Chapter 88
An Improved Weighted Averaging Method for Evidence Fusion

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Abstract D-S evidence theory is an important mathematical tool for uncertainty reasoning. However, it may lead to counterintuitive conclusions when combining conflicting evidences. In order to overcome this disadvantage, one can modify the evidences before Dempster's rule of combination. One representative method is to assign a weight to each evidence according to its credibility degree based on the concept of distance (or similarity) between two evidences. This method can gain more robust fusion results than many other known methods. However, it may fail to correctly converge according to the cardinality of the sets in the evidence. When evidence conflicts with other evidences, the evidence may lose impact on the combination result. Moreover, the combined mass is nonmonotonic even though evidence varies monotonically. Therefore, the method still leads to counterintuitive or confusing results. This paper brings forward an improved weighted averaging method involving a new similarity measure between evidences and a new combination rule. The numerical examples show the proposed method well solves the above problems.

Keywords Data fusion · Evidence theory · Conflicting evidence · Combination rule · Evidence distance · Evidence similarity

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88.1 Introduction

D-S evidence theory is first proposed by Dempster [1] and later developed by Shafer [2]. It can be regarded as a general extension of Bayesian theory that can robustly deal with incomplete data. Due to the capability of uncertain reasoning, it is widely applied in many fields. When there are conflicts among the evidences, however, D-S evidence theory may draw a counterintuitive conclusion [3]. Generally, there are two types of methods for dealing with conflicting evidences. One is to modify Dempster's rule of combination [4–8], while the other is to modify the evidences before using Dempster's rule. Evidence-modifying methods can be further classified into two types, i.e. weighted averaging methods [9–11] and discounting techniques [12–14]. In this paper we study the fusion performance of weighted averaging methods. Murphy's simple averaging method [9] can be viewed as a special case of weighted averaging methods where all the weights of the evidences are identical.

As studied in our previous work [15], compared with rule-modifying methods, weighted averaging methods are more attractive in that they can not only deal with conflicting evidences but converge towards dominant opinion with higher convergence speed. Among the three weighted averaging methods, Deng et al.'s method [10], which is based on Jousselme's measure of distance between two evidences [16], outperform the other two [9, 11]. Since it takes the relationship among the evidences into account, reasonable combination results can be obtained even if some conflicting evidences are collected due to e.g. enemy's disguise or bad weather.

Nevertheless, the convergence of Deng et al.'s method is still imperfect. This paper analyzes the problems and then presents an improved fusion method based on a similarity measure between two evidences and a new combination rule.

88.2 Deng et al.'s Weighted Averaging Fusion Method

In a practical multi-sensors system, the signals may be interfered with by many factors and to different degrees. Besides, some sensors may also be more stable than others. Therefore, the evidences obtained from the sensors are of different credibility degrees and should have different impacts on the fusion result. A reasonable way to handle this problem is to assign a weight to each evidence. When there is no prior knowledge, the relative importance of an evidence can be evaluated by the similarities between it and the other evidences.

Given a finite set of mutually exclusive and exhaustive propositions, i.e. a frame of discernment $\Theta = \{A_1, A_2, \ldots, A_m\}$, where A_i denotes a proposition. All possible subsets of Θ form are a superset $P(\Theta)$ containing 2^N elements. Suppose m_i and m_j be two basic probability assignment functions under the same frame of discernment. Jousselme [16] propose a distance measure between two evidences as.

$$d_{ij} = \sqrt{\frac{1}{2}(m_i - m_j)^T D(m_i - m_j)} \tag{88.1}$$

where D is a $2^N \times 2^N$ matrix with elements $D(A,B) = \frac{|A \cap B|}{|A \cup B|}$, $A, B \in P(\Theta)$.

Then the similarity between two evidences can be defined as

$$sim_{ij} = \frac{1}{2}(\cos(\pi d_{ij}) + 1)$$

The degree of support of an evidence by all the other evidences is defined by.

$$sup_i = \sum_{j=1, j \neq i}^{n} sim_{ij}$$

The normalization of support degree leads to the following credibility degree of evidence

$$cred_i = sup_i / \sum_{j=1}^{n} sup_j \tag{88.2}$$

Accordingly, the weighted average of the evidences is given as

$$MAE(m) = \sum_{i=1}^{n} (cred_i \times m_i)$$

As done in Murphy's method [9], the new BPA is incorporated into Dempster's rule of combination for $n-1$ times in order to offer convergence toward certainty, if there are n evidences.

88.3 Analysis on Deng et al.'s Method

We illustrate the problems of Deng et al.'s weighted averaging method by several numerical examples as follows.

Example 1 Consider the following two groups of evidences under the frame of discernment$\Theta = \{A_1, A_2, A_3, A_4\}$:

Group 1: $m_1(A_1) = 1$, $m_2(A_1) = 1$, $m_3(\{A_1, A_2\}) = 1$, $m_4(\{A_1, A_2\}) = 1$
Group 2: $m_1(A_1) = 1$, $m_2(A_1) = 1$, $m_3(\{A_1, A_2, A_3\}) = 1$, $m_4(\{A_1, A_2, A_3\}) = 1$

The combination results by Deng et al.'s method are shown in Table 88.1. When combining the former three evidences, $m_1 \oplus m_2 \oplus m_3(A_1)$ of Group 2 gains a bigger value than that of Group 1, which is unreasonable. Since the cardinality of

Table 88.1 Combination results of Deng et al.'s method for Example 1

Evidences	$m_1 \oplus m_2$			$m_1 \oplus m_2 \oplus m_3$			$m_1 \oplus m_2 \oplus m_3 \oplus m_4$		
	A_1	$\{A_1, A_2\}$	$\{A_1, A_2, A_3\}$	A_1	$\{A_1, A_2\}$	$\{A_1, A_2, A_3\}$	A_1	$\{A_1, A_2\}$	$\{A_1, A_2, A_3\}$
Group 1	1			0.9937	0.0063		0.9375	0.0625	
Group 2	1			0.9976		0.0024	0.9375		0.0625

$\{A_1, A_2\}$ is less than that of $\{A_1, A_2, A_3\}$, the third evidence of Group 1 contains more certainty information about A_1 than that of Group 2. Therefore, the combined mass on A_1 of Group 1 should be bigger. It is also unreasonable the combined mass on A_1 are equal for both groups when combining all the four evidences.

Example 2 Given the frame of discernment $\Theta = \{A_1, A_2, A_3, A_4\}$ and two groups of evidences, each comprising four conflicting evidences with the following BPAs.

Group 1: $m_1(A_1) = 1$, $m_2(A_1) = 1$, $m_3(\{A_2, A_3\}) = 1$, $m_4(\{A_2, A_3\}) = 1$.
Group 2: $m_1(A_1) = 1$, $m_2(A_1) = 1$, $m_3(\{A_2, A_3, A_4\}) = 1$, $m_4(\{A_2, A_3, A_4\}) = 1$.

Table 88.2 shows the fusion results. Obviously, combining the former three evidences produces counterintuitive results in both the groups. Since A_1 is not a focal element in the third evidence, $m_1 \oplus m_2 \oplus m_3(A_1)$ should be smaller than $m_1 \oplus m_2(A_1)$. In fact, the combination leads to the following distance matrix for both the groups

$$d = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

According to formula (2), there is $Cred = [0.5, 0.5, 0]$. Thus, the third evidence does not have any impact on the combination result.

Example 3 Suppose the first two evidences are the same as in Example 2 and the third one varies as follows.

$$m_3(A_1) = 1 - j/100,\ m_3(A_2) = j/100\ (j = 1, \cdots, 100)$$

As shown in Fig. 88.1, the combined mass on A_1 varies nonmonotonically, which is confusing since $m_3(A_1)$ decreases monotonically.

Table 88.2 Combination results of Deng et al.'s method for Example 2

Evidences	$m_1 \oplus m_2$			$m_1 \oplus m_2 \oplus m_3$			$m_1 \oplus m_2 \oplus m_3 \oplus m_4$		
	A_1	$\{A_2, A_3\}$	$\{A_2, A_3, A_4\}$	A_1	$\{A_2, A_3\}$	$\{A_2, A_3, A_4\}$	A_1	$\{A_2, A_3\}$	$\{A_2, A_3, A_4\}$
Group 1	1			1			0.5	0.5	
Group 2	1			1			0.5		0.5

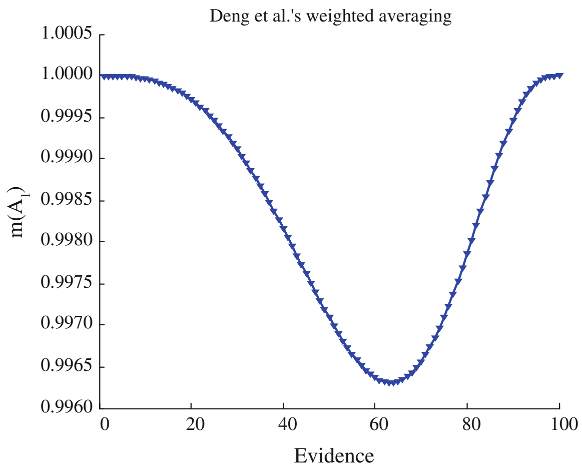

Fig. 88.1 Combined mass on A_1 by Deng et al's weighted averaging method when the third evidence varies

88.4 A New Fusion Method

Let m_i and m_j be two BPAs under the frame of discernment Θ containing N propositions. The similarity between m_i and m_j is defined as

$$sim(\mathbf{m}_i, \mathbf{m}_j) = \frac{\mathbf{m}_i'\mathbf{D}\mathbf{m}_j}{||\mathbf{m}_i||_{\mathbf{D}}||\mathbf{m}_j||_{\mathbf{D}}}$$

where $\mathbf{D}$ is a $2^N \times 2^N$ matrix and $||\mathbf{m}||_{\mathbf{D}} = \sqrt{\mathbf{m}'\mathbf{D}\mathbf{m}}$.

The similarity measure *sim* is a cosine measure which can be categorized into the inner product family of similarity measures [17]. Wen et al. define a cosine similarity measure with $\mathbf{D} = \mathbf{I}$ [18], which does not satisfy any structural property [16]. For the proposed similarity measure, the $\mathbf{D}$ matrix would quantify the interaction between the focal elements of the BPAs. As can be seen from formula (1), Jousselme's distance also satisfied the strong structural property by constructing the matrix via Jaccard index. More choice of such indexes can be found in [19]. However, using any of these indexes still results in the problem described in Example 2. Therefore, a new index is needed.

Let s denote $|A \cap B|$, t refer to $|\Theta - (A \cup B)|$, and p to $|\Theta|$, where $A, B \in P(\Theta)$. The index is defined as

$$D(A,B) = \frac{s+t+p}{2p}$$

Then the degree of support of an evidence by other evidences is defined by

$$sup_i = \sum_{j=1, j\neq i}^{n} sim_{ij} + c$$

where c is a constant which has important influence on the monotonicity of combined mass. In this paper, c takes the value of 2.

Afterwards, the credibility degree of evidence and the weighted average of the evidences can be defined similar to Deng et al.'s method. In order to improve the converging performance, we also integrate structural information into Dempster's rule of combination as follows.

$$\left.\begin{aligned} m(A) = \frac{1}{1-k} \sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j) \frac{|A|^2}{|A_i||B_j|} \\ k = 1 - \sum_{A \in P(\Theta), A \neq \emptyset} m(A) \end{aligned}\right\}$$

We apply the proposed fusion method to the examples discussed in Sect. 88.3 and the combination results are shown in Tables 88.3, 88.4 and Fig. 88.2, respectively.

For Example 1, the combined mass on A_1 of Group 1 gains a bigger value than that of Group 2, no matter when the former three evidences or all the four evidences are combined.

For both the groups in Example 2, the former two evidences are identical and therefore there is $m_1 \oplus m_2(A_1) = 1$. The combined mass on A_1 decreases when combining the former three evidences due to the high conflict among them. Obviously, the third evidence exerts an impact on the combination result as expected. Taking Group 2 as an illustration, when the former three evidences are considered, the credibility degree of the third evidence is 0.3. Besides, it is worthy of notice that the combination results of the former three evidences are different for the two groups. Similar to Example 1, the reason is also related to the cardinalities of focal elements. That is, though $m_3(\{A_2, A_3\})$ and $m_3(\{A_2, A_3, A_4\})$ are equal, the former contains more certainty information than the latter and therefore the combined mass on $\{A_2, A_3\}$ is bigger than that on $\{A_2, A_3, A_4\}$.

For Example 3, it can be observed from Fig. 88.2 that the combined mass on A_1 decreases monotonically when $m_3(A_1)$ decreases. The combination result is much reasonable than as shown in Fig. 88.1.

Table 88.3 Combination results of the proposed method for Example 1

Evidences	$m_1 \oplus m_2$			$m_1 \oplus m_2 \oplus m_3$			$m_1 \oplus m_2 \oplus m_3 \oplus m_4$		
	A_1	$\{A_1, A_2\}$	$\{A_1, A_2, A_3\}$	A_1	$\{A_1, A_2\}$	$\{A_1, A_2, A_3\}$	A_1	$\{A_1, A_2\}$	$\{A_1, A_2, A_3\}$
Group 1	1			0.9454	0.0546		0.8519	0.1481	
Group 2	1			0.9398		0.0602	0.7891		0.2109

Table 88.4 Combination results of the proposed method for Example 2

Evidences	$m_1 \oplus m_2$			$m_1 \oplus m_2 \oplus m_3$			$m_1 \oplus m_2 \oplus m_3 \oplus m_4$		
	A_1	$\{A_2, A_3\}$	$\{A_2, A_3, A_4\}$	A_1	$\{A_2, A_3\}$	$\{A_2, A_3, A_4\}$	A_1	$\{A_2, A_3\}$	$\{A_2, A_3, A_4\}$
Group 1	1			0.9174	0.0826		0.5	0.5	
Group 2	1			0.9270		0.0730	0.5		0.5

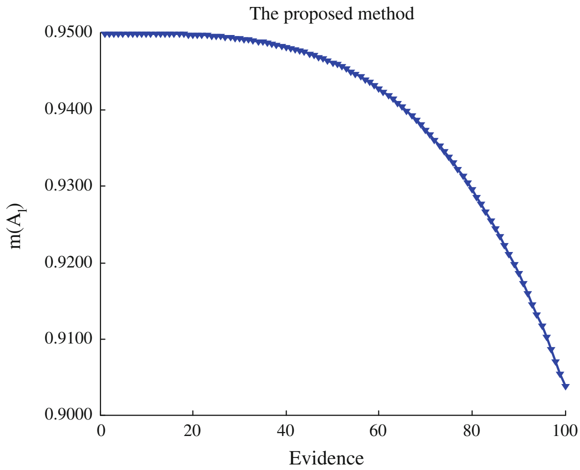

Fig. 88.2 Combined mass on A_1 by the proposed method when the third evidence varies

88.5 Conclusion

Though Deng et al.'s fusion method can gain more robust results than many other known methods, it may still lead to counterintuitive or confusing results. This paper brings forward an improved weighted averaging method involving a new similarity measure between evidences and a new combination rule. The numerical examples show the proposed method well solves the problems.

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