

An EOQ-Based Spare Parts Network Design

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Abstract The aim of this chapter is to combine two types of problems: a facility location problem, that determines where a facility should be set up in order to serve a set of customers, and a EOQ-based spare parts inventory problem, which seeks to determine the optimal stocking levels of a collection of spare parts. The theoretical findings are practiced on a case for projectors of a Belgian Digital Cinema Projector producer, serving customers located all over the world. The goal is to minimize the total costs under four scenarios, which differ mainly in terms of SLA's and modes of transport. The most remarkable difference between these total costs is related to the number of opened facilities, the safety stock that is required, and the transportation modes that can be used in the considered scenario.

1 Introduction

The digital cinema projector (DCP) producer underlying this location problem, wants to offer service contracts which guarantee a service level agreement (SLA) of 4, 6, 8, or 18 h, where the different settings to fulfill the latter differ mainly in terms of the transportation mode. We developed an integrated model to determine the total number of facilities to be set up and to obtain their respective total costs. For the demand information for the spare parts we used historical breakdown data

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provided by the DCP producer. When reviewing the results, it can be concluded that the average costs decrease over the scenarios with the lesser stringent SLA's, mainly due to the location pooling effect, the use of cheaper transportation modes and the smaller number of facilities.

2 Problem Setting

The technology for digital cinema projectors was developed quite some time ago. However, the implementation was a long time coming because of the high expenses that had to be defrayed by the integrators that take care of the distribution, the setup, and the maintenance of post-production technology and digital cinema projection. The movie studios encourage the use of digital equipment because they can realize big savings when customers switch to equipment that can handle digital sources. After some profound discussions, the Virtual Print Fee system was introduced to support the investment of digital projection equipment by the integrators with a fee for each movie performance. When a projector is defective, it is of utmost importance that the number of cancelled movie performances are minimized.

In the USA, a service network was rolled out in 2010 to allow demanding customers to subscribe to a repair service with SLA's of 2 or 4 h. Thirteen facilities were opened initially, but this number was raised to 31 in order to let more customers benefit from the service.

Following the USA example, the DCP manufacturer also wants to set up a similar network in Europe. The European customers are less demanding and until now, the SLA is agreed to be 18 h. In the remainder of this chapter, we will analyze the situation for an SLA of 4, 6, and 8 h and we will compare the total costs and the assignment of customer orders with the current SLA of 18 h.

Some problems came up when developing the model: there is almost no historical data available to analyze because of the recentness of the projectors and the exponentially increasing installed base. The perpetual changes and improvements make it almost impossible to extrapolate the collected data from the past.

3 Literature

Our problem relates to two lines of research, which are combined in this chapter. The first section discusses earlier work where models were developed for the facility location problem and the second section gives an overview of spare parts literature. The last literature section presents rather recent models that combine aspects from the two previous categories.

3.1 The Facility Location Problem

The facility location problem has been discussed in a multitude of studies. The first formal introduction was by Weber and dates back to the beginning of the twentieth century (Weber 1929). A location problem can be defined as a spatial resource allocation problem where one tries to determine the location of one or more service facilities (“servers”) to serve a spatially distributed set of demands (“customers”). The objective is to locate facilities and to allocate customers to the servers to optimize an objective function, such as the minimization of the average travel time (adapted from Brandeau and Chiu 1989). Either a network or a plane (chart) can be used as a basis for the location problem. However, this choice determines the way distances are measured. According to Owen and Daskin (1998) the optimal location of the servers is an important aspect of the company’s strategy because of its long-term nature and its financial impact. Robustness of the network is also desirable to minimize the impact of some unknown factors such as market trends and regulation. More recently, Geographic Information Systems are often used because they can contribute to location analysis and modeling (Murray 2010).

Facility location problems can be subdivided into three categories (Owen and Daskin 1998):

- The first category covers the *static, deterministic problems*, which was the main focus of the earlier work because of computational barriers due to complex formulations. All inputs being considered as fixed, the applicability to real-world situations was quite limited. These problems can further be subdivided into three types of problems. Median problems determine the effectiveness of a facility by considering the average distance (Church and ReVelle 1974). They have been under study since the introduction by Hakimi (1965). Neema et al. (2011) present some new genetic algorithms for the p-Median problem in continuous space. They conclude that GA-based models can be practical when also information of Geographic Information Systems (GIS) can be incorporated. Covering problems consider the maximum distance that can be covered (Current et al. 1990; White and Case 1974). The center problems do not use an exogenous maximal distance but let the model decide the distance endogenously such that the predetermined number of facilities can serve all customers (Tansel et al. 1983).
- *Dynamic location problems* try to take some future uncertainty into account to create a (near-)optimal solution for a particular time horizon. The input parameters are fixed and known or deterministically varying over time. Because investment costs are usually quite large, it is an absolute necessity to consider an extended time horizon. The problem boils down to the selection of a robust set of locations, to serve demand that is subject to changes over time, and the appropriate choice of the timing for facility expansions and relocations over the longer term. Ballou (1968) makes use of static deterministic optimal solutions to locate only one warehouse such that the profits over the considered time period are maximized. Scott (1971) enhanced the model of Ballou by considering

multiple facilities. Contreras et al. (2011) discuss the Dynamic Uncapacitated Hub Location Problem (DUHLP) to minimize the total costs over a finite time horizon while all demand has to be routed through the network for each period and solve it by an exact branch-and-bound algorithm.

- The input parameters of *stochastic location problems* are characterized by random variables. The aim of stochastic location problems is to generate several solutions for each considered scenario to obtain a robust solution that performs well. These problems can be subdivided into probabilistic models and scenario planning models. The former of these two categories consider probability distributions explicitly, while the latter start from an already specified set of possible future values for the variables. Bell et al. (2011) tackle the problem of uncertain demand for the location optimization of alert sites for homeland defense, where response times should be kept as low as possible when highly uncertain events occur.

ReVelle and Eiselt (2005) suggest another way of classifying the models by contemplating the space of location decisions. They distinguish between location problems in the planar spaces and network location problems, which affects the way distances are measured. In planar spaces, distances are often measured by using Minkowski distances, according to $d_{ij}^p = [|a_i - a_j|^p + |b_i - b_j|^p]^{1/p}$ with p indicating the number of dimensions of the solution space. On the other hand, distances in networks are determined from the network characteristics itself and are calculated by making use of the Dijkstra shortest path algorithm (Dijkstra 1959).

3.2 Spare Parts Inventories

According to Kennedy et al. (2002) inventories can serve multiple purposes. On the one hand, raw material, work-in-process, and finished product inventories are directed to smooth out irregularities that are typical of production processes (e.g., machine break downs and different processing speeds) and to cope with changes in consumer demand or supply patterns. On the other hand, spare parts inventories are mandatory when one wants to guarantee the continuity of the production process. The pursued policy determines whether a repair or a replacement is preferred, which is linked with the time interval available to put the machine back into operation. Historical data about part failures is not always readily available and the costs that go with a failure are often difficult to estimate. When machines get out of date, the availability of spare parts also becomes a pressing difficulty. They also distinguish preventive repair, where techniques like just-in-time can be used, from unplanned repair, where it is necessary to keep a stock of the most crucial parts.

A typical issue for spare parts inventories is the distinction that has to be made between single- and multi-echelon inventory control policies (Hausman and Erkip

1994). In a single inventory control policy each warehouse at the lowest echelon is independently responsible for its own stocking policies. Once every individual policy is fixed, an aggregated order is placed at the next higher level, where the next single-echelon inventory control policy is observed. In a multi-echelon inventory control policy, all parameters, and the interaction effects are considered simultaneously which yields a global solution. This type of inventory models are frequently used when customers are geographically dispersed (Lee 1987). Kukreja et al. (2001) analyzed an analytical model to determine the optimal inventory stocking levels for a single-echelon inventory control policy with a number of locations on a continuous-review basis where lateral transshipments are allowed. The emergency lateral transshipments are only considered when customer demand can be met this way and the time that a demand would be backordered is reduced (Lee 1987). Moreover, the complete pooling results in a significant cost reduction. Rappold and Muckstadt (2000) study an inventory system for one product in a two-echelon production and inventory setting in a lean environment. The production quantity, the allocation of the inventory and the order levels are determined in view of the minimization of the long run average system costs. An algorithm was developed to solve large-scale problems. Sun and Zuo (2013) determine the stock level of aircraft spare parts in a multi-echelon inventory control policy. They conclude that it is more obvious to use a negative binomial distribution than a Poisson distribution to characterize a non-stationary demand process.

Most parts are characterized by low demand and high costs. In a recent study, Jalil et al. (2010) stress the potential economic value of installed base data and they consider data quality issues related to installed base data. In a case study at IBM, they conclude that savings up to 58 % could be realized when the installed base for spare parts is tracked properly. Pinçe and Dekker (2011) discuss a continuous review inventory system of a slow moving item that is characterized by a drop in demand at a known future time instance. Cohen and Lee (1990) state that a competitive advantage can be obtained by offering superior after sales services to the customers. Besides the set up of warehouses and the definition of their capacity, the number of echelons, the replenishment policy, the customer allocation, and the granted priorities to the customer groups should also be taken into consideration. Hua et al. (2007) bring the forecasts of demand for spare parts into focus to determine the optimal inventory levels of spare parts in warehouses. Syntetos et al. (2012) discuss which distributional assumptions are appropriate to model demand of spare parts and verify the goodness-of-fit of a number of distributions by using three empirical datasets. Hollier (1980) developed a method to rank the spare parts according to the ratio of the total expected usage to the costs of acquisition and the savings made by having the spare parts close at hand instead of having to obtain them through a normal distribution network. Cohen et al. (1986) consider how to manage low demand products in a multi-echelon inventory system. A formula was developed to decide on the optimal inventory levels while minimizing the total system costs. Cohen et al. (1992) look at the trade-off between a fast response time and the elimination of inventory. This resulted in the classification of customer demand into urgent and standard demand.

3.3 *Integrated Models*

Integrated models cover both the strategic, tactical and operational part of the spare parts management (Miranda and Garrido 2004). These models can be used to analyze realistic scenarios. The Distribution Network Design with Risk Pooling Effect model from Miranda and Garrido combines a facility location model with some crucial elements from inventory management (e.g., the Economic Order Quantity and the risk pooling effect) (Miranda and Garrido 2004). This mixed integer model is nonlinear, for which a heuristic was developed that was based on Lagrangian relaxation and a subgradient method. This model only takes one product and one transportation mode into account. The total cost reduction is higher as the holding costs, ordering costs, lead times, and the service level increase; the beneficial risk pooling effect is easily noticeable. Jayaraman (1998) describes the Facility Location, Inventory, Transportation NETWORK model (FLITNET-model) where different products and transportation modes are taken into account. This linear model does not take into account the safety stock, which makes it relatively easy to generate a solution. Shen et al. (2003) developed a model that takes the safety stock and the risk pooling effect into account. The customers can also serve as a distribution center for customers in their neighborhood. Nozick and Turnquist (2001) take the customer responsiveness as a starting point. They try to find an optimal level of inventory centralization when trying to find a balance between customer responsiveness and cost reductions. Karsten et al. (2012) apply some cooperative game theory concepts to consider several decision makers who independently stock expensive, low-demand, and repairable spare parts and who can collaborate by inventory pooling. Mete and Zabinsky (2010) present a stochastic optimization approach for the storage and distribution problem of medical supplies that are used for disaster management. The decisions in this setting are characterized by uncertainties and incomplete information.

4 Model

4.1 *The Miranda and Garrido Model*

The integrated model discussed in this section incorporates decisions related to the set up of locations and the inventory of the spare parts. The model described by Miranda and Garrido (2004) starts from a given situation where a factory delivers one particular product, which is characterized by high holding costs and a high variability in demand, to a set of facilities. We opt for the Miranda and Garrido model as a basis for the DCP model because it perfectly fits the need for our case study. Some of the arguments will be dealt with in Sect. 5 where the case study is exposed in detail. Each facility f must satisfy the demand of the customers, who

place orders in a stochastic way, with a yearly demand characterized by an average of D_f and a variance U_f . The safety stock for each facility depends on the variance in customers' demand. When the demand of the customers is aggregated, the necessary safety stock decreases by \sqrt{n} due to the effect of location pooling when demand is not correlated (Lambrecht 2006). An assumption is made with regard to the costs, consisting of the costs to open the facilities, the transportation costs and the holding costs, which are born by only one of the contracting parties. The model is based on the Economic Order Quantity Model, in which each facility f observes a stochastic demand pattern characterized by an average yearly demand value D_f and a variance U_f . Every facility follows a continuous review policy (Q_f, RP_f) . The reorder point RP_f indicates when it is obligate to place an order of size Q_f which increases the inventory after LT_f time units. The decisive factors of the reorder point are thus the demand during lead time and the probability that the demand during lead time can be fulfilled. Therefore,

$$RP_f = D_f \cdot LT_f + z_{1-\alpha} \cdot \sigma_f \cdot \sqrt{LT_f} \tag{1}$$

The average holding costs $HC_{f,tot}$ for a facility f are determined by the holding costs of the safety stock and the average cycle stock.

$$HC_{f,tot} = HC_f \cdot z_{1-\alpha} \cdot \sqrt{LT_f} \cdot \sqrt{U_f} + HC_f \cdot \frac{Q_f}{2} \tag{2}$$

The operational costs OK_f are composed of the unit transportation costs RC_f , the ordering costs OC_f , that are incurred for each order placement Q_f , and the elapsed time TP_f between two consecutive orders.

$$OK_f = RC_f \cdot Q_f + OC_f + HC_{f,tot} \cdot TP_f \tag{3}$$

The formula for the total costs also requires the introduction of a binary variable z_f , which is set to one when a facility is set up, zero otherwise, and y_{fc} , which is used to assign customers to facilities. The total costs consist of the fixed costs F_i to set up a facility, the holding costs for the safety stock, and the average cycle stock and the transportation costs, which consist of the shipping costs from the facility to the customer TC_{fc} , the shipment costs between the factory and the facility RC_f and the ordering costs OC_f/Q_f .

$$TC = \sum_f F_i \cdot z_i + \sum_f HC_f \cdot z_{1-\alpha} \cdot \sqrt{LT_f} \cdot \sqrt{U_f} + \sum_f HC_f \cdot \frac{HC_f}{2} + \sum_f \sum_c \left(TC_{fc} + RC_f + \frac{OC_f}{Q_f} \cdot d_c \cdot y_{fc} \right) \tag{4}$$

When taking the derivative of TC with respect to Q_f , the following expression is obtained.

$$\frac{d(TC)}{d(Q_f)} = \frac{HC_f}{2} - \frac{OC_f}{Q_f^2} \cdot \sum d_c \cdot y_{fc} \quad (5)$$

$$Q_f^* = \sqrt{\frac{2 \cdot OC_f \cdot D_f}{HC_f}} \quad (6)$$

We substitute Q_f^* into the total cost formula and assume that d_c is the average demand of customer c . Then we obtain the following expression:

$$\begin{aligned} TC = & \sum_f F_i \cdot z_i + \sum_f \sum_c (TC_{fc} + RC_f) \cdot d_c \cdot y_{fc} + \sum_f \sqrt{2 \cdot HC_f \cdot OC_f} \cdot \sqrt{D_f} \\ & + \sum_f HC_f \cdot z_{1-\alpha} \cdot \sqrt{LT_f} \cdot \sqrt{U_f} \end{aligned} \quad (7)$$

4.2 The DCP Model

The model has to be revisited in order to meet some presumed criteria of the DCP producer. As already described above, the DCP producer wants to meet a specific service level for the on-time delivery of the requested spare parts. A solution to the problem gives the total costs incurred for the DCP producer and furthermore the assignment of the customer orders to a specific facility. We will develop four scenarios. Because the model for the fourth scenario is only slightly different from the model of the first three scenarios, the fourth model will only be discussed briefly. An overview of the model is given in Fig. 1.

In a first step, the spare parts will be sent from the factory to the central hub and during this type of transport, fixed pre-transport costs are incurred for every shipment. In other words, the hub, where all inventory is stored, is the spill and the starting point of the designed network.

The set up of the hub entails a fixed cost which consists of a monthly stand-by fee and yearly insurance charges. When the spare parts are received at the hub, a small quality check is performed to verify the condition of every part after which the parts are stored. The storage of inventory also entails a fixed cost which depends on the number of stored pallet boards and we assume that only one product type is stored on every pallet board. Every 6 months, the stock items are counted at the hub. When items are shipped from the hub, they are delivered either by air transports or by sprinter transports. In the first case, the parcel is prepared at a fixed cost, after which a sprinter transport delivers the parcel at the nearest airport. The cost of air transportation depends on the volumetric weight of the parcel. After the flight, the parcel is transported by a sprinter to the customer. Another way of shipping the spare parts from the hub is by delivering them

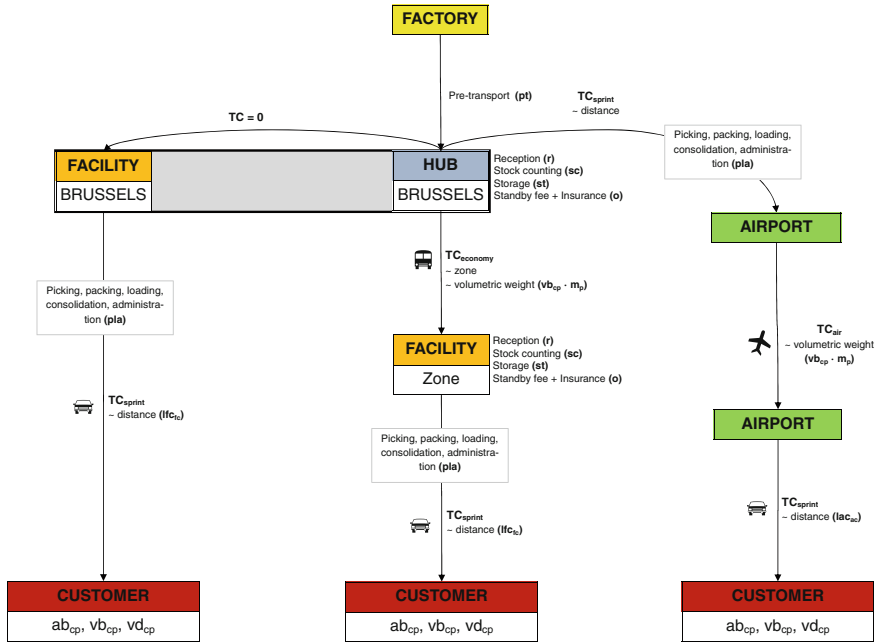


Fig. 1 Overview of the model

directly to the customer via sprinters. In this case, the parcel has also to be prepared at the hub after it is delivered to a dummy facility. This facility is introduced in the model to assign customers to the hub, so no set up costs, transportation costs from the hub to this facility, and no inventory costs are incurred. The last possibility is the delivery of parcels by introducing some extra facilities. These facilities receive their replenishment orders from the hub via economy transport. For these facilities, some fixed costs have to be taken into account: the fixed set up, reception, stock counting, and weekly storage costs are calculated in the same way as for the hub. When a parcel leaves a facility, a call is made to a sprinter transport.

A summary of the used sets can be found in Table 1.

For the first four sets, we keep track of the city name, which will enable us to visualize the solution in the end. The set of facilities and the set of the customers

Table 1 Sets and data

Sets	Data
Hubs (h)	City
Facilities (f)	City, zone
Customers (c)	City, zone, number of orders/year ab , average demand/order vb , variance vd
Airports (a)	City
Products (p)	Name, cost, weight

Table 2 Decision variables

Symbol	Description
a_{fp}	Pallet board for product p in facility f , binary
b_{hp}	Pallet board for product p in hub h , binary
DF_{fp}	Aggregated average demand for product p in facility f
DH_{hp}	Aggregated average demand for product p in hub h
U_{fp}	Aggregated variance of demand for product p in facility f

contain the same elements because a facility can only be set up where the customers are located. For both facilities and customers, we need to keep track of the zone to calculate the shipment rates that have to be applied. Elements of the set of customers are also characterized by their average yearly number of orders and by the average and the variance of their order size. Every element of the products set is characterized by its product name, its cost, and its volumetric weight. These characteristics influence shipment and storage decisions.

An overview of the decision variables in the model can be found in Table 2.

The parameters of the model are briefly described in Table 3.

The model is given below through Eqs. 8–30 and will be discussed below.

$$\min \quad \sum_f o \cdot z_f \quad (8a)$$

$$+ \sum_h o \cdot w_h \quad (8b)$$

$$+ \sum_h \sum_a \sum_c \sum_p pt \cdot ab_{cp} \cdot x_{hacp} \quad (8c)$$

$$+ \sum_h \sum_a \sum_c \sum_p r \cdot ab_{cp} \cdot vc_{cp} \cdot x_{hacp} \quad (8d)$$

$$+ \sum_f \sum_c \sum_p pt \cdot ab_{cp} \cdot y_{fcp} \quad (8e)$$

$$+ \sum_f \sum_c \sum_p 2 \cdot r \cdot ab_{cp} \cdot vb_{cp} \cdot y_{fcp} \quad (8f)$$

$$+ \sum_h \sum_p st \cdot b_{hp} \quad (8g)$$

$$+ \sum_h \sum_p sc \cdot b_{hp} \quad (8h)$$

$$+ \sum_f \sum_p st \cdot a_{fp} \quad (8i)$$

Table 3 Model parameters

Symbol	Description
ab_{cp}	Average yearly number of orders of product p placed by customer c
e	Time margin when products are shipped by air transportation
HC_p	Inventory holding costs for product p
k	Cost price per pallet board
lac_{ac}	Distance from an airport a to a customer c
lfc_{fc}	Distance from a facility f to a customer c
LTf_f	Order lead time for facility f
LTH_h	Order lead time for hub h
m_p	Product weight
mks	Minimum cost level for a sprinter transport
o	Fixed set up costs of a facility f or hub h
OC_f	Order costs when an order is placed at hub h
OC_h	Order costs when a hub places an order at the factory
pla	Fixed costs to collect the items and prepare the shipping documents
pt	Pre-transport costs when transporting from the factory to the hub
r	Reception costs
s	Sprinter rate per kilometre
sc	Stock counting costs
st	Storage costs
T	Time limit
tac_{ac}	Travel time from airport a to customer c
tbl	Travel time from the hub to the airport
tfc_{fc}	Travel time from facility f to customer c
TKE_{fg}	Economy transport costs, depending on the zone of the addressee and the parcel weight
TK_{fg}	Next Day Delivery transport costs
v	Air transportation rate per volumetric weight unit
vb_{cp}	Average order quantity of product p for customer c
vd_{cp}	Demand variance of product p for customer c
vs	Flying speed
vva	Distance between two airports (by using Haversine formula)
$z_{1-\alpha}$	Definition of the service level

$$+ \sum_f \sum_p sc \cdot a_{fp} \tag{8j}$$

$$+ \sum_h \sum_a \sum_c \sum_p pla \cdot ab_{cp} \cdot x_{hacp} \tag{8k}$$

$$+ \sum_h \sum_a \sum_c \sum_p mks \cdot ab_{cp} \cdot x_{hacp} \tag{8l}$$

$$+ \sum_h \sum_a \sum_c \sum_p ab_{cp} \cdot vb_{cp} \cdot m_p \cdot v \cdot x_{hacp} \tag{8m}$$

$$+ \sum_h \sum_a \sum_c \sum_p ab_{cp} \cdot \max(lac_{ac} \cdot s, mks) \cdot x_{hacp} \quad (8n)$$

$$+ \sum_f \sum_c \sum_p \sum_g TKE_{fg} \cdot ab_{cp} \cdot y_{fcp} \quad (8o)$$

$$+ \sum_f \sum_c \sum_p pla \cdot ab_{cp} \cdot y_{fcp} \quad (8p)$$

$$+ \sum_f \sum_c \sum_p ab_{cp} \cdot \max(lfc_{fc} \cdot s, mks) \cdot y_{fcp} \quad (8q)$$

$$+ \sum_f \sum_p \sqrt{2 \cdot HC_p \cdot OC_f \cdot DF_{fp}} \quad (8r)$$

$$+ \sum_h \sum_p \sqrt{2 \cdot HC_p \cdot OC_h \cdot DH_h} \quad (8s)$$

$$+ \sum_f \sum_p HC_p \cdot z_{1-\alpha} \cdot \sqrt{LTF_f} \cdot \sqrt{U_{fp}} \quad (8t)$$

$$+ \sum_h \sum_p HC_p \cdot z_{1-\alpha} \cdot \sqrt{LTH_h} \cdot \sqrt{V_{hp}} \quad (8u)$$

such that

$$\forall c \in C, \forall p \in P : \quad \sum_f y_{fcp} + \sum_h \sum_a x_{hacp} = 1 \quad (9)$$

$$\forall f \in F, \forall c \in C, \forall p \in P : \quad y_{fcp} \cdot t_{fc} \leq T \quad (10)$$

$$\forall h \in H, \forall a \in A, \forall c \in C, \forall p \in P : \quad x_{hacp} \cdot \left(tbl + \frac{vv_a}{vs} + tac_{ac} + e \right) \leq T \quad (11)$$

$$\forall f \in F, \forall p \in P : \quad \sum_c y_{fcp} \leq M \cdot z_f \quad (12)$$

$$\forall f \in F, \forall p \in P, \forall h \in H : \quad \sum_c y_{fcp} \leq M \cdot w_h \quad (13)$$

$$\forall h \in H, \forall a \in A, \forall p \in P : \quad \sum_c x_{hacp} \leq M \cdot \dot{w}_h \quad (14)$$

$$\forall f \in F, \forall p \in P : \quad y_{fcp} \leq M \cdot a_{fp} \quad (15)$$

$$\forall h \in H, \forall a \in A, \forall p \in P : \quad \sum_c x_{hacp} \leq M \cdot b_{hp} \quad (16)$$

$$\forall f \in F, \forall p \in P : \sum_c ab_{cp} \cdot vb_{cp} \cdot y_{fcp} = DF_{fp} \tag{17}$$

$$\forall h \in H, \forall p \in P : \sum_a \sum_c ab_{cp} \cdot vb_{cp} \cdot x_{hacp} = DH_{hp} \tag{18}$$

$$\forall f \in F, \forall p \in P : \sum_c vd_{cp} \cdot y_{fcp} = U_{fp} \tag{19}$$

$$\forall h \in H, \forall p \in P : \sum_a \sum_c vd_{cp} \cdot x_{hacp} = V_{hp} \tag{20}$$

$$\forall f \in F : z_f \in \{0, 1\} \tag{21}$$

$$\forall h \in H : w_h \in \{0, 1\} \tag{22}$$

$$\forall f \in F, \forall c \in C, \forall p \in P : y_{fcp} \in \{0, 1\} \tag{23}$$

$$\forall h \in H, \forall a \in A, \forall c \in C, \forall p \in P : x_{hacp} \in \{0, 1\} \tag{24}$$

$$\forall f \in F, \forall p \in P : a_{fp} \in \{0, 1\} \tag{25}$$

$$\forall h \in H, \forall p \in P : b_{hp} \in \{0, 1\} \tag{26}$$

$$\forall f \in F, \forall p \in P : DF_{fp} \geq 0 \tag{27}$$

$$\forall h \in H, \forall p \in P : DH_{hp} \geq 0 \tag{28}$$

$$\forall f \in F, \forall p \in P : U_{fp} \geq 0 \tag{29}$$

$$\forall h \in H, \forall p \in P : V_{hp} \geq 0 \tag{30}$$

The objective function contains 21 terms, which represent the different costs. Terms (8a) and (8b) represent the set up costs for the hub and the facilities, which consist of the yearly fixed insurance charges and a monthly stand by fee. The pre-transport costs are given in terms (8c) and (8e), which represent the costs for the shipments from the factory to the hub. As all orders that are delivered from facilities passed by the hub, these costs have to be taken into account for this type of transports. Terms (8d) and (8f) give us the reception costs which are proportional to the number of parts sent. When parts are shipped from a facility to the customer, these costs are incurred twice. Terms (8g)–(8j) show us the costs attached to the storage of the goods and the biannual stock counting of the available parts, which are only determined by the number of pallet boards. Terms (8k)–(8n) are associated with the costs to make use of air transportation: the items need to be picked first (8k), after which a sprinter service transports the parcel to the nearest airport (8l). The parcel is loaded into the airplane (8m) and finally, a sprinter delivers the parcel to the end customer (8n). When the requested parts

depart from a facility, the parts need to be available in the facilities, which are in turn supplied by the hub, which entails an economy transport to deliver the parts in advance to the facilities (8o). The parcels need to be prepared, which is represented in (8p), and shipped to the customer, which is shown by (8q). The inventory and the safety stock are both stored in the hub and the facilities, for which the EOQ-model laid the foundation of the calculations (8r)–(8u).

The constraints will be explained below. The first constraint (9) ensures that each order will be handled by either the hub or a facility. Constraints (10) and (11) impose a time limit on the delivery of the requested parts where T represents the requested time limit, fixed for each considered scenario. When a parcel leaves a facility, only the sprinter time needs to be taken into account (10). When a part is shipped directly from the hub to a customer by using air transport, the part must be shipped from the hub to the nearest airport, loaded and unloaded, and shipped to the end customer through a sprinter transport (11). Constraints (12) and (14) are needed to enforce the set up of facilities that supply customers. Constraint (13) demands the set up of the hub even when all parts are shipped to the customers through facilities. Constraints (15) and (16) are needed to ensure that the number of pallet boards at the facilities and the hub equals the number of different requested parts, which is obligate to calculate the costs that are associated with the inventory stock counting and the storage. Constraints (18)–(21) are needed to model the total yearly average demand (D) and variance (U or V) for each requested product in a facility or the hub. These variables are needed to calculate the holding costs for the average cycle stock and the safety stock. The last bunch of constraints (22)–(30) is required to establish the range of the variables, which can be either binary or a positive real number.

5 Case Study

A DCP producer wants to set up a network for the delivery of spare parts to its European customers following the already unfolded US network. At the moment, about 16,000 movie theatres worldwide are equipped with digital projectors originated at the company under study. In the US, a network consisting of 13 facilities ensures that the requested spare parts are delivered within 2 or 4 h to a selected number of customers. In order to offer the same service to more customers, the number of facilities will be increased to 31 in the nearby future. The company has to contend with data shortage with regard to historical failures and an exponentially increasing installed base, which makes it hard to produce meaningful forecasts that determine proper inventory levels for each facility. Based on the argument of the aggregated demand for several closely located customers and the fact that the installed base will increase exponentially in the coming years, a normally distributed demand for these parts will make sense in the future. Therefore, the assumptions of the Miranda and Garrido make sense in our case (Miranda and Garrido 2004). In Europe, SLA's with delivery times of 4, 6, and 8 h

are set against the current delivery time of 18 h. Because no specific potential locations for the set of facilities are given, we assume that every customer can also serve as a proxy for a facility to supply customers in their neighborhood. Note that in many cases just renting a single room, even at a customer's premises may suffice. Many parts are small and do not take much space. The model that is described above in Sect. 3 will be used for the scenarios with a SLA of 4, 6, and 8 h. The model will also be slightly adjusted to be able to compare the desired situation with the current situation with an SLA of 18 h.

5.1 Data Collection and Analysis

All data were extracted from the BAAN ERP system. For each spare part request, a specific order number, the order date, the client identification code and its name and address, and the item and item description were stored. The original data set consisted of 34,552 order lines, and due to incomplete information in some cells, around 95 % of the data was used in the further analysis. The data was obtained from and verified by employees of the DCP producer.

Third party logistics providers take care of the delivery of the requested parts to the customers and offer four types of transport. The first type of transport is economy transport, with a shipping time of one to five days, and is typically used for the shipment of less urgent parcels. Shipments of this type of transport are characterized by the zone of the addressee and the volumetric weight of the parcel. A second type of transport is called Next Day Delivery (NDD) transport. NDD transport is used so far and delivers parcels to the customers within 18 h. The costs are calculated in the same way as for the first type of transport. A third way of transporting goods is when sprinters are used. Sprinters collect the parcel and deliver it to the customer right after. When the transport costs are lower than the postulated minimum costs, the shipment costs equalize that minimum cost. Otherwise, the transport cost amounts the actual costs. The last type of transport under consideration is air transportation. The only determinant for this type of transport is the volumetric weight of the parcel. Due to the small and lightweight of some spare parts, this option is an interesting alternative.

Information regarding the exact flow of the requested parts is rather tacit knowledge and this knowledge was passed on during a number of interviews. During the research, we were in close contact to verify the correctness of the findings.

5.2 Calculation of the Distances

The distance calculation is an important portion of our model input. A first possibility is to calculate the distances by using the coordinates of the cities that are

obtained from Google Maps or similar software packages. When using the Pythagorean Theorem ($a^2 + b^2 = c^2$), the Euclidean distances are easily and quickly calculated. Another way of making use of the coordinates is to calculate the distance with the Haversine formula that factors in the three-dimensional shape of the earth.

$$\begin{aligned}
 R &= 6371 \text{ km} \\
 \Delta\varphi &= \varphi_1 - \varphi_2 \\
 \Delta\lambda &= \lambda_1 - \lambda_2 \\
 a &= \sin\left(\frac{\Delta\lambda}{2}\right) + \cos(\lambda_2) \cdot \cos(\lambda_1) \cdot \sin^2\left(\frac{\Delta\lambda}{2}\right) \\
 c &= 2 \cdot \text{atan2}\left(\sqrt{a}, \sqrt{1-a}\right) \\
 \text{Distance} &= R \cdot c
 \end{aligned}$$

When a new city is considered, the distances can easily be recalculated. The disadvantage, however, is that the distances are rough estimations and they can tremendously deviate from the actual distances. A better way to calculate the actual distances between cities is when the Google Maps API or the Microsoft Mappoint package is used. A Java application that is based on the Google Maps API was implemented and used to read in two lists of cities between which the distances were calculated. When Microsoft Mappoint is used in combination with Winwaed Software Technology LLC's MPMileage, lists can be read in and the distances can be calculated automatically and offline. Moreover, the data can be visualized in maps and the parameters can be set to specific conditions, e.g., the speed of the vehicle on certain types of roads and automatic rest stops. The time and distance matrices are more or less symmetrical. In this paper, we used the Haversine formula to calculate distances between airports, Microsoft Mappoint for sprinter distances, and the Java application for some extra checks.

6 Results

In this section, the generated results will be discussed for the scenarios under study. The academic version of IBM ILOG CPLEX Studio IDE was used to solve the model for a selection of 12 representative parts. On the basis of the results for this basket, conclusions were drawn for the complete set of parts, although full deployment for all spare parts is waiting for some stability in the level of the installed base. Figures 2 and 3 show the allocation of the customers to the hub and the facilities for the considered scenarios. A green dot is placed on the map when a facility is set up and a purple pushpin indicates that customers receive their spare parts from the facility within the blue drawing. The blue outline indicates the drive time zone, enclosing all cities that can be reached within a given time frame which

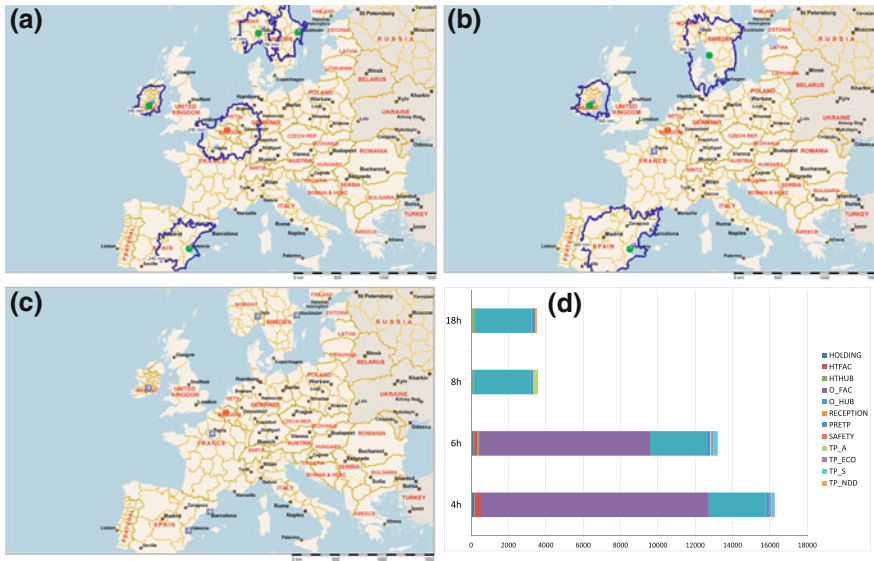


Fig. 2 Results for one single product. **a** Service level agreement of 4 h. **b** Service level agreement of 6 h. **c** Service level agreement of 8 h. **d** Break down analysis of the total costs

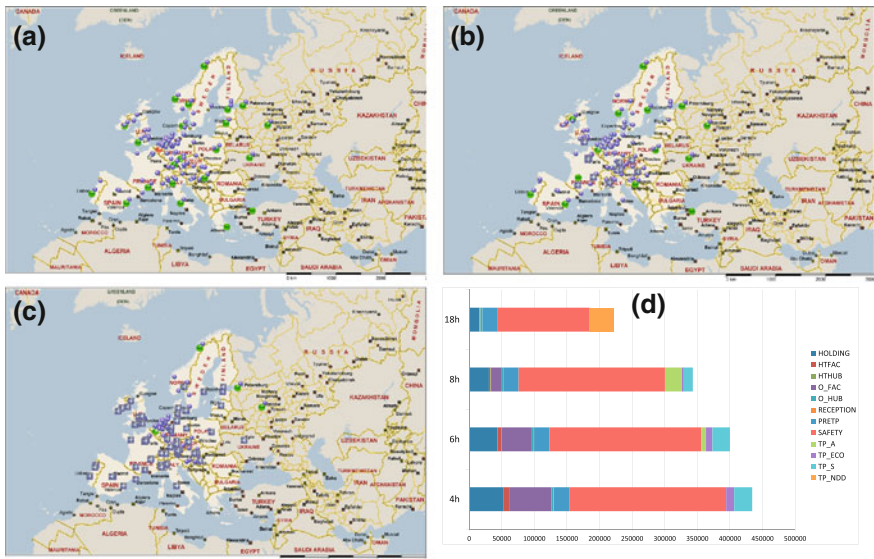


Fig. 3 Results for a Basket of 12 Products. **a** Service level agreement of 4 h. **b** Service level agreement of 6 h. **c** Service level agreement of 8 h. **d** Break down analysis of the total costs

is determined by the SLA of the scenario. When air transportation is used, the addressee is indicated by a symbol of an airplane.

One randomly selected spare part will be analyzed thoroughly to explain the rationale. The selected part has a rather low cost price, a low volumetric weight of 0.67 kg and a rather low average annual demand. The biggest portion of the total costs of the first scenario is related to the set up of four facilities and the hub to supply six customers. The safety stock costs are rather low which can be traced back to the low demand variance of this spare part. When the SLA is relaxed, the number of set up facilities decreases and more air transportation is used to supply the parcels to the customers. The third and fourth scenario have similar total costs.

In Table 4, some numbers and figures are presented for the basket of 12 products. The set up costs for the hub and the pre-transport costs remain unchanged when the SLA changes. Other costs really depend on the considered scenario: as the SLA becomes less stringent, a smaller number of facilities has to be opened. The set up costs for facilities diminish and the holding costs, the storage and stock counting costs, and the safety stock costs decline because of the inventory pooling effect. It is also noticeable that the requested modes of transport hinge on the considered scenario: for the scenario with an SLA of 18 h, only NDD transport is used, while the other types of transport—air transport, economy transport, and sprinter transport—are excluded. For the other scenarios, it is the other way around, and it can be noticed that there is a general trend toward air transport instead of economy and sprinter transport when comparing the scenarios with an SLA of 6 and 8 h and the scenario with an SLA of 4 h.

As the digital cinema market is not yet saturated and still expanding, the future will dictate which of the given scenarios is the best choice. A first step will probably be to open a small number of locations and to deliver the spare parts by using air transport, while in the future, it will be advisable to have a larger number of facilities and to deliver the parts by sprinter transports.

Table 4 Total costs for the basket of twelve products

	4 h	6 h	8 h	18 h
Holding costs	€ 53,019	€ 43,311	€ 30,340	€ 15,599
Stock counting and storage costs facilities	€ 9,399	€ 6,622	€ 2,421	€ 0
Stock counting and storage costs hub	€ 0	€ 499	€ 855	€ 1,709
Opening costs facilities	€ 64,050	€ 45,750	€ 15,250	€ 0
Opening costs hub	€ 3,050	€ 3,050	€ 3,050	€ 3,050
Reception costs	€ 426	€ 426	€ 426	€ 0
Pre-transport costs	€ 23,540	€ 23,540	€ 23,540	€ 23,540
Safety stock	€ 2,40,830	€ 2,33,160	€ 2,24,460	€ 1,41,140
Air transport costs	€ 0	€ 7,103	€ 25,928	€ 0
Economy transport costs	€ 12,318	€ 9,726	€ 3,022	€ 0
Sprinter transport costs	€ 27,620	€ 26,869	€ 13,823	€ 0
NDD transport costs	€ 0	€ 0	€ 0	€ 37,206
Total costs	€ 4,34,252	€ 4,00,056	€ 3,43,115	€ 2,22,244

Given that a lot of the spare parts are small, the definition of a facility should definitely be broadened: a small storage compartment will be sufficient to store the parts. This also implies that the volumetric weight of the packed is low enough to prefer air transport over sprinters when the costs turn out to be lower.

7 Conclusion

In this paper, a model for spare parts was developed to organize the distribution of the parts for a DCP producer. A solution was obtained for four different scenarios, which gives the total costs and the allocation of the customers to the facilities. We can conclude that the number of facilities decreases as the required delivery time increases: every facility supplies more customers which leads to a decrease of the total costs due to the avoidance of set up costs and due to the savings with regard to the average cycle and safety stock. The total costs are also influenced by the transportation costs, which is especially important for this sector. One can notice a shift toward air transportation as the delivery time is relaxed. Our next steps involve getting all the data for all parts as soon as the full range of installed base gets to a more or less stable volume. Subsequently an implementation of the model is foreseen.

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