Modeling a Coordinated Manufacturer–Buyer Single-Item System Under Vendor-Managed Inventory

Fidel Torres, Frank Ballesteros and Marcela Villa

Abstract This work develops a new coordinated manufacturer–buyer model for a single item in a vendor-managed inventory (VMI) context. The proposed model includes the manufacturing uptime and a manufacturer–buyer synchronization scheme. This mechanism makes logistical coordination between manufacturer and buyer much easier. The analysis of the mathematical model of coordination considers production and demand rates, as well as totals of the manufacturer and the buyer's ordering and holding inventory costs. This study is complemented by a sensitivity analysis. It focuses on the effects of parameter variations on proposed performance measurements in the manufacturer–buyer VMI-coordinated scheme. Finally, analytical conditions under which the suggested coordinated implementation of VMI gives benefits to both manufacturer and buyer and to the supply chain are deduced and verified. Results show that it is possible for both the manufacturer and buyer to obtain profits in VMI implementation by selecting satisfactory parameter combinations using our proposed coordination scheme in a win–win relationship.

F. Torres (✉) · F. Ballesteros
Department of Industrial Engineering, Universidad de los Andes, Bogotá, Colombia
e-mail: ftorres@uniandes.edu.co

F. Ballesteros
e-mail: frank.ballesteros@polymtl.ca

M. Villa
School of Industrial Engineering, Universidad Pontificia Bolivariana,
Bucaramanga, Colombia
e-mail: marcela.villa@upb.edu.co

T.-M. Choi (ed.), *Handbook of EOQ Inventory Problems*, International Series in Operations Research & Management Science 197, DOI: 10.1007/978-1-4614-7639-9_12,

1 Introduction

A full integration of the supply chain has become one of industry's greatest dreams, thanks to the success achieved by different businesses working together with their suppliers and customers (Darwish and Odah 2010). Initiatives like "efficient customer response" in the grocery industry and "quick response" in the garment industry (Waller et al. 1999) are good examples of this concept.

In recent years, there has been growing interest in implementing vendor-managed inventory (VMI) initiatives (Emigh 1999), thanks to important recognition from different industrial leaders (Southard and Swenseth 2008). This interest stems from the fact that there are benefits to the whole chain in cost reduction, improved service levels, and supplier performance (Choi et al. 2004).

VMI is a coordination mechanism that improves multi-firm supply chain efficiency (Waller et al. 1999) between a supplier and its customers (Silver et al. 1998). VMI can decrease inventory levels, increase fill rates in the supply chain (Yao et al. 2007), and reduce lead times and inventory stock outs (Daugherty et al. 1999). In spite of this, this tool has not been studied in detail, especially as applied to the systems that exist between manufacturers and purchasers.

The models presented in this paper analyze a two-level supply chain in which external demand for a single item occurs at the purchaser. The paper proposes an analysis of total, ordering, and inventory holding costs for each agent and the supply chain with and without VMI.

The general basis of this work is the classic theory of economic order quantity (EOQ). These models complement some previous works (Yao et al. 2007; Van der Vlist et al. 2007), including research associated to both productive (uptime) and non-productive times. The use of these times facilitates control of inventories in our coordinated manufacturer–buyer single-item VMI-conduced system. Our VMI approach includes a new manufacturer–buyer synchronization scheme that makes logistics coordination in the VMI environment between manufacturer and buyer much easier. The proposed synchronization scheme is a logical extension of previous models studied in other manufacturer–buyer VMI approaches (Dong and Chu 2002; Choi et al. 2004; Yao and Dresner 2008). These models do not include explicit synchronization and coordination mechanisms between buyer and supplier. Analytical conditions under which the suggested coordinated implementation of VMI gives benefits to both manufacturer and buyer and to the supply chain are deduced and verified.

The proposed sensitivity analysis of the involved variables shows the behavior of the parameters—costs, demand, and production rates—over performance measurements related to total cost, inventory holding cost and ordering costs, order quantities, and cycle times, thereby establishing relationships between all relevant manufacturer and buyer parameters and the potential benefits in our proposed manufacturer–buyer synchronized VMI implementation.

This article is divided as follows: Section 2 is a review of the literature. Section 3 describes the proposed model and develops the main results. Section 4 presents a sensitivity analysis for the main parameters of the model. Section 5 presents the main conclusions. Finally, Sect. 6 outlines potential areas for future research.

2 Literature Review

The first VMI models appeared in the late 1980s, when Walmart, K-Mart, and Procter and Gamble implemented major projects relating to supply chain integration (Waller et al. 1999; Blatherwick 1998). However, not until recently was this subject discussed in the academic literature (Southard and Swenseth 2008). To ensure proper classification of the published scientific literature on VMI, we established five categories, divided by focus: strategic, statistical characterization, simulation, deterministic modeling, and stochastic modeling. Summaries of articles with a VMI approach can be seen in Tables 1 and 2.

The first articles published on the subject are those that present a strategic focus. The first work was published in 1994, when Jain (1994) established the basis for VMI implementation between two agents in a chain, outlining VMI's benefits and disadvantages to companies. Cottrill (1997), described some VMI cases and identified some current trends related to this strategy. Cachon and Fisher (1997), reviewed various VMI models, such as synchronized consumer response, continuous replenishment program, efficient consumer response, and rapid replenishment, through the case study of Campbell's Soup.

Table 1 State-of-the-art review in VMI

Category	References
Strategy	Jain (1994), Cottrill (1997), Cachon and Fisher (1997), Holmstrom (1998), Blatherwick (1998), Emigh (1999), Challener (2000), Lee and Chu (2005), Saxena (2009)
Discrete simulation	Waller et al. (1999), Disney and Towill (2002, 2003), Yang et al. (2003), Angulo et al. (2004), White and Censlive (2006), Song and Dinwoodie (2008), Southard and Swenseth (2008), Sari (2008), Ofuoku (2009), Hemmelmayr et al. (2010)
Classical analytical modeling	Cachon and Zipkin (1999), Lee et al. (2000), Achabal et al. (2000), Dong and Chu (2002), Wang et al. (2004), Choi et al. (2004), Yao et al. (2007), Yao and Dresner (2008), Wong et al. (2009), Xu and Leung (2009), Yang et al. (2010), Yao et al. (2010), Darwish and Odah (2010), Hongjie et al. (2011), Kastsian and Mönnigmann (2011), Lee and Ren (2011), Liao et al. (2011), Pasandideh et al. (2011), Chen et al. (2012), Yu et al. (2012), Zanoni et al. (2012), Kristianto et al. (2012)
Statistics	Daugherty et al. (1999), Kuk (2004)
Game theoretical modeling	Yu et al. (2009a), Yu et al. (2009), Yu and Huang (2009b)

Table 2 Chronological state-of-the-art review in VMI

	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Strategy	1			2	2	1	1					1				1			
Simulation						1			1	2	1		1		3	1	1		
Modeling						1	2		1		2			1	1	2	3	5	4
Statistics						1					1								
Game theory																3			

Following this strategic approach, Blatherwick (1998), analyzed some of VMI's benefits and disadvantages to the agents involved in the agreements. He also showed how supply chains have evolved to become co-managed inventories. Around the same time, Holmström (1998), studied and characterized the adaptation of SAP R/3 in a partnership relationship within the context of VMI. Later, Emigh (1999), presented VMI cases in different industrial sectors and analyzed some technological requirements necessary to ensure successful implementation. In this decade, Challener (2000), illustrated VMI implementation in the pharmaceutical industry through detailed presentation of some success stories. Additionally, Lee and Chu (2005), analyzed supply chain interaction and established various control strategies, one of which was VMI. Recently, Saxena (2009), characterized VMI systems and explained the reasons for judging that the system is not always applicable or beneficial to all actors in the chain.

A number of papers have addressed statistical characterization of VMI models, starting with Daugherty et al. (1999), who presented the statistical results of a survey about the implementation of automatic replenishment programs in different industries. Kuk (2004), described the factors that may affect VMI's effectiveness as measured in service improvement and cost reduction in electric enterprises, arguing that the success of VMI programs in some areas cannot be generalized to others.

The articles that cover discrete-event simulation technique application were written before 1999. The first work on this subject was published by Waller et al. (1999), who compared order frequency in different scenarios, facing inventory reduction through experimentation with a VMI strategy. Additionally, Disney and Towill (2002), designed a VMI system with different cost levels and proposed a simulation method to determine the optimal parameters in the chain.

The same authors (Disney and Towill 2003) later compared various supply chains with and without VMI through simulation models and found a substantial reduction in the so-called bullwhip effect. In another article, Yang et al. (2003), analyzed the impacts of different parameters on a supply chain consisting of a single vendor supplying a set of retailers with VMI through discrete-event simulation methods. Angulo et al. (2004), presented the variations of demand and cycle times of a chain with VMI in the simulation of a four-level chain with stochastic demand and lead times.

Following the simulation line, White and Censlive (2006), searched for an appropriate factory production delay for VMI systems and showed that this time depends on the level of aggregation and the representation of the delay, due to production, as a finite or an exponential delay. Shortly after this, Song and Dinwoodie (2008), utilized numerical experiments to show that the politics of VMI and inventory management can be used on uncertain situations, resulting in benefits to the whole chain. Additionally, Southard and Swenseth (2008), showed that VMI can achieve sufficient economic benefits by comparing inventory costs in cooperative farms through discrete-event simulation. Elsewhere, Sari (2008), compared CPFR and VMI models through a simulation of a four-level chain, finding significant influence of uncertain demand. The author shows that benefits are greater in a CPFR than in a VMI environment. Ofuoku (2009), compared total optimal costs obtained for a chain with and without VMI using discrete-event simulation. Finally, Hemmelmayr et al. (2010), developed a technology to plan delivery routes to supply blood to hospitals and blood banks using VMI policies.

We also identified a set of articles using classical mathematical modeling approaches, starting with Cachon and Zipkin (1999), who analyzed a two-level chain with stationary stochastic demand, fixed transport times and cooperative inventory policies. The same approach was developed by Lee et al. (2000), who modeled a chain consisting of a manufacturer and a retailer with stochastic demand and information sharing between agents, thereby reducing inventories and costs. Later, Achabal et al. (2000), described models of demand and inventory forecasting in a VMI environment and found improvement in service level and stock turnover. Dong and Chu (2002), analyzed the ways in which VMI affects a two-level chain with deterministic demand, demonstrating that it reduces total costs, while sometimes also reducing supplier benefits. Wang et al. (2004), analyzed a chain consisting of a supplier and multiple retailers in a non-cooperative environment and with stochastic demand. They demonstrated through a model that coordination is required to achieve an optimal solution. Choi et al. (2004), modeled a system of a supplier and a buyer with independent demand and variables and examined the roles of the service levels and backorders in the system.

More recent mathematical modeling works developed deterministic approaches. Yao et al. (2007), presented an analytical model applied to supply chains of two agents with and without VMI and found inventory cost reductions. Yao and Dresner (2008), planned a model consisting of a manufacturer and a retailer with stochastic demand and examined management practices before and after information-sharing implementation, continuous replenishment, and VMI. Wong et al. (2009), showed how a sales rebate contract helps achieve supply chain coordination through a two-echelon model consisting of a supplier and multiple retailers. Recently, Yang et al. (2010), evaluated the effects of a distribution center on a VMI system consisting of a manufacturer, a distributor, and multiple retailers, analyzing decision strategies of one agent (OSD) and two agents (TSD), which generate different benefit levels. Yao et al. (2010), demonstrated how a manufacturer might use an incentive contract with a distributor under a VMI arrangement to gain market share through an analytical model. This approach models manufacturer–distributor coordination to convert lost sales into backorders. Darwish and Odah (2010), presented a model consisting of a vendor and multiple agents under VMI and proposed a set of theorems and an efficient algorithm to find an optimal total cost solution using KKT. Zanoni et al. (2012), considered a two-level supply chain system with a single vendor and a single buyer at each level, and investigated and compared different policies that the vendor might adopt to exploit the advantages offered by the VMI with a consignment agreement when the vendor's production process is subject to learning effects. Kristianto et al. (2012), proposed an adaptive fuzzy control application to support a VMI. Results showed that the adaptive fuzzy VMI control surpasses fuzzy VMI control and traditional VMI in terms of mitigating the bullwhip effect and lowering delivery overshoots and backorders. Liao et al. (2011), proposed an integrated location-inventory distribution network problem that integrates the effects of facility location, distribution and inventory issues. The problem was formulated under the VMI setup. The paper presented a multi-objective location-inventory problem (MOLIP) model and investigated the use of a multi-objective evolutionary algorithm based on the non-dominated sorting genetic algorithm (NSGA2) to solve MOLIP. Pasandideh et al. (2011), proposed a genetic algorithm to find the order quantities and the maximum backorder levels so that the total inventory cost is minimized over a two-level supply chain system consisting of several products, one supplier and one retailer, in which shortages are backordered under a VMI-controlled system. Chen et al. (2012), studied how a vendor's optimal distribution policies with transshipment combined with the variance of demand affects the optimal policy in a VMI environment. The paper explores a two-echelon supply chain with one supplier and two retailers in a planning horizon of two periods. Kastsian and Mönnigmann (2011), addressed the steady-state optimization of a supply chain that belongs to the class of VMI, automatic pipeline, inventory and order-based production control systems (VMI-APIOBPCS). The supply chain is optimized with the so-called normal vector method. Xu and Leung (2009), focused on a two-party VMI channel in which the vendor operates the basic stocking and delivery functions and makes inventory replenishment decisions, while the retailer is responsible for customer acquisition and in-store services. This book proposed an analytical model for the partners in the supply channel to determine the inventory policy that optimizes net system profit. Hongjie et al. (2011), studied the inventory control of deteriorating items for suppliers under a VMI model, establishing bi-level programming models of integrated delivery strategies and introducing a genetic algorithm to solve the problem. Yu et al. (2012), studied a VMI-conduced supply chain in which the manufacturing vendor decides how to manage the system-wide inventories of its fast-deteriorating raw material and its slowly deteriorating product. The paper proved the convexity of the cost functions and proposed a golden search algorithm to find the model's optimal solution. Lee and Ren (2011), proposed a periodic-review stochastic inventory model to examine the benefits of VMI in a global environment, in which the supplier and the retailer face exchange rate uncertainty and incur different fixed ordering costs. The paper provides some analytical results, including the optimality of a state-dependent (s, S) policy for the supplier.

A recent VMI approach has analyzed the model through game theory. In this category, the work of Yu et al. (2009a), used evolutionary game theory to analyze a strategy of evolutionary stability in supply chains with VMI. An earlier work by Yu et al. (2009c), formulated a model of a manufacturer and multiple retailers and proposed a computational algorithm based on an analysis of a response function and a generic demand function. Additional work by Yu and Huang (2009b), analyzed the interaction between a manufacturer and its retailers to optimize its marketing strategy for a product with VMI by using a Nash game model between agents.

3 Modeling Framework

The supply chain we study consists of a manufacturer and a buyer implementing VMI for a single product. This problem has been studied by Dong and Chu (2002), Choi et al. (2004), and Yao et al. (2007). These approaches propose an implicit coordination strategy between supplier and buyer, but the studied models do not include explicit synchronization and coordination mechanisms between buyer and supplier. In our approach, as an alternative, we propose an unambiguous coordination scheme between manufacturer and buyer by means of which a coherent and realistic VMI implementation can be achieved. A key difference is that we clearly model this coordination strategy by means of a synchronization mechanism between the buyer and manufacturer replenishment cycles.

The proposed coordinated manufacturer–buyer VMI model contains the design parameters of the synchronization scheme between manufacturer and buyer, ordering and holding cost in VMI and non-VMI conditions, and production and demand rates. The decision variables of the model include batch sizes, manufacturer production uptime, manufacturer and buyer inventory replenishment times, and integer coordination and synchronization constants.

The notation used in our model is:

Parameters: $C, c, c', H, h, P, r, d, \delta, g, g'$

Variables: $T, t, Q, q, T_s, k, L, U, \tau_s, I_s$

Where:

C	Setup (ordering) costs for the manufacturer (in \$/setup)
c	Setup (ordering) costs for the buyer without VMI (in \$/setup)
c'	Setup (ordering) costs for the buyer with VMI (in \$/setup)
H	Holding cost of manufacturer inventory (in \$/unit/year)
h	Holding cost of buyer inventory (in \$/unit/year)
P	Manufacturer production rate (in units/year)
r	Demand rate (in units/year)
$d = H/h$	Manufacturer and buyer inventory holding cost ratio
$\delta = r/P$	Demand and production rate
$g = C/c$	Manufacturer and buyer setup (ordering) cost ratio without VMI
$g' = C/c'$	Manufacturer and buyer setup (ordering) cost ratio with VMI
T	Manufacturer replenishment time (in years)
t	Buyer replenishment time (in years)
t	Buyer replenishment time (in years)
Q	Manufacturer lot size or total quantity manufactured over replenishment time T (in units)
q	Buyer lot size or total quantity demanded over replenishment time t (in units)
$T_s = q/P$	Manufacturing time of buyer lot size q (in years)
k	Number of buyer shipments placed during the manufacturer replenishment time (integer)

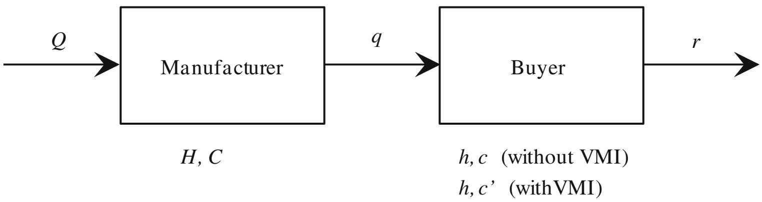

Fig. 1 Modeling framework

L	Number of buyer shipments placed during the manufacturer uptime (integer)
U	Manufacturer uptime (in years)
$\tau_s = U - Lt$	Fractional manufacturer up time (in years)
I_s	Manufacturer average inventory (in units)

The production plant manufactures and distributes a single product to the buyer, who has a known deterministic annual demand rate that is the same for the manufacturer and the buyer and is denoted by r. The system is studied before and after VMI implementation and is presented in Fig. 1. In this article we adopted the convention, used by Yao et al. (2007), that uppercase parameters are for the manufacturer and lowercase parameters are for the buyer.

Annual holding inventory costs per unit are denoted as H for the manufacturer and h for the buyer, in money units per unit per year. Single-order costs are denoted with C for the manufacturer, c' for the buyer with VMI, and c for the buyer without VMI. Production rate is constant and denoted with P and $P \geq r$. The buyer replenishment time is represented by t. The manufacturer replenishment time T is kt (with k integer) and contains L buyer replenishment cycles (with L integer). The time required to produce a lot size required for the buyer (q) is denoted by T_s. The lot size of the manufacturer is $Q = kq$. The explicit synchronization mechanism between buyer and manufacturer consists in sending to buyer from the manufacturer q units during the buyer replenishment period t. These periodical replenishments are planned during the manufacturer replenishment period T. In our model, we explicitly consider the uptime $U = Lt + \tau_s$. This uptime is not taken into consideration in other manufacturer–buyer VMI approaches (Dong and Chu 2002; Choi et al. 2004; Yao et al. 2007).

The explicit replenishment coordination mechanism between manufacturer and buyer is represented in Fig. 2. In this study, we have deduced the mathematical conditions (Eqs. 4a–12) needed to achieve the explicit manufacturer–buyer synchronization, represented with integer coordination constants k and L. In our model, the replenishment cycle of the manufacturer T is exactly k buyer replenishment cycles and contains the uptime $Lt + \tau_s$. From a practical point of view, this mechanism makes logistical coordination between manufacturer and buyer much easier than in the other related VMI approaches (Yang et al. 2003; Dong and Chu 2002; Choi et al. 2004; Yao et al. 2007).

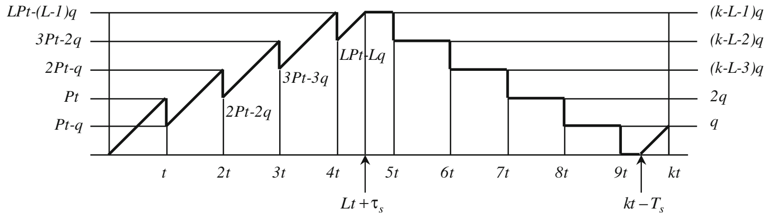

Fig. 2 Manufacturer's inventory levels and the proposed manufacturer–buyer coordination mechanism under VMI

Without VMI, manufacturer and buyer relate to each other following a finite production rate model. Because the buyer average inventory level is driven by a simple EOQ model, his or her average inventory level is $q/2$. As a consequence, the buyer's average total annual holding and setup cost is given by:

$$f(q) = c\frac{r}{q} + h\frac{q}{2} \tag{1}$$

Similarly, without VMI the manufacturer is guided by a finite production rate model. The change in manufacturer inventory level over time is shown in Fig. 3. Feasibility requires that $P > r$. Average inventory level can be deduced as $(Q/21 - r/P)$, according to the economic production quantity (EPQ) model (Silver et al. 1998). In consequence, the manufacturer's average total annual holding and setup cost is:

$$F(Q) = C\frac{r}{Q} + H\frac{Q}{2}\left(1 - \frac{r}{P}\right) \tag{2}$$

It follows that with optimal order quantities q and Q for the buyer and manufacturer, the optimal total costs of the system without VMI are:

$$\mathrm{TC}^{*}_{\mathrm{NON\ \ VMI}} = \sqrt{2r}\left(\sqrt{CH\left(1 - \frac{r}{P}\right)} + \sqrt{ch}\right) \tag{3}$$

Considering our manufacturer–buyer coordinated and synchronized VMI system shown in Fig. 2, manufacturer and buyer replenishment times are related through an integer coordination constant called k. The synchronization scheme

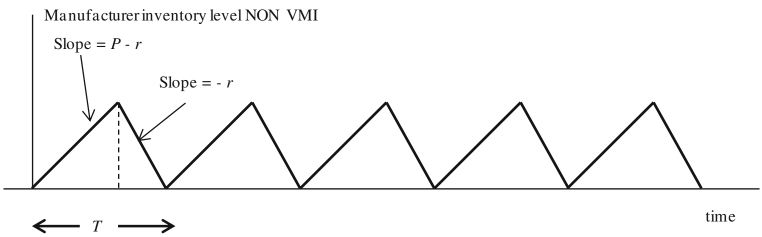

Fig. 3 Manufacturer level without VMI

implies that the manufacturer sends the buyer the lot size q each replenishment time t. In this sense, manufacturer lot size Q is equal to kq. If I_S is the manufacturer's average inventory, the total cost of the manufacturer–buyer coordinated VMI system is given by:

$$\mathrm{TC}_{\mathrm{VMI}} = c'\frac{r}{q} + h\frac{q}{2} + C\frac{r}{Q} + HI_S \tag{4a}$$

In our VMI approach, we can calculate the area under the curve for the manufacturer's inventory over his/her replenishment time $T = kt$. Dividing this value by T, we get the manufacturer's average inventory (denoted by I_S), given by Eq. 4b.

$$I_S = \frac{q}{2}\left[k\left(1-\frac{r}{P}\right) + 2\frac{r}{P} - 1\right] \tag{4b}$$

The proof of Eq. 4b is shown in Appendix 1.

Graphically, it is possible to conclude that:

$$T_s = \frac{q}{P} \tag{5}$$

And by definition, the fractional manufacturer uptime must satisfy:

$$0 \le \tau_s = (k-1)\frac{r}{P}t - Lt \le t \tag{6}$$

From the proposed relationship in (6), we obtain the next relationship between synchronization constants L and k:

$$L = \left\lfloor (k-1)\frac{r}{P} \right\rfloor \tag{7}$$

Using the terms identified above, it is possible to calculate the optimal supply chain total ordering and inventory holding cost, solving the nonlinear model represented in Eq. 8a, as shown in Appendix 2.

$$\begin{aligned} &\text{Min}\left\{c'\frac{r}{q} + h\frac{q}{2} + C\frac{r}{kq} + H\frac{q}{2}\left[k\left(1-\frac{r}{P}\right) + 2\frac{r}{P} - 1\right]\right\} \\ &\text{s.t.} \\ &\quad q \ge 0 \\ &\quad k \in \{1, 2, \ldots,\} \end{aligned} \tag{8a}$$

With k constant, taking the partial derivatives of $\mathrm{TC}_{\mathrm{VMI}}$ with respect to q, and setting the respective equation equal to zero, the optimal cost in terms of k is given in Eq. 8b.

$$\mathrm{TC}^*_{\mathrm{VMI}} = \sqrt{2r}\left[\sqrt{\left(c' + \frac{C}{k}\right)}\sqrt{\left\{h + H\left[k\left(1-\frac{r}{P}\right) + 2\frac{r}{P} - 1\right]\right\}}\right] \tag{8b}$$

Optimizing the expression in Eq. 8b and relaxing the integrality condition on k, the optimal supply chain total ordering and inventory holding cost is calculated in Eq. 8c.

$$\mathrm{TC}^*_{\mathrm{VMI}} = \sqrt{2r}\left[\sqrt{CH\left(1-\frac{r}{P}\right)}+\sqrt{c'\left\{h+H\left(2\frac{r}{P}-1\right)\right\}}\right] \tag{8c}$$

The new optimal order quantities (lot sizes) for the buyer with and without VMI, as shown in Appendix 2, are given by the following equations, respectively:

$$q^*_{\mathrm{VMI}} = \sqrt{\frac{2c'r}{\left(h+H\left[2\frac{r}{P}-1\right]\right)}} \tag{9}$$

$$q^*_{\mathrm{NON\ VMI}} = \sqrt{\frac{2cr}{h}} \tag{10}$$

Due to VMI's savings simplification, we can see that $c' < c$, and, as the relationship between annual holding inventory costs per unit is $h > H$, we cannot conclude a relationship between q^*_{VMI} and $q^*_{\mathrm{non\,VMI}}$, as either one can be larger than the other. This result differs from that in the literature. On the other hand, it is possible to calculate the optimal order quantity for the manufacturer given in Eq. 11. However, in this case we can show that the manufacturer lot size is the same with or without VMI.

$$Q^* = \sqrt{\frac{2Cr}{H\left(1-\frac{r}{P}\right)}} \tag{11}$$

Our analysis can be accomplished by defining the ratios between the demand and production rates ($\delta = r/P$), the manufacturer holding cost and the buyer inventory ($d = H/h$), the ordering cost without VMI ($g = C/c$), and the ordering cost with VMI ($g' = C/c'$). The coordination constant k_{VMI} between the manufacturer and the buyer with VMI is an integer value that we can approach from:

$$k_{\mathrm{VMI}} = \sqrt{\frac{g'\left[(2\delta-1)+\frac{1}{d}\right]}{(1-\delta)}} \tag{12}$$

Values deduced from our coordination model in Eqs. 3 and 8a–12 are taken as the support for the proposed definitions of performance measurement in the sensitivity analysis explained in the next paragraph.

Example:

A manufacturer and a buyer are implementing the coordinated VMI scheme. The operational parameters of the supply chain are:

C \$300

c \$100

c' \$80

Table 3 Results of the manufacturer–buyer coordinated VMI scheme

		Without VMI	With coordinated VMI
Manufacturer	Ordering cost ($/year)	$ 848.53	$ 804.98
	Inventory holding cost ($/year)	$ 848.53	$ 983.87
	Total cost ($/year)	$ 1,697.06	$ 1,788.85
	Q (units)	424.26	447.21
	L	–	2
	t (years)	0.09	0.07
	τ_s (years)	–	0.03
	Uptime (years)	–	0.18
	T (years)	0.35	0.37
	T_s (years)	–	0.04
Buyer	Ordering cost ($/year)	$ 1,095.45	$ 1,073.31
	Inventory holding cost ($/year)	$ 1,095.45	$ 894.43
	Total cost ($/year)	$ 2,190.89	$ 1,967.74
	q (units)	109.54	89.44
Supply chain	Total ordering cost ($/year)	$ 1,943.97	$ 1,878.30
	Total inventory holding cost ($/year)	$ 1,943.97	$ 1,878.30
	Total cost ($/year)	$ 3,887.95	$ 3,756.59
	Coordination constant k	3.87	5

H $10/unit/year
h $20/unit/year
P 2,000 units/year
r 1,200 units/year

Applying our model, Table 3 shows the results of our manufacturer–buyer coordinated VMI scheme.

For the selected parameters, the supply chain receives economic rewards from implementation of the proposed coordinated manufacturer–buyer VMI scheme. In this case, total supply chain costs are reduced by 3.38 %. The buyer receives cost reductions equivalent to 10.19 %, and average inventory level decreases of about 18.35 %. However, the manufacturer sees his or her costs amplified by 5.41 %. The proposed coordinated scheme implies than each manufacturer replenishment cycle (0.37 years) contains five buyer replenishment cycles (0.07 years), with 0.18 years as the manufacturer uptime.

If demand is now increased to $r = 1{,}800$ units/year while the other parameters stay unchanged, the supply chain will not receive economic rewards from implementation of the proposed coordinated manufacturer–buyer VMI scheme, as shown in Table 4. Supply chain total costs increase by 4.21 %. The buyer receives cost reductions equivalent to 9.39 %, and average inventory level decreases by 23.91 %, while the manufacturer's total costs increase by 39.31 %. For the new combination of parameters, the proposed coordinated scheme implies that each manufacturer replenishment cycle (0.57 years) contains 10 buyer replenishment cycles (0.06 years), with 0.46 years as the manufacturer uptime.

Table 4 Results of the manufacturer–buyer coordinated VMI scheme, increasing demand to r = 1800 units/year

		Without VMI	With coordinated VMI
Manufacturer	Ordering cost ($/year)	$ 519.62	$ 528.98
	Inventory holding cost ($/year)	$ 519.62	$ 918.75
	Total cost ($/year)	$ 1,039.23	$ 1,447.73
	Q (units)	1,039.23	1,020.84
	L	–	8
	t (years)	0.07	0.06
	τ_s (years)	–	0.01
	Uptime (years)	–	0.46
	T (years)	0.58	0.57
	T_s (years)	–	0.05
Buyer	Ordering cost ($/year)	$ 1,341.64	$ 1,410.61
	Inventory holding cost ($/year)	$ 1,341.64	$ 1,020.84
	Total cost ($/year)	$ 2,683.28	$ 2,431.44
	q (units)	134.16	102.08
Supply chain	Total ordering cost ($/year)	$ 1,861.26	$ 1,939.59
	Total inventory holding cost ($/year)	$ 1,861.26	$ 1,939.59
	Total cost ($/year)	$ 3,722.51	$ 3,879.18
	Coordination constant k	–	10

The sensitivity analysis presented in the next section studies the effects of combining parameters over different proposed performance measurements. Analytical conditions are derived by explaining the effect of the parameters on the performance of the proposed coordinated VMI scheme.

4 Sensitivity Analysis

4.1 Total Cost Savings Throughout the Supply Chain

This section discusses some details related to our sensitivity analysis associated with the percentage of savings in total costs to the supply chain. In order to compute the system performance, we proposed the indicator V, defined in Eq. 13. The analysis was validated by selecting different values of ratios δ and d ($d = H/h$, $\delta = r/P$), each varying in the (0, 1) interval. The ordering cost parameters were chosen as C = 4,000, c = 100, and c' = 70. As a result, a range of levels for the percentage of savings in total cost was obtained from the implementation of the coordinated VMI scheme, as shown in Fig. 4.

The proposed performance measurement V is:

$$V = \frac{\mathrm{TC}^*_{\mathrm{NON\,VMI}} - \mathrm{TC}^*_{\mathrm{VMI}}}{\mathrm{TC}^*_{\mathrm{NON\,VMI}}} \qquad (13)$$

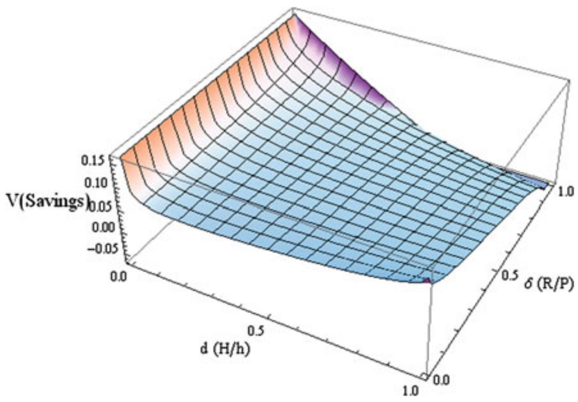

Fig. 4 Sensitivity of V to changes in d and δ ($C = 4000$, $c = 100$, $c' = 70$)

Using the deductions previously obtained in Eqs. 3 and 8, and by simplifying some terms, we arrived at the definition for V described in Eq. 14, as shown in Appendix 3:

$$V = \frac{1 - \sqrt{\frac{g}{g'}(1 - d + 2d\delta)}}{1 + \sqrt{gd(1 - \delta)}} \tag{14}$$

With the selected parameters shown in this paragraph, the sensitivity analysis of the percentage of total cost savings V shows that some d greater than 0.45 and some δ near one generate a negative profit from VMI. In addition, for values of d near zero, the percentage of total cost savings V is always non-negative.

4.2 Savings in Inventory Holding Costs for the Buyer

Figure 5 illustrates a second sensitivity analysis, performed with different values for the ratios δ and d for the percentage of savings from the buyer's holding inventory cost performance indicator (Z) with the selected parameters from Sect. 4.1.

The performance measurement Z presented in Fig. 5 is defined in Eq. 15, where $\text{IHC}^*_{\text{buyer,VMI}}$ and $\text{IHC}^*_{\text{buyer,non VMI}}$ represent the optimal buyer inventory holding costs with and without VMI, respectively.

$$Z = \frac{\text{IHC}^*_{\text{buyer, non VMI}} - \text{IHC}^*_{\text{buyer, VMI}}}{\text{IHC}^*_{\text{buyer, non VMI}}} \tag{15}$$

According to this relationship, it is possible to obtain an equivalent expression in terms of previously known variables, which are represented in Eq. 16, as shown in Appendix 3:

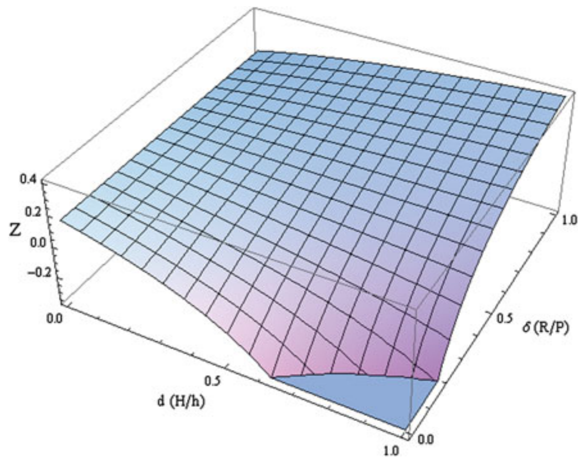

Fig. 5 Sensitivity of Z to changes in d and δ. ($C = 4000$, $c = 100$, $c' = 70$)

$$Z = 1 - \sqrt{\frac{g}{g'[1 + d(2\delta - 1)]}} \tag{16}$$

For this sensitivity ordering, costs remained constant at the level set for the case of V, obtaining the graph presented in Fig. 5, with the same selected parameters from Sect. 4.1.

As Fig. 5 shows, if Z is positive, then VMI implementation results in savings. The combinations of selected variables generate positive benefits with VMI implementation, except for at a few levels, those below $\delta = 0.2$ and higher than $d = 0.6$, which generate negative results for Z.

4.3 Savings in Inventory Holding Costs for the Manufacturer

Figure 6 shows the results of a sensitivity analysis performed with different values d and δ for the percentage of savings on manufacturer inventory holding costs performance indicator Y with the parameters set as defined in Sect. 4.1.

The performance measurement Y presented in Fig. 6 is defined in Eq. 17, where $\text{IHC}^*_{\text{manufacturer,VMI}}$ and $\text{IHC}^*_{\text{manufacturer,non VMI}}$ represent the optimal manufacturer inventory holding costs with and without VMI, respectively.

$$Y = \frac{\text{IHC}^*_{\text{manufacturer, non VMI}} - \text{IHC}^*_{\text{manufacturer, VMI}}}{\text{IHC}^*_{\text{manufacturer, non VMI}}} \tag{17}$$

Substituting terms defined in this article leads to the relationship in Eq. 18, as shown in Appendix 3:

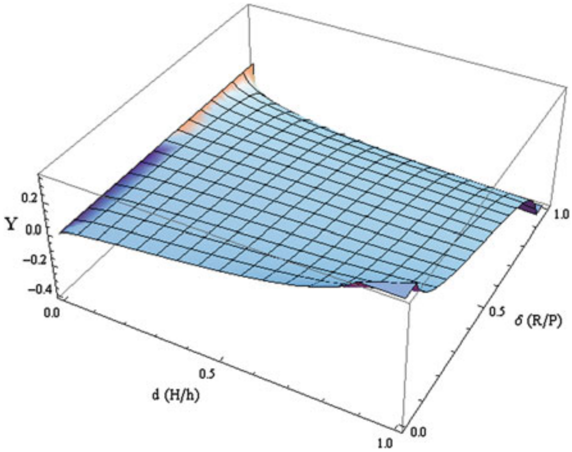

Fig. 6 Sensitivity of Y to changes in δ and d. ($C = 4000$, $c = 100$, $c' = 70$)

$$Y = \frac{(1 - 2\delta)}{\sqrt{1 - \delta}} \sqrt{\frac{d}{g'[1 + d(2\delta - 1)]}} \tag{18}$$

From the above equation, we can perform the same analysis with the previous variables remaining constant, keeping ordering costs at the same levels as those of previous cases.

After this analysis we can see that, in the selected scenario of a VMI system, some values generate positive benefits for the manufacturer's costs, and therefore a δ lower than 0.5 will return a positive value for the performance measurement Y. This observation is also shown mathematically from the ratio of the described variables. Furthermore, we performed a sensitivity analysis on the total cost of placing orders for the manufacturer. In this case, we can show, following Eq. 11, that the cost to the manufacturer is the same when placing an order with or without VMI. Therefore, manufacturer total ordering costs are not affected by the implementation of our coordinated VMI scheme.

4.4 Savings in Buyer Ordering Costs

For the sensitivity analysis of the buyer's ordering cost before and after implementing VMI, we defined the percentage of savings in the implementation of a VMI policy in the buyer's order cost performance indicator (N), according to Eq. 19, where $\mathrm{OC}^*_{\mathrm{buyer,VMI}}$ and $\mathrm{OC}^*_{\mathrm{buyer,non\ VMI}}$ represent the optimal buyer ordering costs with and without VMI, respectively.

$$N = \frac{\mathrm{OC}^*_{\mathrm{buyer,non\,VMI}} - \mathrm{OC}^*_{\mathrm{buyer,VMI}}}{\mathrm{OC}^*_{\mathrm{buyer,non\,VMI}}} \tag{19}$$

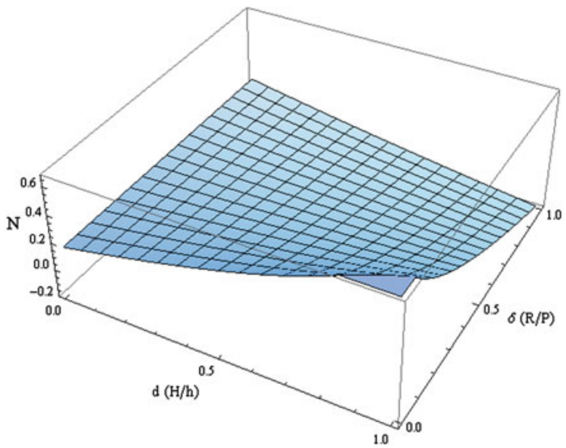

Fig. 7 Sensitivity of N to changes in δ and d. ($C = 4000$, $c = 100$, $c' = 70$)

Dividing these costs into their components and replacing the order quantities from Eqs. 9 and 10, we obtained the expression presented in Eq. 20, as shown in Appendix 3:

$$N = 1 - \sqrt{\frac{g[d(2\delta - 1) + 1]}{g'}} \tag{20}$$

Using the same defined cost levels, there are values for which the VMI implementation process is not beneficial to the buyer, since the relationship $\delta = r/P$ generates outstanding profit at a decreasing rate for values greater than 0.5. In this case, with the selected parameters shown in Sect. 4.1, negative percentages are obtained for certain values of $d = H/h$ greater than 0.7, as shown in Fig. 7.

4.5 Savings in Manufacturer Total Costs

For the sensitivity analysis of manufacturer's total cost before and after implementing VMI, we propose the performance measurement S, which is defined as the percentage of savings in the implementation of the coordinated VMI policy on manufacturer's total costs. With the selected parameters shown in Sect. 4.1, S is defined in Eq. 21 and represented in Fig. 8, where $\mathrm{TC}^*_{\mathrm{manufacturer,VMI}}$ and $\mathrm{TC}_{\mathrm{manufacturer,non\ VMI}}{}^*$ represent the optimal manufacturer total costs with and without VMI, respectively.

$$S = \frac{\mathrm{TC}^*_{\mathrm{manufacturer,non\,VMI}} - \mathrm{TC}^*_{\mathrm{manufacturer,VMI}}}{\mathrm{TC}^*_{\mathrm{manufacturer,non\,VMI}}} \tag{21}$$

Substituting terms defined in this paper result in the relationship shown in Eq. 22, as shown in Appendix 3:

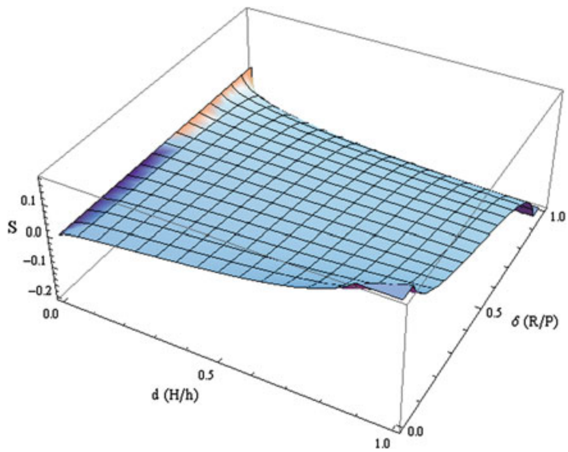

Fig. 8 Sensitivity of S to changes in δ and d ($C = 4000$, $c = 100$, $c' = 70$)

$$S = \frac{(1-2\delta)}{2}\sqrt{\frac{d}{g'(1-\delta)(1+d(2\delta-1))}} \tag{22}$$

From Eq. 22, we can perform our sensitivity analysis for S with all previous parameters remaining constant.

The analysis of manufacturer total costs is performed taking the first derivative of the performance measurement S with respect to g'. The result is shown in Eq. 23.

$$\frac{\partial S}{\partial g'} = \frac{(-1+2\delta)\sqrt{\frac{d}{g'(-1+\delta)(1+d(2\delta-1))}}}{4g'} \tag{23}$$

We can show for this case that S grows as $\delta > 0.5$ and decreases as $\delta < 0.5$.

4.6 Savings in Buyer's Total Costs

Our last sensitivity analysis corresponds to buyer's total costs with and without VMI. The performance measurement T is the percentage of savings in buyer total costs resulting from the implementation of a coordinated VMI policy. T is defined in Eq. 24, where $TC^*_{\text{buyer,VMI}}$ and $TC^*_{\text{buyer,non VMI}}$ represent the optimal buyer total costs with and without VMI, respectively.

$$T = \frac{TC^*_{\text{buyer, non VMI}} - TC^*_{\text{buyer, VMI}}}{TC^*_{\text{buyer, non VMI}}} \tag{24}$$

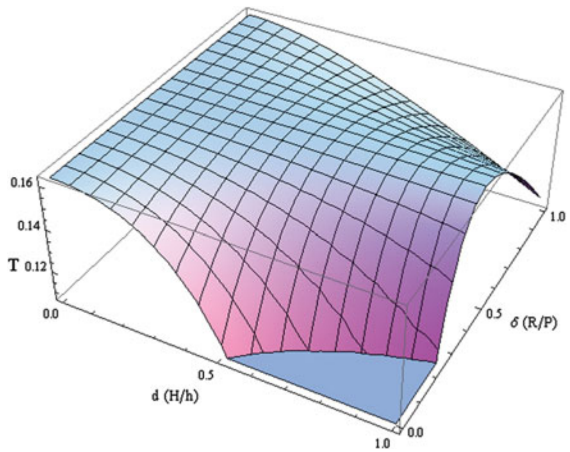

Fig. 9 Sensitivity of T to changes in δ and d ($C = 4000$, $c = 100$, $c' = 70$)

Breaking these costs down into their components and replacing the order quantities from Eqs. 9 and 10, we obtained the expression presented in Eq. 25, as shown in Appendix 3:

$$T = 1 - \frac{1}{2}\sqrt{\frac{g}{g'}}\left\{\frac{2 + d(2\delta - 1)}{\sqrt{1 + d(2\delta - 1)}}\right\} \tag{25}$$

From the above equation, we can perform the same analysis with the previous parameters remaining constant.

Figure 9 shows that a large set of values of d and δ generate positive benefits for the buyer in the VMI agreement, with the selected parameters shown in Sect. 4.1. In addition, when δ is near 0 and d is near 1, T is negative and strongly decreases to $-\infty$. In addition, T is very responsive to changes in d and δ when d is near 1 and δ is near 0 or 1.

From our analysis, we obtain the following findings:

- The potential effect of changes in the g and g' values on the benefits to the buyer in the VMI agreement could be evaluated by computing the first derivative of T with respect to the parameters g and g', as described in Eqs. 26 and 27. As a result, T decreases when the parameter g ($g = C/c$) increases and increases when the parameter g' ($g' = C/c'$) increases.

$$\frac{\partial T}{\partial g'} = \frac{g\left[\sqrt{\frac{1}{1+d(-1+2\delta)}} + \sqrt{1 + d(-1 + 2\delta)}\right]}{4g'^2\sqrt{\frac{g}{g'}}} > 0 \tag{26}$$

$$\frac{\partial T}{\partial g} = -\frac{\sqrt{\frac{1}{1+d(-1+2\delta)}} + \sqrt{1 + d(-1 + 2\delta)}}{4g'\sqrt{\frac{g}{g'}}} < 0 \tag{27}$$

- Savings from VMI implementation in supply chain total costs (V) decrease when the parameter g ($g = C/c$) increases and increase when the parameter g' ($g' = C/c'$) increases, which is checked from the partial derivatives using Eqs. 28 and 29.

$$\frac{\partial V}{\partial g} = \frac{\left[1+\sqrt{\frac{2\delta-1+\frac{1}{d}}{(1-\delta)g'}}\right]}{\left[1+\sqrt{\frac{1}{gd(1-\delta)}}\right]^2}\frac{1}{\sqrt{d(1-\delta)}}\left(-\frac{1}{2}g^{-3/2}\right) < 0 \tag{28}$$

$$\frac{\partial V}{\partial g'} = -\frac{1}{1+\sqrt{\frac{1}{gd(1-\delta)}}}\sqrt{\frac{2\delta-1+\frac{1}{d}}{(1-\delta)}}\left(-\frac{1}{2}g^{-3/2}\right) > 0 \tag{29}$$

- The percentage of savings in manufacturer inventory holding costs (Y) is analyzed from the partial derivatives with respect to g' shown in Eq. 30.

$$\frac{\partial Y}{\partial g'} = \frac{(-1+2\delta)\sqrt{\frac{d}{1+(-1+2\delta)d}}}{2g^{-3/2}\sqrt{1-\delta}} \tag{30}$$

- In this case, values of the first derivative of Y with respect to g' depend on values of δ, as shown in Eqs. 31 and 32.

$$\frac{\partial Y}{\partial g'} < 0 \text{ if } 0 < \delta < 0,5 \tag{31}$$

$$\frac{\partial Y}{\partial g'} > 0 \text{ if } 0,5 < \delta < 1 \tag{32}$$

- The performance measurement Z, related to buyer inventory holding costs, is analyzed from the partial derivatives with respect to the parameters g and g', in Eqs. 33 and 34. In the same way, results show that Z decreases when the parameter g ($g = C/c$) increases and increases when the parameter g' ($g' = C/c'$) increases.

$$\frac{\partial Z}{\partial g'} = \frac{\left[\frac{1}{g'}\right]^{3/2}}{2\sqrt{\frac{1-d+2d\delta}{g}}} > 0 \tag{33}$$

$$\frac{\partial Z}{\partial g} = -\frac{\sqrt{\frac{1}{g'}}}{2g\sqrt{\frac{1-d+2d\delta}{g}}} < 0 \tag{34}$$

- From the partial derivatives of the percentage of savings in the implementation of a VMI policy in buyer's order cost performance indicator (N) with respect to the parameters g and g', represented in Eqs. 35 and 36, we deduce similarly that

N decreases when the parameter g ($g = C/c$) increases and increases when the parameter g' ($g' = C/c'$) increases.

$$\frac{\partial N}{\partial g} = -\frac{\sqrt{1 + d(-1 + 2\delta)}}{2\sqrt{g'g}} < 0 \tag{35}$$

$$\frac{\partial N}{\partial g'} = \frac{\sqrt{g(1 + d(-1 + 2\delta))}}{2g'^{3/2}} > 0 \tag{36}$$

5 Conclusions

This article analyzes the performance of a supply chain under a coordinated manufacturer–buyer VMI approach. The main contribution of the proposed model is that it includes an explicit coordinated manufacturer–buyer VMI scheme related to the manufacturer's uptime and non-productive time. The model proposes synchronization between the manufacturer's and the buyer's replenishment cycles. The realistic manufacturer–buyer coordination scheme makes VMI logistics implementation much easier, which is not the case in other related VMI approaches. Our proposed synchronization scheme was modeled and optimized. Studying the supply chain under ordering and holding cost optimization, mathematical expressions were deduced for:

- Buyer and manufacturer lot sizes
- Inventory replenishment times
- Buyer and manufacturer average inventory levels
- Manufacturer uptimes
- Integer coordination constants
- Buyer and manufacturer total, holding and ordering costs

In conclusion, we have presented and developed a comprehensive model showing an explicit manufacturer–buyer coordination mechanism for a VMI implementation.

5.1 Managerial Implications

The objective of this study was to compare the behavior of the total supply chain costs, individual buyer costs, and manufacturer costs in a synchronized VMI implementation, according to a sensitivity analysis on the model parameters. We have proposed different performance measurements to evaluate the benefits of our coordinated scheme. The proposed performance measurements evaluate costs to the manufacturer, buyer, and supply chain. Our analysis of the savings to total supply chain costs with the implementation of VMI has shown from the partial derivatives of performance indicators that total savings increase at an increasing rate in $g' = C/c'$ (the ratio between manufacturer and buyer of ordering cost with VMI) and decrease at an increasing rate in $g = C/c$ (the ratio between the manufacturer and buyer ordering cost without VMI). Furthermore, savings to total buyer costs and buyer inventory holding and ordering costs with the implementation of VMI also increase at an increasing rate in g' and decrease at an increasing rate in g. When $\delta = r/P$ (the ratio between demand and production rates) is lower than 0.5, the savings to manufacturer inventory holding costs decrease at an increasing rate in g'. When δ is greater than 0.5, the savings to manufacturer inventory holding costs increase at an increasing rate in g'.

Savings to the manufacturer's inventory holding cost and to the total cost with VMI implementation will be non-negative when δ is lower than 0.5. In general, our sensitivity analysis determined that there is a set of values for the levels of c, c', d, and δ that generates non-negative benefits with the implementation of a VMI system for both the manufacturer and buyer. Table 5 shows a summary of the set of conditions that must satisfy all parameters to accomplish non-negative benefits with the implementation of the VMI-coordinated approach for both the manufacturer and buyer. From a practical point of view, these results have clear managerial implications and can explain the general buyer and manufacturer expectations in the implementation of a coordinated VMI approach under different supply chain conditions. The analysis presented in Table 5 shows that it is completely possible and realistic that both manufacturer and buyer obtain positive benefits in VMI implementations using the proposed coordination scheme in a win–win relationship. Our model shows that this can be accomplished by selecting particular combinations of c, c', d, and δ parameters as shown in the last column of Table 5. This finding differs from that in the literature. The main result of the study is to show how manufacturer and buyer can both obtain profits under a coordinated VMI implementation.

Table 5 Set of conditions to achieve non-negative benefits with the implementation of the VMI-coordinated approach, according to the performance measurements for each agent

Performance measurement	Cost	Agent	Agent obtains a non-negative benefit from VMI implementation when
Z	Inventory holding cost	Buyer	$\delta > \frac{1}{2}$ or $\left[\delta < \frac{1}{2} \text{ and } d(2\delta - 1) > \frac{c' - c}{c}\right]$
Y	Inventory holding cost	Manufacturer	$\delta < \frac{1}{2}$
N	Ordering cost	Buyer	$0 < \delta < \frac{1}{2}$ or $\left[\frac{1}{2} < \delta < 1 \text{ and } d(2\delta - 1) < \frac{c - c'}{c'}\right]$
M	Ordering cost	Manufacturer	Equal with and without VMI
T	Total cost	Buyer	$\sqrt{\frac{c'}{c}} < \frac{\sqrt{1 + 2d(\delta - \frac{1}{2})}}{1 + d(\delta - \frac{1}{2})}$
S	Total cost	Manufacturer	$\delta < \frac{1}{2}$
V	Total cost	Chain	$\frac{c}{c'} > d(2\delta - 1) + 1$

A non-empty intersection of c, c', d, and δ values will give a general condition need to achieve non-negative benefits to both the manufacturer and the buyer with the implementation of VMI, when performance measurements T and S accomplish the simultaneous settings shown in the last column of Table 5. In addition, we can deduce two independent sufficient conditions required to achieve non-negative benefits to the buyer's inventory holding cost. These conditions are described by the performance measurement Z in Table 5. The first one is associated with values of δ greater than 0.5, while the second corresponds to values of δ lower than 0.5 with $d(2\delta - 1) > \frac{c'-c}{c}$. In the same way, a non-negative benefit in the manufacturer's inventory holding cost is described by the performance measurement Y and is associated with values of δ lower than 0.5. Also, there are two sets of values for c, c', d, and δ that give non-negative benefits with the implementation of VMI for the buyer's ordering cost. These sets are described by $0 < \delta < \frac{1}{2}$ and $\left[\frac{1}{2} < \delta < 1 \text{ and } d(2\delta - 1) < \frac{c-c'}{c'}\right]$. Finally, the whole supply chain will achieve non-negative benefits from VMI implementation when $\frac{c}{c'} > d(2\delta - 1) + 1$.

From the models developed in this work, under our proposed manufacturer–buyer synchronization scheme, it follows that the buyer's optimal order quantity under the VMI-coordinated approach is lower than that without VMI when $\frac{c'}{c} < d(2\delta - 1) + 1$, according to Appendix 4. This result differs from and complements previous findings in the literature (Dong and Chu 2002; Choi et al. 2004; Yao et al. 2007). In our approach, the buyer's optimal order quantity under the VMI-coordinated approach is not always lower than that without VMI. However, Table 5 shows that it is even possible to select a combination of the parameters$c$, c', d, and δ that generates non-negative cost benefits for buyer, manufacturer, and the supply chain, with the buyer's optimal order quantity lower under the VMI-coordinated approach than that without VMI. From our models, we can show that these conditions are accomplished when $0 < \delta < \frac{1}{2}$ and $\frac{c'}{c} < d(2\delta - 1) + 1$, as shown in Appendix 4.

6 Future Work

The next step in this research topic will be the extension of this type of analysis to other models of supply chains with stochastic demands, including VMI, systems formed by a manufacturer and multiple buyers, multi-product supply chains, and delivery-time links between manufacturers and buyers, including transportation costs.

We also expect that some real applications of this model to the industry will be undertaken, possibly together with system simulations using specialized software.

Acknowledgments The authors would like to thank to the anonymous referee and to the editor for their valuable comments.

Appendix 1

To calculate the area under the curve for inventory levels of the manufacturer, the geometry of the Fig. 2 is considered as follows:

First we analyzed the manufacturer's inventory levels to obtain the equation for the area under the curve:

$$\begin{aligned} A_S = {} & Pt\frac{t}{2} + \frac{t}{2}(Pt - q + 2Pt - q) + \frac{t}{2}(2Pt - 2q + 3Pt - 2q) \\ & + \cdots + \frac{t}{2}([L-1]Pt - [L-1]q + LPt - [L-1]q) \\ & + \frac{\tau_s}{2}(LPt - Lq + [k - L - 1]q) + (t - \tau_s)[k - L - 1]q \\ & + t([k - L - 2]q + [k - L - 3]q + \cdots + q) + PT_s\frac{T_s}{2} \end{aligned} \tag{37}$$

Figure 2 gives the average inventory level as follows:

$$\begin{aligned} I_S = {} & \left[\frac{Pt^2}{2}\left(\sum_{i=1}^{L} i + \sum_{i=1}^{L-1} i\right) - qt\sum_{i=1}^{L-1} i + [(k - L - 1)q + Lpt - Lq]\frac{\tau_s}{2}\right. \\ & \left. + (k - L - 1)q(t - \tau_s) + qt\sum_{i=1}^{k-L-2} i + P\frac{T_s^2}{2}\right]/kt \end{aligned} \tag{38}$$

Replacing it with the relationships included in Eqs. 5 and 6, we can turn this equation into:

$$\begin{aligned} I_S &= \left[\frac{Pt^2L^2}{2} - \frac{qtL(L-1)}{2} + [-kq + q + Lpt]\frac{\tau_s}{2} + \frac{qt}{2}(k - L - 1)(k - L) + P\frac{T_s^2}{2}\right]/kt \\ &= \left[\frac{Pt^2L^2}{2} - \frac{qtL(L-1)}{2} + \frac{\tau_s}{2}P\left(Lt + \frac{q}{P} - k\frac{q}{P}\right) + \frac{qt}{2}(k - L - 1)(k - L) + P\frac{T_s^2}{2}\right]/kt \\ &= \left[\frac{Pt^2L^2}{2} - \frac{qtL(L-1)}{2} - \frac{P}{2}\left(Lt + \frac{q}{P} - k\frac{q}{P}\right)^2 + \frac{qt}{2}(k - L - 1)(k - L) + P\frac{T_s^2}{2}\right]/kt \\ &= \left[\frac{Pt^2L^2}{2} - \frac{qtL(L-1)}{2} + \frac{P}{2}\left[\frac{q^2}{P^2} - \left(Lt + \frac{q}{P} - k\frac{q}{P}\right)^2\right] + \frac{qt}{2}(k - L - 1)(k - L)\right]/kt \\ &= \left[\frac{\mathrm{Pt}^2L^2}{2} - \frac{qtL(L-1)}{2} + \frac{P}{2}\left[-L^2t^2 + Lt\left(2\frac{kq}{P} - 2\frac{q}{P}\right) + \frac{kq^2}{P^2}(2 - k)\right] + \frac{qt}{2}(k - L - 1)(k - L)\right]/kt \\ &= \left[\frac{kq}{2}\left(2\frac{q}{P} - k\frac{q}{P}\right) + \frac{q}{2}tk^2 - \frac{q}{2}tk\right]/kt \\ &= \left[\frac{krt}{2}\left(2\frac{rt}{P} - k\frac{rt}{P}\right) + \frac{rt}{2}tk^2 - \frac{rt}{2}tk\right]/kt \\ &= \frac{\mathrm{q}}{2}\left[2\frac{r}{P} - 1 + k\left(1 - \frac{r}{P}\right)\right] \end{aligned} \tag{39}$$

And finally:

$$I_S = \frac{q}{2}\left(-1 + \frac{2r}{P} + k\left(1 - \frac{r}{P}\right)\right) \tag{40}$$

Appendix 2

With VMI, the total costs of the system are:

$$\begin{aligned}\mathrm{TC}_{\mathrm{VMI}} &= c'\frac{r}{q} + h\frac{q}{2} + C\frac{r}{Q} + HI_S \\ &= c'\frac{r}{q} + h\frac{q}{2} + C\frac{r}{kq} + H\frac{q}{2}\left[k\left(1 - \frac{r}{P}\right) + 2\frac{r}{P} - 1\right]\end{aligned}$$

Taking the first derivative with respect to q and equaling to zero to minimize $\mathrm{TC}_{\mathrm{VMI}}$:

$$\frac{\partial \mathrm{TC}_{\mathrm{VMI}}}{\partial q} = 0 = -c'\frac{r}{q^2} + \frac{h}{2} - C\frac{r}{kq^2} + \frac{H}{2}\left[k\left(1 - \frac{r}{P}\right) + 2\frac{r}{P} - 1\right]$$

Therefore:

$$q^2_{\mathrm{VMI}} = \frac{2r\left(c' + \frac{C}{k}\right)}{\left[h + H\left[k\left(1 - \frac{r}{P}\right) + 2\frac{r}{P} - 1\right]\right]} \tag{41}$$

And:

$$TC_{\mathrm{VMI}} = \sqrt{\left[2r\left(c' + \frac{C}{k}\right)\right]\left[h + H\left[k\left(1 - \frac{r}{P}\right) + 2\frac{r}{P} - 1\right]\right]}$$

Taking the first derivative of the last expression with respect to k and equaling to zero to minimize $\mathrm{TC}_{\mathrm{VMI}}$:

$$0 = \left(c' + \frac{C}{k}\right)H\left(1 - \frac{r}{P}\right) - \frac{C}{k^2}\left[h + H\left[k\left(1 - \frac{r}{P}\right) + 2\frac{r}{P} - 1\right]\right]$$

As a result:

$$k^2 = \frac{C\left[h + H\left(2\frac{r}{P} - 1\right)\right]}{c'H\left(1 - \frac{r}{P}\right)}$$

Replacing the last expression in Eq. 41, we obtain:

$$q_{VMI}^2 = \frac{2r(kc' + C)}{\left[hk + H\left[k^2\left(1 - \frac{r}{P}\right) + k\left(2\frac{r}{P} - 1\right)\right]\right]}$$
$$= \frac{2r(kc' + C)}{\left[hk + \frac{C\left[h + H\left(2\frac{r}{P} - 1\right)\right]}{c'} + HK\left(2\frac{r}{P} - 1\right)\right]}$$
$$= \frac{2rc'(kc' + C)}{\left[h(kc' + C) + \left(2\frac{r}{P} - 1\right)H(kc' + C)\right]} = \frac{2rc'}{\left[h + H\left(2\frac{r}{P} - 1\right)\right]}$$

As a consequence:

$$Q_{\mathrm{VMI}} = kq_{\mathrm{VMI}} = \sqrt{\frac{C\left[h + H\left(2\frac{r}{P} - 1\right)\right]}{c'H\left(1 - \frac{r}{P}\right)} \frac{2rc'}{\left[h + H\left(2\frac{r}{P} - 1\right)\right]}} = \sqrt{\frac{2Cr}{H\left(1 - \frac{r}{P}\right)}}$$

And:

$$\mathrm{TC}_{\mathrm{VMI}} = q_{\mathrm{VMI}}\left[h + H\left[k\left(1 - \frac{r}{P}\right) + 2\frac{r}{P} - 1\right]\right]$$
$$= \sqrt{\frac{2rc'}{\left[h + H\left(2\frac{r}{P} - 1\right)\right]}}\left[h + H\left[\sqrt{\frac{C\left[h + H\left(2\frac{r}{P} - 1\right)\right]}{c'H\left(1 - \frac{r}{P}\right)}}\left(1 - \frac{r}{P}\right) + 2\frac{r}{P} - 1\right]\right]$$
$$= \sqrt{2r}\left[\sqrt{CH\left(1 - \frac{r}{P}\right)} + \sqrt{c'\left[h + H\left(2\frac{r}{P} - 1\right)\right]}\right]$$

Appendix 3

The percentage of savings in total costs for the supply chain is defined according to performance measurement V as:

$$V = \frac{\mathrm{TC}^*_{\mathrm{NON\,VMI}} - \mathrm{TC}^*_{\mathrm{VMI}}}{\mathrm{TC}^*_{\mathrm{NON\,VMI}}} = 1 - \frac{\mathrm{TC}^*_{\mathrm{VMI}}}{\mathrm{TC}^*_{\mathrm{NON\,VMI}}}$$

Replacing:

$$V = 1 - \frac{\sqrt{2r}\left[\sqrt{CH\left(1-\frac{r}{P}\right)} + \sqrt{c'\left[h + H\left(2\frac{r}{P}-1\right)\right]}\right]}{\sqrt{2r}\left[\sqrt{CH\left(1-\frac{r}{P}\right)} + \sqrt{ch}\right]}$$

$$= 1 - \frac{1 + \sqrt{\frac{\left(2\frac{r}{P}-1\right)+\frac{h}{H}}{\frac{C}{c'}\left(1-\frac{r}{P}\right)}}}{1 + \sqrt{\frac{1}{\frac{CH}{ch}\left(1-\frac{r}{P}\right)}}} = 1 - \frac{1 + \sqrt{\frac{(2\delta-1)+\frac{1}{d}}{g'(1-\delta)}}}{1 + \sqrt{\frac{1}{gd(1-\delta)}}}$$

$$= \frac{1 - \sqrt{\frac{g}{g'}(1 - d + 2d\delta)}}{1 + \sqrt{gd(1-\delta)}}$$

The percentage of savings in inventory holding cost for the buyer is defined according to performance measurement Z as:

$$Z = \frac{\text{IHC}^*_{\text{buyer, non VMI}} - \text{IHC}^*_{\text{buyer, VMI}}}{\text{IHC}^*_{\text{buyer, non VMI}}} = 1 - \frac{\text{HC}^*_{\text{buyer, VMI}}}{\text{IHC}^*_{\text{buyer, non VMI}}}$$

$$= 1 - \frac{h\frac{q_{\text{VMI}}}{2}}{h\frac{q_{\text{non VMI}}}{2}} = 1 - \frac{q_{\text{VMI}}}{q_{\text{non VMI}}} = 1 - \sqrt{\frac{c'h}{c\left[h + \left(2\frac{r}{P}-1\right)H\right]}}$$

$$= 1 - \sqrt{\frac{g}{g'[1 + d(2\delta - 1)]}}$$

The percentage of savings in inventory holding cost for the manufacturer is defined according to performance measurement Y as:

$$Y = \frac{\text{IHC}^*_{\text{manufacturer,non VMI}} - \text{IHC}^*_{\text{manufacturer,VMI}}}{\text{IHC}^*_{\text{manufacturer,non VMI}}} = 1 - \frac{\text{IHC}^*_{\text{manufacturer,VMI}}}{\text{IHC}^*_{\text{manufacturer,non VMI}}}$$

$$= 1 - \frac{H\frac{q_{\text{VMI}}}{2}\left[k\left(1 - \frac{r}{P} + 2\frac{r}{P} - 1\right)\right]}{H\frac{Q_{\text{non VMI}}}{2}\left(1-\frac{r}{P}\right)} = \frac{-\left(2\frac{r}{P}-1\right)}{\left(1-\frac{r}{P}\right)\sqrt{\frac{C\left[h+H\left(2\frac{r}{P}-1\right)\right]}{c'H\left(1-\frac{r}{P}\right)}}}$$

$$= \frac{(1-2\delta)}{\sqrt{1-\delta}}\sqrt{\frac{d}{g'[1 + d(2\delta-1)]}}$$

The percentage of savings in buyer ordering cost is defined according to performance measurement N as:

$$N = \frac{\text{OC}^*_{\text{buyer, non VMI}} - \text{OC}^*_{\text{buyer, VMI}}}{\text{OC}^*_{\text{buyer, non VMI}}} = 1 - \frac{\text{OC}^*_{\text{buyer,VMI}}}{\text{OC}^*_{\text{buyer,non VMI}}}$$

$$= 1 - \frac{c'\frac{r}{q_{\text{VMI}}}}{c\frac{r}{q_{\text{non VMI}}}} = 1 - \sqrt{\frac{c'}{c}\left[1 + \frac{H}{h}\left(2\frac{r}{P}-1\right)\right]} = 1 - \sqrt{\frac{g[1 + d(2\delta-1)]}{g'}}$$

The percentage of savings in manufacturer total costs is defined according to performance measurement S as:

$$\begin{aligned} S &= \frac{\mathrm{TC}^*_{\text{manufacturer,non VMI}} - \mathrm{TC}^*_{\text{manufacturer,VMI}}}{TC^*_{\text{manufacturer,non VMI}}} = 1 - \frac{\mathrm{TC}^*_{\text{manufacturer,VMI}}}{\mathrm{TC}^*_{\text{manufacturer,non VMI}}} \\ &= 1 - \frac{\sqrt{2CrH\left(1-\frac{r}{P}\right)} + \frac{H}{2}\left(2\frac{r}{P}-1\right)\sqrt{\frac{2rc'}{\left[h+H\left(2\frac{r}{P}-1\right)\right]}}}{\sqrt{2CrH\left(1-\frac{r}{P}\right)}} \\ &= \frac{1}{2}\left(1-2\frac{r}{P}\right)\sqrt{\frac{Hc'}{C\left(1-\frac{r}{P}\right)\left[h+H\left(2\frac{r}{P}-1\right)\right]}} \\ &= \frac{1}{2}(1-2\delta)\sqrt{\frac{d}{g'(1-\delta)[1+d(2\delta-1)]}} \end{aligned}$$

The percentage of savings in buyer's total costs is defined according to performance measurement T as:

$$\begin{aligned} T &= \frac{\mathrm{TC}^*_{\text{buyer, non VMI}} - \mathrm{TC}^*_{\text{buyer, VMI}}}{TC^*_{\text{buyer, non VMI}}} = 1 - \frac{\mathrm{TC}^*_{\text{buyer, VMI}}}{\mathrm{TC}^*_{\text{buyer, non VMI}}} \\ &= 1 - \frac{\left(c'\frac{r}{q_{\text{VMI}}} + h\frac{q_{\text{VMI}}}{2}\right)}{hq_{\text{VMI}}} = 1 - \frac{1}{\sqrt{2crh}}\left[\sqrt{\frac{rc'}{2}\left[h+H\left(2\frac{r}{P}-1\right)\right]} + \frac{h}{2}\sqrt{\frac{2rc'}{\left[h+H\left(2\frac{r}{P}-1\right)\right]}}\right] \\ &= 1 - \frac{1}{2}\sqrt{\frac{c'}{c}}\left\{\sqrt{1+\frac{H}{h}\left(2\frac{r}{P}-1\right)} + \frac{1}{\sqrt{1+\frac{H}{h}\left(2\frac{r}{P}-1\right)}}\right\} \\ &= 1 - \frac{1}{2}\sqrt{\frac{g}{g'}}\left\{\sqrt{1+d(2\delta-1)} + \frac{1}{\sqrt{1+d(2\delta-1)}}\right\} = 1 - \frac{1}{2}\sqrt{\frac{g}{g'}}\left\{\frac{2+d(2\delta-1)}{\sqrt{1+d(2\delta-1)}}\right\} \end{aligned}$$

Appendix 4

Given that:

$$q_{\text{non VMI}} = \sqrt{\frac{2rc}{h}}, \qquad q_{\text{VMI}} = \sqrt{\frac{2rc'}{\left[h+H\left(2\frac{r}{P}-1\right)\right]}}$$

Therefore, if $\frac{q_{\text{non VMI}}}{q_{\text{VMI}}} = \sqrt{\frac{c}{c'}(1+d(2\delta-1))} > 1$ then $\frac{c'}{c} < d(2\delta-1)+1$
Furthermore,

If $0<\delta<\frac{1}{2}$, then $\sqrt{\frac{c'}{c}} < \frac{\sqrt{1+2d\left(\delta-\frac{1}{2}\right)}}{1+d\left(\delta-\frac{1}{2}\right)}$ because the function $F(x) = \frac{1+2dx}{(1+dx)^2}$ is non-decreasing in the interval $\left[-\frac{1}{2}, 0\right]$ and $c'<c$.

References

Achabal, D., McIntyre, S., Smith, S., Kalyanam, K., 2000, A decision support system for vendor managed inventory, Journal of Retailing 76 (4), 430.

Angulo, A., Nachtmann, H., Waller, M., 2004, Supply chain information sharing in a Vendor-managed inventory partnership, Journal of Business Logistics 25 (1), 101-120.

Blatherwick, A., 1998, Vendor-managed inventory: Fashion fad or important supply chain strategy?, Supply Chain Management 3 (1), 10-11.

Cachon, G., Fisher, M., 1997, Campbell soup's continuous replenishment program: evaluation and enhanced inventory decision rules, Production and Operations Management 6 (3), 266-276.

Cachon, G., Zipkin, P., 1999, Competitive and cooperative inventory policies in a two-stage supply chain, Management Science 45 (7), 936-947.

Challener, C., 2000, Taking the VMI step to collaborative commerce, Chemical Market Reporter 258 (21), 11-12.

Chen, X., Hao, G., Li, X., Yiu, K.F.C., 2012, The impact of Demand variability and transshipment on vendor's distribution policies under vendor managed inventory strategy. International Journal of Production Economics 139 (1), 42-48.

Choi, K., Dai, J., Song, J., 2004, On measuring supplier performance under vendor-managed inventory programs in capacitated supply chains, Manufacturing & Service Operations Management 6 (1), 53-72.

Cottrill, K., 1997, Reforging the supply chain, Journal of Business Strategy 81 (6), 35-39.

Darwish, M., Odah, O., 2010, Vendor-managed inventory model for single-vendor multi-retailer supply chains, European Journal of Operational Research 204 (3), 473-484.

Daugherty, P., Myers, M., Autry, C., 1999, Automatic replenishment programs: An empirical examination, Journal of Business Logistics 20 (2), 63-82.

Disney, S., Towill, D., 2002, A procedure for the optimization of the dynamic response of a Vendor Managed Inventory system, Computers & Industrial Engineering 43 (1-2), 27-58.

Disney, S., Towill, D., 2003, The effect of vendor-managed inventory (VMI) dynamics on the bullwhip effect in supply chains, International Journal of Production Economics 85 (2), 199-215.

Dong, Y., Chu, K., 2002, A supply chain model of vendor managed inventory, Transportation Research Part E 38 (2) 75–95.

Emigh, J., 1999, Vendor-managed inventory, Computerworld, 33 (34), 52.

Hemmelmayr, V., Doerner, K., Hartl, R., Savelsbergh, M., 2010, Vendor managed inventory for environments with stochastic product usage, European Journal of Operational Research 202 (3), 686-695.

Holmström, J., 1998, Business process innovation in the supply chain – a case study of implementing vendor managed inventory, European Journal of Purchasing & Supply Management 4 (2-3) 127-131.

Hongjie, L., Ruxian, L., Zhigao L., Ruijiang W., 2011, Study on the inventory control of deteriorating items under VMI model based on bi-level programming, Expert Systems with Applications 38 (8), 9287–9295.

Jain, C., 1994, Ready or not, here comes vendor-managed inventory program, Journal of Business Forecasting 13 (2), 2-44.

Kastsian, D., Mönnigmann, M., 2011, Optimization of a vendor managed inventory supply chain with guaranteed stability and robustness, Int. J. Production Economics 131 (2), 727–735.

Kristianto, Y., Helo, P., Jiao, J., Sandhu M., 2012, Adaptive fuzzy vendor managed inventory control for mitigating the Bullwhip effect in supply chains, European Journal of Operational Research 216 (2), 346–355.

Kuk, G., 2004, Effectiveness of vendor-managed inventory in the electronics industry: determinants and outcomes, Information & Management 41 (5) 645–654.

Lee, C., Chu, W., 2005, Who should control inventory in a supply chain, European Journal of Operational Research 164 (1), 158–172.
Lee, H., So, K., Tang, C., 2000, The value of information sharing in a two-level supply chain, Management Science 46 (5), 626-643.
Lee, J., Ren, L., 2011, Vendor-managed inventory in a global environment with exchange rate uncertainty, Int. J. Production Economics 130 (2), 169–174.
Liao, S., Hsieh C., Lai, P., 2011, An evolutionary approach for multi-objective optimization of the integrated location–inventory distribution network problem in vendor-managed inventory, Expert Systems with Applications 38 (6), 6768–6776.
Ofuoku, M., 2009, Stochastic vendor managed inventory: impact on non-identical retailers, New Mexico State University, PhD. Thesis, 2009, 121 pages, AAT 3400350.
Pasandideh, S.H.R., Niaki, S.T.A., Nia, A.R., 2011, A genetic algorithm for vendor managed inventory control system of multi-product multi-constraint economic order quantity model, Expert Systems with Applications 38 (3), 2708–2716.
Sari, K., 2008, On the benefits of CPFR and VMI: a comparative simulation study, International Journal of Production Economics 113 (2) 575-586.
Saxena, R., 2009, Vendor-managed inventory, Industrial Engineer 41 (7), 20.
Silver, E., Pyke, D., Peterson, R., 1998, Inventory management and production planning and scheduling, third edition, John Wiley & sons.
Song, D., Dinwoodie, J., 2008, Quantifying the effectiveness of VMI and integrated inventory management in a supply chain with uncertain lead-times and uncertain demands, Production Planning and Control 19 (6), 590-600.
Southard, P., Swenseth, S., 2008, Evaluating vendor-managed inventory (VMI) in non-traditional environments using simulation, International Journal of Production Economics, 116 (2), 275-287.
Van der Vlist, P., Kuik, R., Verheijen, B., 2007, Note on supply chain integration in vendor-managed inventory, Decision Support Systems 44 (1), 360-365.
Waller, M., Johnson, E., Davis, T., 1999, Vendor-Managed Inventory in the retail supply chain, Journal of Business Logistics 20 (1), 183-203.
Wang, H., Guo, M., Efstathiou, J., 2004, A game-theoretical cooperative mechanism design for a two-echelon decentralized supply chain, European Journal of Operational Research 157 (2), 372–388.
White, A., Censlive, M., 2006, Observations on modeling strategies for vendor-managed inventory, Journal of Manufacturing Technology Management 17 (4), 496-512.
Wong, W., Qi, J., Leung, S., 2009, Coordinating supply chains with sales rebate contracts and vendor-managed inventory, International Journal of Production Economics 120 (1), 151-161.
Xu, K., Leung M. T., 2009, Stocking policy in a two-party vendor managed channel with space restrictions, Int. J. Production Economics 117 (2), 271–285.
Yang, K., Ruben, R., Webster, S., 2003, Managing vendor inventory in a dual level distribution system, Journal of Business Logistics 24 (2), 91-108.
Yang, L., Ng, C., Cheng, T., 2010, Evaluating the effects of distribution centers on the performance of vendor-managed inventory systems, European Journal of Operational Research 201 (1), 112-122.
Yao, Y., Dong, Y., Dresner, M., 2010, Managing supply chain backorders under vendor managed inventory: An incentive approach and empirical analysis, European Journal of Operational Research 203 (2), 350-359.
Yao, Y., Dresner, M., 2008, The inventory value of information sharing, continuous replenishment and vendor-managed inventory, Transportation Research Part E 44 (3) 361–378.
Yao, Y., Evers, P., Dresner, M., 2007, Supply chain integration in vendor-managed inventory, Decision Support Systems 43 (2), 663-674.
Yu, H., Zeng, A., Zhao, L., 2009a, Analyzing the evolutionary stability of the vendor-managed inventory supply chains, Computers & Industrial Engineering 56 (1), 274–282.

Yu, Y., Huang, G., 2009b, Nash game model for optimizing market strategies, configuration of platform products in a Vendor Managed Inventory (VMI) supply chain for a product family, European Journal of Operational Research 206 (2), 361.

Yu, Y., Huang, G., Liang, L., 2009c, Stackelberg game-theoretic model for optimizing advertising, pricing and inventory policies in vendor managed inventory (VMI) production supply chains, Computers & Industrial Engineering 57 (1) 368–382.

Yu, Y., Wang, Z., Liang, L., 2012, A vendor managed inventory supply chain with deteriorating raw materials and products. International Journal of Production Economics 136 (2), 266-274.

Zanoni, S., Jaber M.Y., Zavanella, L.E., 2012, Vendor managed inventory (VMI) with consignment considering learning and forgetting effects, International Journal of Production Economics 140 (2), 721-730.