A Century of the EOQ

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Abstract When Ford W. Harris published his short three-page article developing the Economic Order Quantity (EOQ) model in 1913, he likely did not foresee that it would still be discussed and used 100 years later. Harris' EOQ model was one of the first applications of mathematical modeling to guide managers in making business decisions, and it has spawned thousands of related studies over the past century that have built on its major foundations and insights. In this chapter we present a short history of the EOQ model by discussing the model itself, some practical issues about implementing the model, and major extensions to the basic model grouped by the dominant foci of each subsequent decade.

1 Introduction

Every organization must determine the number of items or units to order every time it acquires stock from its suppliers. Perhaps it is this universal application to every type of business that has kept the Economic Order Quantity (EOQ) model relevant for 100 years. First published by Ford W. Harris in 1913, the EOQ model prescribes the optimal order quantity for organizations that minimizes the total ordering and holding cost under a relatively restrictive set of assumptions.

Even with these restrictions, it is impossible to overstate the influence that the EOQ model has had on a century of researchers and practitioners in the fields of operations management and operations research. This is largely because the model is the foundation for literally *thousands* of later studies that relaxed a subset of its

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assumptions to create a modified model that better fits a particular decision environment. The remaining chapters in this book highlight cutting-edge research in the field by including today's contemporary extensions of the original model. The EOQ model is also an essential part of the history of operations research because it represents one of the first published applications of a mathematical model to business decision making.

In addition, the original EOQ model in its unadjusted form remains relevant because it is widely used in practice. Undoubtedly, some organizations apply the model incorrectly to situations where it is not the best practical solution, but it still works well in practice because of its simplicity and robustness (Ptak 1988), which are discussed in detail in the next section of the chapter. The model is also taught in a majority of academic courses that cover inventory control to even a small degree. It proves to be an effective method to expose students and practitioners to the major cost trade-off in inventory control—ordering costs versus holding costs.

The remainder of the chapter is organized as follows. The next section discusses the original EOQ model itself along with its assumptions and the practical issues with implementing the model in a business setting. The subsequent five sections detail the development of EOQ-based research decade by decade from the 1950s to the present. Concluding remarks are provided in the final section of the chapter.

2 The Original EOQ Model

2.1 Model Assumptions

Harris' original EOQ model in 1913 was established to provide a guideline for managers to use when ordering items from their suppliers. Technically, his article says that it determines manufacturing quantities, which is evident in the refreshingly straightforward title of the paper ("How Many Parts to Make at Once"); however, his model really only applies to *batch* manufacturing where all units become available to satisfy customer demand all at once in contrast to items produced one at a time on an assembly line. The Economic Production Quantity (EPQ) model, which determines the optimal batch size for this one-at-a-time type of production where the first units in a batch can be used to fulfill customer orders while the rest of the batch is still being produced, is one of the first examples of extensions to the original EOQ model (Taft 1918). The assumption of simultaneous availability of the entire order quantity is appropriate for situations where organizations purchase items from suppliers because they usually arrive in complete transportation batches, but the EPQ model is more applicable to single-unit, assembly line production environments.

In addition to the simultaneous availability assumption, the original EOQ model assumes the following conditions. The notation used in the model is also defined in the list below.

- Annual demand for the item, D, is deterministic and occurs at a constant rate over time. This assumption is especially problematic in situations where demand varies from month to month or from season to season throughout the year. Silver et al. (1998) argue that if the variation of demand from period to period is sufficiently low (i.e., the squared coefficient of variation is less than 0.2), the original EOQ model with constant demand can be used without a large degradation in results; when the variation in demand from period to period is high, firms should utilize a model that considers this variation such as the Wagner-Whitin algorithm discussed in Sect. 3.
- The unit cost of the item, p, is known and fixed over the length of the planning horizon. This is a reasonable assumption in practice when firms have negotiated a long-term, fixed-price contract for the item. Extensions of the original EOQ model exist for quantity discount situations or items subject to significant inflation. Many of these models are discussed in the subsequent sections of the chapter.
- Lead times for receiving orders, L, are known and constant. This assumption applies in practice when the firm has a high-quality supplier that fulfills orders consistently within the same period of time. This situation becomes less applicable in practice when suppliers are located far away from the company because the shipments spend more time in transit and are subject to more uncertainty with respect to transportation conditions and customs scrutiny at international borders.
- The firm's ordering cost, S, is fixed and independent of the size of the order quantity.
- The firm's annual holding cost rate, r, is fixed and independent of the size of the order quantity. Thus, the cost of holding a unit in inventory for an entire year, H, can be computed as $H = r^*p$.
- No capacity or financial limitations apply for the firm or its supplier. This is especially applicable for make-to-stock products that are available immediately in a supplier's distribution center as well as for cheap items for which the firm has ample cash reserves to pay for orders.
- No stockouts are allowed; that is, the firm orders enough items to satisfy all of the demand when it occurs. It can be shown, however, that if the cost of backordering is sufficiently low, the firm can reduce its total cost by planning to backorder some of the demand (Zipkin 2000).

2.2 Model Derivation

In light of these assumptions, the firm seeks to determine the optimal order quantity, Q^*, which minimizes its total annual relevant costs. No quantity discounts apply to the original model; as a result, the amount that the firm must pay the supplier for the annual supply of the item, p^*D, is irrelevant to the order quantity decision. Similarly, stockout costs are not relevant either because the firm is assumed to satisfy all of the demand when it occurs. The two remaining components of the total annual relevant cost are the annual ordering costs and the annual holding costs. We can set up the following function representing the total annual relevant cost as a function of any order quantity, Q.

$$\mathrm{TARC}(Q) = S^*D/Q + H^*Q/2$$

The first term above represents the annual ordering cost by multiplying the cost per order, S, by the number of orders per year, D/Q. The second term is the annual holding cost, which is the product of the cost of holding a unit in inventory for a year, H, and the average Inventory level, $Q/2$. The holding cost is applied on the basis of the average inventory level because the number of units in inventory is constantly changing; some units spend a long time in inventory while others spend only a short time.

The total annual relevant cost function above is convex (meaning that the second derivative is positive for all values of Q); thus, the cost-minimizing value of Q can be found by setting the first derivative equal to zero and solving for Q^*. While Harris (1913) used this calculus-based approach to determine the optimal quantity, subsequent researchers have used other methods to derive the same optimal solution (e.g., Minner 2007 and Teng 2009). Each of these solution approaches yields the following optimal order quantity, which is commonly referred to as the EOQ.

$$Q^* = \sqrt{\frac{2DS}{H}}$$

2.3 Model Implementation

A quick scan of the EOQ model's foundational assumptions suggests that they are so restrictive that this model would only apply to very few, if any, products in practice. Additional criticism has been levied upon the EOQ over the years due to the fact that many of its parameters are difficult for companies to estimate exactly (Woolsey 1988). Holding costs are often particularly nebulous because they include elements such as obsolescence and pilferage that must be estimated from historical inventory control performance statistics. Burnham and Mohanty (1988) also criticize the EOQ model because it considers items independently and separate from their logistics and distribution operations throughout the supply chain. They propose that firms should instead utilize a Uniform Order Quantity that determines the best integrated order quantity for all parties in the entire supply chain.

These scathing criticisms of the original EOQ model may lead some readers to think that the model is never used; and if it is used in practice, companies are making a significant mistake in doing so. The criticisms are valid, but their focus is misguided. Just because some (or all) of the EOQ model's assumptions may be violated in a technical sense, there are still many situations where the model's performance is good enough to help companies reduce their inventory costs. We are reminded of the classic quote, "All models are wrong. Some are useful." This is certainly the case for the original EOQ model in practice.

The major reason that the original EOQ model is still used so often in practice is the difference between the added cost that firms incur by applying the EOQ model to situations instead of more complex models that better capture the circumstances in the particular business setting and the increased cost of implementing one of the more complex models compared to the original EOQ. Everyone agrees that the EOQ formula itself is relatively straightforward to compute and implement. Studies (e.g., see Fulbright (1979)) have shown that the EOQ model is relatively robust to errors in estimating the model's cost parameters (which was one of Woolsey's 1988 major criticisms of the EOQ). This is due to the fact that the total annual relevant cost function is relatively flat around the optimal quantity. Therefore, using an order quantity that differs from the EOQ by X% yields an increase in total cost that is less than X%.

It is also important to note that every item a firm keeps in inventory is not necessarily very important to the firm's overall performance. Many items can be classified as insignificant from a competitive standpoint; as a result, they do not warrant application of more complex models even if they can generate a large cost savings percentage. For example, consider a product that costs a firm $1.50 each, and the firm's customers order 50 of these products, on average, each year for $3.00 each. If the firm had no units of this product in stock at any point in the year, it loses out on, at most, $75 in gross profit. Even if the firm managed inventory of this item perfectly, the benefits would be minimal at best. Organizations should apply more complex models to their most important products. Simple methods such as the EOQ are appropriate for the rest of the items, which for most companies represent the vast majority of their items. In fact, even calculating the EOQ is probably more work than some of these products warrant!

Practitioners should be cautioned if they believe that the EOQ formula derived above provides the one single order quantity that they *must* use when they place their orders. The EOQ formula instead provides a *guideline* for a reasonable order quantity that balances the ordering and holding costs for the item. Wilson (1977) discusses many instances including batch size capacity restrictions, obsolescence risk, and production scheduling coordination where the EOQ model provides a good starting value but requires modification to be appropriate for practice in light of the pertinent real-world constraints. Cannon and Crandall (2004) note that many of the more sophisticated methods that outperform the EOQ in practice were developed with the same goal that is at the foundation of the EOQ—balancing the ordering and the holding costs. They echo Wilson's opinion that the EOQ provides a good first-pass solution to this inventory cost trade-off. Even Harris (1913) himself did not envision the original EOQ model as a panacea for control of all inventories when he wrote, “In conclusion, it may be well to say that the method given is not rigorously accurate, for many minor factors have been purposely left out of the consideration…. The general theory as developed here is reasonably correct and will be found to give good results.”

Cannon and Crandall (2004) make the following suggestions about when it is beneficial to use the EOQ directly, when it should be modified, and when it should not be used at all. The EOQ is useful for make-to-stock products with stable demand and when ordering and holding costs are relatively stable, conditions that largely mimic the underlying model assumptions. The EOQ provides a good starting point and should be modified under conditions of quantity discounts and when the organization’s order must be split into multiple shipments in light of production and/or transportation constraints. The EOQ should not be used in a make-to-order environment that requires the firm to ship orders complete in a single shipment.

Academics and researchers have spent a century modifying the original EOQ model by relaxing one or many of its underlying assumptions to develop models that address particular business situations more appropriately. Many published studies compare the new model’s performance to that of the original EOQ and determine the conditions whereby the new model improves performance significantly. In the next five sections of this chapter, we discuss the major developments in EOQ models from the 1950s to the present day.

3 The 1950s and 1960s

In the late 1950s, research on the EOQ started appearing more frequently in academic journals. One of the earliest works emphasized the scheduling difficulties that may arise when using the EOQ as the economic lot size. Vazsonyi (1957) addressed this issue and suggested a technique to take into account the available production hours on each machine and labor hours available, when the holding cost, setup cost, and production time are known. The formulation also allowed for the ability to plan parts separately in the case that labor and machine capacity were available. Nonlinear mathematical programming was used to solve this problem.

Research on the EOQ model with quantity discounts began appearing in journals in the early 1960s. Crowther (1964) was the first to consider this issue from both the buyer’s and seller’s viewpoint. His formulation was based on the EOQ model as the breakpoint quantity prior to receiving a quantity discount. Through his formulation, cost reductions to the buyer and seller were determined. During this decade, others began to address the limitations of the EOQ by pointing out its shortcomings under conditions that were more complex than its simplest assumptions. These conditions included situations when there is an overlap in time between production and demand as well as an allowance for inventory and backorders (Diegel 1966) or dollar stockout penalties and desired service levels (Herron 1967).

This decade also marked the beginning of a research stream devoted to comparing the performance of the EOQ to other methods of inventory control. Kaimann (1968a, b, 1969) wrote a series of articles addressing questions regarding the traditional EOQ formulation. He pointed out a "fallacy" of the EOQ—that it is often used in situations that are not appropriate, which lead to suboptimal policies. He introduced a new methodology that uses the Wagner-Whitin dynamic programming algorithm and contrasted his results with the traditional EOQ. He believed the Wagner-Whitin methodology was more representative of a typical production scenario where demand widely changes between periods. When compared with the EOQ, the total cost was significantly lower, and he urged readers to be more selective of using the EOQ in situations where its underlying assumptions may be violated (Kaimann 1968a, b, 1969).

Additional research on the EOQ during this time period focused on developing extensions to the original EOQ model to make it more applicable in different production or service settings. Philips and Dawson (1968) addressed how retailers could use Bayesian statistics to calculate their order quantities and reorder points more accurately. By using a priori probabilities of inventory factors, rather than averages, managers could address the impact of upcoming events in their inventory management strategies. Hoffmann (1969) addressed the optimal interest rate useful in determining EOQ values using exchange curve and optimal policy curve concepts. Schussel (1968) expanded on the EOQ formulation in his development of an Economic Lot Release Size model by determining the least expensive lot sizes for repetitive parts and subassemblies. This decade marked the beginning of an overwhelming amount of research dedicated to using the EOQ as a foundation of lot sizing.

4 The 1970s

4.1 Analysis of EOQ Performance

Research studies comparing the performance of the EOQ and other inventory models continued throughout the next decade. Kaimann (1970) continued his research on addressing the impact of demand fluctuations on the EOQ model by examining three types of variation—random, cyclical, and seasonal. In his results, he concluded that as variation increases, the Wagner-Whitin method is superior; however, he conceded that the EOQ still has the advantages of being easy to compute and comprehend, and robust even as variability increases. Others arrived at a similar conclusion when comparing the two methods, finding the Wagner-Whitin algorithm superior and more representative of reality (Phillippakis 1970; Gleason 1971). Kaimann (1972) also performed a similar comparison of approaches while addressing lead time variability resulting in safety stock. Bechtold and Nast (1978) compared performance of the EOQ to the ROQ, a model developed by

Schroeder and Krishan (1976) that maximizes return on investment and found that the EOQ was appropriate for inventory decisions related to both finished goods and work-in-process inventory.

4.2 Development of EOQ Extensions

This decade marked the beginning of research on the extensions of the EOQ model that grew exponentially over the next 40 years. Researchers used the EOQ model as a foundation and then relaxed a subset of assumptions and added costs or constraints to apply the model to a variety of situations. Some examples of extensions of the EOQ model include the consideration of multiple setup costs (Lippman 1971), an all-time requirement which takes into account the cumulative demand to improve forecasting in replacement part items (Moore 1971), the development of a present value formulation of the EOQ that takes into account an infinite time horizon (Trippi and Lewin 1974), inclusion of a working capital constraint (Ram Mohan 1978), and consideration of the EOQ under stochastic lead time (Liberatore 1979). Langley (1976) discussed the implications of relaxing the assumption of *certainty* for the EOQ model parameters. He considered the optimistic, likely, or pessimistic alternatives of the EOQ by using maximax, maximin, and minimax regret strategies as well as the Laplace criterion as a way of enabling decision-makers to make more appropriate decisions dependent on workplace conditions. In contrast, others discussed their issues with models that relax assumptions of the EOQ which create *unrealistic* production scenarios and urged researchers to make their EOQ extensions more compatible with present conditions and readily available production data (Soyster and Enscore 1975).

4.3 EOQ and MRP

Research on the relationship between Materials Requirement Planning (MRP) and the EOQ model became more prevalent in the 1970s as the performance of the EOQ within an MRP system was examined. Chamberlain (1977) referred to MRP as the "salvation of production and inventory control management" and cautioned on using the EOQ as an ordering policy due to its creation of inventory and inability to respond to plans aimed at reducing inventory. Others examined how the EOQ lot sizing rule compares to other methods (e.g., L4L, POQ, FOQ, LTC, etc.). Yelle (1978a, b) examined this issue in the context of a multi-level lot sizing problem and suggested which lot sizing sequences utilizing these rules should be used to achieve the lowest inventory costs as demand patterns differed (i.e. lumpy, increasing, decreasing, etc.). Kropp et al. (1979) suggested that the EOQ was one way to deal with MRP *nervousness* due to the fact that it is relatively unaffected when cost and demand estimates are incorrect but found that a dynamic lot-sizing method was preferred due to its ability to strike a balance between all relevant inventory costs.

5 The 1980s

Research on the EOQ in the 1980s can be primarily classified into three areas of interest: (1) the performance of the EOQ against other lot-sizing rules, (2) the introduction of additional extensions of the EOQ model, and (3) the role of the EOQ in logistics. In addition, the number of articles devoted to the EOQ increased significantly during this time period, which is evidence that the formulation maintained its position as a robust and relevant lot sizing method even under changing environments.

5.1 Analysis of EOQ Performance

Evaluation of the performance of EOQ versus other lot sizing rules continued in this decade under different contexts. Choi et al. (1984) examined the EOQ versus eight other rules in multi-echelon MRP systems using FORTRAN, with the EOQ performing in the lower third. Rubin et al. (1983) compared the classical EOQ approach to the total setup lot sizing model (T-S) developed by Kuzdrall and Britney (1982) in the case of quantity discounts and found that a modified EOQ performed better than the T-S method. The performance of the EOQ was also examined by Boucher (1984), and this study found that a modified EOQ (GTOQ) was superior in the context of group technology systems. Melnyk and Piper (1985) examined the effects of lead time errors on different lot sizing rules. Similar comparisons were made with a fixed charge heuristic (Bahl and Zionts 1986) and the incremental part-period algorithm (Patterson and LaForge 1985) and under conditions of serially correlated demand sequences (Williams et al. 1985) and linear and increasing demand (Ritchie and Tsado 1986).

5.2 Development of EOQ Extensions

Some of the extensions of the EOQ that appeared in this decade included the consideration of a temporary one-time price discount on the EOQ model (Tersine and Price 1981), the addition of nonlinear (increasing) holding costs (Weiss 1982), the consideration of discounting rates (Gurani 1983; Clarke 1987), the addition of multiple setup costs (Aucamp 1984), and the inclusion of constant inflation and simple interest (Kanet and Miles 1985). Others included the consideration of future price increases (Markowski 1986; Tersine and Tersine 1986), increasing step-function ordering costs (Bigham 1986) and all units price discounts (Gupta 1988). Another set of models addressed the impact of variation in the sizes of production loads when determining the EOQ, considering the impact of overtime when regular capacity is exceeded (Axsater 1980, 1981; Goyal and Evans 1981).

Other researchers extended the EOQ model to different contexts including Dyl and Keaveny's (1983) application of the fundamentals of the EOQ model to a human resources problem in an effort to minimize the costs associated with hiring and training, as well as managing an excess or shortage of labor. Also, the impact of EOQ systems on deteriorating items was first examined by taking into account variations in deterioration rates and product shortages under probabilistic and deterministic demand conditions (Elsayed and Teresi 1983). Das (1984) suggested modifications to the traditional EOQ under conditions inherent in developing countries (e.g., changing rates of price and supply, capacity expansion issues, etc.).

Lee and Rosenbatt (1986) addressed the significance of the EOQ model developed by Subramanyam and Kumaraswamy (1981) that took into account the effects of advertising and the impact of price elasticity, along with potential damages among ordered items and economies of scale. They expanded on the earlier work and developed three different EOQ formulas to account for the different cases that may arise. The impact of learning on setup costs was addressed by Replogle (1988). He developed a modified EOQ that takes into account the impact of reducing setup costs to gain competitiveness.

As research on JIT became more prevalent in this decade, authors began to initiate discussions on how an inventory system based on the EOQ differed from policies in a JIT environment. Schonberger and Schneiderjans (1984) addressed the fundamental differences between JIT and EOQ ordering and pointed out the strengths of JIT ordering over EOQ ordering. These included an ability to cut holding costs and lessen setup costs through process improvements and the importance of factors such as quality rework and work motivation. Fox (1984) compared EOQ fundamentals to the Optimized Production Technology (OPT) rules developed by Goldratt, particularly focusing on the EOQ notion of a single uniform batch size versus the OPT idea of multiple batch sizes or varying quantities.

In contrast to the significant number or articles that attempted to make the EOQ more complicated, Banks and Hohenstein (1981) prescribed a simplification of the EOQ by dividing it into two parts: (1) the value "2" and the ordering/holding cost ratio and (2) the demand to cost ratio. The first part was to be considered a "constant" or "index number" that would be applicable to a group of similar items in inventory for a firm, while the other constant would fluctuate. This enabled managers to choose an index number and utilize this over a variety of different items to simplify the inventory management process.

5.3 Application in Transportation and Logistics

Research that addressed the use of EOQ models in transportation and logistics first appeared in the 1980s. Tanchoco et al. (1980) considered the impact of material handling and the transportation of unit loads on lot sizing. They modified the EOQ model to include the transportation cost which was dependent on the quantity of unit loads necessary for a specified production lot. Burns et al. (1985) examined the optimal lot sizes using different types of distribution strategies—direct shipping (direct from supplier to customer) and peddling (more than one customer per load)—and found that the EOQ value was the optimal shipping size for direct shipping. In addition, Landeros and Lyth (1989) studied the impact of lot sizing in purchasing and logistics management decision-making.

6 The 1990s

In the 1990s, researchers continued to address EOQ performance (e.g., Gupta and Brennan 1992; Ho 1993; Toklu and Wilson 1995), to compare and contrast the method with JIT strategies (e.g., Chyr et al. 1990a; Grant 1993; Baker et al. 1994; Fazel 1997; Fazel et al. 1998), and to study the transportation implications (e.g., Russell and Krajewski 1991; Tersine and Barman 1991; Ha and Kim 1995). In addition, new methodologies to solve EOQ problems and technologies that use EOQ models were introduced here. For example, Stockton (1993) used a genetic algorithm methodology to solve EOQ problems by capitalizing on the methodology's ability to "generate populations of solutions," giving managers the ability to choose from among the batch sizes (even those that are non-optimal) if they provide a better fit for the current operational environment. Also, Pullin (1995) discussed the importance of the EOQ model to Electronic Data Interchange (EDI) to improve supply chain coordination.

However, the area of research that received the most attention continued to be extensions to the EOQ model. These extensions included an application to retail cycle stock inventories (Bassin 1990), the addition of cost changes under a finite or infinite time horizon (Lev and Weiss 1990), the inclusion of storage size considerations (Rao and Bahari-Kashani 1990), and the addition of damage costs (Chyr et al. 1990b). Others addressed pricing considerations (Cheng 1990), inspection delays (Porteus 1990), supplier credit (Wilson 1991), cash float (Bregman 1992), conditions of a temporary sale for a buyer (Chen and Min 1995), and supply credit using a discounted cash flow approach (Carlson and Miltenburg 1996).

In the 1990s, several streams of research that would continue throughout the next decade began to surface. These include research on continuously deteriorating products (Fujiwara and Perera 1993), forgetting and learning effects along with the time value of money (Chiu and Chen 1997), permissible delay in payments (Chung 1998), and deteriorating and ameliorating items (Hwang 1999). Researchers used the EOQ model as a basis while they considered situations of cyclical demand (Specht and Kagan 1994), multiple stocking points (Meller 1995), random fluctuations of demand (Bill and Chaouch 1995), product substitutions (Drezner and Gurnani 1995), obsolescence for fast moving spare parts (Cobbaert and van Oudheusden 1996), conditions of variable capacity (Wanga and Gerchak 1996), imprecise estimation of parameters (Vujošević et al. 1996), random supplier capacity (Hariga and Haouari 1999), and dollar cost averaging (Khouja and Lamb

1999). In addition, Tersine (1996) developed a composite EOQ model that could be broken down into separate deterministic models. These extensions are representative of the decade, although not exhaustive. However, they illustrate the importance of the EOQ model not only to inventory management but also to supply chain management. Research in many of these streams continued in the twenty-first century.

7 The 2000s to the Present

Moving into the new millennium, the number of articles dedicated to the EOQ model more than doubled. In this time period many authors continued to focus on extensions to the original EOQ model, specifically to those presented in the 1990s, (e.g., variations in parameters, inflation, JIT, deteriorating items, effects of learning, supplier credits, Quantity discounts, capacity constraints). Other research streams that were studied extensively during this time include using the EOQ model for items of imperfect quality (e.g., Salameh and Jaber 2000; Maddah and Jaber 2008; Khan et al. 2010, 2011; Hsu and Hsu 2012) and a renewed interest in using the EOQ under conditions where partial backordering occurs. Pentico and Drake (2011) provided an exhaustive summary of the research on EOQ models that handle partial backordering, and some even more current models with partial or full backordering include those by Zhang et al. (2011), Toews et al. (2011), Chung and Cárdenas-Barrón (2012), and Taleizadeh et al. (2013). However, the two areas of research that became more prominent during this time period were EOQ models and their impact on supply chain applications, performance, and coordination and the EOQ's role with respect to sustainability applications. We highlight these areas below.

7.1 Supply Chain Models

As companies looked to consolidate warehouses and other stocking locations to increase competitive advantage, research in this area became more prevalent. For example, Lim et al. (2003) determined the cost savings of warehouse consolidation by modeling each warehouse as a single-stage EOQ system. Ng et al. (2009) also approached this issue by treating the capacity of the warehouse as a decision variable. They also included in their model the condition that the warehouse cost is greater than other relevant holding costs. In his comparison of continuous and periodic review systems, Cachon (2001) found the EOQ model to yield the minimum order quantity in his study of retailer shelf space availability using different transportation dispatching policies. Balakrishnan et al. (2004) extended the EOQ model to the context of a retailer to understand how inventory levels can stimulate demand. EOQ and inventory models were also used in facility design decisions by

Miranda and Garrido (2004) to solve a distribution network design problem. Zinn and Charnes (2005) compared Quick Response (QR) models and EOQ models and concluded that the EOQ was superior to the QR under many conditions including low-risk conditions and when demand or product value is low. E-commerce inventory decisions were examined by Bhargava et al. (2006). They developed an EOQ-based model to determine the impact of offering customers compensation when they have to wait for products due to stockouts.

Supply chain models relating to perishable items were also examined in recent years. Li et al. (2007) examined postponement strategies of perishable items using an EOQ model, while Ferguson et al. (2007) applied Weiss (1982)'s nonlinear holding cost model in the context of perishable items. The use of EOQ models in determining how contracts in Vendor Managed Inventory (VMI) systems can improve system performance was discussed by Nagarajan and Rajagopalan (2008). Battini et al. (2010) compared EOQ lot sizing to a consignment stock approach of managing inventory. Lau et al. (2008) examined supply chain performance using a computer simulation of four different lot sizing policies—EOQ, POQ, Silver-Meal, and Part-Period Balancing. Supply chain performance was measured in terms of costs and service levels. Their results showed that the EOQ costs were lowest for the retailers and for the supply chain as a whole. Lastly, the importance of coordination among supply chain partners continued to be an important area of research (e.g., Sucky 2005; Jain et al. 2006; Chiou et al. 2007). Some examples include Chan et al. (2010), who considered this issue under the context of delayed payments, and Chan and Lee's (2012) examination of supply chain coordination with incentive schemes that entice partners to alter policies in order to achieve monetary savings.

7.2 Sustainability

Although research on waste and repair models appeared in the previous decade (e.g., Richter 1996; Richter and Dobos 1999), this area of research expanded greatly due to the importance of sustainability to firm performance. Teunter (2001) built off of Schrady's (1967) original work on reverse logistics by adding in a disposal option and using different holding cost rates for recovered and manufactured items to determine the optimal batch quantities for both manufacturing and recovery. Extensions to the original model were presented by Dobos and Richter (2000) along with new perspectives using the EOQ model as a foundation in the context of production recycling models (Dobos and Richter 2003, 2004, 2006).

Other extensions included determining optimal lot sizes for the recovery of items that were returned (Teunter 2004) and the consideration of entropy cost (Jaber and Rosen 2008) and switching costs (El Saadany and Jaber 2008). Gou (2008) modified the EOQ model to find the optimal delivery batch size in his research on open-loop reverse supply chains including one centralized returns center and multiple local collection points. Alinovi et al. (2012) developed an EOQ system for a mixed manufacturing/remanufacturing system as a framework to aid managers in how to effectively use EOQ policies in other reverse logistics environments. An excellent summary of the integration of the EOQ model into sustainability research was provided by Bouchery et al. (2012).

8 Conclusion

This chapter merely scratches the surface when it comes to chronicling the evolution of EOQ-type models over the past century. A truly comprehensive effort would likely span the length of this entire book (or even longer!). We have discussed the major developments in EOQ-type research specifically since the 1950s, and we have classified the types of extensions by their major foci in each decade.

In light of a century's worth of extensions and applications, the impact that Harris' little three-page article has had on an entire field of management is truly staggering. The model, even in its original restrictive form, is still presented in every book and every course that introduces inventory control concepts to new generations of students. Criticisms aside, the EOQ model's longevity can likely be traced to its fundamental goal that still holds true today—illustrating the trade-off that exists between ordering costs and holding costs in inventory control. Like all models, the EOQ is a simplification of reality, and many times it performs well enough in practice. But like all models, one single formula should not necessarily be immune to the managerial modifications that can adjust its recommendation to apply more directly to a given business situation. After all, Harris (1913) knew this better than anyone when he wrote, "The writer … does not wish to be understood as claiming that any mere mathematical formula should be depended upon entirely for determining the amount of stock that should be carried or put through on an order. This is a matter that calls, in each case, for a trained judgment, for which there is no substitute."

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