Chapter 27
Real-Time Damage Identification in Nonlinear Smart Structures Using Hyperchaotic Excitation and Stochastic Estimation

Shahab Torkamani, Eric A. Butcher, and Michael D. Todd

Abstract Among numerous damage identification techniques, those which are used for online data-driven damage identification have received considerable attention recently. One of the most widely-used vibration-based time-domain techniques for nonlinear system identification is the extended Kalman filter, which exhibits a good performance when the parameter to be identified is a constant parameter. However, it is not as successful in identification of changes in time-varying system parameters, which is essential for real-time identification. Alternatively, the extended Kalman-Bucy filter has been recently proposed due to its enhanced capabilities in parameter estimation compared with extended Kalman filter. On the other hand, when applied as the excitation in some attractor-based damage identification techniques, chaotic and hyperchaotic dynamics produce better outcomes than does common stochastic white noise. The current study combines hyperchaotic excitations and the enhanced capabilities of extended Kalman-Bucy filter to propose a real-time approach for identification of damage in nonlinear structures. Simulation results show that the proposed approach is capable of online identification and assessment of damage in nonlinear elastic and hysteretic structures with single or multiple degrees-of-freedom using noise-corrupted measured acceleration response.

Keywords Smart structures • Real-time damage identification • Hyperchaos • Stochastic estimation • Kalman-Bucy filter

27.1 Introduction

Damage identification in structures can be divided into active (on-line) and passive (off-line) approaches. The active approach needs actuation (or excitation) of monitored structures and real-time measurements and analysis of the resulting response. While the passive approach is satisfactory for traditional structures, it is not as desirable for some modern structures, and in particular, "smart structures" which contain active components or layers. For such smart structures, the identification algorithm needs to provide instantaneous updates of the mechanical properties of the structure to the active components to adaptively perform functions of sensing and actuation required. For the online identification of damage, various time-domain approaches have been used in the literature with different degree of success. A few examples include least-squares estimation [1–3], different filter approaches including the extended Kalman filter [4–7], H_∞ filter [8], Monte Carlo filter [9], etc. The Monte Carlo method is capable of dealing with nonlinear systems with even non-Gaussian uncertainties. However, it is computationally expensive due to requiring a large number of sample points. Since the application of least squares estimation (LSE) for nonlinear structural system identification requires displacement and velocity measurements, which may not always be readily available. The extended Kalman filter (EKF) is perhaps the most widely-used vibration-based time-domain techniques for identification of nonlinear systems. While the EKF has good performance when the parameter to be identified is a constant parameter, it is not as successful in identification of changes in time-varying system parameters

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[10]. A common technique used in the literature for identification of time-varying parameters is an extension of the LSE approach. This technique makes use of a constant [11, 12] or time-dependent [13] forgetting factor in LSE. This approach has some drawbacks and shows good performance in some cases; however, it exhibits poor results when the stiffness of the structure has an abrupt change [10]. An adaptive tracking technique based on EKF to identify structural parameters is proposed in [10], which is particularly suitable for tracking the abrupt changes of the system parameters with the purpose of online evaluation of the structural damages.

The extended Kalman-Bucy filter is an alternative filtering approach that has been recently reintroduced due to its enhanced capabilities in parameter estimation compared with the extended Kalman filter [14]. On the other hand, an aspect of damage identification which is shown to be crucial from a detectability standpoint is the excitation. When applied as the excitation in some attractor-based damage identification techniques, chaotic and hyperchaotic dynamics can often produce better outcome rather than the common stochastic white noise [15–21]. The current study combines hyperchaotic excitations and the enhanced capabilities of extended Kalman-Bucy filter to develop a feasible real-time technique for identification of damage which can be used in nonlinear smart or self-healing structures. Simulation results show that the proposed approach is capable of real-time identification and assessment of damage in nonlinear elastic and hysteretic structures with single or multiple degree-of-freedom using noise-corrupted acceleration response.

27.2 Methodology

27.2.1 Stochastic Estimation Problem

In a general parametric identification problem (cf. non-parametric identification) it is assumed that the form of the model is known only approximately due to imperfect knowledge of the dynamical model that describes the motion and/or imperfect knowledge of parameters. The goal is to obtain the best estimate of the state as well as of model parameters based on measured data that has a random component due to observation errors. In addition, a second source of stochastic excitation typically appears in the state dynamics as so-called process noise. Both the stochastic excitation and the measurement noise are assumed to be additive in this paper. The optimal continuous-time filtering problem in the general form considered in this paper can be written as a set of Ito stochastic differential equations as

$$\begin{aligned} d\mathbf{x}(t) &= \boldsymbol{f}(\mathbf{x}(t), \mathbf{a}(t), t)\, dt + \mathbf{G}(\mathbf{x}(t), t)\, d\boldsymbol{\beta}(t) \\ d\mathbf{z}(t) &= \boldsymbol{h}(\mathbf{x}(t), \mathbf{a}(t), t)\, dt + \mathbf{J}(t)\, d\boldsymbol{\eta}(t), \end{aligned} \tag{27.1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state process, $\mathbf{z}(t) \in \mathbb{R}^q$ is the measurement process, $\mathbf{a}(t) \in \mathbb{R}^r$ is a vector of unknown parameters, $\boldsymbol{f}$ is the drift coefficient, $\boldsymbol{G}$ is the diffusion coefficient, $\boldsymbol{h}$ is the measurement model function, $\mathbf{J}(t)$ is an arbitrary time-varying functions independent of $\mathbf{x}$, and $\boldsymbol{\beta}(t)$ and $\boldsymbol{\eta}(t)$ are independent Brownian motion additive stochastic processes with $E[d\boldsymbol{\beta}(t)] = \mathrm{E}[d\boldsymbol{\eta}(t)] = 0,\ E[d\boldsymbol{\beta}(t) d\boldsymbol{\beta}^T(t)] = \mathbf{Q}dt$ and $E\left[d\boldsymbol{\eta}(t)\, d\mathbf{j}^T(t)\right] = \mathbf{R}dt$ where $E[\ ]$ represents the expectation operator. Note that in this paper the system is considered to be excited by additive noise only. Therefore, hereafter we will treat stochastic differential equations with the diffusion coefficient only depending on t and not the process $\mathbf{x}$. Under this condition the filtering problem can also be formulated in terms of the stationary zero-mean Gaussian white noise processes formally defined as $\mathbf{v}(t) = d\boldsymbol{\beta}(\mathrm{t})/dt, \mathbf{w}(t) = d\boldsymbol{\eta}(\mathrm{t})/dt$ and differential measurement $\mathbf{y}(t) = d\mathbf{z}(t)/dt$ as [22]

$$\begin{aligned} \dot{\mathrm{x}}(t) &= \boldsymbol{f}(\mathbf{x}(t), \mathbf{a}(t), t) + \mathbf{G}(t)\,\mathbf{v}(t) \\ \mathbf{y}(t) &= \boldsymbol{h}(\mathbf{x}(t), \mathbf{a}(t), t) + \mathbf{J}(t)\,\mathbf{w}(t), \end{aligned} \tag{27.2}$$

where $\mathbf{v}(t)$ and $\mathbf{w}(t)$ are assumed to be both mutually independent and independent from the state and observation with constant covariance matrices of $\boldsymbol{Q}$ and $\boldsymbol{R}$, respectively, i.e. $\mathbf{v} \sim N(0, \boldsymbol{Q})$ and $\mathbf{w} \sim N(0, \boldsymbol{R})$. Here in this paper, the stochastic term $\mathbf{v}(t)$ (the "process noise") functions as an approximation for the influence of the unknown dynamics of the process model. The time evolution of the states of the system and the unknown parameters of the stochastic model are to be identified using measurements of the output corrupted by the measurement noise term $\mathbf{w}(t)$.

The Ito and standard forms in Eqs. (27.1) and (27.2) are only equivalent, however, because the process and measurement noise are restricted to be additive and not multiplicative. Also, note that $\mathbf{a}(t)$ can be a constant or time-varying vector and it is assumed to be Heaviside function later in this study. Equation (27.2) defines a continuous-time state-space optimal filtering model. The purpose of the optimal continuous-time filtering problem is to recursively obtain estimates of the states and parameters from the mean, median, or mode of the time-varying conditional probability density

$$p(\mathbf{x}\,(\mathrm{t})\,|\,\{\mathbf{y}\,(\tau)\ :\ 0 \leq \tau \leq t\}). \tag{27.3}$$

To see how the filtering problem can be represented in the context of system identification, once again consider Eq. (27.2). As mentioned before, the assumed model of the system consists of the nonlinear function f which is a function of the state vector $\mathbf{x}(t)$ and parameters $\mathbf{a}(t)$. Suppose that $\mathbf{a}(t)$ is unknown but is assumed to be piecewise constant. The Kalman-Bucy filter can be used to simultaneously estimate the states $\mathbf{x}\,(t)$ and the parameters $\mathbf{a}\,(t)$. The standard method employs the so-called state augmentation method, in which the parameter vector $\mathbf{a}\,(t)$ is included in an augmented state vector $\mathbf{X}\,(t) = [\mathbf{x}\,(t)\,,\mathbf{a}\,(t)]^T$ while being constrained to have a predefined rate of change (zero here), i.e.

$$\begin{aligned}\dot{\mathbf{X}} &= \mathcal{F}\,(\mathbf{X},t) + \mathcal{G}\,(t)\,\mathbf{v}(t) = \begin{Bmatrix} f\,(\mathbf{x}\,(t)\,,\mathbf{a}\,(t)\,,t) \\ 0 \end{Bmatrix} + \begin{Bmatrix} \mathbf{G}\,(t) \\ 0 \end{Bmatrix}\mathbf{v}(t) \\ \mathbf{y}\,(t) &= \mathcal{H}\,(\mathbf{X},t) + \mathbf{J}\,(t)\,\mathbf{w}(t).\end{aligned} \tag{27.4}$$

The parameter vector $\mathbf{a}\,(t)$ is assumed to initially have a Gaussian distribution with mean $\mathbf{a}_0$ and covariance $\mathbf{P}_0$. Note that there is no noise term in the equation for the unknown parameter dynamics. The reason is that the parameters are already assumed to be stationary. Therefore, the augmented state method along with optimal filtering problem provides a pertinent approach for simultaneous estimation of the state and parameters of a (possibly) nonlinear system.

27.2.2 The Extended Kalman-Bucy Filter

The nonlinear optimal filtering problem described via the Ito differential form of Eq. (27.1) is considered, where the nonlinear process and measurement function are now functions of the augmented state $\mathbf{X}$ (as in Eq. (27.4)), and $\mathbf{J}(t)$ is the identity matrix. In order for the Kalman-Bucy filter to be applicable to the nonlinear system, the dynamics need to be locally linearized. Rather than linearizing about a reference trajectory, the extended Kalman-Bucy filter employs a linearization about the state estimate itself. It can be derived by taking the expectation of the dynamic model and adding a feedback term consisting of the measurement residual times an (as yet) unknown gain matrix, i.e.

$$d\hat{\mathbf{X}}\,(t) = E\,[\mathcal{F}\,(\mathbf{X},t)]\,dt + \boldsymbol{K}\,(t)\,[d\,\mathbf{z}\,(t) - E\,[\mathcal{H}\,(\mathbf{X},t)]\,dt]\,. \tag{27.5}$$

Defining the observer error as $\boldsymbol{e}(t) = \mathbf{X}(t) - \hat{\mathbf{X}}(t)$, the differential observation error is obtained as

$$d\boldsymbol{e}\,(t) = d\mathbf{X}\,(t) - d\,\hat{X}\,(t) = \mathcal{F}\,(\mathbf{X},t)\,dt - E\,[\mathcal{F}\,(\mathbf{X},t)]\,dt - \boldsymbol{K}\,(t)\,[\mathcal{H}\,(\mathbf{X},t)\,dt - E[\mathcal{H}\,(\mathbf{X},t)]dt] + d\boldsymbol{B}(t), \tag{27.6}$$

where $d\boldsymbol{B}\,(t) = \mathcal{G}\,(t)\,d\boldsymbol{\beta}(t) - \boldsymbol{K}\,(\mathrm{t})\,d\boldsymbol{\eta}\,(t)$ is a Brownian motion process with

$$E\left[d\boldsymbol{B}\,(t)\,d\boldsymbol{B}\,(t)^T\right] = \left[\mathcal{G}\,(t)\,\boldsymbol{Q}\,(t)\,\mathcal{G}\,(t)^T + \boldsymbol{K}\,(t)\,\boldsymbol{R}\,(t)\,\boldsymbol{K}\,(t)^T\right]dt. \tag{27.7}$$

Defining $\tilde{\mathbf{F}}(\mathrm{t})$ and $\tilde{\mathbf{H}}\,(\mathrm{t})$ to be the Jacobian matrices

$$\tilde{\mathbf{F}}\,(t) \sim:= \left.\frac{\partial\mathcal{F}\,(\mathbf{X,}\boldsymbol{t})}{\partial\mathbf{X}}\right|_{X=\hat{X}}, \qquad \tilde{\mathbf{H}}\,(t) := \left.\frac{\partial\mathcal{H}\,(\mathbf{X,}\boldsymbol{t})}{\partial\mathbf{X}}\right|_{X=\hat{X}}. \tag{27.8}$$

and linearizing about the current estimate yields

$$\begin{aligned}\mathcal{F}\,(\mathbf{X},t) &= \mathcal{F}\left(\hat{\mathbf{X}},t\right) + \tilde{F}\,(t)\left(\mathbf{X}-\hat{\mathbf{X}}\right) + \boldsymbol{r_f}(\mathbf{X},\hat{\mathbf{X}},t) \\ \mathcal{H}\,(\mathbf{X},t) &= \mathcal{H}\left(\hat{\mathbf{X}},t\right) + \tilde{H}\,(t)\left(\mathbf{X}-\hat{\mathbf{X}}\right) + \boldsymbol{r_h}(\mathbf{X},\hat{\mathbf{X}},t),\end{aligned} \tag{27.9}$$

from which $E\left[\mathcal{F}(\mathbf{X},t)\right] = \mathcal{F}\left(\hat{\mathbf{X}},t\right) + E[\boldsymbol{r}_f\left(\mathbf{X},\hat{\mathbf{X}},t\right)]$ and $E\left[\mathcal{H}(\mathbf{X},t)\right] = \mathcal{H}\left(\hat{\mathbf{X}},t\right) + E\left[\boldsymbol{r}_h(\mathbf{X},\hat{\mathbf{X}},t)\right]$: $\boldsymbol{r}_f(\mathbf{X},\hat{\mathbf{X}},t)$ and $r_h(\mathbf{X},\hat{\mathbf{X}},t)$ represent the remaining higher order terms. Truncating the Taylor series after the first order terms yields the differential observation error as

$$d\boldsymbol{e}(t) = \left[\tilde{\boldsymbol{F}}(t) - \boldsymbol{K}(t)\tilde{\boldsymbol{H}}(t)\right]\boldsymbol{e}(t)\,dt + d\boldsymbol{B}(t). \tag{27.10}$$

The error covariance matrix can be obtained by differentiating $\mathrm{E}\left[\boldsymbol{e}(t)\,\boldsymbol{e}(t)^T\right]$ using the Ito differential rule to obtain

$$d\boldsymbol{P}(t) = \left[\tilde{\boldsymbol{F}}(t) - \boldsymbol{K}(t)\tilde{\boldsymbol{H}}(t)\right]\boldsymbol{P}(t)\,dt + \boldsymbol{P}(t)\left[\tilde{\boldsymbol{F}}(t) - \boldsymbol{K}(t)\tilde{\boldsymbol{H}}(t)\right]^{\boldsymbol{T}} dt + \mathcal{G}(t)\,\boldsymbol{Q}(t)\,\mathcal{G}(t)^T\,dt + \boldsymbol{K}(t)\,\boldsymbol{R}(t)\,\boldsymbol{K}(t)^T\,dt. \tag{27.11}$$

The optimal gain matrix $\boldsymbol{K}$ (t) which leads to a minimum variance estimator can be obtained by minimizing the cost function $J = \mathrm{Trace}(d\boldsymbol{P}(t))$ with respect to $\boldsymbol{K}(t)$ as

$$\frac{\partial}{\partial \boldsymbol{K}(\mathrm{t})}\left[\mathrm{Trace}(d\boldsymbol{P}(t))\right] = -2\boldsymbol{P}(t)\,\tilde{\boldsymbol{H}}(t)^{\mathrm{T}} + 2\boldsymbol{K}(t)\,\boldsymbol{R}(t)^{\mathrm{T}} = 0, \tag{27.12}$$

which yields the $\boldsymbol{K}(t)$ matrix as

$$\boldsymbol{K}(t) = \boldsymbol{P}(t)\,\tilde{\boldsymbol{H}}^{\mathbf{T}}(t)\,\boldsymbol{R}(t)^{-1}. \tag{27.13}$$

Therefore the propagation of the estimate is obtained as

$$d\hat{\mathbf{X}}(t) = \mathcal{F}\left(\hat{\mathbf{X}},t\right)dt + \boldsymbol{K}(\mathrm{t})\left[dz(t) - \mathcal{H}\left(\hat{\mathbf{X}},t\right)dt\right], \tag{27.14}$$

while the following Riccati differential equation is obtained which propagates the error covariance $\boldsymbol{P}(t)$.

$$d\boldsymbol{P}(t) = \tilde{\boldsymbol{F}}(t)\,\boldsymbol{P}(t)\,dt + \boldsymbol{P}(t)\,\tilde{\boldsymbol{F}}(t)^{\mathrm{T}}\,dt + \mathcal{G}(t)\,\boldsymbol{Q}(t)\,\mathcal{G}(t)^{\mathrm{T}}\,dt - \boldsymbol{K}(t)\,\tilde{\boldsymbol{H}}(t)\,\boldsymbol{P}(t)\,dt. \tag{27.15}$$

The estimator so obtained is the extended Kalman-Bucy filter. Unlike the discrete-time extended Kalman filter, the prediction and measurement update steps are combined in the continuous-time extended Kalman filter.

27.2.3 Tuned Hyperchaotic Excitation

Traditionally, broadband random signals have been widely used for exciting structures. The reason is that the broadband nature of noise ensures a full modal response, ideal for frequency domain approaches to system identification or feature extraction. The motivation for the use of a chaotic signal as the excitation mechanism in damage detection is due to various unique features intrinsic to a chaotic signal. Chaotic signals also tend to possess broadband frequency spectra. However, unlike noise, chaos is deterministic and intrinsically low-dimensional (a stochastic process is infinite-dimensional). In fact many chaotic systems can be as low as three-dimensional when described as a continuous time process. In addition, a chaotic system is defined by a positive Lyapunov exponent (LE) implying extreme sensitivity to small changes in system parameters. The subtlety of damage-induced changes to a structure further motivates this choice as the mechanism of excitation.

Hyperchaos can be defined as chaotic behavior where at least two Les are positive. Having all the advantages that make a chaotic signal suitable for being used as an excitation, it is shown in [20,21] that a hyperchaotic signal is even more sensitive to subtle changes in damage severity as a result of the trajectory being permitted to more fully explore the entire phase space. Thus, hyperchaotic oscillators can be an alternative excitation mechanism in damage detection when extra sensitivity to damage is required. However, in order for hyperchaotic excitation to have the best performance the excitation should be tuned for the structure. There are two tuning criteria based on attractor dimensionality. First, the Lyapunov spectrum of the oscillator must overlap that of the structure. This ensures that changes to the Les of the structure, i.e. by damage, will alter the dimension of the filtered signal. Second, the dominant exponent associated with the oscillator must be minimized for a given degree of overlap in order to maintain the lowest possible dimensionality. By employing the Kaplan-York conjecture in attractor dimensionality, these criteria become

$$\left|\lambda_M^C\right| > \left|\lambda_1^L\right|$$
$$\left|\lambda_1^L\right| > \sum_{r=1}^{p} \lambda_r^C, \tag{27.16}$$

where λ_i^C are the exponents associated with the M-dimensional hyperchaotic system, λ_j^L are the exponents of the N-dimensional structure, and p is the number of positive Lyapunov exponents of the M-dimensional oscillator.

As mentioned before, the proposed approach takes advantage of the optimal filtering problem as the estimation technique for real-time identification of damage in structural systems. Therefore, the process equation in the optimal filtering problem of Eq. (27.4) is considered as

$$\boldsymbol{f}\left(\mathbf{x}\left(t\right),\mathbf{a}\left(t\right),t\right) = \boldsymbol{\varphi}\left(\mathbf{x}\left(t\right),\mathbf{a}\left(t\right)\right) + \mathbf{bu}(t), \tag{27.17}$$

where $\boldsymbol{\varphi}()$ describes the nonlinear structure of interest, $\mathbf{u}()$ is the hyperchaotic excitation force, and the constant coefficient $\mathbf{b}$ determines which component of $\mathbf{u}()$ to be used as the excitation and which degree-of-freedom of the structure is to be excited. Any of the hyperchaotic nonlinear systems may be used as an excitation. We use a hyperchaotic version of the well-known Lorenz oscillator shown below, i.e.

$$\begin{aligned}
\dot{u}_1 &= (\sigma(u_2 - u_1) + u_4)\delta \\
\dot{u}_2 &= (ru_1 - u_2 - u_1u_3 - u_5)\delta \\
\dot{u}_3 &= (u_1u_2 - bu_3)\delta \\
\dot{u}_4 &= (du_4 - u_1u_3)\delta \\
\dot{u}_5 &= (ku_2)\delta,
\end{aligned} \tag{27.18}$$

which exhibits hyperchaotic behavior with three positive LEs for $\sigma = 10, \rho = 28, \beta = \frac{8}{3}, d = 2, k = 10$ [23]. Note that if we eliminate states u_4 and u_5 from the first three states of the oscillator above, the resulting 3-dimensional oscillator is the well-known Lorenz oscillator which exhibits chaotic behavior (one positive LE) for $\sigma = 10,\ \rho = 28,\ \beta = \frac{8}{3}$ The bandwidth control parameter δ is used to tune the LEs of the excitation based on the tuning criteria of Eq. (27.16). Values of δ that are less than unity decrease the bandwidth of the input, while values greater than unity increase the bandwidth. In order to eliminate transient dynamics when using the hyperchaotic oscillator as an excitation, the oscillator is initiated at a point on the attractor. The matrix $\mathbf{b}$ throughout this paper is chosen in a way that the first component u_1 of the chaotic/hyperchaotic Lorenz oscillators is used for the excitation.

27.3 Simulation Results

27.3.1 S-DOF Hysteretic Nonlinear Structure

Consider a single degree of (SDOF) nonlinear hysteretic Bouc–Wen system subject to the excitation $f_{exc}(t)$

$$\mathrm{m}\ddot{x}(t) + \mathrm{c}\dot{x}(t) + \mathrm{k}r(t) = f_{exc}(t) \tag{27.19}$$

where $r(t)$ is the Bouc–Wen hysteretic component with

$$\dot{r} = \dot{x} - \beta|\dot{x}||r|^{\alpha-1}r - \gamma\dot{x}|r|^{\alpha} \tag{27.20}$$

The system parameters $\mathrm{m} = 1, \mathrm{c} = 0.3, \mathrm{k} = 9, \beta = 2, \gamma = 1, \alpha = 2$ are chosen for the simulation. Considering $\mathbf{z} = [x, \dot{x}, r]^T$ the $\boldsymbol{\varphi}$ function in Eq. (27.17) that forms the process function $\boldsymbol{f}$ of the filtering problem for this system is

$$\boldsymbol{\varphi}\left(\mathbf{z}\left(t\right)\right) = \left\{\begin{array}{c} z_2(t) \\ (1/\mathrm{m})\left(-\mathrm{k}z_3\left(t\right) - \mathrm{c}z_2\left(t\right)\right) \\ z_2(t) - \beta\left|z_2(t)\right|\left|z_3\left(t\right)\right|^{\alpha-1} z_3\left(t\right) - \gamma z_2(t)\left|z_3\left(t\right)\right|^{\alpha} \end{array}\right\}, \tag{27.21}$$

Time-varying damage is implemented as a 50% abrupt reduction in the stiffness and damping coefficients of the system at time t = 50 s. The mass of the system is assumed to be known throughout this simulation. The filtering sequence is initiated with values of the state and parameters (k,c) which are 50% deviated from the true values. Three types of excitation $\mathbf{u}(t)$ including white noise, chaotic Lorenz excitation and hyperchaotic Lorenz excitation (Eq. 27.18) are applied to the system under identical measurement and process noise covariance $(\boldsymbol{R}, \boldsymbol{Q})$ with identical initial error covariance $\boldsymbol{P}$. The measurement function $\boldsymbol{h}$ is considered to be the identity function i.e. both displacement r and velocity $\dot{x}$ are measured. The extended Kalman-Bucy filter is used for real-time identification of the stiffness k and the damping coefficient c of the system. The value of $\delta = 0.542$ can be shown to satisfy the tuning criteria for both the chaotic Lorenz oscillator and hyperchaotic Lorenz oscillator of Eq. (27.18) and is used for this simulation. The results of the identified parameters with each of the three excitations are shown in Fig. 27.1. As is clear from the figure, in the case of random excitation the change in system parameters is not sensed. However, when chaotic and hyperchaotic excitations are applied the approach successfully identifies the change. Note that in the case of hyperchaotic excitation the filter converges to the true value of the parameter faster than for the case of chaotic excitation.

In the second simulation the proposed approach is applied for real-time identification of a 10% stiffness reduction in the hysteretic system of Eq (27.19). The system parameters are considered as mentioned previously and the hyperchaotic Lorenz oscillator of Eq. (27.18) is used for the excitation. The value of $\delta = 0.1$ is used for this simulation which can be shown to satisfy the tuning criteria of Eq. (27.18) for the hyperchaotic Lorenz oscillator and the nonlinear system of Eq. (27.19). Since measurements of displacement r and velocity $\dot{\mathrm{x}}$ may not always be readily available, a more common acceleration measurement is considered here. Acceleration measurements in this simulation are provided by using the measurement function $\boldsymbol{h}$ based on the Eq. (27.19) as

$$\boldsymbol{h}(x(t), t) = \frac{1}{\mathrm{m}} \left(f_{\mathrm{exc}}(t) - \mathrm{c}\dot{x}(\mathrm{t}) - \mathrm{k}\, r(t) \right) \tag{27.22}$$

Note that the excitation force in Eq. (27.22) is assumed to be easily measurable via force transducers. The filtering sequence is initiated with values of the state and unknown parameters (k,c) which are 50% deviated from the true values. As is clear from Fig. 27.2, the approach is capable of real-time identification of the 10% stiffness reduction at time $t = 250$ s with good accuracy and fast convergence from measurements of acceleration in the presence of noise.

Although measuring acceleration is more realistic and practical than measuring both velocity $\dot{x}$ and displacement r, the $\boldsymbol{h}$ function that is used for measuring acceleration is still not quite realistic for some modern structures. The arguable part is that the values of system parameters m,k and c used in the $\boldsymbol{h}$ function are assumed to be known a priori. This is only realistic in the case that the mechanical properties of the structure can be measured or identified before the occurrence of damage. However, if the structure under consideration is a smart structure, then the identification algorithm needs to provide instantaneous updates of the mechanical properties of the structure to some embedded or layered actuators in order for the structure to adaptively perform functions of sensing and actuation. Therefore, the measurement function of Eq. (27.22) is not applicable to a smart structures. Consequently, in the third simulation the measurement function $\boldsymbol{h}$ is modified to incorporate the estimated values of the system parameters instead of the true values. Assuming the vector of unknown parameters in Eq. (27.2) to be composed of parameters m,k and c, i.e. $\mathbf{a} = [\mathrm{m}, \mathrm{k}, \mathrm{c}]$, the modified measurement function is

$$\boldsymbol{h}(x(t), \mathbf{a}(t), t) = \frac{1}{\boldsymbol{a}_1(\boldsymbol{t})} \left(f_{exc}(t) - a_3(t)\, \dot{x}(t) - a_2(t) r(t) \right), \tag{27.23}$$

where $\mathbf{a}(t)$ is identical to what used as the augmented state $\mathbf{X}(t)$ in Eq. (27.4) when forming the process function $\boldsymbol{\mathcal{F}}(\mathbf{X}, t)$ in the process of using the EKB filter. The EKB filter estimates are thus simultaneously used in the measurement function $\boldsymbol{h}$ to relate the measured acceleration to the states of the system ($\dot{x}$ and r). Note that in this simulation m,k and c are all assumed to be unknown.

Again, the value of $\delta = 0.1$ is used for this simulation and the filtering sequence is initiated with values of the state and parameters (m,k and c) 50% deviated from the true values. Figure 27.3 shows the simulation results. As is clear from the figure, in this simulation a stiffness reduction of 10% at time $t = 250$ is accurately identified online in the nonlinear hysteretic system without any prior knowledge of the system parameters and by sole measurement of the acceleration response and the excitation force. In fact, the proposed technique first accurately identifies the values of the parameters from noise-corrupted measurements of the acceleration response within the first 250 s of excitation, and then successfully monitors the change in the system by identifying the parameter that has changed, the amount of change, and the instant of occurrence of the change. Therefore, the proposed adaptive identification technique is capable of real-time sensing of mechanical properties (m,k and c) in a S-DoF nonlinear smart structure.

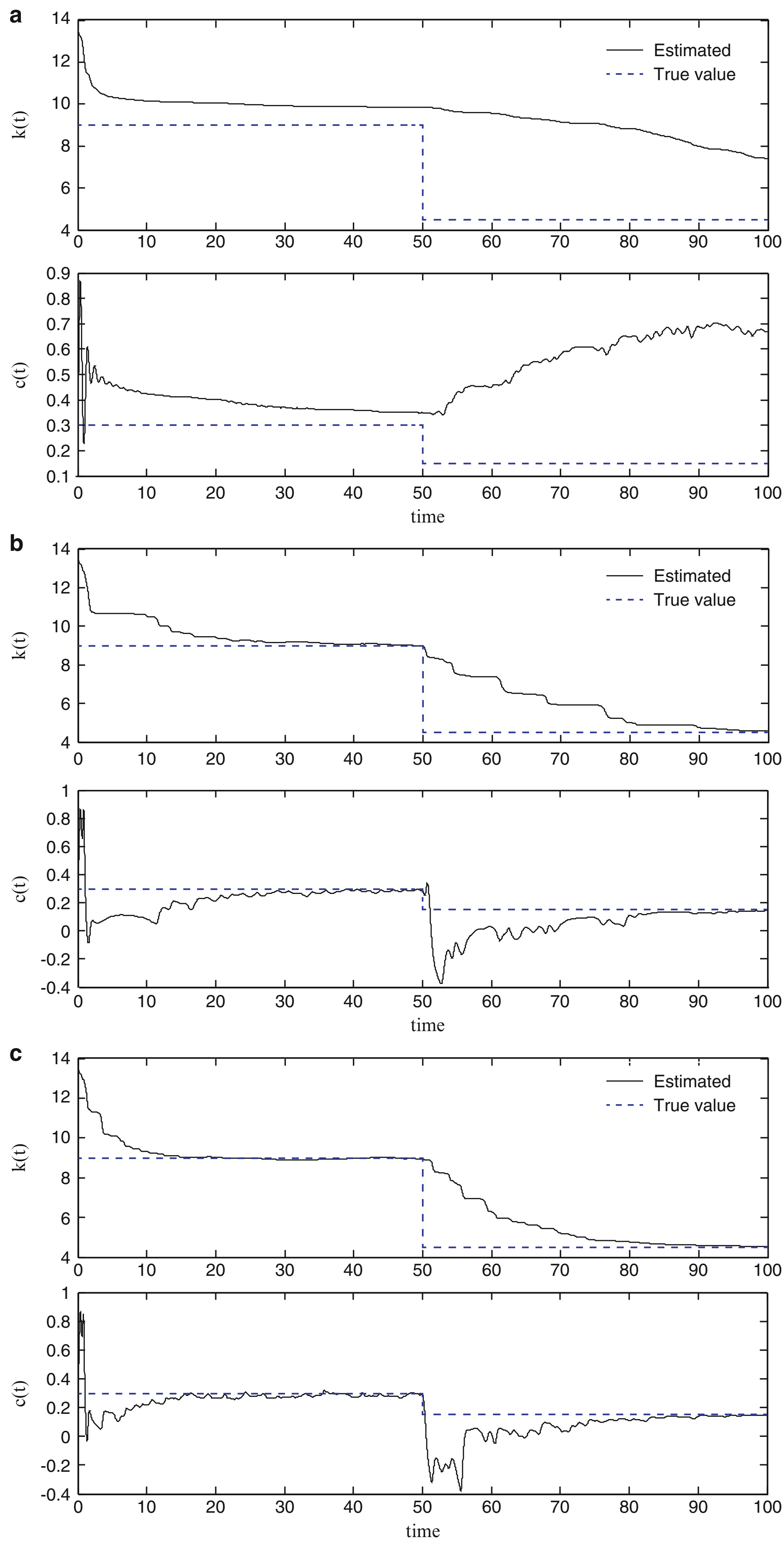

Fig. 27.1 Comparing damage identification with (**a**) random, (**b**) chaotic and (**c**) hyperchaotic excitations in nonlinear system of Eq. (27.19) estimated from noise-corrupted measurement using extended Kalman-Bucy filter (damage defined as a 50% stiffness and damping reduction at $t = 50$ s). $\boldsymbol{Q} = 1 \times 10^{-8}\boldsymbol{I}$, $\boldsymbol{R} = 1 \times 10^{-8}\boldsymbol{I}, \boldsymbol{P}_0 = 1 \times 10^{-6}\boldsymbol{I}$

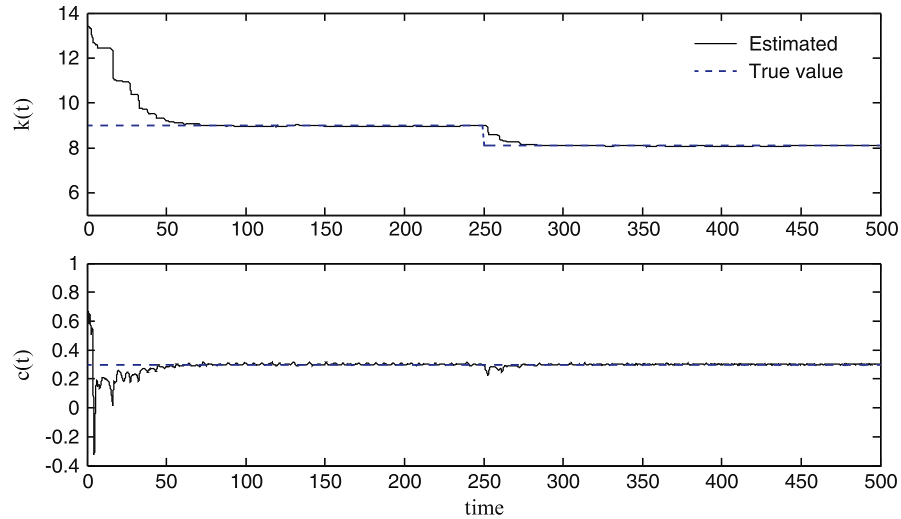

Fig. 27.2 Identification of 10% change in the stiffness of the nonlinear hysteretic system of Eq. (27.19) from acceleration measurements (Eq. 27.22) using hyperchaotic excitation and extended Kalman-Bucy (EKB) filter ($\boldsymbol{Q} = 1 \times 10^{-4}\boldsymbol{I}$, $\boldsymbol{R} = 1 \times 10^{-4}\boldsymbol{I}, \boldsymbol{P}_0 = 1 \times 10^{-4}\boldsymbol{I}$)

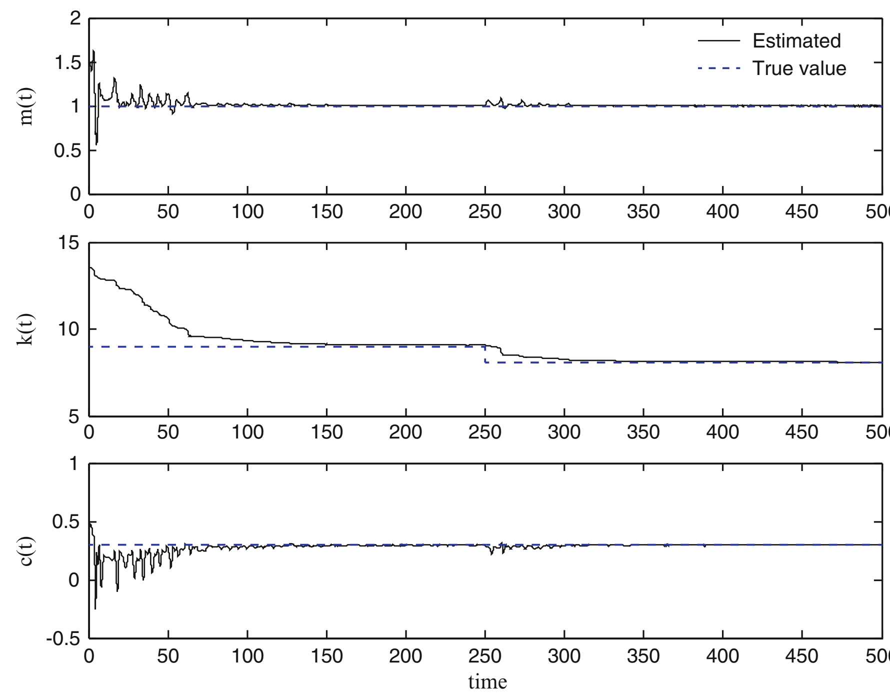

Fig. 27.3 Identification of 10% change in the stiffness of the nonlinear hysteretic system of Eq. (27.19) from acceleration measurements (Eq. 27.22) using hyperchaotic excitation and extended Kalman-Bucy (EKB) filter ($\boldsymbol{Q} = 1 \times 10^{-4}\boldsymbol{I}$, $\boldsymbol{R} = 1 \times 10^{-4}I, \boldsymbol{P}_0 = 1 \times 10^{-4}\boldsymbol{I}$)

27.3.2 Four-Story Shear-Beam Structure

Consider an idealized four-story linear shear-beam type building with floor masses m_i, inter-story stiffnesses k_i, and inter-story viscous damping coefficients c_i where $i = 1, \ldots, 4$. The structure is modeled with a linear spring–mass–damper system where the first spring is connected to the ground. The masses, spring stiffnesses and damping coefficients forming the $\boldsymbol{M}, \boldsymbol{K}$ and $\boldsymbol{C}$ matrices are set to $m_i = 5$, $k_i = 8$ and $c_i = 0.5$. Considering the state-space vector $\mathbf{z} = [\mathbf{x}(t), \dot{\mathbf{x}}(t)]^{\mathrm{T}}$, the $\boldsymbol{\varphi}$ function of Eq. (27.17) for this system is the linear function

$$\boldsymbol{\varphi}(\mathbf{z}(t)) = \boldsymbol{A}\mathbf{z}(t) = \begin{bmatrix} 0 & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix} \mathbf{z}(t) \tag{27.24}$$

Damage is introduced as a 10% stiffness reduction in the third spring of the system occurring at time $t = 350$ s. In order to implement the proposed real-time damage identification technique, the fourth mass is excited with the hyperchaotic Lorenz excitation of Eq. (27.18) with a value of $\delta = 0.1$ which can be shown to satisfy the tuning criteria of Eq. (27.16). It is assumed

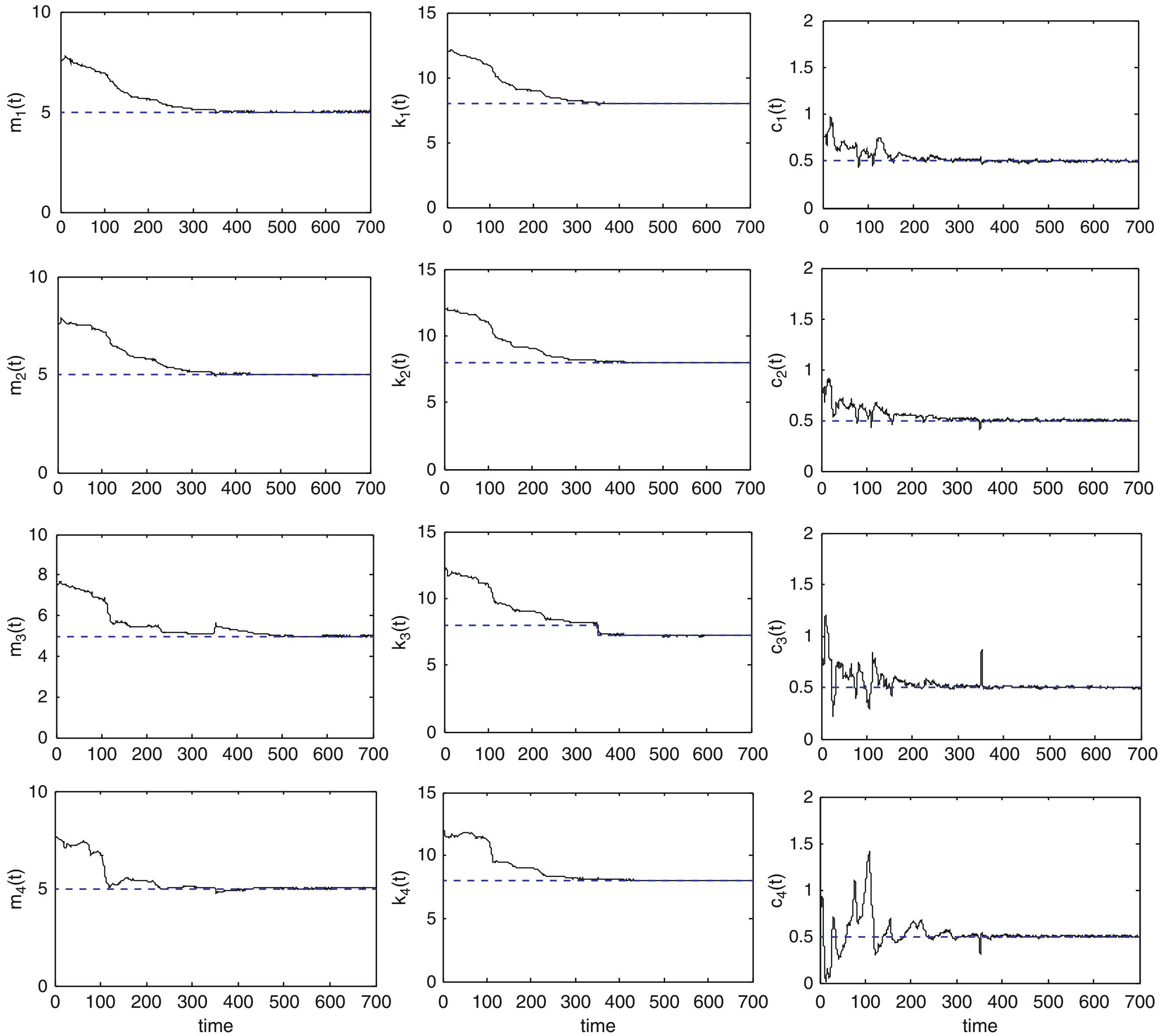

Fig. 27.4 Identification of 10% change in the stiffness 4-DoF linear system of Eq. (27.24) from acceleration measurements (Eq. 27.23) using hyperchaotic excitation and extended Kalman-Bucy (EKB) filter $\left(\boldsymbol{Q} = 1 \times 10^{-3}\boldsymbol{I}, \boldsymbol{R} = 1 \times 10^{-2}\boldsymbol{I}, \boldsymbol{P}_0 = 1 \times 10^{-2}\boldsymbol{I}\right)$

that only the excitation force and the acceleration of the masses are measured. Thus, since the states of the system consist of velocity $\dot{\mathbf{x}}$ and displacement $\mathbf{x}$, an appropriate measurement function is required to enable acceleration measurement. For the first simulation, a measurement function similar to that of Eq. (27.22) is considered by replacing m,k and c respectively with $\boldsymbol{M}, \boldsymbol{K}$ and $\boldsymbol{C}$ matrices. All 12 components of the $\boldsymbol{M}, \boldsymbol{K}$ and $\boldsymbol{C}$ matrices are considered as unknown parameters to be identified. The filtering sequence is initiated with values of the state and unknown parameters 50% deviated from the true values. The system is excited for 700 s by the hyperchaotic Lorenz excitation and the parameters are identified using the current approach. The real-time values of all 12 parameters of the 4-DoF system are depicted in Fig. 27.4. The identified parameters converge to the true value within the first 300 s. Upon the occurrence of damage, the identified values of some parameters experience a disturbance without losing convergence. The identified stiffness of the third spring clearly monitors the 10% reduction at time $t = 350$ s.

In the fourth simulation, a measurement function similar to that of Eq. (27.23) is used. Again, all 12 components of the $\boldsymbol{M}, \boldsymbol{K}$ and $\boldsymbol{C}$ matrices are considered as unknown parameters and the estimated values of those parameters are employed in the measurement function to relate the acceleration measurements to the states of the process. The same damage scenario occurring at time $t = 300$ s is applied with the same excitation strategy as used in the previous simulation. The filtering

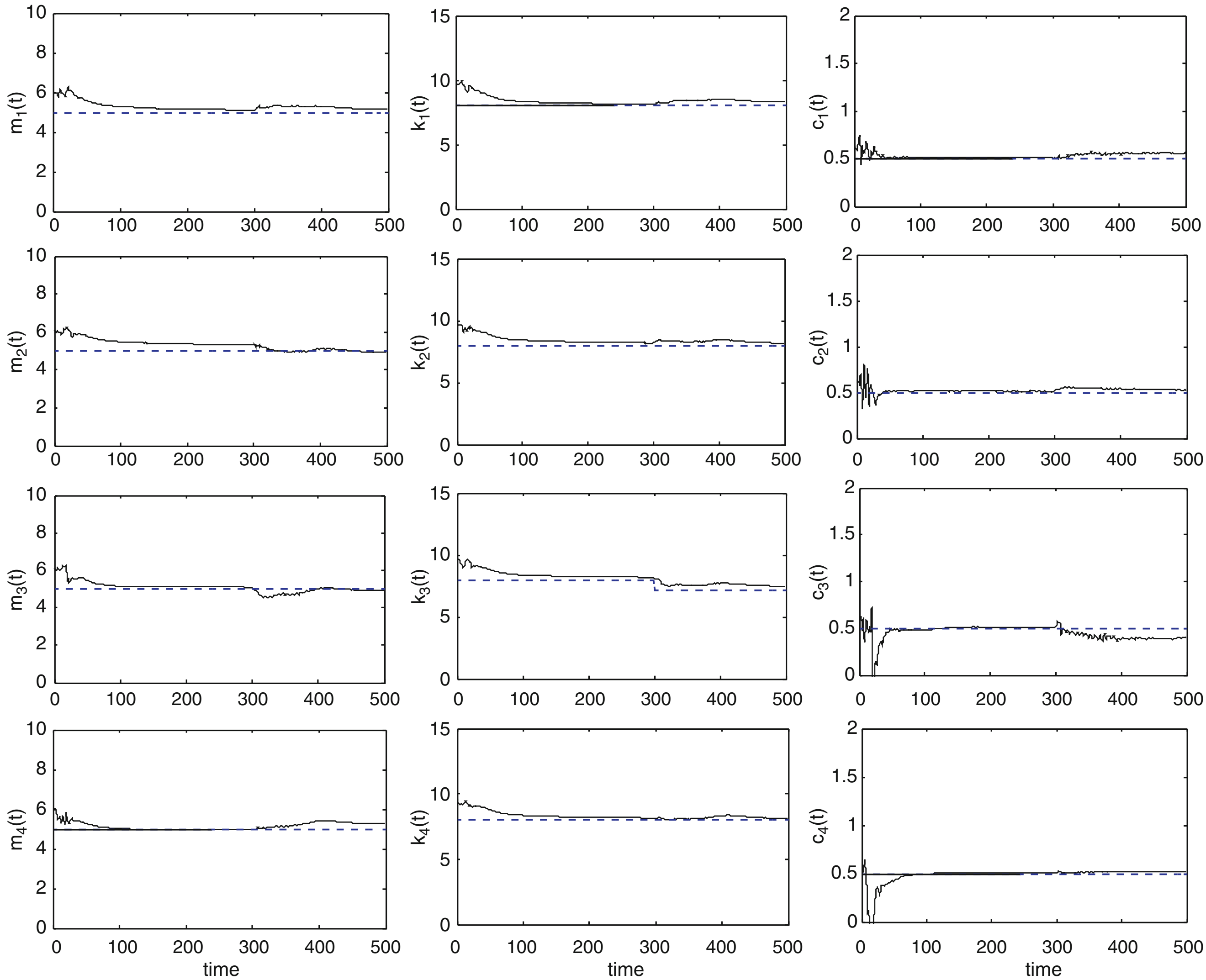

Fig. 27.5 Identification of 10% change in the stiffness 4-DoF linear system of Eq. (27.24) from acceleration measurements (Eq. 27.23) using hyperchaotic excitation and extended Kalman-Bucy (EKB) filter $\left(\boldsymbol{Q} = 1 \times 10^{-3}\boldsymbol{I}, \boldsymbol{R} = 1 \times 10^{-2}\boldsymbol{I}, \boldsymbol{P}_0 = 1 \times 10^{-2}\boldsymbol{I}\right)$

sequence is initiated with values of the state and unknown parameters 20% deviated from the true values. Figure 27.5 demonstrates the results. As seen in the figure, the identified parameters converge to the true value with good accuracy (except for m_2 which only has an acceptable accuracy) within the first 300 s. Upon the occurrence of damage, the identified values of some parameters experience a disturbance and, given enough time they will retrieve convergence. The identified stiffness of the third spring monitors 10% reduction at time $t = 300$ s. The simulation results show that the technique needs a longer excitation than just 500 s for a better accuracy. Therefore, using the proposed technique, a stiffness reduction of the 10% at time $t = 300$ s is identified in the 4-DoF system without any prior knowledge of the system parameters and by sole measurement of the acceleration response and the excitation force.

27.4 Conclusions

The current study combines hyperchaotic excitation, which has been previously shown to produce improved outcome when applied as the excitation in some attractor-based damage identification techniques, with the stochastic estimation technique of extended Kalman-Bucy filter, which has been recently reintroduced due to its enhanced estimation capabilities over similar filtering techniques. As a result, a novel feasible technique for identification of damage in nonlinear structures is developed that can be used in smart or self-healing structures for real-time identification of damage. Simulation results performed on single- and multi-DoF linear and nonlinear structural systems show that the proposed technique is capable of identifying modal parameters of structures from noise-corrupted measurements of acceleration response. The current technique is also capable of monitoring changes in the identified parameters by determining the parameter that has changed, the amount of change, and the instant of occurrence of the change. Therefore, the proposed adaptive identification technique is capable of real-time monitoring of damage based on the response of the system to a hyperchaotic probe. The results in this study are obtained with a fixed integration time step of 0.01 s. Considering the continuous-time nature of the filtering algorithm used in this approach, more accurate estimates are expected to be achieved by using smaller time steps for integration.

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