Chapter 5
Forward Look at Research Perspectives

Abstract This chapter presents some on research perspectives. Various topics are treated focusing on the following issues: further analysis of the modeling of welfare policy in the case of interactions in a network and in open systems; generalization of the modeling approach to various systems of social sciences, for instance opinion formation; modeling the interplay of different types of dynamics also viewed as a tool for predicting rare events; and analytic problems posed by the application of models to the study of social phenomena.

5.1 Introduction

The mathematical tools presented in Chap. 2 have been applied to modeling the dynamics related to social and economic policies that a government can develop to affect the trend of wealth distribution toward a planned direction. The specific case study, presented in Chaps. 3 and 4, has shown that models can predict various aspects of the dynamics and analyze the influence of the parameters of the model on the aforesaid trend. This case study should be regarded as a preliminary attempt, to be further generalized to enlarge the variety of phenomena described. This exercise can possibly improve the predictive ability of models. In addition, applications generate interesting analytic problems that offer applied mathematicians challenging goals to be properly achieved within a program of research.

This chapter refers to the identification of research perspectives related to the aforementioned topics. The style will be somewhat different from that of preceding chapters. In fact, rather than dealing exhaustively with the issues presented, it focuses on new models and mathematical problems generated by the study of real socio-economic problems. Some hints toward research directions follow. Moreover, additional bibliography is reported to offer interested readers a sufficiently broad list of references. In particular, we wish to mention the following web pages, which

G. Ajmone Marsan et al., *Complex Systems and Society: Modeling and Simulation*, SpringerBriefs in Mathematics, DOI 10.1007/978-1-4614-7242-1_5,

offer profitable suggestions for developing mathematical approaches to studying socio-economic systems:

http://www.oecd.org/about/secretarygeneral

http://ineteconomics.org/research_note

In this chapter we also revisit the fact that one of the goals, perhaps the most important one, is the design of models that have the ability to predict rare, seemingly unpredictable, events such as so-called *black swans* [149, 150]. The idea of pursuing this challenging goal is given in a recent paper [39], which has shown that extreme events can be generated by an interplay of different types of dynamics; in the specific case studied, the interplay is between welfare policies and support or opposition to a certain political regime.

Reader may be somewhat disappointed by the very introductory stage at which the contents of this chapter are presented. Nevertheless, it is useful to recall that this monograph is presented according to the spirit of "Springer Briefs", hence as an introduction, by no means exhaustive, to possible research lines currently not yet thoroughly developed, which might stimulate specific research programs.

Bearing all of the above in mind, the following topics have been selected, according to the authors' preferences, for looking ahead at research perspectives:

- Further analysis on the modeling of welfare dynamics involving interactions over networks and within open systems.
- Generalizations of the modeling approach to a variety of new studies, including opinion formation, democratic transitions, and political instability.
- Modeling the interplay of different types of dynamics, also viewed as a source that can generate rare events.
- Analytic problems related to the applications of models.

These topics are treated in the following sections, and the last section concludes the monograph.

5.2 Welfare and Well-being Policies

As already mentioned, the contents of Chaps. 3 and 4 cannot be considered exhaustive; indeed various specific problems and generalizations are omitted. A few of these are presented in this section, followed by some hints for pursuing them.

5.2.1 Interactions Over Networks

Active particles interact, in most cases, over networks [27, 29, 32, 160]. In general, the identification of the functional subsystems playing the game depends on the localization of the nodes. The simplest case is when each functional subsystem corresponds to a node.

Interactions over networks can induce substantial modifications to the dynamics studied in Chaps. 3 and 4. For instance, the prevalence of selfishness or altruism, as well as the overall wealth distribution, may be affected by the network topology of interaction. In addition, the modeling of the interaction rate depends entirely on the structure of the network. In the case of welfare dynamics, the analysis of Chap. 4 provides evidence that the trend of the system depends on the overall wealth of a society. Therefore it may happen that the same rules cannot be applied with the same advantage to every society—for example, every country of a given continent. As an example, imposing the rules of the wealthiest country to other countries can have a negative effect. Rules and laws must account for individual characteristics of each place, and therefore must be adapted accordingly [68, 129].

An interesting research perspective consists of the development of models in which functional subsystems localized in a node interact over networks of living systems having a self-organizing ability; see e.g., [99]. Moreover, the study of networks can involve the modeling of other interesting types of dynamics such as migration phenomena from less developed countries to wealthier ones [111].

The mathematical tools proposed in Chap. 2 can be technically generalized, while still maintaining a low computational complexity at least for small networks. The advantage is that nonlinearly additive interactions can be taken into account, thereby improving mean field descriptions when necessary. On the other hand, the need for reducing computational complexity arises in the case of large networks. Perhaps the clustering of nodes that exhibit the same features can contribute to tackling this delicate problem.

5.2.2 Modeling Open Systems

All models mentioned so far refer to closed systems, such as countries and networks of countries closed to any external action. However, it can be argued that economic and political phenomena can be subject to important modifications induced by external actions, which modify the interaction rules at the microscopic scale. Particularly interesting is the case of external actions causing quantitative modifications of time-asymptotic configurations and qualitative dynamical behaviors.

The mathematical structures presented in Sect. 2.4 provide the tools to deal with this problem. Specific applications are known [51] in the case of binary interactions. However, the modeling approach should be complemented with a deep analysis of the generation of internal forces. The book by Helbing [95] offers a valuable contribution to this issue. The modeling approach should take into account the action of the outer environment on the whole system, both at the macroscopic and at the particle scales. Some relevant studies, for instance in the field of opinion formation [52], can be found in the literature. However, a systematic study able to model the influence of real external actions on the dynamics of the system is still missing. Transferring the mathematical tools of Chap. 2 to these new concepts appears to be a challenging research problem.

5.2.3 Understanding Ethical and Unethical Behaviors

A recent paper [135] has analyzed to what extent unethical behaviors can be sources of richness. This interesting topic can be approached using the model presented in Chap. 3, with the expectation of further and deeper analysis of the interplay between the critical threshold γ and the average wealth of the society. In fact, the various simulations produced in Chaps. 3 and 4 have clearly shown that the resulting asymptotic configurations are sensitive to both of them. Again, this suggests that the rules valid for a rich country cannot be applied to countries with less wealth.

5.3 Toward Additional Applications

The mathematical approach under consideration has been developed, since the pioneer paper [48], in various fields of social sciences. Applications have been made not only to the dynamics of wealth redistribution but also in other contexts, such as opinion formation [50], taxation systems [46,53], competition for secession [7], and behavioral economics [6]. Additional studies make use of methods of mathematical kinetic theory, specifically mean field equations. We note, among other applications, decision making [65], market dynamics [55,116], opinion formation [12], and more recently migration phenomena [23,111].

Therefore, we can state that there exists a quite extensive literature in the field applying similar, though technically different, approaches. In general, understanding a certain system contributes to generalizing the approach to other systems. This line of thought suggests also examining systems in other fields of life sciences, such as mathematical biology [44] or epidemics with virus mutations [73].

As we have seen, the modeling approach first requires assessment of the functional subsystems and the types of active particles that play the game, along with the specific activities they express. Subsequently, the developed strategy has to be identified, in such a way that interactions at the microscopic scale can be properly modeled. In most cases, it is also useful to speculate about the lower and higher scales, when they can be identified, as well as on conceivable networks related to the specific system under consideration. This process can be analyzed in the case of some specific applications briefly reported in the following paragraphs.

However, before dealing with technical issues it is worth stressing that the various fields of application reported in the following paragraphs have been selected among several conceivable ones according to the authors' preferences. The aforementioned screening can be viewed as a very preliminary step toward modeling. Obviously, the derivation of models should constantly face the complexity features of the system under consideration. The remarks concluding each specific presentation are possibly valid for all of them.

5.3.1 Voting Dynamics

Voting dynamics have been widely studied in the literature by political scientists and economists [30,76,117], with a wide range of techniques borrowed from qualitative analysis, statistics, game theory, and mathematical modeling. Understanding the way in which different groups of individuals interact in making decisions about whom to vote for in national or local elections is crucial in democratic regimes. A wide range of voting mechanisms exist, strongly dependent on various countries and political systems. In addition, different voting systems (such as, for example, electronic voting and mail-in voting) [45] and news and information media [86] may affect election outcomes, making voting dynamics a complex phenomenon, whose aggregate outcomes, namely the results of elections, depends on the sub-dynamics of several concurrent factors.

Consistent with the approach of Chap. 2, we now identify some basic modeling features for this kind of system.

- *Microscopic entities*: individuals in a nation.
- *Microscopic state*: inclination to vote for a certain party in an election.
- *Lower scale*: individual political opinions.
- *Higher scale*: aggregates of opinions in a nation.
- *Networks of interaction*: spatial interactions among voters in cities, districts, and regional areas.
- *External actions for open systems*: actions of parties by means of various types of media.

Remark 5.1. Voting dynamics are generally studied in the framework of closed systems. However, international networks can play a role in the game by driving public opinion toward the trend of economically stronger countries. In some cases these interactions can generate a domino effect.

5.3.2 Diffusion of Technological Innovations

Knowledge transfer has largely been studied in the economics literature related to innovation adoption and diffusion [107]. Knowledge spill-overs and the diffusion of new ideas are clearly linked to interactions of individuals in firms and other institutions, over time and space [4,22,91]. The diffusion of innovations may also be induced by networks of firms located in different geographical areas, as well as by market structures and dynamics, and external actions related to the kinds of policies implemented by governments either domestically or internationally.

- *Microscopic entities*: firms.
- *Microscopic state*: technological stage in firms.
- *Lower scale*: staff desires to improve the quality of their activity.
- *Higher scale*: population of firms in a given area.
- *Networks of interaction*: networks of firms at a regional or national level.
- *External actions for open systems*: innovation policy.

Remark 5.2. The diffusion of technology has a relevant interplay with other social dynamics, starting with economic growth or decay. Therefore, it is important that the modeling approach takes into account the hints of the following section.

5.3.3 Migration Phenomena

Migration flows have great socio-economic impacts on countries and regions from which and to which they occur. Migrations have been extensively studied in the literature by sociologists and economists [56, 63, 71]. They occur as a consequence of several phenomena: level of wealth and safety of the country of origin of migrants, wars and discrimination, and natural and environmental disasters, to mention but a few examples. Migrants may modify the socio-economic and cultural context of cities, regions, and countries, and therefore a better understanding of the dynamics generating and generated by migrations would help us understand how this phenomenon contributes to shape socio-economic development.

- *Microscopic entities*: individuals and/or families.
- *Microscopic state*: tendency to migrate.
- *Lower scale*: level of wealth of individuals and families.
- *Higher scale*: level of development of a country.
- *Networks of interaction*: networks of displacements.
- *External actions for open systems*: social policies fostering and/or preventing migration.

Remark 5.3. The modeling approach of [111] is based on the mathematical approach presented in this monograph, whereas a different technique is used in [23], where suitable developments of Hamiltonian mechanics are applied.

5.3.4 Democratic Transitions

The dynamics of the transition from a dictatorship to a democracy are still not clear as they can be influenced by a very complex interplay of factors. Political transitions have often occurred in the past century especially in political regimes outside the Western World, which have been studied from several points of view by scholars in political science and economics. The latter studied, in particular, which economic variables and factors could either foster or prevent the occurrence of political transitions.

These complex dynamics can lead to unpredictable events that cause sudden shocks in political regimes (as in the recent case of the “Arab Spring”), and whose origins still need to be understood clearly. For a comprehensive overview of these topics, one can refer to [2, 3, 131].

- *Microscopic entities*: supporters or opponents of a dictatorship.
- *Microscopic state*: political attitude.
- *Lower scale*: level of support or dissent toward a dictatorship.
- *Higher scale*: average collective behaviors.
- *Networks of interaction*: small areas of a town or a region.
- *External actions for open systems*: international support for or dissent toward a regime, media impact.

Remark 5.4. The modeling approach can be developed by either using one functional system only or partitioning the whole system into different subsystems characterized by different styles in pursuing their strategies. When the strategy is different, possibly even antagonistic, the identification of several functional subsystems is often mandatory, as happens for instance for the system briefly described in the following paragraph.

5.3.5 Spread and Evolution of Criminality

Criminality may originate because of multiple factors related to social, economic, and political conditions and it clearly has a great impact on people’s well-being. Social segregation can affect criminality rates and their patterns. The problem of criminality in urban agglomerations has been studied by sociologists interested in understanding the dynamics of crime and insecurity [60, 137, 157]. More recently, some mathematical models on this topic have emerged [85]. Criminality and social segregation are interconnected complex phenomena, which could be better understood by means of sophisticated mathematical modeling intended to capture their patterns of evolution.

- *Microscopic entities*: individuals subdivided into different functional subsystems; for example, criminals and police officers.
- *Microscopic state*: criminal ability in the first subsystem and ability to apprehend criminals in the second one.
- *Lower scale*: psychological attitude to criminality.
- *Higher scale*: average collective behaviors.
- *Networks of interaction*: small areas of a town or of a region.
- *External actions for open systems*: international crime legislation.

Remark 5.5. The modeling approach might divide the whole system into several functional subsystems characterized by different levels of criminal and detective ability. Possible transitions across the levels should be included in the model.

5.4 On the Interplay Among Different Dynamics

The modeling approach proposed in previous chapters is based on the idea that the activity variable is a scalar or, when it is a vector, that the whole system can be decomposed into subsystems such that each of them is characterized by a scalar activity variable only. However, this type of decomposition is not always technically feasible. Moreover, systems where the interplay involves different activity variables appears to be an interesting topic worthy of investigation.

It has been shown [39] how welfare dynamics based on a selfish attitude of specific social classes can lead, for certain parameter values, to the clustering of the population in the extreme wings of support/opposition to a regime. Moreover, in these extreme situations, even subgroups of the wealthy population become doubtful about the regime, though they were initially in favor of it.

The interplay between different factors appears to be crucial in several types of dynamics. For instance, in [3] the interplay between dictatorial and democratic trends linked to economic issues is analyzed; in [33] the evolution of biodiversity related to mutualistic networks is modeled (also see [136]). Similarly, it is possible to look at the influence of social policies, including welfare policies, on the growth of criminal behaviors [143].

More generally, an overview of social sciences shows that the interplay of different dynamics can have an important influence on the collective behaviors of social systems and, more specifically, on their asymptotic trends. Indeed, this is what has been shown in [39]. Therefore, a natural question arises: how can this interesting topic be further studied and understood? A straightforward application of the methods presented in Chap. 2 suggests that the use of vector activity variables gives, at a technical level, the desired result. On the other hand, this approach significantly increases the difficulty of modeling individual interactions. A useful alternative is offered in [39]. It consists of assuming the sequentiality of the dynamics; for instance first welfare policy and then political dynamics. In this way the output of the first-level dynamics becomes an input for the second-level dynamics. Future activity in the field may clarify which is the most appropriate strategy.

Developing these perspectives requires deeper insight into game theory and evolution. A fundamental reference in this context is [124], which proposes qualitative analogies between biological and social systems. The role of Darwinian selection is deeply analyzed in [113]; also see [125, 130].

The case studies addressed in the previous section indicate, specifically, some conceivable interplays. For instance, the study of opinion formation has an important interplay with the expression of political preferences in voting dynamics, whereas diffusion of technology may have an immediate influence on the dynamics of wealth distribution and life conditions. Heterogeneity needs to be carefully taken into account, also considering that small groups of individuals (representing a minority of the whole population) can have an important impact on the global dynamics of social systems, as documented in [84].

In general, it can be stated that the selection of interplays that effectively have a role in the game depends on the specific goal of the modeling approach. Different choices correspond to different goals. Interplays should provide *early-warning signals*, which indicate changes in the trend of the collective behaviors exhibited by the systems under consideration [127, 140]. A quantitative example is offered in [39] for the interplay between welfare policy and support/opposition to a certain political regime. Specifically, it is shown that when welfare policies tend to be oriented against the well-being of citizens, and when this trend is not controlled by the government but is simply left to spontaneous competition within the population, early signals can be detected which anticipate a radicalization of opposition to the regime.

5.5 Analytical Problems

The application of models to real social phenomena generates interesting analytical problems, which can stimulate further challenging investigations for applied mathematicians.

As we have seen, mathematical problems are stated, in the case of discrete activity variables, as initial-value problems for a nonlinear system of ordinary differential equations. If the activity variable is continuous, initial-value problems refer to systems of integro-differential equations. The qualitative analysis can focus on the following issues:

- Well-posedness of initial-value problems.
- Existence and uniqueness of equilibrium configurations and their stability properties.
- Dependence of the qualitative behaviors of solutions on the parameters of the model, in particular on the initial conditions.
- Analytical problems for open systems.
- Multiscale issues.
- Models with spatial structure.
- Further developments of game theory.

Some results are already known in the literature, generally for models featuring linearly additive interactions. Their extension to the case of nonlinearly additive interactions may not be immediate but is necessary due to the much greater interest of this class of models.

Bearing this in mind, let us briefly sketch some aspects of the issues mentioned above.

5.5.1 Existence of Solutions

This problem was first addressed in [16] for systems of integro-differential equations with linear interactions and in the absence of external actions. It is not a difficult problem, considering that the interaction operator is locally Lipschitz continuous and that, due to the conservation of mass, the L^1 norm of the distribution function is preserved in time. More recent works involve generalization to open systems [18] and qualitative analysis in the case of nonlinear interactions [19].

The generalization of these results to the case of systems of ordinary differential equations generated by discrete activity variables is immediate as documented in [48]. Additional difficulties have to be tackled if the model includes proliferative events, which however have not been treated in this monograph.

5.5.2 Equilibrium Configurations and Their Dependence on the Model Parameters

Existence, but not uniqueness, of equilibrium solutions has already been studied in [16] and further generalized in [19]. On the other hand proof of uniqueness seems to be a difficult problem, although simulations presented in Chap. 3 suggest that the equations, at least in the case of closed systems, show a trend toward a unique asymptotic configuration, which appears to be numerically stable. If the activity variable is discrete then the proof of uniqueness and stability has been obtained for models with a relatively small number of activity classes [49]; however, the proof has not yet been extended to the general case.

Moreover, as shown by the specific examples treated in Chaps. 3 and 4, although the equations always show a trend to an asymptotic equilibrium configuration, the shape of such a configuration depends on initial conditions. More precisely, it depends, in the specific model dealt with in this monograph, on the initial mean value of the wealth but apparently not on the shape of the initial distribution. This amazing result is not well understood and analytical proofs are not available to support such a numerical insight. Partial results are known for models of opinion formation with discrete states [47]; on the other hand, further analysis is welcome for understanding the role played by various parameters on the aforementioned equilibrium configurations.

5.5.3 Open Systems

Most of the literature concerned with analytical problems, such as those briefly sketched above, is limited to closed systems. On the other hand, the role of external actions can be of paramount importance if it refers either to actions at the macroscopic scale or to the influence of agents acting at the microscopic scale. The formal structure to be used in this type of modeling approach was given in Chap. 2.

Since very limited activity has been developed in this field, we simply bring this topic to the attention of readers and stress its importance for applications. The main difficulty consists in modeling external actions in terms of agents and of the games they play with active particles. In some cases interactions can modify the outer environment. This issue is well documented in earth sciences [163] and should probably also be accounted for in the case of social dynamics.

5.5.4 Multiscale Problems

The modeling approach has shown that the analysis of dynamics at the microscopic scale can be transferred to a statistical description of collective behaviors. The conceivable applications summarized in the previous section have shown that for each system it is possible to look at a lower submicroscopic scale and at a higher macroscopic scale. The interplay between different scales generates modeling and analytical problems of great interest for applications. The link between submicroscopic and microscopic scales implies the necessity to model the interplay between the games at the level of individuals and the inner dynamics of the latter. On the other hand, looking for collective dynamics at the macroscopic scale means obtaining an aggregate characterization of the system behavior, for instance via suitable asymptotic approaches or averaging techniques, stemming from, but not necessarily focused on, individualities.

5.5.5 Models with Spatial Structure

The mathematical models studied in this monograph have been derived by assuming that the dynamics in space were limited to interactions involving different nodes of a network, whereas spatial dynamics within each node were neglected. This assumption is not always valid. In fact, social interactions produce, in some cases, aggregation and fragmentation phenomena that are localized in space.

An example is offered by the study of criminal behaviors: the knowledge of the localizations of aggregation spots can contribute to organizing the fight against criminals [143]. Furthermore, as observed in [111], in the case of migrations the spatial distribution of communities of migrants can also be a useful detail for the study of the system.

The modeling of spatial dynamics can take advantage of kinetic-type descriptions, where the localization of active particles is included in the distribution function as a further microstate. The derivation of models at higher scales then needs to be obtained from the underlying description delivered by such kinetic models. It is possible that the approach reviewed in [36] can be properly developed in this direction for addressing the study of social systems. The conceptual difficulty consists in modeling spatial dynamics related to nonlocal games [113]. Some hints toward this specific goal might be extracted from the study of swarms [25].

5.5.6 Further Developments of Game Theory

The modeling approach proposed in this monograph has been constantly referred to game-theoretical ideas, which have been used to model nonlinear interactions within a general framework of generalized kinetic equations. Recent literature reflects different approaches according to different ways of treating individual-based interactions and of inserting them in different classes of evolution equations. Among others, we mention here evolutionary games in the framework of statistical mechanics [95], differential games [57, 58] in the framework of controlled differential equations, and mean field games [92, 114]; see also the recent special issue [61].

5.6 Conclusions

This monograph has shown how suitable generalizations of the kinetic theory for active particles can be applied to model a variety of social and economic systems. The first part of the monograph has focused on the derivation of mathematical tools, and the second part on applications and research perspectives. We feel confident in stating that the indications given as possible research perspectives will generate interesting results from the point of view of both modeling and analytical problems.

The application of the mathematical tools discussed in this monograph to a broader set of socio-economic systems appears to be quite a natural perspective, whereas analysis of the interplay involving different activity variables is more challenging. The indication that the latter can lead to extreme radicalization suggests continuing along the research line outlined in [39] in other fields of life sciences as well. Concerning this, it is worth stressing again that interest in new analytical problems generated by such a modeling approach, some of which are definitely challenging to tackle, is not only due to their intrinsic technical difficulty but, first and foremost, to their immediate applicability.

Some concluding arguments can address the big problem of looking for a mathematical theory of social systems. The first step toward this challenging goal should be the development of mathematical tools suitable for capturing the most relevant general complexity features of such systems. Addressing such an issue in a satisfactory way is, by itself, an extremely challenging task. We certainly do not claim that the search for mathematical tools is completed with the contents of Chap. 2. We simply claim that a preliminary approach has been proposed, which is waiting for further refinements and improvements.

Fig. 5.1

The main positive aspect of the proposed approach is that it has introduced, within a unified framework, a class of equations that includes the following specific features:

- Heterogeneous distribution of the ability of individuals to pursue specific goals. Heterogeneity can have an important influence in determining the output of interactions and hence the overall dynamics.
- Nonlinearly additive interactions, along with related learning processes, treated in terms of stochastic games. This opens up the possibility of going beyond the limitations of rational players and classical game theory.
- The ability to describe social behaviors within a multiscale perspective, which includes, in particular, interaction rules at the microscale and collective trends at a more aggregate statistical level.

The various arguments presented in Chap. 1 motivate the search for a unified mathematical structure, to be regarded as a first step toward the derivation of a mathematical theory of social systems. Such a structure is required to include all paradigms of the complexity of the class of systems under consideration, so that it can compensate, at least partially, for the lack of fundamental background theories that is currently typical of living systems.

These structures can be technically improved by including the ability to describe additional phenomena. Nevertheless, the validity of a model is related to its success in modeling interactions at the microscopic scale by an appropriate phenomenological interpretation of social reality. It is possible that mathematical methods such as those reviewed in [36] can derive macroscopic averaged behaviors from the underlying description delivered by the kinetic theory for active particles.

However, the most significant step toward a mathematical theory of social systems might be a deeper understanding of the dynamics at the submicroscopic scale, which are responsible for the games played by individuals at the microscopic scale. This implies in turn a deeper understanding of the psychological mechanisms that generate individual strategies.

Even if this is an extremely difficult goal to be achieved, intermediate results are interesting and the analytical problems generated by this attempt are definitely challenging, and hence worth tackling.