EOQ Models with Supply Disruptions

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Abstract Most of the early research in inventory theory concentrates purely on demand uncertainty. However, models which aim to capture the dynamics of real-world systems must also take uncertainties in the supply side into consideration. One type of supply uncertainty that has attracted considerable attention during the past decade is supply disruptions, such as those that arise as a result of customs delays, labor strikes, and natural disasters. Over the past several years, companies have developed many strategies to mitigate the effects of such disruptions. One strategy is to hold more inventory with the additional amount serving as a buffer against disruptions. Since it is among the most basic inventory models, the EOQ model features prominently in the earliest work on disruptions, as well as many subsequent models. This chapter summarizes the studies on EOQ models with supply disruptions.

1 Introduction

An Icelandic volcano eruption in 2010 resulted in the shutdown of Europe's airspace for a number of days, causing delays in air freight shipments for many multi-national companies. A number of factories flooded due to the Japanese earthquake and tsunami in 2011, causing production to be halted. Floods in Thailand, in 2011, severely affected high-tech supply chains, resulting in shortages

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of some key components and shutdowns of significant processes. These are some recent examples of *supply disruptions* that have caused supply chains to stop functioning properly for considerable period of time.

Snyder et al. (2012) define supply disruptions as random events that cause a supplier or other elements of the supply chain to stop functioning either completely or partially, for a random duration of time. As the examples above suggest, these random events can have significant operational effects resulting in severe financial loses (Hendricks and Singhal 2003, 2005a, b). In order to avoid or reduce the overall impact, supply chain practitioners need to improve the way in which they run their supply chains.

There are multiple disruption mitigation strategies that companies can choose from (Tomlin 2010). One of them is to hold more safety stock with the additional inventory serving as a buffer against disruptions. Most research on evaluating this strategy considers single-location systems (although some more recent papers consider multi-echelon systems). Since it is among the most basic inventory models, the economic order quantity (EOQ) model serves as ideal starting point. The assumption of deterministic demand enables us to isolate the pure effects of supply disruptions and to obtain results and insights that can assist in solving more complex problems.

In this chapter, we discuss the EOQ model with disruptions. The exact formulation and an approximation for the simplest model, with external disruptions only, are explained in Sect. 2. In Sect. 3, we present the EOQ model with both external and internal disruptions. We briefly discuss other extensions of the EOQ model with disruptions in Sect. 4.

2 The EOQ Model with External Supply Disruptions

2.1 The Exact Model

We first consider the classical EOQ model with a single retailer and a single item. Demand is deterministic and continuous (with a rate of d items per unit time) and production or delivery is instantaneous. There is a fixed cost, K, per order and a holding cost, h, per item per unit time. In the absence of disruptions, the average cost per unit time, the optimal order quantity and the corresponding Optimal average cost per unit time are as in the following proposition.

Proposition 1 *The average cost per unit time, the optimal order quantity and the optimal average cost for the classical EOQ model are*

$$C(Q) = \frac{Kd}{Q} + \frac{hQ}{2}, \quad Q^* = \sqrt{\frac{2Kd}{h}}, \quad C(Q^*) = \sqrt{2Kdh}$$

One implicit assumption of the classical EOQ model is the perfect reliability of the supplier. Parlar and Berkin (1991) are the first to relax this assumption by considering the possibility of supply disruptions. They assume that the supplier functions normally for a certain amount of time and then is disrupted for a certain amount of time. We refer to the disrupted times as *dry* intervals and times during which the supplier functions normally as *wet* intervals (Other authors sometimes refer to these as *off* and *on* intervals, *down* and *up* intervals, etc.). The retailer cannot receive any items from the supplier during dry intervals. Unlike the classical EOQ, the retailer will stock out on customer demands that occur when the retailer does not have any inventory and its supplier is in a dry interval. We assume that unmet demands are lost, and that lost sales incur a cost of p per item.

It is possible to make different assumptions about the transitions from one state to the other, but for the sake of simplicity, we assume that the transitions between dry and wet intervals are governed by a continuous-time Markov chain (CTMC). The duration of dry and wet intervals is exponentially distributed with rates μ (known as the *recovery rate*) and λ (known as the *disruption rate*), respectively.

The EOQ problem with disruptions is known as the EOQD. The inventory curve for it is as pictured in Fig. 1.

Parlar and Berkin (1991) derive an expression for the expected cost per unit time and prove its convexity. However, their analysis contains some errors. First, they assume that stockouts occur every time the supplier is disrupted, but in fact, it is possible for a disruption to begin and end entirely during an interval in which the retailer has positive inventory. The other error is that, they account for the lost sales cost as though it is incurred per item per unit time, rather than simply per item. Berk and Arreola-Risa (1994) correct these mistakes and present the results that we discuss below.

Define the time between successive orders as a *cycle* with a random length, T. If the supplier is in a wet interval when the retailer places an order, we have $T = \frac{Q}{d}$. Otherwise, the retailer needs to wait for a positive duration of time until the disruption is over. Define β to be the probability that the supplier is in a dry

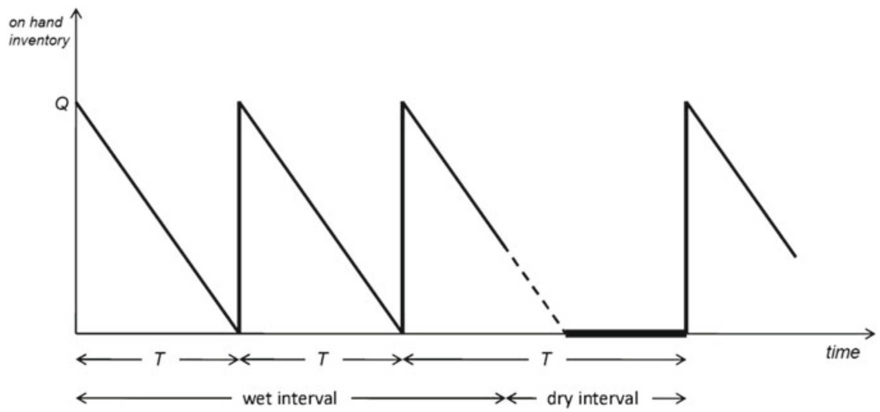

Fig. 1 Inventory curve for EOQD model (Parlar and Berkin 1991)

interval when the retailer places a replenishment order. Using the properties of the underlying CTMC it can be shown that

$$\beta = \frac{\lambda}{\lambda+\mu}\left(1 - e^{-(\lambda+\mu)\frac{Q}{d}}\right)$$

The probability density function of T, $f(t)$, is the following:

$$f(t) = \begin{cases} 0, & \text{if } t < Q/d \\ 1-\beta, & \text{if } t = Q/d \\ \beta\mu e^{-\mu\left(t-\frac{Q}{d}\right)}, & \text{if } t > Q/d \end{cases}$$

Based on this function, we can say that each cycle lasts for at least $\frac{Q}{d}$ time units and after that, with probability β, it lasts, on average, an additional $\frac{1}{\beta}$ time units. This implies that the expected cycle length is $E[T] = \frac{Q}{d} + \frac{\beta}{\mu}$.

As in the classical EOQ model, we want to find an expression for the expected cost per unit time and then determine the order quantity Q that minimizes this cost. Given that we know $E[T]$ we can determine an expression for the expected cost per cycle and then make use of the Renewal Reward Theorem to find the expected cost per unit time.

The total order cost and the Holding cost per cycle are the same as in the classical EOQ model: $K + cQ$ and $h\frac{Q^2}{2d}$, respectively. In addition, we have a penalty cost for lost sales arising when the inventory level is zero and the supplier is in a dry interval. Due to the memoryless property of the exponential distribution, the remaining duration of a dry interval after a replenishment order is given by $\frac{1}{\mu}$. Hence, the expected penalty cost per cycle is $p\frac{d\beta}{\mu}$. As a result, the overall expected cycle cost and the expected cost per unit time are given via the following proposition.

Proposition 2 *The expected cycle cost for the EOQ model with exponential disruption and recovery rates* (λ *and* μ, *respectively*) is:

$$K + cQ + h\frac{Q^2}{2d} + p\frac{d\beta}{\mu}$$

The corresponding expected cost per unit time is given by

$$C(Q) = \frac{K + cQ + h\frac{Q^2}{2d} + p\frac{d\beta}{\mu}}{\frac{Q}{d} + \frac{\beta}{\mu}}$$

Next, we want to determine Q^*, which is the order quantity that minimizes $C(Q)$. It is not known whether $C(Q)$ is convex. It is, however, quasiconvex, which implies that it has a single local minimum. Nevertheless, there is, unfortunately, no closed-form expression for Q^*, primarily due to the exponential terms within β (Recall that β is itself a function of Q). Numerical techniques must be used to determine Q^*.

Consistent with the classical EOQ model, Berk and Arreola-Risa (1994) demonstrate numerically that Q^* is nondecreasing in $K, p,$ and d. In addition, it is nondecreasing in the *availability ratio* $\frac{\lambda}{\mu}$, which implies that the retailer orders more when its supplier is disrupted more frequently and/or for longer intervals.

2.2 An Approximation

Although the EOQD can be solved numerically, an approximate closed-form solution is still attractive since it can be used in solving other problems that require the optimal order quantity or cost as an input. In addition, a closed-form solution can provide insights that might be difficult to obtain from numerical methods. To this end, Snyder (2011) introduces a simple method that approximates the cost function by a convex function. In particular, the author approximates β with a new term β' which ignores the exponential term:

$$\beta' = \frac{\lambda}{\lambda + \mu}$$

In fact, β' is the probability that the supplier is in a dry interval at an arbitrary point in time, while β is the probability that the supplier is in a dry interval when a replenishment order is placed. By replacing β with β', the transient behavior of the system is ignored and it is assumed that the system approaches steady state very quickly. This approximation performs quite well when the Cycle length is relatively long, i.e., $\frac{Q}{d}$ is relatively large.

As pointed out above, the approximation leads to an expected cost function that is convex, and whose minimizer can be expressed in closed-form by setting the derivative of the cost function to 0 and solving for Q. The following proposition summarizes the approximate result.

Proposition 3 *Approximating* β *with* $\frac{\lambda}{\lambda+\mu}$, *the order quantity to minimize* $C(Q)$ *becomes*

$$Q' = \sqrt{\frac{2Kd}{h} + A^2 + B} - A$$

where

$$A = \frac{\beta' d}{\mu} \quad \text{and} \quad B = \sqrt{\frac{2d^2 p \beta'}{h\mu}}$$

Recall that the optimal order quantity for the classical EOQ model is $\sqrt{\frac{2Kd}{h}}$, implying that—Q' is larger. The same relation holds for the optimal cost. Snyder (2011) also demonstrates that ignoring the possibility of supply disruptions and using the order quantity from the classical EOQ model can be very costly if the disruption risk is nontrivial.

3 The EOQ Model with External and Internal Supply Disruptions

The simplest EOQD model takes into consideration only disruptions at the supplier, i.e., external disruptions. Next, we consider a retailer that faces random disruptions both internally and externally. In case of an internal disruption all inventory at the retailer is destroyed, and the retailer cannot place a new order until the disruption is over. Examples of these types of disruptions include fires, machine breakdowns resulting in damaged items, and so on. The transitions between dry and wet intervals for the internal disruptions are similarly governed by a CTMC. The duration of these dry and wet intervals is exponentially distributed with recovery rate γ and disruption rate α, respectively.

We retain all the assumption in the previous section regarding the external disruptions and system parameters. Our objective, as in Sect. 2, is to find the optimal order quantity. This problem is studied by Qi et al. (2009) and the following results are based on their analysis.

We again define the time between two successive orders as a cycle and develop an expression for the expected cost function using the Renewal Reward Theorem. The expected cost function $C(Q)$ is the sum of the expected ordering, holding, and lost-sales costs, divided by the expected cycle length.

Proposition 4 *The expected cost function for the EOQ model with external and internal Disruptions is*

$$C(Q) = pd + \frac{K + \left(c + \frac{h}{\alpha}\right)Q - \left(1 - e^{-\alpha\frac{Q}{d}}\right)\left(\frac{hd}{\alpha^2} + \frac{pd}{\alpha}\right)}{E[T]}$$

Here, $E[T]$ is the expected cycle length which is itself a messy function depending on the disruption parameters, as well as on Q and d. It can be shown that $C(Q)$ is quasiconvex in Q. As a result, like the EOQD, the optimal order quantity can be found using any method for solving single-dimensional unconstrained quasiconvex optimization problems, such as bisection or golden section search. However, one cannot derive a closed-form expression for Q^*.

Using a similar idea as that of Snyder (2011), Qi et al. (2009) propose an effective approximation for the average cost function. They derive an approximate but closed form expression for the optimal order quantity by replacing one exponential term in the objective function with zero and another with its second-order Taylor-series expansion. The corresponding approximate Optimal order quantity, Q', is given by

$$Q' = d\frac{-\bar{A} + \sqrt{\bar{A}^2 + \frac{2\alpha(\bar{A}+\bar{B})\left(\frac{\alpha K\bar{B}}{d} + \bar{A}(p-c)\right)}{c\alpha+h}}}{\alpha(\bar{A}+\bar{B})}$$

where

$$\bar{A} = \frac{\lambda(\alpha + \gamma)}{\gamma\mu(\alpha + \lambda + \mu)} \quad \text{and} \quad \bar{B} = \frac{1}{\alpha} + \frac{1}{\gamma}$$

Qi et al. (2009) show that, when the retailer is never disrupted, i.e., when $\alpha = 0$, Q' reduces to the approximate solution derived by Snyder (2011) (see Sect. 2.2), and when neither the retailer nor the supplier is disrupted, Q' reduces to the classical EOQ solution.

Qi et al. (2009) compare the optimal order quantity of the EOQ model, Q_{EOQ} and Q'. In fact, the difference $Q' - Q_{EOQ}$ can be defined as the safety stock that the retailer holds to protect against both types of disruptions. This safety stock increases with the supplier's disruption probability and it decreases with the supplier's recovery probability. On the other hand, the retailer tends to keep small or even negative safety stock when the retailer is often disrupted. The reason is that the internal disruptions destroy the retailer's inventory. In fact, Q' is small when the retailer is disrupted very often or the supplier has high availability.

The authors also compare the effects of both types of disruptions on the fill rate and conclude that internal disruptions have a greater impact than external ones. This result is in line with the conclusion by Atan and Snyder (2012), who state that in one-warehouse, multiple-retailer (OWMR) systems with disruptions, uncertainty in the part of the supply chain closer to the customers has a more significant negative impact than uncertainty farther upstream. As a result, one can conclude that when both the retailer and its supplier are subject to disruptions, although both disruption types have significant effects and one needs to consider both to achieve cost savings, disruptions at the retailer have a much larger impact on the fill rate at the retailer than disruptions at the supplier do.

The approximation by Qi et al. (2009) is used by Qi et al. (2010) in the context of a joint location-inventory model with disruptions. The approximation enables the optimal inventory cost to be a concave function of the demand, and this property allows Qi et al. (2010) to apply an effective algorithm in solving their optimization problem. This is analogous to the way in which Daskin et al. (2002) embed the cost of the classical EOQ model into a joint location-inventory model without disruptions.

4 Extensions of the EOQD Model

The EOQD is the simplest continuous-review model with Supply disruptions, but its solution allows practitioners to have a basic understanding of the effects of supply disruptions on inventory management decisions. This model is extended in multiple ways. In this section, we discuss a few such extensions with fewer mathematical details.

4.1 Disruptions in Manufacturing Environments

In addition to Supply chains, manufacturing environments are also subject to disruptions, resulting from machine breakdowns or maintenance requirements. Compared to disruptions caused by natural disasters, labor strikes, etc., disruptions on the manufacturing floor tend to be more minor. Keeping inventory buffers to mitigate the effects of these disruptions is the commonly employed strategy.

In this section, we discuss an unreliable manufacturing process studied by Groenevelt et al. (1992a, b). The demand is deterministic, continuous and constant with rate d items per unit time. The production process is continuous with rate P items per unit time. The classical economic manufacturing quantity (EMQ) model assumes that inventory accumulates during production intervals and is depleted until the inventory reaches level zero. Then, production begins again. However, in the unreliable process, we assume that when a machine breakdown takes place the interrupted lot is aborted and the next production interval begins when the inventory is depleted. Figure 2 depicts the on-hand inventory for the classical EMQ problem and the EMQ problem with breakdowns.

We consider two maintenance processes for the EMQ with machine breakdowns. The first one is *corrective maintenance*. It is performed after every breakdown and it costs $K + M$. The second one is *regular maintenance*. It is performed at the end of each production interval and it costs K. Both corrective and regular maintenances are instantaneous.

When the manufacturing process is functioning properly, i.e., the system is in a wet interval, the next machine breakdown is assumed to happen at time B, which is a random variable with density and distribution functions $f(b)$ and $F(b)$, respectively. Given that the system also incurs a linear holding cost of h per item per unit time, the objective is to find the lot size, Q^*, that minimizes the average cost per unit time.

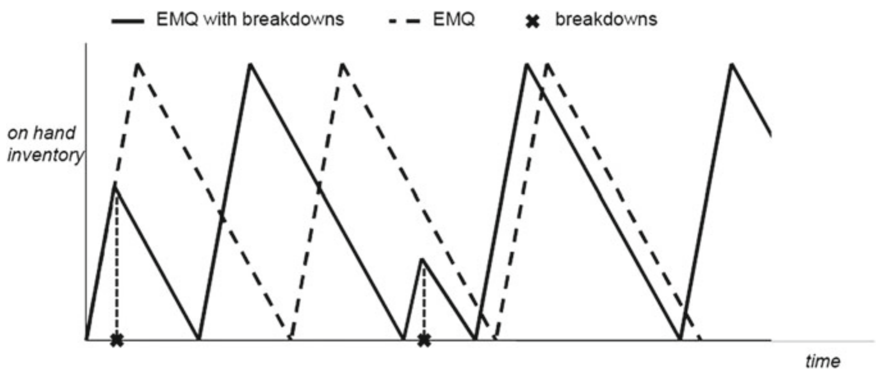

Fig. 2 Inventory curves for the classical EMQ model and the EMQ model with disruptions (Groenevelt et al. 1992a)

Groenevelt et al. (1992a) define a cycle as the time between starts of successive production runs and obtain the following expression for the expected cost function:

$$E[C] = \int_{0}^{Q/P} \left(K + M + \frac{1}{2}h(P-d)\frac{P}{d}b^2\right)f(b)db + \int_{Q/P}^{\infty} \left(K + \frac{1}{2}h(P-d)\frac{P-d}{pd}Q^2\right)f(b)db$$

The first integral is the expected cycle cost if the time to machine breakdown is shorter than $\frac{Q}{P}$. This means that a breakdown happens that requires a corrective maintenance costing $K + M$. The third component of the first integral is the expected holding cost per unit time for the cycle during which a machine breakdown happens. The second integral is the expected cycle cost if the time to machine breakdown is longer than $\frac{Q}{P}$. That means a breakdown does not occur and the regular maintenance is enough. As in the first integral, the expected cost for this type of cycle is the sum of maintenance and inventory holding costs.

The expected cycle length is given by

$$E[T] = \int_{0}^{Q/P} \frac{P}{d}bf(b)db + \int_{Q/P}^{\infty} \frac{Q}{d}f(b)db$$

As in the expected cycle cost function, the first and the second parts are the expected cycle lengths if the time to the next disruption is shorter and longer than $\frac{Q}{P}$, respectively.

Using Renewal Theory, the long-run average cost per unit of time can be calculated as $\frac{E[C]}{E[T]}$. As in the EOQD model, the cost expression is complex and it cannot be solved in closed-form when B has a general distribution. On the other hand, if B is exponential with rate λ, Q^* is the unique nonnegative solution of the following nonlinear equation:

$$e^{-\left(\frac{\lambda Q}{P}\right)} + \frac{\lambda Q}{P} = 1 + \frac{d\lambda^2 K}{hP(P-d)}$$

This rather simple equation allows Groenevelt et al. (1992a) to obtain some basic insights. They prove that the long-run average corrective maintenance cost does not depend on Q. They also show that Q^* and the optimal cost values are increasing functions of λ. Via Proposition 1, the authors prove that when the system approaches perfect reliability Q^* approaches the classical EMQ.

Proposition 5 *When the rate λ goes to zero, Q^* approaches the classical EMQ, i.e.*

$$\lim_{\lambda \to 0} Q^* = \sqrt{\frac{2KdP}{h(P-d)}}$$

Surprisingly, Groenevelt et al. (1992a) prove that when the system is subject to disruptions, using the classical EMQ disruptions instead of Q^* results in an average cost increase of at most 2 %. Although, the difference between the EMQ and Q^* can be very large, the reason for the small cost difference is that when a machine breakdown takes place, the interrupted lot is aborted. Hence, the difference in the average lot sizes is much smaller than the difference in the optimal lot sizes.

An important assumption of the EMQ model with disruptions is that the corrective maintenance times are negligible. In reality, repairs might be time consuming. Safety stocks are required to satisfy the customer demands arising during disruptions lasting long enough to deplete the entire inventory. Under these conditions, one needs to maintain separate cycle and safety stocks, the latter to be used only when a machine breakdown occurs. In fact, Groenevelt et al. (1992b) show that the optimal safety stock level increases with the disruption rate, required service level, demand rate, and setup and repair times.

4.2 Disrupted Demand Process

Disruptions do not only affect supply side of the inventory systems, but they can also result in intermittent demand. Weiss and Rosenthal (1992) study an EOQ model with a single disruption that can happen in either the supply or demand process. In this section, we discuss the latter case only.

Assume that a demand disruption happens at a single known time in the future, S. The disruption lasts for a random length $D \geq 0$ with distribution function $F_D(t)$ and incurs a cost at a constant rate, p, per disrupted time. When the demand process is interrupted, demand does not arrive and the inventory level stays the same from the beginning until the end of the disruption. Otherwise, the demand is continuous and constant with rate d. Each order incurs a setup cost of K and there is a unit holding cost h per unit time. The objective is to determine the structure of the optimal policy and develop an algorithm for finding the optimal order quantity.

Weiss and Rosenthal (1992) consider two cases, first with the disruption occurring just as the inventory is depleted and second with the disruption happening when the inventory level is positive. In both cases, the inventory level remains the same from the beginning to the end of the disruption.

Weiss and Rosenthal (1992) show that for the first case, the optimal policy is to place $n^*(S)$ orders of size $\frac{dS}{n^*(S)}$ before the disruption happens. Here, $n^*(S)$ is then that satisfies

$$n(n-1) \leq \frac{hdS^2}{2K} \leq n(n+1)$$

After the disruption is over at time $S + D$, the ordinary EOQ policy needs to be employed.

For the second case, suppose that at time S the inventory level is a. The optimal order, Q^*, before the disruption is given by

$$Q^* = \frac{C^*}{h} - dE[D]$$

where $C^* = \sqrt{2dKh}$, i.e., the optimal average cost of the classical EOQ model. Like the first case, after the disruption the optimal order quantity is given by the EOQ.

4.3 EOQD Model with Demand Uncertainty

Bar-Lev et al. (1993) extends the EOQD model of Parlar and Berkin (1991) by considering Stochastic demand. The inventory process is assumed to be a Brownian motion with negative drift implying that customers can return items. Assuming finite capacity, the objective is to find the order quantity and the capacity. Cost parameters include fixed and variable ordering costs, linear holding and stockout costs, as well as a cost that is linear in the capacity. Using Renewal Theory, the authors derive the expected cost function and minimize it numerically.

4.4 Phase-Type Disruption Parameters

Ross et al. (2008) study the EOQD problem with durations of dry and wet intervals having phase-type distributions. The authors model this problem as a non-homogeneous continuous-time Markov chain (CTMC) and solve it numerically. They propose several ordering policies and compare the costs of these policies under different parameter settings. They conclude that nonstationary policies not only provide some cost benefit but are also robust to errors in estimating the system parameters.

5 Conclusions and Future Research Directions

In this chapter, we summarized the studies on EOQ models with supply disruptions. For the EOQ model with disruptions at the external supplier only, we showed the derivation of the exact expression for the expected cost and claimed the impossibility of obtaining a closed-form solution for the optimal order quantity. Then, we mentioned an approximation which results in a closed-form expression and shows how much more inventory is needed in order to be buffer against the uncertainty introduced by supply disruptions. In addition, we summarized EOQ models with both external and internal disruptions. As in the previous case, the exact expression for the expected cost does not yield a closed-form expression for the optimal order quantity but an approximation exists. An important finding of this model is that internal disruptions have a greater impact than external ones. Finally, we studied extensions of EOQ models with disruptions to manufacturing environments. We also studied extensions with disrupted demand processes, demand uncertainty and phase-type disruption parameters.

All the studies summarized in this chapter suggest that if inventory is chosen as a disruption mitigation strategy, keeping extra inventory, the amount of which depends on disruption parameters, is required. These studies make an assumption regarding the structure of the inventory replenishment policy and do not try to find the optimal policy structure. Although there are some studies on optimal policy structures for single-location systems subject to disruptions, for multi-echelon systems this is rather difficult. De Croix (2012) proves the optimality of state-dependent base-stock policies for serial systems with linear holding and backordering costs and extends this result to assembly systems. Future research should be conducted on identifying optimal policies for distribution and more general systems.

There are many other directions for future research on the subject of inventory models subject to supply disruptions. Objective functions other than minimizing the expected cost should be explored. For example, worst-case analysis is an attractive alternative to reflect decision makers' risk-averse attitudes toward disruptions. In addition, more general models of disruption processes are likely to reflect the complexities of real-world consequences more realistically. The literature has a huge gap in this respect. Another important question is how to estimate the parameters on these processes. supply disruptions are random events and historical data might not provide accurate enough information. More research is needed to develop methods for parameter estimation.

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