the buyer's warehouse before shipping them together after N buyer's cycles. Hence, the total inventory of the defective items per shipment can be determined as follows:

$$I_{dv} = Q\mathbf{E}[\gamma]\mathbf{E}[T] + 2Q\mathbf{E}[\gamma]E[T] + \dots + NQ\mathbf{E}[\gamma]\mathbf{E}[T]. \quad (12)$$

Therefore, the expected holding cost per unit time for defective items can be written as

$$H_{v_2} = \frac{h_d Q\mathbf{E}[\gamma](1+N)}{2}. \quad (13)$$

2.2.2 Transportation Cost

As discussed in [Sect. 2.1.1](#), in order to ship the defective items accumulated at the buyer's warehouse, we consider a fixed and a variable volumes of the gasoline that will be burnt during the shipment. Let f_v and λ_v be the fixed and variable volumes (in liters) of gasoline, respectively. Since the vendor receives $NQ\mathbf{E}[\gamma]$ defective items in N cycles of the buyer, the transportation cost per unit time is

$$U_v = \frac{(f_v + \lambda_v NQ\mathbf{E}[\gamma])e^{(\mu + \frac{\sigma^2}{4\beta})}}{N\mathbf{E}[T]}. \quad (14)$$

2.2.3 Carbon Emission and Setup Costs

The vendor has a fixed setup cost of S_v per vendor's cycle. The vendor produces Q items per buyer's cycle and pays a carbon emission cost of K_v per unit of production. The vendor also pays a carbon emission cost for shipping back the defective items from the buyer. As discussed in [Sect. 2.1.4](#), we consider fixed and variable carbon emission costs for shipping back the defective items; and let ϑ_v and Γ_v be the fixed and variable carbon emission costs, respectively. Since the number of defective items in each shipment from the buyer to the vendor is $NQ\mathbf{E}[\gamma]$, the total emission cost is $E_v = (\vartheta_v + \Gamma_v NQ\mathbf{E}[\gamma])$ for every N cycles of the

buyer. Therefore, the vendor’s carbon emission and setup costs per unit time can be expressed as

$$\frac{S_v}{n\mathbf{E}[T]} + \frac{K_v Q}{\mathbf{E}[T]} + \frac{(\vartheta_v + \Gamma_v N Q \mathbf{E}[\gamma])}{N \mathbf{E}[T]}. \tag{15}$$

Finally, the vendor’s expected total cost per unit time is given by

$$\begin{aligned} \mathbf{E}[TC(Q, n, N)]_v &= \frac{h_v Q^2}{\mathbf{E}[T]} \left[\frac{1}{P} \left(1 - \frac{n}{2} \right) + \frac{1}{2D} (n - 1)(1 - \mathbf{E}[\gamma]) \right] + \frac{h_d Q \mathbf{E}[\gamma](1 + N)}{2} \\ &+ \frac{(f_v + \lambda_v N Q \mathbf{E}[\gamma]) e^{(\mu + \frac{\sigma^2}{4\beta})}}{N \mathbf{E}[T]} + \frac{1}{\mathbf{E}[T]} \left[\frac{S_v}{n} + \frac{\vartheta_v}{N} + \Gamma_v Q \mathbf{E}[\gamma] + K_v Q \right]. \end{aligned} \tag{16}$$

2.3 The Total Expected Cost Per Unit Time

The total cost per unit time for the coordinated supply chain is the sum of the total cost per unit time for the buyer and the vendor and can be written as

$$\mathbf{E}[TC(Q, n, N)] = \mathbf{E}[TC(Q, n)]_b + \mathbf{E}[TC(Q, n, N)]_v. \tag{17}$$

Expressions for $\mathbf{E}[TC(Q, n)]_b$ and $\mathbf{E}[TC(Q, n, N)]_v$ can be obtained from Eqs. (9) and (16), respectively, and substituted in Eq. (17) as follows:

$$\begin{aligned} \mathbf{E}[TC(Q, n, N)] &= \frac{1}{\mathbf{E}[T]} \left[\frac{1}{n} (S_b + S_v) + \left(\vartheta_b + \frac{\vartheta_v}{N} \right) \right] + \frac{Q}{\mathbf{E}[T]} (\chi + \Gamma_b + \Gamma_v \mathbf{E}[\gamma] + K_v) \\ &+ \frac{h_b}{\mathbf{E}[T]} \left(\frac{\mathbf{E}[\gamma] Q^2}{2\pi} + \frac{Q^2 \mathbf{E}[(1 - \gamma)^2]}{2D} \right) \\ &+ \frac{h_v Q^2}{\mathbf{E}[T]} \left[\frac{1}{P} \left(1 - \frac{n}{2} \right) + \frac{1}{2D} (n - 1)(1 - \mathbf{E}[\gamma]) \right] \\ &+ \frac{h_d Q \mathbf{E}[\gamma](1 + N)}{2} + \frac{1}{\mathbf{E}[T]} \left[\left(f_b + \frac{f_v}{N} \right) e^{(\mu + \frac{\sigma^2}{4\beta})} \right] \\ &+ \frac{Q}{\mathbf{E}[T]} \left[(\lambda_b + \lambda_v \mathbf{E}[\gamma]) e^{(\mu + \frac{\sigma^2}{4\beta})} \right]. \end{aligned} \tag{18}$$

Substituting $\mathbf{E}[T] = (Q/D)(1 - \mathbf{E}[\gamma])$, Eq. (18) can be written as

$$\begin{aligned}
\mathbf{E}[TC(Q, n, N)] &= \frac{D}{Q(1 - \mathbf{E}[\gamma])} \left[\frac{1}{n} (S_b + S_v) + \left(\vartheta_b + \frac{\vartheta_v}{N} \right) \right] \\
&+ \frac{D}{(1 - \mathbf{E}[\gamma])} (\chi + \Gamma_b + \Gamma_v \mathbf{E}[\gamma] + K_v) \\
&+ \frac{QD}{1 - \mathbf{E}[\gamma]} \left[\frac{h_b \mathbf{E}[\gamma]}{2\pi} + \frac{h_b \mathbf{E}[(1 - \gamma)^2]}{2D} + \frac{h_v}{P} \left(1 - \frac{n}{2} \right) \right] + \frac{h_v Q}{2} (n - 1) \\
&+ \frac{h_d Q \mathbf{E}[\gamma] (1 + N)}{2} + \frac{D}{Q(1 - \mathbf{E}[\gamma])} \left[\left(f_b + \frac{f_v}{N} \right) e^{(\mu + \frac{\sigma^2}{4\beta})} \right] \\
&+ \frac{D}{(1 - \mathbf{E}[\gamma])} \left[(\lambda_b + \lambda_v \mathbf{E}[\gamma]) e^{(\mu + \frac{\sigma^2}{4\beta})} \right].
\end{aligned} \tag{19}$$

Now, the objective is to determine optimal order quantity, Q^* , the optimal number of shipments to the buyer, n^* , and the optimal number of cycles that the defective items are accumulated at the buyer's warehouse, N^* , so that the total expected cost per unit time is minimized. Taking the partial derivative of $\mathbf{E}[TC(Q, n, N)]$ with respect to Q and setting it as zero, we have

$$\begin{aligned}
\frac{\partial \mathbf{E}[TC(Q, n, N)]}{\partial Q} &= -\frac{D}{Q^2(1 - \mathbf{E}[\gamma])} \left[\frac{1}{n} (S_b + S_v) + \left(\vartheta_b + \frac{\vartheta_v}{N} \right) \right] + \frac{h_v}{2} (n - 1) \\
&+ \frac{D}{(1 - \mathbf{E}[\gamma])} \left[\frac{h_b \mathbf{E}[\gamma]}{2\pi} + \frac{h_b \mathbf{E}[(1 - \gamma)^2]}{2D} + \frac{h_v}{P} \left(1 - \frac{n}{2} \right) \right] \\
&+ \frac{h_b \mathbf{E}[\gamma] (1 + N)}{2} - \frac{D}{Q^2(1 - \mathbf{E}[\gamma])} \left(f_b + \frac{f_v}{N} \right) e^{(\mu + \frac{\sigma^2}{4\beta})} = 0.
\end{aligned} \tag{20}$$

After rearranging the above equation, we obtain the following expression for the optimal shipment size:

$$Q^* = \sqrt{\frac{2\pi PD \left\{ N[(S_b + S_v) + n(\vartheta_b + \frac{\vartheta_v}{N})] + n(f_b N + f_v) e^{(\mu + \frac{\sigma^2}{4\beta})} \right\}}{nN[A_1 + A_2 + A_3]}} \tag{21}$$

where, $A_1 = Ph_b \{ D\mathbf{E}[\gamma] + \pi\mathbf{E}[(1 - \gamma)^2] \}$, $A_2 = \pi h_v \{ D(2 - n) + P(n - 1) (1 - \mathbf{E}[\gamma]) \}$, and $A_3 = h_d \pi P \mathbf{E}[\gamma] (1 - \mathbf{E}[\gamma]) (1 + N)$.

Taking the partial derivative of $\mathbf{E}[TC(Q, n, N)]$ with respect to n and setting it as zero, we obtain the following:

$$\frac{\partial \mathbf{E}[TC(Q, n, N)]}{\partial n} = -\frac{D(S_b + S_v)}{Qn^2(1 - \mathbf{E}[\gamma])} + \frac{h_v Q}{2} - \frac{h_v D Q}{2P(1 - \mathbf{E}[\gamma])} = 0. \tag{22}$$

After rearranging the above equation, we obtain the following expression for the optimal number of shipments from the vendor to the buyer:

$$n^* = \sqrt{\frac{2DP(S_b + S_v)}{h_v Q^2 (P(1 - \mathbf{E}[\gamma]) - D)}}. \quad (23)$$

Taking the partial derivative of $\mathbf{E}[TC(Q, n, N)]$ with respect to N and setting it as zero, we can write the following equation:

$$\frac{\partial \mathbf{E}[TC(Q, n, N)]}{\partial N} = \frac{h_d Q \mathbf{E}[\gamma]}{2} - \frac{D(f_v e^{(\mu + \frac{\sigma^2}{4p})} + \vartheta_v)}{QN^2(1 - \mathbf{E}[\gamma])} = 0. \quad (24)$$

This can be rearranged to obtain the optimal N^* as follows:

$$N^* = \sqrt{\frac{2D(f_v e^{(\mu + \frac{\sigma^2}{4p})} + \vartheta_v)}{h_b Q^2 (1 - \mathbf{E}[\gamma]) \mathbf{E}[\gamma]}}. \quad (25)$$

Substituting the values of n^* and N^* into Eq. (21), we rewrite the expressions for Q^* as follows:

$$Q^* = \sqrt{\frac{2\pi DP(f_b e^{(\mu + \frac{\sigma^2}{4p})} + \vartheta_b)}{B_1 + B_2 + B_3}}. \quad (26)$$

where $B_1 = Ph_b\{D\mathbf{E}[\gamma] + \pi\mathbf{E}[(1 - \gamma)^2]\}$, $B_2 = h_v\pi\{2D - P(1 - \mathbf{E}[\gamma])\}$, and $B_3 = h_d P\pi\mathbf{E}[\gamma](1 - \mathbf{E}[\gamma])$.

Substituting the value of Q^* into Eqs. (23) and (25), we rewrite the expressions for n^* and N^* as follows:

$$n^* = \sqrt{\frac{(S_b + S_v)(B_1 + B_2 + B_3)}{h_v \pi (f_b e^{(\mu + \frac{\sigma^2}{4p})} + \vartheta_b) (P(1 - \mathbf{E}[\gamma]) - D)}}. \quad (27)$$

$$N^* = \sqrt{\frac{(f_v e^{(\mu + \frac{\sigma^2}{4p})} + \vartheta_v)(B_1 + B_2 + B_3)}{h_d P \pi (f_b e^{(\mu + \frac{\sigma^2}{4p})} + \vartheta_b) \mathbf{E}[\gamma] (1 - \mathbf{E}[\gamma])}}. \quad (28)$$

Lastly, as it is given below, it can easily be shown that the expected total cost per unit time $\mathbf{E}[TC(Q, n, N)]$ is a convex function on Q^* , n^* , and N^* simultaneously. The Hessian matrix is given as:

$$\begin{bmatrix} \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial Q^2} & \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial Q \partial n} & \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial Q \partial N} \\ \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial Q \partial n} & \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial n^2} & \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial n \partial N} \\ \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial Q \partial N} & \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial n \partial N} & \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial N^2} \end{bmatrix}.$$

The elements of the Hessian matrix are:

$$\begin{aligned} \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial Q^2} &= \frac{2D}{Q^3(1 - \mathbf{E}[\gamma])} \left[\frac{1}{n}(S_b + S_v) + (\vartheta_b + \vartheta_v) \right] \\ &+ \frac{2D}{Q^3(1 - \mathbf{E}[\gamma])} \left(f_b + \frac{f_v}{N} \right) e^{(\mu + \frac{\sigma^2}{4\beta})} > 0, \end{aligned} \quad (29)$$

$$\frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial n^2} = \frac{2D(S_b + S_v)}{n^3 Q(1 - \mathbf{E}[\gamma])} > 0, \quad (30)$$

$$\frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial N^2} = \frac{(f_v e^{(\mu + \frac{\sigma^2}{4\beta})} + \vartheta_v)}{N^3 Q(1 - \mathbf{E}[\gamma])} > 0. \quad (31)$$

$$\frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial Q \partial n} = \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial n \partial Q} = h_v \left(1 - \frac{D}{(1 - \mathbf{E}[\gamma])P} \right) > 0, \quad (32)$$

$$\frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial Q \partial N} = \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial N \partial Q} = (1 - \mathbf{E}[\gamma])h_d > 0, \quad (33)$$

and

$$\frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial n \partial N} = \frac{\partial^2 \mathbf{E}[TC(Q, n, N)]}{\partial N \partial n} = 0, \quad (34)$$

and the second order condition is positive:

$$\frac{D \mathbf{E}[\gamma]^2 h_d^2 Q (S_b + S_v) \left(f_b e^{\left(\mu + \frac{\sigma^2}{4\beta} \right)} + \vartheta_b \right) \sqrt{\frac{D}{\mathbf{E}[\gamma](1 - \mathbf{E}[\gamma])h_d} \left(f_v e^{\left(\mu + \frac{\sigma^2}{4\beta} \right)} + \vartheta_v \right)}}{(1 - \mathbf{E}[\gamma]) \left(\frac{DP(S_b + S_v)}{h_v(D + (1 - \mathbf{E}[\gamma])P)} \right)^{3/2} \left(f_v e^{\left(\mu + \frac{\sigma^2}{4\beta} \right)} + \vartheta_v \right)} > 0$$

The algorithm to solve the above model is as follows:

Step 0. Determine Q using Eq. (26)

Step 1. Determine the optimal number of shipments n^* and N^* using Eqs. (27) and (28)

- Step 2. If both n^* and N^* are integers, go to Step 5
- Step 3. If both n^* and N^* are not integers, or either of them is not an integer, go to Step 4
- Step 4. For $n = \lfloor n \rfloor$ and $\lceil n \rceil$; and for $N = \lfloor N \rfloor$ and $\lceil N \rceil$ determine Q using Eq. (21), and then go to Step 5
- Step 5. Choose $n^* = n$, $N^* = N$, and $Q^* = Q$ as the optimal ones that give the minimum cost from Eq. (19), and then stop

3 A Numerical Example

In this section, a numerical example is presented to illustrate the model. Let production rate, $P = 200000$ units/year, demand rate, $D = 70000$ units/year, holding cost for vendor, $h_v = \$2/\text{unit}/\text{year}$, setup cost for vendor, $S_v = \$300/\text{cycle}$, fixed gasoline consumption for vendor, $f_v = 15$ l per shipment, variable gasoline consumption, $\lambda_v = 0.25$ l per unit of item in a shipment, fixed carbon emission cost for

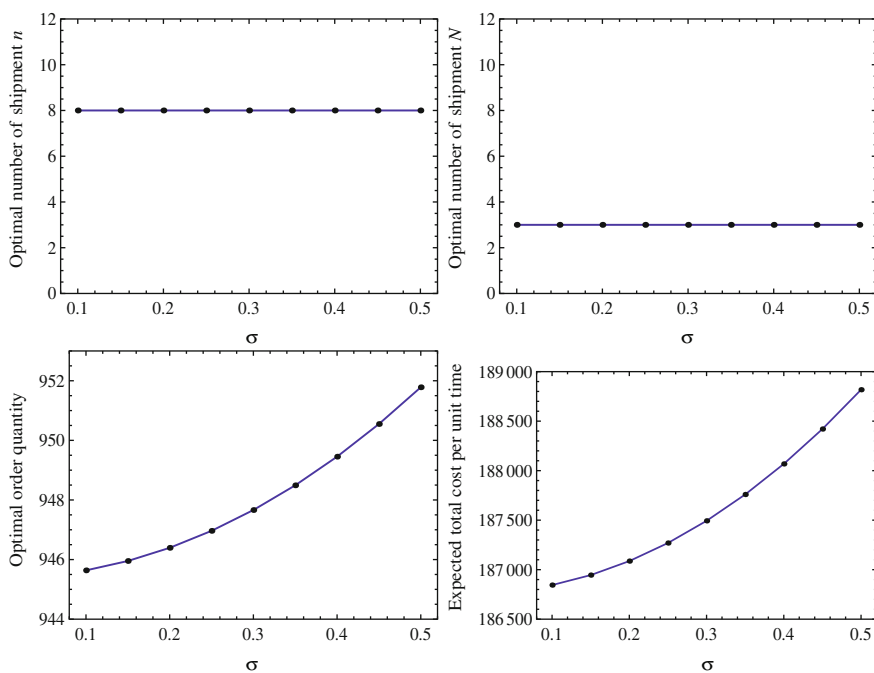


Fig. 2 The behavior of n^* , N^* , Q^* , and the expected total cost as the volatility of gasoline price, σ , changes

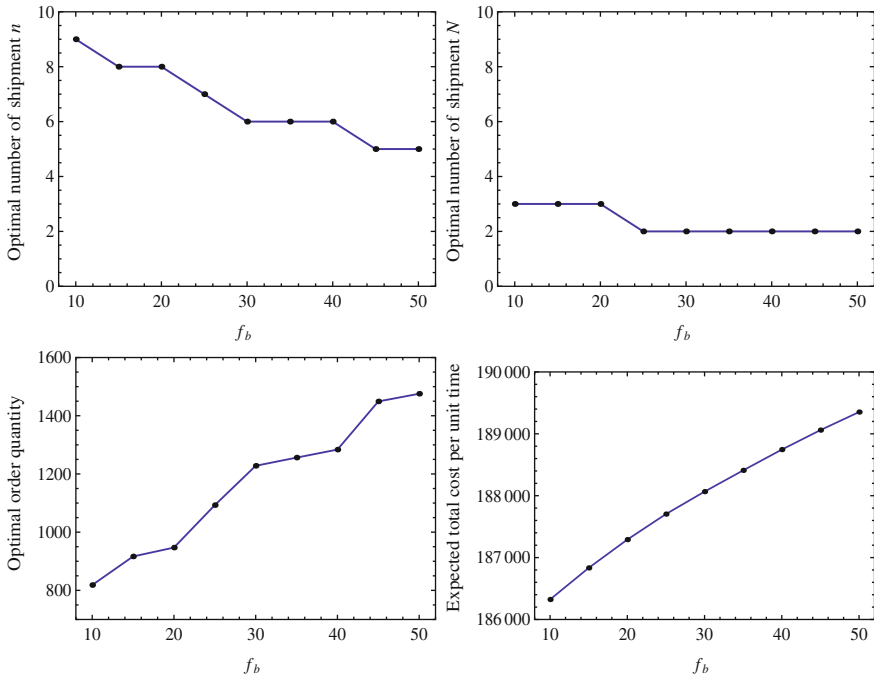


Fig. 3 The behavior of n^* , N^* , Q^* , and the expected total cost as the fixed gasoline consumption for the buyer, f_b , changes

vendor, $\vartheta_v = \$2.5$ per shipment, variable carbon emission cost for vendor, $\Gamma_v = \$0.3$ per unit of item in a shipment, carbon emission cost due to production, $K_v = \$0.4$ per unit, setup cost for buyer, $S_b = \$100/\text{order}$, holding cost for buyer, $h_b = \$4.5/\text{unit}/\text{year}$, screening rate, $\pi = 195200$ units/year, screening cost, $\chi = \$0.6/\text{unit}$, fixed gasoline consumption for buyer, $f_b = 20$ l per shipment, variable gasoline consumption, $\lambda_b = 0.5$ l per unit of item in a shipment, fixed carbon emission cost for buyer, $\vartheta_b = 5$ per shipment, variable carbon emission cost for buyer, $\Gamma_b = 0.5$ per unit of item in a shipment, and holding cost of defective items at buyer’s warehouse, $h_d = \$3/\text{unit}/\text{year}$. The percentage of defective items, γ , is uniformly distributed, i.e., $\gamma \sim U[c, d]$, where $E[\gamma] = (c + d)/2$, $E[1/(1 - \gamma)] = [1/(c - d)] \ln[(1 - d)/(1 - c)]$, and $E[(1 - \gamma)^2] = (1/3)(c^2 + cd + d^2) + (1 - c - d)$. We consider $c = 0$ and d is varied from 0.025 to 0.225.

We first analyze the effect of the gasoline price uncertainty on the decision variables. Gasoline price volatility, σ , is varied from 0.1 to 0.5 while assuming that the percentage of defective items, $E[\gamma]$, is 0.15. Figure 2 shows that, as the volatility of the gasoline price increases, n^* and N^* do not change but Q^* and the expected total cost increase.

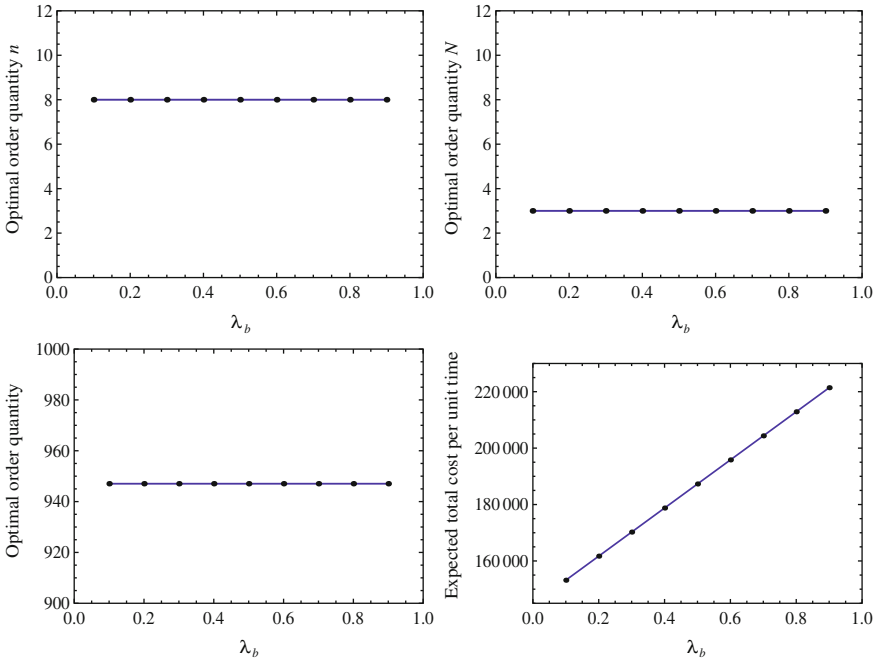


Fig. 4 The behavior of n^* , N^* , Q^* , and the expected total cost as the variable gasoline consumption per item for the buyer, λ_b , changes

We then study the influence of the fixed volume of gasoline, f_b , which will be burnt per shipment when the items are transported from the vendor to the buyer. It is considered that f_b depends on distance traveled, type of the vehicle type, age of the vehicle, etc. For a given vehicle, the vendor, and the buyer f_b are constant, but f_b will change when any of these factors change. For example, if the vendor’s location changes the distance between the vendor and the buyer will change, or when a different type of vehicle is used for shipment f_b will change. In this analysis, the value of f_b is varied from 10 to 50 l. As expected, Fig. 3 shows that, when the value of f_b increases, the optimal number of shipments, n^* , decreases. This indicates that as more gasoline is going to be burnt in shipping, one should decrease the frequency of transportation to minimize the cost. When the frequency of shipping decreases, shipment size, Q^* , will increase. As Q^* increases the proportion of the defective items will increase and this leads to a higher frequency (a lower N^*) of shipments of the defective items. Overall the expected total cost increases. Similarly, as the variable gasoline consumption per item for the buyer is varied from 0.1 to 0.9 liters, Fig. 4 shows that it only influences the expected total cost.

Next, as shown in Fig. 5, when the value of the fixed carbon emission cost for the buyer, ϑ_b , increases, the number of shipments decreases so as to reduce the

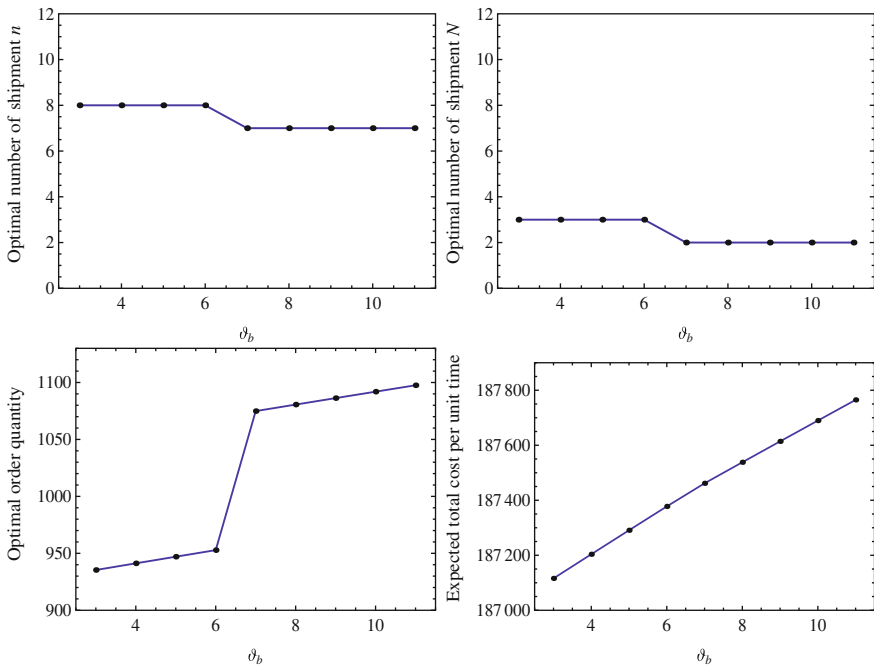


Fig. 5 The behavior of n^* , N^* , Q^* , and the expected total cost as the fixed carbon emission cost for the buyer, ϑ_b , changes

cost; this leads to a smaller value of N^* , a larger value of Q^* , and a higher expected total cost. Likewise, the influence of the variable carbon emission cost for the buyer, Γ_b , is investigated. Figure 6 indicates that it only effects the expected total cost.

Finally, how the percentage of defective items affects the supply chain decisions is also studied. From Fig. 7, it can be seen that when the percentage of defective items increases, the number of shipments increases to compensate for the defective items so that the demand can be met. When the number of shipments remains the same at 6, the order quantity increases, because more items are needed to compensate for the increasing defective items. However, when the number of shipments shifts from 7 to 8, the order quantity drops because of a larger number of shipments. Again, when the number of shipments remains at 8, the order quantity keeps increasing to compensate for a higher percentage of defective items. A higher percentage of defective items leads to a higher number of defective items and consequently the frequency of shipments of the defective item increases. In other words, N^* , which is the number of buyer’s cycles during which the defective items are stored at the buyer’s warehouse, decreases. Again, as one would expect, the expected total cost increases when the percentage of defective items increases.

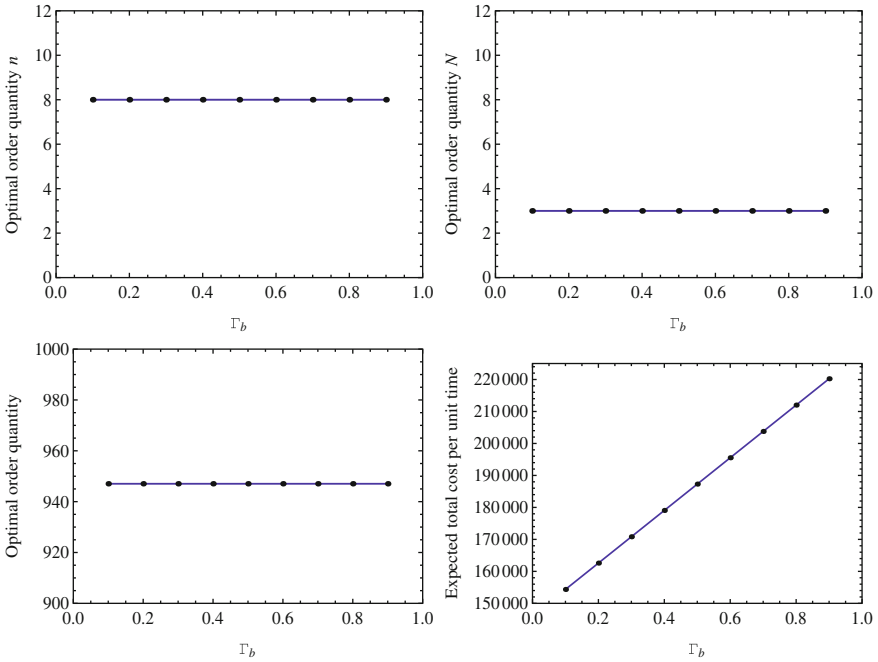


Fig. 6 The behavior of n^* , N^* , Q^* , and the expected total cost as the variable carbon emission cost for buyer, Γ_b , changes

4 Conclusion

A coordinated supply chain is studied under energy (gasoline) price uncertainty and defective items in transshipment. The expression for the optimal order quantity, the optimal number of shipments, the number of buyer’s cycles during which the defective items are stored at the buyer’s warehouse, and the expected total cost per unit time are derived. Examples are presented to show the effects of gasoline price uncertainty, fixed and variable carbon emission costs, and percentage of defective items on the supply chain coordination.

Transportation is always a part of the supply chain coordination and the transportation cost depends on the gasoline price. However, the literature has ignored how recent gasoline price uncertainty influences the supply chain coordination. The results show that, in a coordinated supply chain, when the gasoline price uncertainty increases, the shipment size increases and hence the total supply chain cost increases. Moreover, it can also be seen that the increment in the shipment size is not good enough to trigger a higher number of shipments. Since the gasoline price plays a role in the supply chain coordination, another aspect that has been investigated in this chapter is how the gasoline consumption of the vehicle used for transportation affects the supply chain coordination. With respect

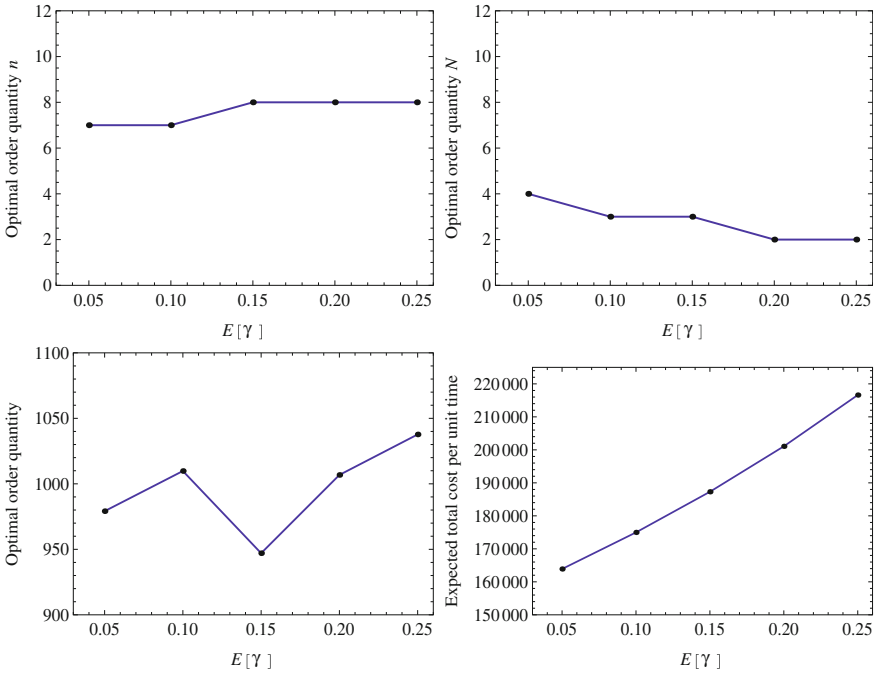


Fig. 7 The behavior of n^* , N^* , Q^* , and the expected total cost as the percentage of defective items, $E[\gamma]$, changes

to the gasoline consumption, fixed and variable gasoline consumptions are considered. The fixed gasoline consumption basically depends on the vehicle type, size, age, etc. The results show that when the vehicle used for transportation changes, the decisions on the supply chain coordination change. For example, if a vehicle with a higher fixed gasoline consumption is used, the number of shipments from the vendor to the buyer becomes smaller and this leads to a larger shipment size. Moreover, the larger shipment size increases the amount of defective items at the buyer’s warehouse and hence the frequency of returning the defective items from the buyer to the vendor increases. With respect to the variable gasoline consumption, it only increases the total cost of the supply chain. Lastly, effect of carbon emission cost, a factor related to the gasoline consumption, is also investigated using fixed and variable carbon emission costs. It is observed that the effect of the fixed carbon emission cost is similar to that of the fixed gasoline consumption. Likewise, the effect of the variable carbon emission cost is similar to that of the variable gasoline consumption.

The future research can be directed in many ways. For example, demand uncertainty, lead time uncertainty, and multistage supply chain model can be investigated in this framework.

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