The Utility of EOQ in Supply Chain Design and Operation

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Abstract Supply chain design and operation problems are complicated in many fronts due to intervened decision making. One of the main complications is related to inventory decisions and costs. In many cases, considering inventory in the overall supply chain domain introduces stochastic and nonlinear formulations. Ignoring inventories, as in the traditional approach, results in inferior supply chain designs and operations in terms of cost performance. Economic order quantity (EOQ) models, with their simplicity, intuitive explanation, and clear implementation, aid in the resolution of these issues in a number of integrated supply chain problem contexts. In this chapter, we summarize the role and utility of EOQ models in these integrated supply chain design and operation problems. Specifically, we consider the three pillars of supply chain management: location, transportation, and inventory. We discuss how EOQ models ease the analysis of integrated supply chain models under each pillar in detail including inventory-location models, inventory-routing models, and multi-echelon inventory models.

1 Introduction

The supply chain (SC) encompasses all activities associated with the flow and transformation of goods from the raw material procurement stage to production, from storage to distribution, from markets to end users for demand satisfaction. Supply chain management (SCM) is the integration of these activities, through

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improved supply chain relationships, to achieve a sustainable, competitive advantage (Handfield and Nichols 1999). SCM is a term that has emerged in recent years that captures the essence of integrated logistics and even goes beyond it. It emphasizes the logistics interactions that take place among the functions of purchasing, production, logistics, and marketing within a firm and those interactions that take place between separate firms within the product-flow channel. Three major problem areas of supply chain management include facility location, inventory decisions, and transportation decisions. Although it is common to treat them separately, these problem areas are interrelated and should be planned as a unit (Ballou 2004). Each has an important impact on the system design, and ultimately, on operating costs.

Inventories, in particular, are essential to supply chain management since it is usually not possible or practical to provide instant production or ensure delivery times to customers. Inventories serve as a buffer between supply and demand so that needed product availability is maintained for customers while providing flexibility for production and distribution in seeking efficient methods to manufacture and transport the product. Inventory decisions refer to the manner in which inventories are managed. The particular inventory policy used by the firm affects the facility location and transportation decisions, and, therefore, the policy should be considered in overall logistics strategy. The economic order quantity (EOQ) model is the simplest and most fundamental of all inventory models. It describes the important trade-off between fixed order costs and holding costs and it is the basis for the analysis of more complex settings.

In this chapter, we review a number of recent research papers that use EOQ as a tool to resolve complicated supply chain trade-offs. In particular, we examine the integrated decision making within three areas of supply chain addressing location, transportation, and other inventory decisions and demonstrate the utility of EOQ models. In the remainder of this chapter, in Sect. 2, we discuss the integrated inventory-location models within discrete and continuous facility location models. Also, in Sect. 2, reflecting on the similarities between vendor selection and facility location problems, we present integrated inventory-sourcing models. In Sect. 3, we consider the integrated inventory transportation models including freight transportation and routing problems. In Sect. 4, we provide a summary of other inventory problems that consider EOQ as a subproblem such as multi-product constrained systems, joint replenishment problem, and multi-echelon inventories. Finally, in Sect. 5, we conclude by discussing some of the future research directions in integrated SCM.

2 Location Models

The relationship between inventory and location has long been revealed by Eppen (1979). He was the first to discuss the "impact of inventories" on locations by exploiting risk pooling effects. Assuming that each facility operates under the

EOQ assumptions with individual inventory management, Eppen show that when n identical facilities are consolidated at a single location, the cost ratio of independent facilities versus consolidated facility would be $1\sqrt{n}$. This is known as the *square root law of inventory centralization.* Building on this knowledge, there are several new studies that combine inventory management and location decisions including discrete location models (Erlebacher and Meller 2000; Keskin and Üster 2012; Keskin et al. 2012; Romeijn et al. 2007; Shu et al. 2005) and continuous location models (Drezner et al. 2003; McCann 1993; Üster et al. 2008).

The relationship between inventory and location has been characterized through the modeling of transportation. In many facility location problems, the decision for selecting a facility depends on the resolution of fixed facility location costs and transportation costs. Since changing the order quantities implies a modification of the ordering frequencies, the transportation costs are directly influenced by the inventory policy. Some researchers consider this impact explicitly (Keskin et al. 2010b, 2012; Keskin and Üster 2012; Üster et al. 2008) and some implicitly. On the other hand, some others consider the relationship between inventory and location problems from a lead time perspective and evaluate the impact of lead times on backorders (Drezner et al. 2003). In the rest of this section, we discuss how these models utilize EOQ to simplify the analysis and draw insights from complicated trade-offs.

2.1 Discrete Location Models

Daskin et al. (2002), Shen et al. (2003) and Shen and Daskin (2005) develop location models with risk pooling that explicitly incorporate inventory decisions into the uncapacitated facility location problem (UFLP). Ozsen (2004) and Ozsen et al. (2008, 2009) expand these models to consider the capacitated warehouse location model with risk pooling which captures the interdependence between capacity issues and the inventory management at the distribution centers. In these problems, it is common to assume a three-tiered supply chain consisting of one or more suppliers, distribution centers, and retailers. Furthermore, the locations of suppliers at the first tier and retailers at the third tier are known. The problem is to determine the optimal number of distribution centers (DC), their locations, the retailers assigned to each distribution center, and the optimal ordering policy at the distribution centers. In all of these models, the costs at each distribution center exhibit economies of scale, especially for capacitated models. These integrated inventory-location problems are formulated as mixed integer, nonlinear programs in which the objective function is neither concave nor convex. The typical solution approaches are based on Lagrangian relaxation as well as set covering reformulation and column generation algorithms.

The inventory problem faced by the distribution centers is modeled using a (Q, r) inventory model with type I service (Hopp and Spearman 1996; Nahmias

2009). It is common to approximate the (Q, r) model using two steps, where in the first step the order quantity is determined using an EOQ model in which the mean demand is used to represent the stochastic demand process and in the second step the reorder point is determined (Axsater 1996; Zheng 1992). Axsater (1996) shows that the maximum relative error incurred by using the EOQ instead of the optimal (Q, r) quantity is 0.118; and Zheng (1992) argues that in most cases, the relative increase is much less than the worst-case bounds. Therefore, in many of the inventory-location problems, the DC orders inventory from the supplier using an EOQ model. Next, we present the formulation of this model under these assumptions. For the sake of the technical discussion, we introduce the following notation given in Table 1.

For a given DC j with a particular set of assigned retailers $S \subset \mathscr{I}$, the total cost of ordering inventory at a DC j is approximated by the EOQ formula:

$$K_j n_j + (p_j + r_j Q_j) n_j + 0.5 h_j Q_j. \tag{1}$$

Given that $n_j = D_j / Q_j$ due to the EOQ approximation assumptions, we can restate the cost as

Table 1 Discrete inventory-location problem: notation

Parameters		
	$\mathscr{I}$	Set of retailers, indexed by i
	$\mathscr{J}$	Set of candidate DC sites, indexed by j
	$N(\mu_i, \sigma_i^2)$	Normally distributed daily demand at retailer i, $\forall i \in \mathscr{I}$
	D_j	Expected annual demand observed by DC j from its assigned retailers $i, \forall i \in S \subset \mathscr{I}, D_j = \sum_{i \in S} \chi \mu_i$, where χ is the number of days per year
	h_i	Holding cost of retailer i per item per year, $\forall i \in \mathscr{I}$
	K_j	Fixed cost of placing an order at DC j, $\forall j \in \mathscr{J}$
	p_j	Fixed cost of transportation from the supplier to DC j, $\forall j \in \mathscr{J}$
	r_j	Volume-dependent cost of transportation from the supplier to DC j, $\forall j \in \mathscr{J}$
	$p_j + r_j x$	Total transportation cost of an order size x from the supplier to DC j, $\forall j \in \mathscr{J}$
	f_j	Fixed (annual) cost of locating a distribution center at candidate site j, $\forall j \in \mathscr{J}$
	d_{ij}	Cost per unit to ship between retailer i and candidate DC j, $\forall i \in \mathscr{I}$, $\forall j \in \mathscr{J}$
	L_{ij}	Leadtime between retailer i and candidate DC j, $\forall i \in \mathscr{I}$, $\forall j \in \mathscr{J}$
	z_α	A standard Normal deviate such that $P(z \leq z) =$
Decision variables		
	n_j	Number of orders per year by DC j, $\forall j \in \mathscr{J}$
	Q_j	Order quantity of DC j, $\forall j \in \mathscr{J}$
	X_j	1, if we locate a DC at candidate site j; 0, if not, $\forall j \in \mathscr{J}$
	Y_{ij}	1, if demands at retailer i are assigned to a DC at candidate site j, $\forall i \in \mathscr{I}$, $\forall j \in \mathscr{J}$

$$r_j D_j + (K_j + p_j)\frac{D_j}{Q_j} + 0.5 h_j Q_j,$$

where the optimal quantity Q_j and the corresponding number of orders n_j at DC j are given as follows, respectively:

$$Q_j = \sqrt{\frac{2(K_j + p_j)D_j}{h_j}} \qquad \text{and} \qquad n_j = \sqrt{\frac{h_j D_j}{2(K_j + p_j)}}. \tag{2}$$

Unfortunately, the derivation of the working inventory cost in (1) assumes that we know the set $S \subset \mathscr{I}$, assignments of retailers to selected DC. The contents of this set is not known a priori and must be determined using the following integer program:

$$\begin{aligned} \text{Min} \ & \sum_{j \in \mathscr{J}} f_j X_j + \sum_{i \in \mathscr{I}} \sum_{j \in \mathscr{J}} d_{ij} \chi \mu_i Y_{ij} \\ & + \left\{ \sum_{j \in \mathscr{J}} p_j \chi \mu_i Y_{ij} + \sum_{j \in \mathscr{J}} \sqrt{2 h_j (K_j + p_j) \sum_{i \in \mathscr{I}} \chi \mu_i Y_{ij}} \right\} \\ & + z_\alpha \sum_{j \in \mathscr{J}} h_j \sqrt{\sum_{i \in \mathscr{I}} L_{ij} \sigma_i^2 Y_{ij}} \end{aligned} \tag{3}$$

S.t.

$$\sum_{j \in \mathscr{J}} Y_{ij} = 1, \quad \forall i \in \mathscr{I}. \tag{4}$$

$$\sum_{j \in \mathscr{J}} Y_{ij} \leq X_j, \quad \forall i \in \mathscr{I}, \ \forall j \in \mathscr{J}. \tag{5}$$

$$X_j \in \{0,1\}, Y_{ij} \in \{0,1\}, \quad \forall j \in \mathscr{J}. \tag{6}$$

In the objective function (3), the first term is the cost of opening DCs, the second term is the total unit transportation cost, the third term is the working inventory cost at DCs, and the last term is the safety stock cost at DCs. The impact of inventories on the transportation cost is captured using the EOQ, as the EOQ-based cost clearly appears as the second term of the total transportation cost. Constraints (4) ensure that each retailer must be assigned to a DC. Constraints (5) stipulate that assignments can only be made to open DCs. Finally, constraints (6) are the standard integrality constraints. This problem and its variants are solved using Lagrangian Relaxation based heuristics (Daskin et al. 2002; Ozsen et al. 2008, 2009) and set covering and column generation based algorithms (Romeijn et al. 2007; Shen et al. 2003).

2.2 Continuous Location Models

As opposed to the discrete location problem where DCs are located at one of the pre-determined candidate facilities, in the continuous facility location problem the goal is to determine *the coordinates* of a DC on *a plane* such that the weighted sum of the distances to given retailers on the plane are minimized. Finding the optimal location of this new facility is equivalent to solving the following optimization problem (Love et al. 1988), also known as the Weber problem:

$$\min_{\mathbf{X}} W(\mathbf{X}) = \sum_{i \in \mathcal{I}} w_i d(\mathbf{X}, \mathbf{A}_i),$$

where

$\mathcal{I}$	the set of existing facilities (or, "demand points", "retailers"),
w_i	weight that converts the distance between the new DC facility and existing retailer facility i into cost, and $w_i > 0$,
$\mathbf{X} = (x_1, x_2)$	coordinates of the new DC facility,
$\mathbf{A}_i = (a_{i1}, a_{i2})$	coordinates of the existing retailer facility,
$d(\mathbf{X}, \mathbf{A}_i)$	distance between the new facility and retailer facility i, $i \in \mathcal{I}$.

The limited existing research by Drezner et al. (2003), Keskin and Üster (2012) and McCann (1993) considers joint optimization of *continuous* facility location and inventory decisions while considering EOQ models as subproblems. In particular, McCann (1993) considers a two-stage supply chain that consists of a DC and *two* markets (e.g., retailers) where the only inventory keeping point is the DC and its location is unknown. Hence, the problem is to find the optimum location and the optimum order quantity of the DC while minimizing total inventory and transportation costs in the system. The inventory subproblem in this problem is a single-facility lot-sizing problem that is solved using the EOQ formula. McCann shows that the location of the DC, obtained using constant transportation costs, does not coincide with the location obtained using total logistics costs. As an extension of McCann's work, Drezner et al. (2003) consider the problem of locating a central DC given the locations of a fixed number (≥ 2) of multiple local DCs where the central DC does not keep inventory, but the local DCs do. In fact, the local DCs operate under the assumptions of EOQ with backorders. At a local DC i, $i \in \mathcal{I}$, the optimal order quantity and reorder point as a function of the central DC location X are given as

$$Q_i(X) = \sqrt{\frac{2D_i K_i(X)}{h_i \theta_i}} \quad \text{and}$$

$$R_i(X) = -(1-\theta)Q_i(X) + D_i L_i(X),$$

where

- $K_i(X) = \kappa_i + r_i\|X - A_i\|$, where κ_i is fixed order cost, and r_i is the transportation cost per truck per mile for DC $i \in \mathscr{I}$, and $\|X - A_i\|$ is the distance between new facility X and existing facility A_i;
- θ_i is equal to $\frac{b_i}{b_i + h_i}$ with b_i is the backorder cost for local DC i; and
- $L_i(X) = \tau_i + \beta_i\|X - A_i\|$, where τ_i is order processing time, and β_i is a non-negative constant that can be considered as the inverse of the average speed for local DC i, $i \in \mathscr{I}$.

Drezner et al. (2003) show that the solution determined by the traditional approach, that minimizes the total transportation costs only, differs from the one determined by the approach that also takes into account the inventory and service costs.

Keskin and Üster (2012) investigate a similar problem under three transportation cost functions and two different distance modeling. In particular, Keskin and Üster (2012) state the total average annual cost for the integrated location-inventory problem as:

$$\min_{\mathbf{X},\mathbf{T}} Z(\mathbf{X}, \mathbf{T}) = \sum_{i=1}^{n} \frac{\alpha_i}{T_i} + \sum_{i=1}^{n} \left\{ \frac{K_i}{T_i} + \frac{1}{2} h_i T_i D_i \right\}, \tag{7}$$

where α_i represents the transportation cost between the central DC to the local DCs and T_i represents the cycle time of DC i, i.e., $T_i = Q_i/D_i$, for $i \in \mathscr{I}$. For different forms of α, the key results are summarized as follows:

Quantity Based Transportation Costs: $\alpha_i(Q_i) = p_i^q + r_i^q Q_i$, where p_i^q and r_i^q are the fixed and variable portions of the transportation cost, respectively, the integrated location-inventory problem becomes

$$\min_{\mathbf{X},\mathbf{Q}} Z(\mathbf{X}, \mathbf{Q}) = \sum_{i=1}^{n} \frac{(p_i^q + r_i^q Q_i) D_i}{Q_i} + \sum_{i=1}^{n} \left\{ \frac{K_i D_i}{Q_i} + \frac{1}{2} h_i Q_i \right\}.$$

That is, the central DC location and the reorder quantities of the local DCs are independent of each other. The central DC can be located anywhere on the plane. The reorder quantity of each local DC i, $i \in \mathscr{I}$, is given by

$$Q_i = \sqrt{\frac{2(K_i + p_i^q) D_i}{h_i}}, \qquad \forall i \in \mathscr{I}.$$

Quantity and Distance Based Transportation Costs: $\alpha_i(Q_i, d_i) = p^{qd} + r^{qd} Q_i d_i$, where p_i^{qd} and r_i^{qd} are the fixed and variable portions of the transportation cost, respectively, as before and d_i is the distance between the central DC location $\mathbf{X}$ and the existing local DC location $\mathbf{A}_i$. Then, the integrated location-inventory problem becomes

$$\min_{\mathbf{X},\mathbf{Q}} Z(\mathbf{X},\mathbf{Q}) = \sum_{i=1}^{n} w_i d_i + \sum_{i=1}^{n} \left\{ \frac{(K_i + p_i^{qd})D_i}{Q_i} + \frac{1}{2} h_i Q_i \right\},$$

where $w_i = r_i^{qd} D_i$, the weight of each facility in the location problem. Therefore, the location and inventory problems become separable. Furthermore,

- The location of the central DC depends on the solution of the following Weber problem:

$$\min_{\mathbf{X}} \sum_{i=1}^{n} w_i d_i,$$

where $w_i = r_i^{qd} D_i$ and $d_i = d(\mathbf{A}_i, \mathbf{X})$ for $i \in \mathscr{I}$; and
- the order quantity of each local DC i is given by

$$Q_i = \sqrt{\frac{2(K_i + p_i^{qd})D_i}{h_i}}, \qquad \forall i \in \mathscr{I}.$$

Distance-Based Transportation Costs: $\alpha_i(d_i) = p^d + r^d d_i$, where, as before, where p_i^d and r_i^d are the fixed and variable portions of the transportation cost, respectively. Then, the problem is

$$\min_{\mathbf{X},\mathbf{Q}} Z(\mathbf{X},\mathbf{Q}) = \sum_{i=1}^{n} \frac{(p_i^d + r_i^d d_i)D_i}{Q_i} + \sum_{i=1}^{n} \left\{ \frac{K_i D_i}{Q_i} + \frac{1}{2} h_i Q_i \right\}. \tag{8}$$

Even after reorganization, the facility location and the inventory problems are not separable due to the first term. Solution of (8) depends on how the distance is modeled. Under squared Euclidean distances, convexity is preserved and an iterative algorithm similar to Weiszfeld Algorithm solves the problem (Brimberg and Love 1993; Kuhn 1973; Morris 1981; Ostresh 1978; Üster and Love 2000). Using the Euclidean distances, the convexity is not guaranteed, but the iterative algorithm provides decent solutions (Keskin and Üster 2012).

2.3 Sourcing Models

In many industries, the cost of raw materials and component parts constitutes the main cost of a product. Therefore, vendor selection decisions have been one of the most important traditional functions of the purchasing department of a firm (Dobler et al. 1990). Unfortunately, analytical models addressing the importance of vendor selection problem are limited. Furthermore, research that is placing an emphasis on the impact of vendor selection on the *supply chain cost efficiency* is even more sparse. Several researchers (Ghodsypour and O'Brien 2001; Keskin et al. 2010a, b) study generalized vendor selection models aimed at optimizing the *total logistical costs* including not only the vendor-specific fixed management and purchasing costs considered in traditional models, but also the transportation, inventory replenishment, and holding costs.

Ghodsypour and O'Brien (2001) propose a single buyer, multi-supplier problem with inventory considerations. They also consider supplier-specific fixed ordering cost, unit cost, and quality levels. The main decisions are how much the buyer orders and which supplier fulfills what percentage of the order quantity. The total annual cost consists of purchasing cost, inventory holding cost, and ordering cost. The optimal order quantity formulation for the buyer is similar to the EOQ formulation. The major difference is that instead of using a single holding cost and a single setup cost, a weighted average of combined holding and setup costs is used considering the different parameters of the selected suppliers.

In the solution, Ghodsypour and O'Brien first determine the total order quantity of the buyer using an EOQ formulation:

$$Q = \sqrt{\frac{2DK}{rC}},$$

where r is the inventory holding cost rate, K is the sum of all of the ordering costs, and C is the generalized unit cost. In the next step, the total economical order quantity is distributed among suppliers. They assign each supplier a certain percentage, X_j of Q. Let Y_j be 1 if supplier j is selected, otherwise, 0. The total annual ordering cost is then equal to

$$\left(\sum_{j}^{n} K_j Y_j\right)\frac{D}{Q}.$$

Furthermore, the annual purchasing and holding costs are given as

$$\sum_{j}^{n} X_j c_j D \quad \text{and} \quad \frac{rQ}{2}\left(\sum_{j}^{n} X_j^2 c_j\right),$$

respectively. Then, the total annual cost is given as

$$\sum_{j}^{n} X_j c_j D + \left(\sum_{j}^{n} K_j Y_j\right)\frac{D}{Q} + \frac{rQ}{2}\left(\sum_{j}^{n} X_j^2 c_j\right).$$

The optimal order quantity is calculated as

$$Q = \sqrt{\frac{2D\sum_j^n (K_j Y_j)}{r\left(\sum_j^n X_j^2 c_j\right)}}, \tag{9}$$

and the total annual cost corresponding to order quantity Q is

$$\sqrt{2Dr\left(\sum_{j}^{n} K_j Y_j\right)\left(\sum_{j}^{n} X_j^2 c_j\right)} + \sum_{j}^{n} c_j X_j D. \tag{10}$$

Ghodsypour and O'Brien (2001) solve this mixed integer nonlinear program by branching over Y_j variables and solving a number of pure nonlinear models for each fixed Y_j using a general purpose nonlinear programming software package. Although there could be as many as 2^n potential pure nonlinear models for n suppliers, the solution is still obtained quite quickly by limiting the number of suppliers to 12 and eliminating the cases that cannot satisfy the demand constraint.

Keskin et al. (2010b) generalize this problem to a multi-buyer firm where the goal is the simultaneous determination of (i) the set of vendors the firm should work with and (ii) how much each buyer should order from the selected vendors. By exploiting the relationship between facility location applications and the problem at hand, Keskin et al. approach the problem as an *integrated* location-inventory optimization model with dispersed buyer stores where each store faces store-specific deterministic and stationary demand. The vendor selection decisions for each buyer store are conducted at the firm level, considering a pool of vendors with vendor-specific fixed management and purchasing costs that meet initial quality and delivery performance criteria. Each store operates under the assumptions of the classical Economic Order Quantity (EOQ) model (Zipkin 2000). That is, each store is replenished by a single vendor and holds inventory to meet the deterministic stationary demand.

In their mathematical model, they introduce three sets of variables representing the vendor selection and inventory decisions for the buyer stores. The first set relates to the vendor selection decisions. For each vendor $j \in \mathcal{J}$,

$$X_j = \begin{cases} 1, & \text{if vendor } j \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases}$$

The second set of decision variables pertains to the assignment of stores to vendors. For store $i \in \mathcal{I}$ and vendor $j \in \mathcal{J}$,

$$Y_{ij} = \begin{cases} 1, & \text{if store } i \text{ is assigned to vendor } j, \\ 0, & \text{otherwise.} \end{cases}$$

These binary assignment variables ensure that each store receives shipments from only one (dedicated) vendor, i.e., a *single-sourcing* strategy. Finally, the third set of decision variables relates to the inventory policies of the stores, Q_i, $i \in \mathcal{I}$. The generalized vendor selection problem *GVSP* is then formulated as the following MINLP.

$$\text{Min} \quad \sum_{j\in\mathcal{J}} f_j X_j + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} c_j D_i Y_{ij} + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} \frac{(p_{ij} + r_{ij} d_{ij}) D_i}{Q_i} Y_{ij} + \sum_{i\in\mathcal{I}} \left(\frac{K_i D_i}{Q_i} + \frac{h_i Q_i}{2}\right) \tag{GVSP}$$

subject to

$$\sum_{j \in \mathscr{J}} Y_{ij} = 1, \qquad \forall i \in \mathscr{I}, \tag{11}$$

$$Y_{ij} \leq X_j, \qquad \forall i \in \mathscr{I},\ \forall j \in \mathscr{J}, \tag{12}$$

$$\sum_{i \in \mathscr{I}} D_i Y_{ij} \leq P_j X_j, \qquad \forall j \in \mathscr{J}, \tag{13}$$

$$\sum_{i \in \mathscr{I}} \frac{D_i}{Q_i} Y_{ij} \leq R_j X_j, \qquad \forall j \in \mathscr{J}, \tag{14}$$

$$X_j \in \{0, 1\}, \qquad \forall j \in \mathscr{J}, \tag{15}$$

$$Y_{ij} \in \{0, 1\}, \qquad \forall i \in \mathscr{I},\ \forall j \in \mathscr{J}, \tag{16}$$

$$Q_i \in \mathbb{R}_+, \qquad \forall i \in \mathscr{I}. \tag{17}$$

The objective function of *GVSP* is aimed at minimizing the *annual* total cost which includes (i) fixed management costs associated with the selected vendors, (ii) purchasing costs, (iii) fixed dispatch and distance-based transportation costs from the selected vendors to the stores, and (iv) inventory replenishment and holding costs of the stores. Constraints (11) dictate that the annual demand of each store must be satisfied. Constraints (12) ensure that each store is assigned to a selected vendor. Constraints (13) and (14) represent the throughput and dispatch capacities at the vendors, respectively. Finally, constraints (15) and (16) ensure integrality, whereas constraints (17) ensure nonnegativity. The resulting model is a major extension of the fixed charge facility location problem with explicit inventory decisions at the stores and generalized transportation costs and capacity constraints. It is solved using an algorithm based on the Generalized benders decomposition (GBD).

To develop a GBD approach here, a critical property of *GVSP* is used: when the vendor selection and assignment decisions are known (i.e., when the binary $\mathbf{X}$ and $\mathbf{Y}$ vectors are fixed), the resulting problem is separable for each (selected) vendor, and, more importantly, each such problem is a nonlinear program in $\mathbf{Q}$ representing a multi-store capacitated EOQ problem. When the values of binary variables are fixed, say as $\hat{\mathbf{X}}$ and $\hat{\mathbf{Y}}$, *GVSP* reduces to

$$\text{Min} \quad SP(\mathbf{Q}|\hat{\mathbf{X}}, \hat{\mathbf{Y}}) = \sum_{i \in \mathscr{I}} \sum_{j \in \mathscr{J}} \left(\frac{p_{ij} + r_{ij} d_{ij}}{Q_i} \right) D_i \hat{Y}_{ij} + \sum_{i \in \mathscr{I}} \left(\frac{K_i D_i}{Q_i} + \frac{1}{2} h_i Q_i \right)$$

subject to

$$\sum_{i\in\mathcal{I}}\frac{D_i}{Q_i}\hat{Y}_{ij}\le R_j\hat{X}_j,\quad \forall j\in\mathcal{J},$$
$$Q_i\in\mathbb{R}_+,\qquad \forall i\in\mathcal{I},$$

which is the subproblem of interest, denoted by $SP(\mathbf{Q}|\hat{\mathbf{X}},\hat{\mathbf{Y}})$. Examining the above formulation, it is easy to see its equivalence to a multi-vendor, multi-store EOQ problem with dispatch limitations for the selected vendors. Furthermore, letting $\hat{\mathcal{J}}=\{j\in\mathcal{J}:X_j=1\}$ denote the set of selected vendors and $\mathcal{I}_j=\{i\in\mathcal{I}:Y_{ij}=1\}$, $\forall j\in\hat{\mathcal{J}}$ denote the set of stores (uniquely) assigned to a selected vendor j, it is also easy to verify that $SP(\mathbf{Q}|\hat{\mathbf{X}},\hat{\mathbf{Y}})$ is separable for each selected vendor $j\in\hat{\mathcal{J}}$. This observation, in turn, implies that an optimal solution to $SP(\mathbf{Q}|\hat{\mathbf{X}},\hat{\mathbf{Y}})$ can be obtained by solving

$$\text{Min}\quad SP_j(\mathbf{Q})=\sum_{i\in\mathcal{I}_j}\left(\frac{(p_{ij}+r_{ij}d_{ij})D_i}{Q_i}+\frac{K_iD_i}{Q_i}+\frac{1}{2}h_iQ_i\right)\tag{18}$$

subject to

$$\begin{aligned}&\sum_{i\in\mathcal{I}_j}\frac{D_i}{Q_i}\le R_j,\\ &Q_i\in\mathbb{R}_+,\qquad \forall i\in\mathcal{I}_j,\end{aligned}\tag{19}$$

for each $j\in\hat{\mathcal{J}}$. Each such problem is essentially a single-vendor, multi-store EOQ problem with a simple dispatch capacity constraint. At every iteration of the GBD algorithm, A Benders cut is generated by solving the subproblem $SP(\mathbf{Q}|\hat{\mathbf{X}},\hat{\mathbf{Y}})$ for given values of $\hat{\mathbf{X}},\hat{\mathbf{Y}}$. As highlighted before, the EOQ problem is extremely useful in exploiting complex trade-offs in distribution system design, especially in the integrated decision making associated with sourcing and inventory.

3 Transportation Models

The integration of inventory and transportation decisions has received increasing attention both from academia and practice (Çetinkaya 2004). This line of research (i) investigates the impact of inbound and/or outbound transportation costs and decisions on inventory optimization and (ii) demonstrates that significant cost savings are realizable through simultaneous consideration of inventory and transportation costs and decisions. For this purpose, in this section, we present two important problem domains under transportation models. In the first group of work, the transportation among different facilities assumes direct shipment whereas in the second group of work, the transportation is done via routing.

3.1 Integrated Inventory and Transportation Models

In this subsection, we assume that the transportation is conducted as *direct shipments* as in Drezner et al. (2003), Keskin et al. (2010b), Keskin and Üster (2012), Toptal et al. (2003) and Üster et al. (2008). Keskin et al. (2012) propose a model that minimizes the total annual cost that includes the fixed facility location costs associated with the open DCs, the transportation cost from DCs to retailers, and the inventory replenishment and holding costs at the retailers. The decision variables of model are opening DCs, assigning retailers to DCs, and order quantity for each retailer. A main difference of this work from others is that the transportation cost is subject to *cargo capacity*. That is, the number of trips/trucks required by retailer i, $i \in \mathscr{I}$ for a replenishment quantity of Q_i is given by $\lceil Q_i/C_T \rceil$ where C_T is the cargo capacity. Then, the total transportation cost from DC j, $j \in \mathscr{J}$ to retailer i, $i \in \mathscr{I}$ for a replenishment quantity of Q_i is $(p_{ij} + r_{ij}d_{ij})\lceil Q_i/C_T \rceil$.

Consequently, the integrated location-inventory problem with cargo costs can be formulated as the following mixed integer nonlinear program denoted by *IFLP*

$$\text{Minimize} \quad \sum_{j \in \mathscr{J}} f_j X_j + \sum_{i \in \mathscr{I}} \sum_{j \in \mathscr{J}} \left(\frac{(p_{ij} + r_{ij}d_{ij}) \left\lceil \frac{Q_i}{C_T} \right\rceil}{Q_i} \right) D_i Y_{ij} + \sum_{i \in \mathscr{I}} \left(\frac{K_i D_i}{Q_i} + \frac{1}{2} h_i Q_i \right) \tag{IFLP}$$

subject to

$$\sum_{j \in \mathscr{J}} Y_{ij} = 1, \qquad \forall i \in \mathscr{I}. \tag{20}$$

$$Y_{ij} \leq X_j, \qquad \forall i \in \mathscr{I} \text{ and } \forall j \in \mathscr{J}. \tag{21}$$

$$X_j \in \{0, 1\}, \qquad \forall j \in \mathscr{J}. \tag{22}$$

$$Y_{ij} \in \{0, 1\}, \qquad \forall i \in \mathscr{I} \text{ and } \forall j \in \mathscr{J}. \tag{23}$$

$$Q_i \geq 0, \qquad \forall i \in \mathscr{I}. \tag{24}$$

The objective function of *IFLP* minimizes the total annual costs: (i) the fixed facility location costs associated with the open DCs, (ii) the cargo-based transportation costs from DCs to retailers, and (iii) the inventory replenishment and holding costs at the retailers. Constraints (20) ensure that the demand of each retailer is satisfied. Constraints (21) establish that each retailer will be assigned to an open DC. Finally, constraints (22) and (23) ensure integrality, and constraints (24) ensure nonnegativity. Observe that the cargo capacity is modeled in the objective function rather than in the constraints. This is because the replenishment quantities of the retailers imply the number of trucks needed along with the resulting transportation costs. Hence, one can argue that the overall transportation capacity is *installable* which is true in many practical settings due to the abundance of contract and for-hire carriers in the truckload industry.

Using the structural properties of the problem, Keskin et al. (2012) show that for given retailer-DC assignment, the optimal order quantity could be found by using modified EOQ formulation, considering transportation cost as a part of fixed order cost. Now, observe that if $\mathbf{X}$ and $\mathbf{Y}$ are known, then the remaining problem is a multi-retailer EOQ-model with a generalized replenishment cost structure. Further, given $\mathbf{X}$ and $\mathbf{Y}$, the *IFLP* is decomposable for each retailer $i \in \mathscr{I}$. More specifically, let $\mathscr{I}_j = \{i \in \mathscr{I} : Y_{ij} = 1\}$, for any $j \in \mathscr{J}$. Then, for each DC $j \in \mathscr{J}$ and each retailer $i \in \mathscr{I}_j$, we have the following EOQ problem with a generalized replenishment cost structure:

$$\min_{Q \geq 0} g_{ij}(Q) = \frac{\left(K_i + (p_{ij} + r_{ij}d_{ij})\left\lceil \frac{Q}{C_T} \right\rceil\right) D_i}{Q} + \frac{1}{2} h_i Q. \tag{25}$$

This problem can be solved using the Generalized EOQ Algorithm, given below, developed by Toptal et al. (2003) (see Algorithm 1 on p. 991).

Generalized EOQ Algorithm:

For retailer $i \in \mathscr{I}$ and DC $j \in \mathscr{J}$:

Step 1: Compute $\sqrt{2K_iD_i/h_i}$.

Step 2: Let N denote the integer multiple of C_T such that $NC_T < \sqrt{2K_iD_i/h_i} \leq (N+1)C_T$. Compute

$$Q_{ij}^{N+1} = \sqrt{\frac{2D_i(K_i + (N+1)(p_{ij} + r_{ij}d_{ij}))}{h_i}}.$$

If $Q_{ij}^{N+1} \geq (N+1)C_T$, then go to Step 3. Otherwise, go to Step 4.

Step 3: $Q_{ij}^* = \arg\min\{g(NC_T), g((N+1)C_T)\}$. Stop.

Step 4: $Q_{ij}^* = \arg\min\left\{g(NC_T), g(Q_{ij}^{N+1})\right\}$. Stop.

Note that the optimal Q_{ij}^* resulting from the above algorithm is the preferred order quantity of retailer $i \in \mathscr{I}$ and DC $j \in \mathscr{J}$ under single sourcing, and it is given by

$$\arg\min\left\{g_{ij}(Q_{ij}^{N+1}), g_{ij}(NC_T), g_{ij}((N+1)C_T)\right\}.$$

Swenseth and Godfrey (2002) approach a similar problem from a different perspective. Dating to the origination of economic order quantity (EOQ) models, the objective of inventory replenishment decisions has centered on the minimization of total annual logistics cost. Swenseth and Godfrey (2002) note that accurate solutions require that all of the relevant costs be appropriately incorporated into the total annual logistics cost function to determine purchase quantities. Furthermore, depending on the estimates used, upwards of 50 % of the total annual logistics cost of a product can be attributed to transportation. Any consideration of purchase quantities should therefore consider transportation costs. To appropriately represent the *true* total annual logistics cost function, transportation cost functions that emulate reality and simultaneously provide a straightforward representation of actual freight rates must be identified first.

Swenseth and Godfrey (2002) explain that there are technically three ways to find a good order quantity: (i) shipments that result in true truck-load (TL) shipping quantities; (ii) shipments that are likely to be over-declared as TL; and (iii) shipments that are less-than-truck-load (LTL) rates. In addition to the constant charge per unit of the EOQ model, two freight rate functions, the inverse and the adjusted inverse, were incorporated into the total annual cost. The inverse transportation rate, a constant charge *per shipment*, models the freight rates exactly when TL shipping weights are transported. On the other hand, the adjusted inverse transportation rate takes on the same characteristics as the inverse function but emulates the LTL rates.

With the inverse transportation rate, the company is assumed to ship everything with a full truck load. Then the total cost is:

$$TC = \frac{DK}{Q} + \frac{Qh}{2} + \left[\frac{F_x W_x}{Qw}\right] Dw,$$

where $F_x W_x$ is the total charge for a TL shipment for a given route and w is per unit weight. The corresponding optimal order quantity is

$$Q = \sqrt{\frac{2D(K + F_x W_x)}{h}}.$$

This model, by accounting for the TL cost, sets the shipping weight as a TL or would be over-declared as a TL.

On the other hand, the adjusted inverse rate provides a means of emulating freight rates without further complicating the order quantity decision. With the inverse adjusted rate, the cost per pound is calculated as

$$F_y = F_x + \alpha F_x \left[\frac{W_x - Qw}{Qw}\right],$$

where F_y is the freight rate for the given order quantity, F_x is TL freight rate, and α is a constant between 0 and 1. The revised total cost and the corresponding order quantity are

$$TC = \frac{DK}{Q} + \frac{Qh}{2} + \left[F_x + \alpha F_x \left[\frac{W_x - Qw}{Qw}\right]\right] Dw, \qquad \text{and}$$

$$Q = \sqrt{\frac{2D(K + \alpha F_x W_x)}{h}}, \qquad \text{respectively} .$$

One of the challenges of the adjusted inverse model is finding a value for α. To predict α, Swenseth and Godfrey (2002) utilize real LTL and TL rates from actual carriers in a linear regression formula. From their analysis, they set α as 0.173050.

3.2 Inventory Routing Problems

A typical problem in supply chain management is the coordination of product and material flows between locations. These main activities involve distribution decisions among the facilities of the distribution. The task is often performed by a fleet of vehicles either directly controlled by the firm or the management of the fleet is assigned to a third party logistics provider (Anily and Bramel 1999). In the inventory routing problem (IRP), a central warehouse with unlimited supply serves a set of retailers distributed in a given area. The retailers experience a fixed demand per unit of time for the items, and the vehicles of limited capacity must be dispatched to replenish the retailer inventories. Each retailer incurs a holding cost per item per unit of time and a fixed cost per order placed. The objective is to schedule the vehicle departures and specify the loads destined for each retailer such that the total cost per unit time is minimized. This includes transportation cost, fixed ordering cost, and inventory holding cost at the retailers.

Bramel and Simchi-Levi (1995) propose an algorithm that can solve an IRP with the characteristics stated above. The main issue is clustering customers so that the total inventory holding cost and transportation cost are minimized. The algorithm first selects m seeds. Then, it assigns each retailer to one of the seeds. The main challenge in this method is related to calculating the retailer assignment cost to the seed. Bramel and Simchi-Levi (1995) use an EOQ approximation to estimate this assignment cost. This similar approach is later used by other researchers including Natarajarathinam et al. (2012) and Stacey et al. (2007).

Assume that a set S of customers, assigned to a particular seed, is served every $t(S)$ units of time. The optimal $t^*(S)$

$$t^*(S) = \sqrt{\frac{2(L_O(S) + K(S))}{\sum_{i \in S} h_i D_i}},$$

where $K(S) = \sum_{i \in S} K_i$, $D(S) = \sum_{i \in S} D_i$, and $L_0(S)$ is cost of optimal variable route cost. However, the vehicle capacity, Cap may be violated with this frequency. Therefore, the frequency needs to be adjusted to $t(S) = \min\left\{t^*(S), \frac{Cap}{D(S)}\right\}$. Then, the total annual cost $\phi(S)$ for set S is given as

$$\phi(S) = \frac{L_0(S) + K(S)}{t(S)} + \frac{1}{2} t(S) \sum_{i \in S} h_i D_i.$$

The algorithm calculates this cost for each potential seed $j = 1, \ldots, m$ and sets it as *seed assignment cost*, i.e., $v_j = \phi(T_j), \forall j = 1, 2, \ldots, m$, where T_j is the seed set j. In the next step, the algorithm calculates the assignment cost of a particular retailer to a seed:

$$c_{ij} = \phi(T_j \cup \{x_i\}) - \phi(T_j), \forall i = 1, 2, \ldots, n \quad \text{and} \quad \forall j = 1, 2, \ldots, m. \tag{26}$$

In the final stage, the algorithm uses an assignment model to find assignment. That is, let y_j be 1, if a seed is located at site j, 0, otherwise, and let x_{ij} be 1 if retailer i is assigned to seed j, 0, otherwise. Then this assignment is formulated as

$$\begin{aligned} &\min \sum_{j=1}^{m} v_j y_j + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \\ &\text{subject to} \\ &\sum_{j=1}^{m} x_{ij} = 1, \qquad i = 1, \ldots, n. \end{aligned} \tag{27}$$

$$\sum_{i=1}^{n} w_i x_{ij} \leq Q_j, \qquad j = 1, \ldots, m. \tag{28}$$

$$x_{ij} \leq y_j, \qquad i = 1, \ldots, n, \quad j = 1, \ldots, m. \tag{29}$$

$$x_{ij} \in \{0, 1\}, \qquad i = 1, \ldots, n, \quad j = 1, \ldots, m. \tag{30}$$

$$y_j \in \{0, 1\}, \qquad j = 1, \ldots, m. \tag{31}$$

Constraints (27) ensure that each retailer is assigned exactly to one seed location. Constraints (28) ensure that the seed capacity is not violated. Constraints (29) guarantee that a retailer is only assigned to a selected seed. Finally, constraints (30) and (31) ensure the integrality. Even though this is an NP-hard formulation, it is considerably easier to solve it in the sense of finding a "good" solution in a "reasonable" amount of time.

4 Inventory Models

The EOQ appears as a subproblem to many advanced inventory models including multi-product constrained systems, joint replenishment problem, and multi-echelon inventories. Many of these models have already appeared as mainstream course materials in various books including Ballou (2004), Ghiani et al. (2003), Nahmias (2009) and Zipkin (2000). Additionally, there is extensive literature on the coordination of supply chains via supply contracts that utilize EOQ models or analysis as a major part of the contribution. We refer the interested readers to the research in Banerjee (1986), Banerjee and Burton (1994), Chan and Lee (2012), Chan et al. (2010), Chen and Mushaluk (2013), Goyal (1976) and Goyal and Gupta (1989). In this section, we summarize the inventory coordination problem associated with a multi-echelon supply chain, studied by Khouja (2003), and we showcase how an EOQ model appears as a subproblem at every step and simplifies the overall analysis.

Khouja (2003) considers a three-stage supply chain model with multiple firms at each stage, and each firm can supply two or more customers. In formulation, it is assumed that the product is processed on a single system at each firm. Production and usage rates at each firm are deterministic and uniform. The holding cost is linear in the inventory held. Furthermore, the per unit annual holding cost is the same for firms in the same stage. Similarly, the setup/ordering cost is the same for firms in the same stage. The downstream firms in the chain are retailers or assemblers. He analyzes three inventory coordination mechanisms (CM) between chain members and solves a cost minimization model for each mechanism. Specifically, he considers

CM 1: Equal Cycle Time	The same cycle time is used throughout the supply chain.
CM 2: Integer Policies	The cycle time at each stage of the chain is an integer multiple of the cycle time of the adjacent downstream stage.
CM 3: Power-of-two Policies	The cycle time of each firm is an integer power of two multiples of a basic cycle time.

We review the respective coordination mechanisms as follows.

4.1 CM 1: Equal Cycle Time

All firms in the supply chain use the same cycle time which implies $T_{ij} = T$, for all i and j. Let $i = 1$, 2, and 3, be an index denoting the stage in the supply chain, where 1 denotes upstream suppliers, 2 denotes manufacturers, and 3 denotes retailers or assemblers. The total annual cost for a downstream firm is:

$$TC_{3,j} = \frac{TD_{3,j}}{2}h_3 + \frac{K_3}{T},$$

where j is the index for firms within each stage such that $j = 1, 2, \ldots, J_3$.

The total annual cost for the manufacturers is

$$TC_{2,j} = \frac{TD_{2,j}^2}{2P_{2,j}}(h_1 + h_2) + \frac{K_2}{T},$$

where $P_{2,j}$ is annual production rate for $j = 1, 2, \ldots, J_2$, and h_1 and h_2 are the holding costs for the raw materials and finished good of manufactured product, respectively.

Similarly, the total annual cost for a supplier stage firm is

$$TC_{1,j} = \frac{TD_{1,j}^2}{2P_{1,j}}(h_0 + h_1) + \frac{K_1}{T}.$$

The total cost TC of the whole supply chain is $TC = \sum_{i=1}^{3} TC_i$ is nothing but a modified EOQ formulation. The optimal order frequency of the supply chain is then

$$T = \sqrt{\frac{2(J_1K_1 + J_2K_2 + J_3K_3)P}{\sum_{i=1}^{2}\left[(h_{i-1} + h_i)\sum_j(D_{i,j}^2\bar{P}_{i,j})\right] + h_3DP}} \tag{32}$$

In Eq. (32), J_i is the number of firms in each stage, $P = \prod_{i,j} P_{i,j}$ is the product of the production rates for all firms in the supply chain, $\bar{P}_{i,j} = P - P_{i,j}$ is the product of production rates for all firms in the chain except for firm j in stage i, $D = \sum_{j=1}^{J_1} D_{1,j} = \sum_{j=1}^{J_2} D_{2,j} = \sum_{j=1}^{J_3} D_{3,j}$ is the total demand at each stage.

4.2 CM 2: Integer Policies

In this method, the cycle time at each stage is an integer multiplier of the cycle time at the adjacent downstream stage, which implies $T_{3,j} = T$ for $j = 1, 2, \ldots, J_3$, $T_{2,j} = S_2T$ for $j = 1, 2, \ldots, J_2$, and $T_{1,j} = S_1S_2T$ for $j = 1, 2, \ldots, J_1$, where S_i is an integer multiplier at stage i, $i = 1$ and 2.

The total cost for the retailers in stage 3 stays the same as the equal cycle time coordination mechanism. On the other hand, the total cost for stage 2 firm, manufacturer, is given by an augmented EOQ formulation:

$$TC_{2,j} = \frac{S_2TD_{2,j}^2}{2P_{2,j}}h_1 + \frac{TD_{2,j}}{2}(S_2(1 + D_{2,j}/P_{2,j}) - 1)h_2 + \frac{K_2}{S_2T},$$

where the first term is holding cost of raw materials, the second term is annual holding cost of finished goods for the production portions of the cycle and annual holding cost for the non-production portions of the cycle, and the third term is the annual setup cost. Similarly, the total annual cost for a supplier stage firm is:

$$TC_{1,j} = \frac{S_1S_2TD_{1,j}^2}{2P_{1,j}}h_0 + \frac{S_2TD_{1,j}}{2}(S_1(1 + D_{1,j}/P_{1,j}) - 1)h_1 + \frac{K_1}{S_1S_2T}.$$

Let $S_t = S_1 \times S_2$. For any value of S_1 and S_2, the optimal cycle time T is

$$T = \sqrt{\frac{2(J_1K_1 + S_1J_2K_2 + S_tJ_3K_3)P}{S_t\left[(h_1+h_2)S_2\sum_j D_{2,j}^2\bar{P}_{i,j} + (h_0+h_1)S_t\sum_j D_{1,j}^2\bar{P}_{i,j} + DP[(h_3-h_2)+(h_2-h_1)S_2+h_1S_t]\right]}} \tag{33}$$

To minimize TC, the solution to $\partial TC/\partial S_1 = 0$ gives an S_1 that minimizes the total cost:

$$S_1 = \sqrt{\frac{2J_1K_1P_1}{T^2S_2^2\left[(h_0+h_1)\sum_j\left(D_{1,j}^2\omega_{1,j}\right) + h_1DP_1\right]}}, \tag{34}$$

where $P_i = \prod_{j=1}^{j=J_1} P_{i,j}$, and $\omega_{i,g} = P_i - P_{i,g}$. Substituting for S_1 to total cost function and finding S_2 in a similar fashion gives:

$$S_2 = \sqrt{\frac{2J_2K_2P_2}{T^2\left[(h_1+h_2)\sum_j\left(D_{2,j}^2\omega_{2,j}\right) + (h_2-h_1)DP_2\right]}}. \tag{35}$$

Finally, substituting both S_1 and S_2 in Eq. (33) provides the optimal T :

$$T = \sqrt{\frac{2J_3K_3}{(h_3-h_2)D}}.$$

After finding the optimal T, simply back tracking equation (34) and (35) and rounding them to the closest integer, helps set the integer multiples S_1 and S_2.

4.3 CM 3: Power-of-Two Policies

To further generalize the inventory policy of the integer multipliers mechanism (CM2), the cycle times for firms within each stage can be unequal. To achieve that, the cycle time for each firm is assumed to be an integer powers-of-two multiplier of a basic cycle time, T. In addition, to guarantee feasibility, the powers of two multipliers of each firm is assumed to be equal to or greater than the largest powers of two of any of the firm's customers at the adjacent downstream stage.

The total annual cost for a stage 3 firm, i.e., a retailer, is

$$TC_{3,j} = \frac{2^{S_{3,j}}TD_{3,j}}{2}h_3 + \frac{K_3}{2^{S_{3,j}}T},$$

where $S_{i,j}$ is an integer power of two associated with firm j in stage i.

The total annual cost for a manufacturer stage firm is:

$$TC_{2,j} = \frac{K_2}{2^{S_{2,j}}T} + \frac{2^{S_{2,j}}TD_{2,j}^2}{2P_{2,j}}(h_1 + h_2) + h_2T\sum_{g\in A_{2,j}}\left[2^{S_{2,j}} - 2^{S_{3,g}} - \frac{2^{2S_{3,g}}}{2^{S_{2,j}}}\left(\sum_{v=1}^{2^{S_{2,j}}/2^{S_{3,g}}-1} v\right)\right]D_{3,g}, \tag{36}$$

where $A_{i,j}$ is the set of firms at stage $i+1$ that satisfies their demand from firm j in stage i. In Eq. (36), the first term is the annual setup cost; the second term is the annual holding cost of raw material and finished goods for the production portions of the cycles; and the last term is the annual holding cost of finished goods during the non-production portion of the cycle.

The total annual cost for a supplier stage firm is:

$$TC_{1,j} = \frac{K_1}{2^{S_{1,j}}T} + \frac{2^{S_{1,j}}TD_{1,j}^2}{2P_{1,j}}(h_0 + h_1) + h_1T\sum_{g\in A_{1,j}}\left[2^{S_{1,j}} - 2^{S_{2,g}} - \frac{2^{2S_{2,g}}}{2^{S_{1,j}}}\left(\sum_{v=1}^{2^{S_{1,j}}/2^{S_{2,g}}-1} v\right)\right]D. \tag{37}$$

Solving $\partial TC/\partial S_{i,j} = 0, i = 1,2,3$ gives:

$$S_{3,j} = \frac{1}{2} - 0.721348Ln[(h_3 - h_2)T^2D_{3,j}/K_3],$$
$$S_{2,j} = \frac{1}{2} + 0.721348Ln\left[\sqrt{\frac{K_2P_{2,j}}{T^2D_{2,j}[(h_1+h_2)D_{2,j} + (h_2 - h_1)P_{2,j}]}}\right],$$
$$S_{1,j} = \frac{1}{2} + 0.721348Ln\left[\sqrt{\frac{K_1P_{1,j}}{T^2D_{1,j}[(h_0+h_1)D_{1,j} + h_1P_{1,j}]}}\right].$$

Let

$$C_1 = \sum_{i=1}^{2}(h_{i-1} + h_i)\sum_j D_{i,j}^2\bar{P}_{i,j}2^{2S_{i,j}+\bar{S}_{i,j}};$$
$$C_2 = \sum_{i=2}^{3}(h_{i-1}P)\sum_j D_{i,j}(2^{2S_{i-1,\rho(i,j)}+\bar{S}_{i-1,\rho(i,j)}} - 2^S);$$
$$C_3 = (h_3P)\sum_{i=3,j} D_{i,j}2^{2S_{3,j}+\bar{S}_{3,j}};$$

where $\bar{S}_{i,g} = S - S_{i,g}$ the sum of power of two for all firms in the chain except for firm g in stage i, $S = \sum_{i,j} S_{i,j}$ the sum of powers of two for all firms in the chain, $\rho(i,j)$ is the index of the firm at stage $i-1$ which supplies firm j in stage i. Solving $\partial TC/\partial T = 0$ gives:

$$T = \sqrt{\frac{P \sum_{i=1}^{3} K_i \sum_j 2^{1+\bar{S}_{i,j}}}{C_1 + C_2 + C_3}}. \tag{38}$$

To find optimal or near-optimal cycle time and integer powers of two, Khouja (2003) proposes a heuristic. The analysis shows that the integer multipliers mechanism has a lower total cost than the equal cycle time mechanism, and the integer powers of two multipliers mechanism has a lower cost than the integer multipliers mechanism. While the savings in total cost a supply chain realizes in moving from the equal-cycle time mechanism to the integer multipliers mechanism may be considerable, the savings in moving from the integer multipliers mechanism to the integer powers of two multipliers mechanism are less considerable.

5 Conclusions

This chapter demonstrates the use and utility of EOQ models in analyzing complex supply chain problems related to integrated inventory-location, integrated inventory-transportation, and higher level inventory problems. A number of recent research papers are reviewed and summarized with regard to how EOQ models aid in resolving the complicated supply chain problems. Given the increased importance of big data and increased pressure to do well with respect to all of the supply chain performance metrics, the importance of EOQ-based models continue to increase in the near future due to their simplicity and intuitiveness. Important extensions of the research presented in each one of the pillar areas include the following:

- **Location Models**: Since location decisions are inherently strategic and long-term in nature, there is a strong need for supply chain models that account for the inherent uncertainty surrounding future conditions and possible disruptions. Scenario-based location models, reliability-based location models, as well as models that account for robust optimization are areas worthy of considerable additional research.
- **Transportation**: There are two important trends in transportation models that will need immediate attention. One of the trends is related to sustainability and environmental issues in transportation. Models that reduce carbon foot-print and that incorporate environmental costs are interesting extensions of the proposed works. The second trend is about availability and visibility of data, in particular related to tracking shipments, providing information at every step of the transportation process, and re-optimizing given new information. In regard to this trend and to meet the growing real-time requirements of customers, fast online optimization algorithms are needed.
- **Inventory**: In the area of inventory, supply chain coordination and the mechanisms for coordination will continue to be the focus of research in this area. To

coordinate the whole supply chain, the aggregation of the impact of all coordination mechanisms on the performance of supply chain is required. Various combinations may be explored with the help of simulation. Supply chain contracts have proved to coordinate single period supply chains. The research is required to explore the utility of contracts in multi-period cases. In multi-period model, the supply chain members are exposed to the uncertainty more as they are dealing with supply chain members frequently. It is important to evaluate how various coordination mechanisms can be developed in multi-period problems.

Some of the models presented in this chapter will be useful in formulating and solving such advanced models.

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