# Chapter 11
# P-Hub Airline Network Design Incorporating Interaction Between Elastic Demand and Network Structure

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**Abstract** This paper innovates the p-hub airline network median design method. Researchers present a new mathematical programming model, which incorporates the interaction between elastic demand in air passenger market and airline network structure. The model optimizes both the ticket prices and the profit of airline company, and subsequently determines the passenger volume influenced by different network structure. The effectiveness and practicability of the model are demonstrated by a realistic example of Chinese airline network which includes 15 major airports. Numerical analysis result indicates that hub locations tend to select the airports which have bigger passenger volume.

**Keywords** Traffic planning · Hub-and-spoke network · Elastic demand · Profit maximization

## 11.1 Introduction

The hub-and-spoke (HS) airline network, which includes hub and non-hub airports simultaneously, has already become major operational mode in mature aviation markets of developed countries. Hubs are special facilities that serve as switching, transshipment and sorting. When we design a HS airline network, we need to choose a fixed number P hub from all airports and allocate the remaining airports to these hubs. This design problem is known as p-hub median problem. The

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research of p-hub began with the pioneering work of O'Kelly, which gave a programming formulation of the single allocation p-hub median problem [1]. Campbell formulated the multiple allocation p-hub median problems firstly as a linear integer programming [2]. Skorin-Kapov et al. demonstrated that the LP relaxation of Campbell formulation leads to highly fractional solutions [3]. Mingguo Bai et al. developed an appropriate attribute index system to select spare hubs, and then applied the shortest path algorithm to design the HS network of fifteen Chinese airports [4]. Carello et al. investigated the cost of installing routes on the edge [5]. Yaman studied a problem which she named the uncapacitated hub location problem with modular arc capacities [6]. Yaman et al. investigated the capacitated version of problem, which the capacity of a hub is defined as the amount of traffic passing through the hubs [7].

These researches all attempted to find the best network design which has minimal total cost, based on the hypothesis that the volume of travelers were fixed. They have not considered that HS networks are absolutely different from traditional point-to-point (PP) networks in the transportation cost, flying routes, travel time and route distance. These differences will certainly affect the amount of travelers, which contradicts with the hypothesis of fixed number of travelers. For the reason to remedy this critical drawback, an original mathematic optimization model of p-hub median problem has been developed in this paper, which considers the interaction between elastic demand and network structure. The effectiveness and practicability of this model is proved by an example constructing a HS airline network containing 15 major airports in China.

## 11.2 Basic Assumptions of Airline Markets

In the airline market, every O-D pair is an independent submarket. The amount of travelers are dependent on the total travel cost including ticket price and travel time cost. Without loss of generality, we assume that the amount of travelers can be calculated by

$$q_{ij} = \theta_{ij} + \lambda_{ij}\left(p_{ij} + t_{ij}\right), \tag{11.1}$$

where $q_{ij}$ is the amount of travelers in the submarket between airport $i$ and $j$ ($ij$ submarket). $\lambda_{ij}$ is the elastic demand coefficient, and $\lambda_{ij} < 0$. $\theta_{ij}$ is the number of possible customers in $ij$ submarket. $p_{ij}$ is the ticket price between airport $i$ and $j$, $t_{ij}$ is the travel time cost. Let $\pi_{ij}$ denote the profit of airline company in $ij$ submarket, which can be expressed by

$$\pi_{ij} = \left(p_{ij} - d_{\partial ij}\right)q_{ij}, \tag{11.2}$$

where $d_{ij}$ is the straight distance between airport $i$ and $j$. Because the cost of transporting, a single traveler is correlative with the length of travel route. So $d_{ij}$

can measure the cost of transporting a single traveler. And $p_{ij}$ is measured by unit of length, either. When $t_{ij} + c_{ij} \geq -\frac{\theta_{ij}}{\lambda_{ij}}$ in $ij$ submarket, the airline company cannot obtain any profit from this submarket, and they will abandon this submarket. In this case, $\pi_{ij} = p_{ij} = q_{ij} = 0$.

## 11.3 Parameters of the Point-to-point Airline Network

We use superscript 0 to denote PP network. The time cost in PP network $t_{ij}^0$ can be expressed by

$$t_{ij}^0 = \gamma d_{ij}, \tag{11.3}$$

where $\gamma$ is the translation coefficient between time cost and travel length. If there are some travelers in $ij$ submarket, namely $q_{ij}^0 > 0$, then substituting Eqs. (11.1), (11.3) into Eq. (11.2) yields

$$\pi_{ij}^0 = \left(p_{ij}^0 - d_{ij}\right)\left(\theta_{ij} + \lambda_{ij}\left(\pi p_{ij}^0 + \gamma d_{ij}\right)\right), \tag{11.4}$$

Solving the maximization of Eq. (11.4) with variable $p_{ij}^0$, i.e., the first-order condition $\partial\pi_{ij}^0/\partial p_{ij}^0 = 0$, yields that when the profit reaches its peak value, we have

$$p_{ij}^0 = d_{ij} - \frac{q_{ij}^0}{\lambda_{ij}}, \tag{11.5}$$

And substituting Eq. (11.5) into Eq. (11.2), the maximal profit for the airline company is

$$\pi_{ij}^0 = -\frac{1}{\lambda_{ij}}(q_{ij}^0)^2, \tag{11.6}$$

In the case of travel demand in $ij$ submarket $q_{ij}^0 = 0$, we can simply assume that there is not any potential customers in this submarket, i.e. $\theta_{ij} = 0$. Substituting Eqs. (11.5), (11.6) into Eq. (11.4) and rewriting the equation, parameter $\theta_{ij}$ can be expressed by

$$\theta_{ij} = \begin{cases} 2q_{ij}^0 - \lambda_{ij}(\gamma + 1)d_{ij}, & if \quad q_{ij}^0 > 0, \\ 0, & if \quad q_{ij}^0 = 0, \end{cases} \tag{11.7}$$

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The total profit of whole PP airline network $\pi^0$ can be calculated by

$$\pi^0 = \sum_i \sum_j \pi^0_{ij} = -\sum_i \sum_j \frac{1}{\lambda_{ij}} (q^0_{ij})^2, \tag{11.8}$$

## 11.4 Parameters of the Hub-and-Spoke Airline Network

We use superscript 1 to denote HS networks and we adopt the strict uncapacitated multiple allocation HS network structure. There are n airports in which the set of origins, destinations and potential hub locations are identified.

Because the HS network concentrates the traveler flow through the hubs, which generates the economy of scale, there are the cost discounts in HS network. $\alpha (0 < \alpha \leq 1)$ is the discount factor between hubs, and $\beta (\alpha \leq \beta \leq 1)$ is the discount factor between non-hub and hub. As general setting of p-hub research, $\alpha$ and $\beta$ are known as parameters. The transportation cost in HS networks is

$$c^1_{ijkm} = \beta d_{ik} + \alpha d_{km} + \beta d_{mj} \tag{11.9}$$

The time cost of each traveler in HS networks is

$$t^1_{ijkm} = \gamma \left( d_{ik} + d_{km} + d_{mj} \right) \tag{11.10}$$

In the HS networks, travelers of *ij* submarket transship at hub *k* and *m*. Combining Eqs. (11.1), (11.2), the airline company's profit in the *ij* submarket $\pi^1_{ijkm}$ is expressed by

$$\pi^1_{ijkm} = \left( p^1_{ijkm} - c^1_{ijkm} \right) q^1_{ijkm} = \left( p^1_{ijkm} - c^1_{ijkm} \right) \left( \theta_{ij} + \lambda_{ij} \left( p^1_{ijkm} + t^1_{ijkm} \right) \right), \tag{11.11}$$

Solving the maximization of Eq. (11.11) with variable $p^1_{ijkm}$, i.e., the first-order condition $\partial \pi^1_{ijkm} / \partial p^1_{ijkm} = 0$, we obtains that when the profit reaches its peak value, the ticket prices are calculated by

$$p^1_{ijkm} = \begin{cases} -\dfrac{\theta_{ij}}{2\lambda_{ij}} - \dfrac{1}{2} t^1_{ijkm} + \dfrac{1}{2} c^1_{ijkm}, & if \quad c^1_{ijkm} + t^1_{ijkm} < -\dfrac{\theta_{ij}}{\lambda_{ij}} \\ 0, & if \quad c^1_{ijkm} + t^1_{ijkm} \geq -\dfrac{\theta_{ij}}{\lambda_{ij}} \end{cases} \tag{11.12}$$

Substituting Eqs. (11.9), (11.10), (11.12) into Eq. (11.1) yields that the amount of travelers is calculated by

$$q^1_{ijkm} = \begin{cases} \dfrac{\theta_{ij}}{2} + \dfrac{\lambda_{ij}}{2} t^1_{ijkm} + \dfrac{\lambda_{ij}}{2} c^1_{ijkm}, & if \quad c^1_{ijkm} + t^1_{ijkm} < -\dfrac{\theta_{ij}}{\lambda_{ij}} \\ 0, & if \quad c^1_{ijkm} + t^1_{ijkm} \geq -\dfrac{\theta_{ij}}{\lambda_{ij}} \end{cases} \tag{11.13}$$

## 11.5 Model and Numerical Example

On the basis of above analysis, we provide our mathematic optimization model for designing the HS network:

$$\max \quad \pi^1 = \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{m \in N} X_{ijkm} \left( p^1_{ijkm} - c^1_{ijkm} \right) q^1_{ijkm}, \tag{11.14}$$

$$s.t. \quad \sum_{k \in N} H_k = P, \tag{11.15}$$

$$\sum_{k \in N} \sum_{m \in N} X_{ijkm} = 1, \quad \forall i, j \in N, \tag{11.16}$$

$$X_{ijkm} \le H_k, \quad \forall i, j, k, m \in N, \tag{11.17}$$

$$X_{ijkm} \le H_m, \quad \forall i, j, k, m \in N, \tag{11.18}$$

$$H_k \in \{0, 1\}, \quad \forall k \in N, \tag{11.19}$$

$$X_{ijkm} \in \{0, 1\}, \quad \forall i, j, k, m \in N. \tag{11.20}$$

where $N = \{1, 2, 3, \ldots, n\}$, is the set of all n airports in the airline network. $p^1_{ijkm}, c^1_{ijkm}, q^1_{ijkm}$ in Eq. (11.14) are calculated by Eqs. (11.9), (11.10), (11.12), (11.13). $H_k$ is a $0-1$ variable, when node $k$ is a hub airport, $H_k = 1$, otherwise $H_k = 0.X_{ijkm}$ is a $0-1$ variable, when the flying route from node $i$ to node $j$ need to tranship at hub $k$ and $m$, $X_{ijkm} = 1$, otherwise $X_{ijkm} = 0$. According to the features of the air passenger transportation, we limit that transshipment times are less than or equal to 2.

The objective function (11.14) is the total profit of the HS network. Constraint (15) ensures the number of hubs is P. Constraint (11.16) ensures that there is only one flying route between airport $i$ and $j$. Constraint (11.17) and (11.18) ensure all flying routes tranship at hubs only. Constraint (11.19), (11.20) are $0-1$ variable constraints.

To prove the effectiveness and practicability of this optimization model, we select 15 Chinese airports to design the HS airline network. The set of airports is quoted from the Ref. [4], which is (1) Beijing, (2) Shanghai, (3) Shenyang, (4) Zhengzhou, (5) Xi'an, (6) Wulumuqi, (7) Nanjing, (8) Hangzhou, (9) Changsha, (10) Wuhan, (11) Chengdu, (12) Guangzhou, ( 13) Haikou, (14) Kunming, (15) Xiamen. The model (11.14)–(11.20) is a NP-hard problem. We adopt the software GAMS to solve the model directly. The initial data of the traveler volume in point-to-point network $q^0_{ij}$ and straight distances of each O–D pair $d_{ij}$ is quoted from the Ref. [8]. We report the calculation results in Table 11.1.

We calculated several sets of results under a series of parameters to analyze the influence of different parameters. By observing the calculation results in Table 11.1, several principles are obviously revealed. Firstly, when parameter $\lambda_{ij}$

**Table 11.1** The calculation results

| $P$ | $\alpha$ | $\beta$ | $\gamma$ | $\forall\lambda_{ij}$ | Hub airports | $\pi^0$ ($10^8$ yuan) | $\pi^1$ ($10^8$ yuan) | $\pi^1/\pi^0$ (%) |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.8 | 0.9 | 0.1 | −0.1 | 10 | 2611.87 | 2205.86 | 84.46 |
| 2 | 0.8 | 0.9 | 0.1 | −0.1 | 1,9 | 2611.87 | 2518.97 | 96.44 |
| 3 | 0.8 | 0.9 | 0.1 | −0.1 | 1,2,12 | 2611.87 | 2640.04 | 101.08 |
| 4 | 0.8 | 0.9 | 0.1 | −0.1 | 1,8,11,12 | 2611.87 | 2781.12 | 106.48 |
| 5 | 0.8 | 0.9 | 0.1 | −0.1 | 1,2,10,11,12 | 2611.87 | 2834.18 | 108.51 |
| 6 | 0.3 | 0.4 | 0.1 | −0.1 | 10 | 2611.87 | 3598.85 | 137.79 |
| 7 | 0.3 | 0.4 | 0.1 | −0.1 | 1,9 | 2611.87 | 3825.23 | 146.46 |
| 8 | 0.3 | 0.4 | 0.1 | −0.1 | 1,4,12 | 2611.87 | 3876.71 | 148.43 |
| 9 | 0.3 | 0.4 | 0.1 | −0.1 | 1,7,11,12 | 2611.87 | 4056.01 | 155.29 |
| 10 | 0.3 | 0.4 | 0.1 | −0.1 | 1,2,10,11,12 | 2611.87 | 4112.73 | 157.46 |
| 11 | 0.8 | 0.9 | 0.1 | −0.5 | 1 | 522.37 | 413.60 | 79.18 |
| 12 | 0.8 | 0.9 | 0.1 | −0.5 | 1,12 | 522.37 | 580.54 | 111.14 |
| 13 | 0.8 | 0.9 | 0.1 | −0.5 | 1,2,12 | 522.37 | 687.32 | 131.58 |
| 14 | 0.8 | 0.9 | 0.1 | −0.5 | 1,2,11,12 | 522.37 | 759.59 | 145.41 |
| 15 | 0.8 | 0.9 | 0.1 | −0.5 | 1,2,11,12,15 | 522.37 | 785.27 | 150.33 |

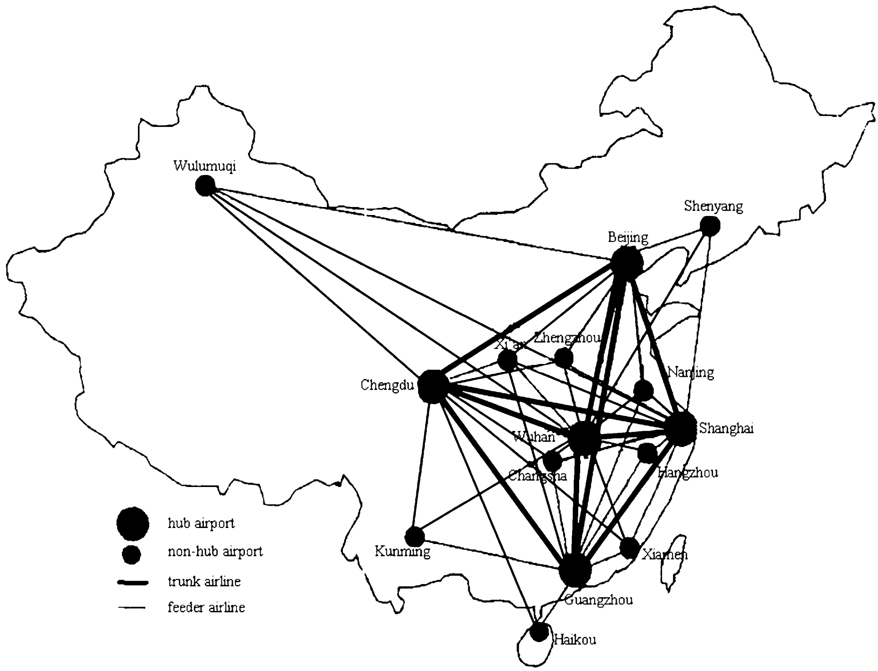


**Fig. 11.1** The hub-and-spoke airline network of 15 Chinese cities

increases, namely, the elasticity of demand intensifies, the hubs tend to select the airports which have bigger amount of travelers. Because in this case, travelers are more sensitive to total cost. The hubs with more travelers may save more transportation costs. Secondly, when the elasticity of demand intensifies, the profit of HS network is remarkably bigger than PP network. Finally, when the discount factors become small, namely, the economy of scale becomes more obvious, the hubs tend to select the airports which positions are closer to geographic center of gravity. And reduced discount factor results in less total transport cost.

On the basis of calculation results, the best structure of airline network can be obtained. Figure 11.1 shows the design of Chinese airline network by above method with $P = 5$, $\alpha = 0.3$, $\beta = 0.4$, $\forall \lambda_{ij} = -0.1$, $\gamma = 0.1$. In this case, hubs are Beijing, Shanghai, Wuhan, Chengdu and Guangzhou.

## 11.6 Conclusion

Although HS airline networks take advantage of economies of scale and scope, foster hub airports, optimize resource of aviation industry, HS airline networks can also influence the transportation cost, flying routes and travel time, and further influence the airline company's profit and customer demand. When airline companies construct their hub-and-spoke networks, they must trade off various factors for maximizing the net profit.

This paper merges operational behavior of airline company and p-hub median problem together, presents a effective design method of HS network, which not only accords with the real environment, but also reflects the process of decision making. By applying 15 Chinese airports, researchers construct a HS airline network, which can provide reference for Chinese aeronautic transportation planning.

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