Chapter 7
Quantification of Parametric Model Uncertainties in Finite Element Model Updating Problem via Fuzzy Numbers

Yildirim Serhat Erdogan, Mustafa Gul, F. Necati Catbas, and Pelin Gundes Bakir

Abstract Analytical and numerical models that simulate the physical processes inevitably contain errors due to the mathematical simplifications and the lack of knowledge about the physical parameters that control the actual behavior. In this sense, parametric identification of civil engineering structures using uncertain numerical models should be subject to a particular interest in terms of accuracy and reliability of identified models. In this study, model uncertainties are modeled by fuzzy numbers and quantified using fuzzy model updating approach. In order to find the possible variation range of the response parameters (e.g. natural frequencies, mode shapes and strains) using uncertain finite element model, successive updating is employed. A simplified approach is proposed in order to facilitate the time consuming successive model updating phase. The identified variation range of the response parameters is employed to construct the fuzzy membership functions for each response parameter. Finally, fuzzy finite element model updating method (FFEMU) is used to obtain the membership functions of the model parameters. Different sets of model parameters are chosen to represent different models in terms of accuracy and these parameters are identified in the same way to investigate the model complexity. A two span laboratory grid structure developed for simulating bridge structures is used to validate and demonstrate the proposed approaches. The results show that the proposed approaches can efficiently be utilized to quantify the modeling uncertainties for more realizable and quantitative condition assessment and decision making purposes.

Keywords Finite element model updating • Fuzzy numbers • Model uncertainties • Optimization • Inverse fuzzy problems

7.1 Introduction

Identification of civil structures using structural health monitoring data has gained considerable attention with the development of measurement technologies [1–3]. Model based methods such as Finite Element Model Updating (FEMU) provide calibrated models, which better represent the actual behavior of structures [4, 5]. These models might be used for different analysis like damage detection, reliability analysis, load rating calculation etc. in order to make reliable decisions for the remaining life of the structure or retrofitting purposes [6–8]. However, the model and the measurement uncertainties that directly affect the updating process have still to be considered in order to obtain reliable updated models, which provide a variation range for model prediction in probabilistic sense. Quantification of these uncertainties is not a straightforward task, especially when the degree of freedom of the structure is too high since it requires too many model calculations [9, 10].

Y.S. Erdogan • P.G. Bakir
Department of Civil Engineering, Istanbul Technical University, Maslak, 34469, Istanbul, Turkey
e-mail: yserdogan@itu.edu.tr; gundesbakir@yahoo.com

M. Gul
Department of Civil and Environmental Engineering, University of Alberta, Edmonton, AB, Canada
e-mail: mustafa.gul@ualberta.ca

F.N. Catbas (✉)
Department of Civil, Environmental and Construction Engineering, University of Central Florida, Orlando, FL, 32816-2450, USA
e-mail: catbas@ucf.edu

T. Simmermacher et al. (eds.), *Topics in Model Validation and Uncertainty Quantification, Volume 5*, Conference Proceedings of the Society for Experimental Mechanics Series 41,
DOI 10.1007/978-1-4614-6564-5_7, © The Society for Experimental Mechanics, Inc. 2013

The main objective of this study is to investigate and then demonstrate a model uncertainty quantification approach first in the laboratory assuming that the measurement noise is time invariant and also less than certain real life civil engineering structures since the experiments are conducted in a controlled laboratory environment. The impact of modeling uncertainties on the response predictions are quantified by Fuzzy Finite Element Model Updating (FFEMU) method. Several measurement data sets (e.g. strain readings, modal parameters) and updating model parameter sets are generated and used to update the baseline FEM model of the structure. By this way, uncertainty amount in structural responses are compared for different data sets in order to quantify the uncertainty effect on model predictions. A benchmark grid structure designed for investigating structural health monitoring technologies and St-Id strategies at the University of Central Florida is used as the test structure [11, 12]. A damage case is adopted in which the boundary conditions are turned to be flexible to simulate the damage at the supports. In order to extract modal parameters for initial and damage cases, Complex Mode Indicator Function (CMIF) is employed [11, 13]. However, many other methods can be used effectively for experimental modal analysis like subspace based system identification techniques [14].

FFEMU method together with some proposed constraints which is required for the uniqueness of the inverse solution is employed to quantify uncertainties in the model. A Gaussian Process model, which is the surrogate for the numerical model is used to tackle with the computational issues. The results show that appropriate measurement sets should be generated instead of involving the complete measurement in order to keep the uncertainty at certain levels. This is due to the fact that the response parameters, which are more affected by the model uncertainties, cause the total uncertainty in the updated model to increase. Hence, the uncertainty quantification methods should be employed for different measurement sets and compared with each other to determine the appropriate data sets required for reliable models, particularly when both static and dynamic data are utilized together.

7.2 Fuzzy Finite Element Model Updating

Fuzzy set theory [15] is one of the efficient ways to quantify parametric uncertainty involved in the input and output parameters. Fuzzy numbers have been used as a tool in order to investigate the effect of uncertainties in different engineering input/output systems [16, 17]. A general illustration of fuzzy forward and inverse analysis for an I/O system is given in Fig. 7.1. In Fig. 7.1, the inputs and the outputs are the fuzzy parameters with different membership functions. Those membership functions can be chosen depending on the assumed and/or the quantified uncertainty in the parameters with the help of expert knowledge and the past experiences.

The equations for objective functions and related constraints that have to be strictly applied in order to make inverse problem to have a unique solution and capture all uncertainty in model responses are given in Eqs. (7.1), (7.2), (7.3), (7.4) and (7.5).

$$\min f\left(\theta^{\text{int}}\right) = \underline{r}\left(\theta^{\text{int}}\right)^T \mathrm{W}\underline{r}\left(\theta^{\text{int}}\right) + \bar{r}\left(\theta^{\text{int}}\right)^T W\bar{r}\left(\theta^{\text{int}}\right) \tag{7.1}$$

$$\theta^{\text{int}} = \left[\underline{\theta}, \overline{\theta}\right] \tag{7.2}$$

$$\underline{r}\left(\theta^{int}\right) = \frac{\underline{\gamma}\left(\theta^{int}\right) - \underline{\gamma}^e}{\underline{\gamma}^e} \tag{7.3}$$

$$\bar{r}\left(\theta^{int}\right) = \frac{\bar{\gamma}\left(\theta^{int}\right) - \bar{\gamma}^e}{\bar{\gamma}^e} \tag{7.4}$$

$$\left[\lambda^{int}, \phi^{int}, \varepsilon^{int}\right] = f^{model}\left(\theta^{int}\right) \tag{7.5}$$

$$\underline{\theta}_i^{(j+1)} \leq \underline{\theta}_i^{(j)} \tag{7.6}$$

$$\bar{\theta}_i^{(j+1)} \geq \bar{\theta}_i^{(j)} \tag{7.7}$$

$$\underline{\gamma}_k^{num} \leq \underline{\gamma}_k^{\exp} \forall j \in [0, 1]\, k = 1, \ldots .nm \tag{7.8}$$

$$\bar{\gamma}_k^{num} \geq \bar{\gamma}_k^{\exp} \forall j \in [0, 1]\, k = 1, \ldots .nm \tag{7.9}$$

$$f^{\text{infeasible}}(\theta) = f(\theta) + H \tag{7.10}$$

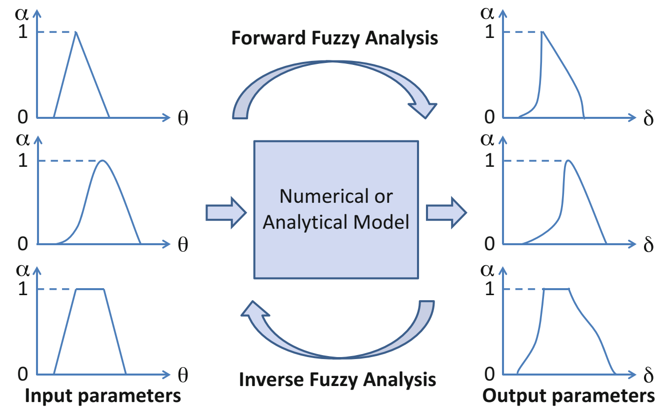

Fig. 7.1 General structure of the forward and the inverse fuzzy problem

where, θ^{int} is the interval valued updating parameter vector, W is the weighting matrix, which might be determined intuitively considering the relative accuracy of the measurements, $\gamma = [\lambda, \phi, \varepsilon]$ is the response vector, $f(\theta)$ is the objective function and $f^{\text{model}}(\theta)$ is the model function that governs the physical process. In Eq. (7.5), frequency, mode shape and strain vectors are denoted by λ, φ and ε, respectively. The bar above and below the response quantities denotes the upper and lower quantities. In addition to the objective function provided by Eqs. (7.1), (7.2), (7.3), (7.4) and (7.5), some additional constraints have to be introduced to preserve the monotonic behavior of the fuzzy set. It should also be noted that the optimization problem given Eqs. (7.1), (7.2), (7.3), (7.4) and (7.5) has to be solved for some specified numbers of α-level in order to capture the nonlinear relationship between inputs and outputs. The equations (7.8) and (7.9) can effectively be satisfied by assigning some penalties to the infeasible regions in the output space domain. In Eq. (7.10), the term H is a number in which its value is very high compared to the objective function value. By this way, infeasible regions can be disregarded.

7.3 Gaussian Process Model for FFEMU

Gaussian Process (GP) models are very efficient to develop approximate I/O computer models which require less computational effort to calculate. Since FFEMU is a computationally expensive method, it is not allow the computer simulation to be used directly due to time and cost constraints. Hence, surrogate models are efficient solutions to this problem with the limitation in the number of input parameters. However, [18] states that the GP can effectively be implemented to problems with 50 input parameters. This number is sufficient in most cases to identify a real life civil engineering structure.

Let x be the vector valued input parameter and the $[Y(X_1), Y(X_2), \ldots Y(X_n)]$ be the outputs based on n observations. In order to choose appropriate observations for the input parameter x, Central Composite Design (CCD) can be utilized as second-order design method [19]. If Y(x) denotes a Gaussian Process the mean and the covariance can be expressed by Eqs. (7.11) and (7.12).

$$E\left[Y(x)\right] = q(x)^T\beta \tag{7.11}$$

$$Cov\left[Y(x),\ Y(x*)\right] = \sigma^2 C\left(x, x^* \,|\xi\right) \tag{7.12}$$

Where (x) are trend functions given by $[1x^T]^T$ for linear trend and 1 for constant trend; β is the vector of regression coefficients, x^* is the untested input, σ^2 and $C(x, x*|\xi)$ are the variance of overall process and the correlation function, respectively. The governing parameters of the correlation function, which have to be determined together with β and σ^2 to create the GP model, are represented by ξ. The correlation function and the joint distribution function for Y expending on n observation are given by Eqs. (7.13) and (7.14).

$$C\left(x, x*\right) = exp\left(-\sum_{i=1}^{m} \xi_i \left(x_i - x_i^*\right)^2\right) \tag{7.13}$$

$$\boldsymbol{Y} \sim \boldsymbol{N}_n\left(q^T(x)\beta,\, \sigma^2\boldsymbol{R}\right) \tag{7.14}$$

If we know all the parameters governing the overall Gaussian Process, the mean and the covariance of the output can be calculated by Eqs. (7.15) and (7.16) instead of solving linear system of equations or eigenvalue problems in which large system matrices has to be handled.

$$E\left[\boldsymbol{Y}(x^*)\,|Y\right] = \boldsymbol{q}^T(x^*)\,\beta + r^T(x^*)\,\boldsymbol{R}^{-1}\left(\boldsymbol{Y} - \boldsymbol{F}\beta\right) \tag{7.15}$$

$$Cov\left[Y(x)Y(x^*)\,|\boldsymbol{Y}\right] = \sigma^2\left(C(x,x^*) - r^T\boldsymbol{R}^{-1}\boldsymbol{r}\right) \tag{7.16}$$

In Eqs. (7.15) and (7.16), r is the vector of correlations between x and each of the observation points, r^* is the vector of correlations between x* and each of the observation points. F is a matrix with rows $q^T(x_i)$. In this study, there is no need to calculate covariance of the predictions since this information will not be used in the scope of our methodology. In order to obtain the governing parameters ξ, β and the variance of the process σ^2 of GP, maximum likelihood estimation (MLE) might be employed which is required to solve optimization problem for each response variable only once. More details can be found in [20].

7.4 Numerical Verification

The benchmark grid structure developed for bridge health monitoring studies is used to investigate the model uncertainties for different experimental data sets. The grid structure has been designed to enable researchers to explore the use of different sensor technologies, St-Id and damage detection algorithms under different conditions, which offer promise cases for real life bridge structures. The girders and the columns of the grid have been constructed using steel sections $S3 \times 5.7$ and $W12 \times 26$, respectively. The 3D cad model and plan view of the structure are given in Fig. 7.2. More details about the grid structure can be found in [12].

Several static and dynamic tests have been conducted on the grid structure. The natural frequencies, mode shapes and displacements obtained from different loading cases are involved in FFEMU procedure. The natural frequencies and mode shapes are obtained by using Complex Mode Indicator Function (CMIF) method [11,13]. In static case, five different loading conditions are considered. Single loads (671.6 N) are applied to different nodes for each static loading case. The nodes where the single static loads are applied are as follows: (1) N3-N6-N9-N12 (2) N3-N9 (3) N6-N12 (4) N3-N6 (5) N9-N12. In order to demonstrate the methodology, a damage case in which two supports located at nodes N7 and N14 are replaced with Duro50 elastomeric pads to simulate flexible boundaries. The experimental and the deterministically updated natural frequencies (the natural frequencies which correspond to the α-cut level 1) are presented in Table 7.1.

The fuzzy response parameters obtained from updated fuzzy model are investigated for two main cases. In the first case, different sets of experimental data which contain several numbers of measured dynamic and static responses are considered. In the second case, different sets of updating parameters are used in fuzzy updating procedure in order to investigate the effect of uncertainties in structural response parameters (e.g. natural frequencies, static displacements). The flexible boundary conditions case with elastomeric pads is considered to be the damage case and use to demonstrate the fuzzy updating and analysis concepts. In the first case, column stiffnesses are chosen as the updating parameters. The data sets used in FFEMU is given in Table 7.2. Only results for the natural frequencies obtained from updated fuzzy model are presented in this paper. However, same results can be deducted for both strain and mode shapes.

After employing Fuzzy FEM to the updated model, fuzzy valued response parameters are obtained for each data sets. Membership functions of four predicted natural frequencies using updated fuzzy model are given in Fig. 7.3. The fourth and the fifth frequencies are not illustrated in order to make the figure more tractable. As apparent in Fig. 7.3, given frequencies are capable of illustrating the trend in uncertainty distribution for the increasing frequencies. The uncertainty in the frequencies is increasing when the amount of data involved in the updating is increased. The deterministic values for the first natural frequency which correspond to α-level 1 are very close to experimental one when the first six natural frequencies and first mode shapes (f + 1 m) are included. The amount of imprecision is also very low for this frequency. This is same for the f + 3 m together with slight increase in the uncertainty. This means that the effect of modeling uncertainties on the first natural frequency is low and this frequency can be predicted more accurate using the updated model as expected. However, the accuracy of predictions decreases and amount of imprecision increases when the strain data is included. This is due to fact that the uncertainty sources coming from dynamic and static tests are different. In the static tests, there might be some additional uncertainties due to loading. As opposed to static case, in dynamic tests there are some additional uncertainties arising from slight nonlinearity in boundaries due to the loss of contact and lack of tension stiffness in supports. In static tests, these problems are surpassed by adding some additional weights on support. However, first two natural frequencies and

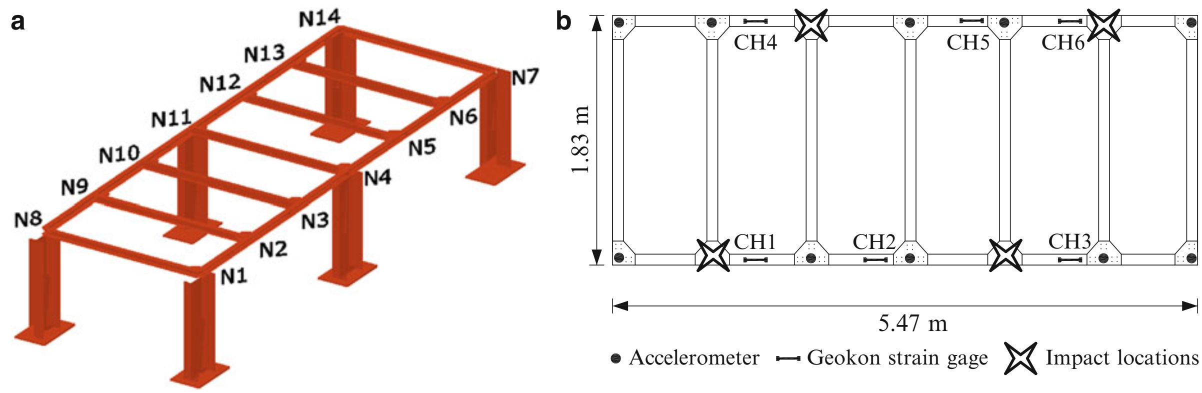

Fig. 7.2 (**a**) 3D view (**b**) plan view and sensor configuration

Table 7.1 Comparison of natural frequencies obtained from updated FEM and experiments

Mode number	Initial FEM (Hz)	Deterministically updated FEM (Hz)	Experimental frequencies (Hz)	
			Initial case	Damage case
1	21.52	22.65	22.72	14.31
2	26.22	27.60	27.67	18.50
3	32.69	34.10	34.13	26.12
4	40.72	42.57	42.15	32.25
5	62.73	64.54	65.18	37.68
6	66.52	67.99	68.27	47.68
7	93.98	94.02	95.36	61.18
8	96.47	96.70	97.90	65.50

Table 7.2 Data sets involved in the updating procedure

Data set	Abbreviation	Explanation
1	$f+1\text{m}$	First six natural frequencies and first mode shape values
2	$f+3\text{m}$	First six natural frequencies and first three mode shape values
3	$f+\varepsilon$ (1 loading)	First six natural frequencies and strain values from first loading
4	$f+\varepsilon$ (3 loading)	First six natural frequencies and strain values from the first three loading
5	ε (3 loading)	Only strain values from first three loadings
6	Full data	First six natural frequencies, first three mode shape values and strain values from first three loadings

mode shapes are affected less from these uncertainties and they can be predicted more precisely by means of FEMU through the first few modal parameters.

The imprecision amounts given in Table 7.4 demonstrate the statements made in the previous paragraph. The imprecision amount is low and the deterministic values are close to measurements for the data set f + 1 m. In Table 7.3, it is also apparent that the imprecision increases dramatically for the first two frequencies with the increase in data due to the uncertainty coming from other response variables. It should also be mentioned that while total imprecision may be less with fewer data and more imprecision with more data, models calibrated with more data better represent the overall structural characteristics.

In the second case three sets of updating parameter are used to update the FEM of the benchmark grid structure. The updating parameter sets are as follows: (1) Axial stiffness of the columns and the moment of inertia of the connections. The connections are grouped as N2,N3 – N5,N6 – N9,N10 – N12,N13 – N11,N4 (Total 11 parameters) (2) Axial stiffness of the columns and moment of inertia of the connections at nodes N2,N3,N4,N5,N6,N9,N10,N11,N12,N13. For this set, the connections are grouped as N2,N3,N4,N5,N6 and N9,N10,N11,N12,N13 (Total 8 parameters) (3) Axial stiffness of the columns (Total 6 parameters). Full data set is used as the measurement data for all updating parameter set. The fuzzy natural frequencies obtained from fuzzy models updated using each sets of updating parameters and amount of imprecision are given in Fig. 7.4 and Table 7.1, respectively.

As apparent in Fig. 7.4 and Table 7.1, the amount of uncertainty is decreased as the number of updating parameters is increased. In addition, the deterministic values of response parameters which correspond to the α-cut level 1 are close to the

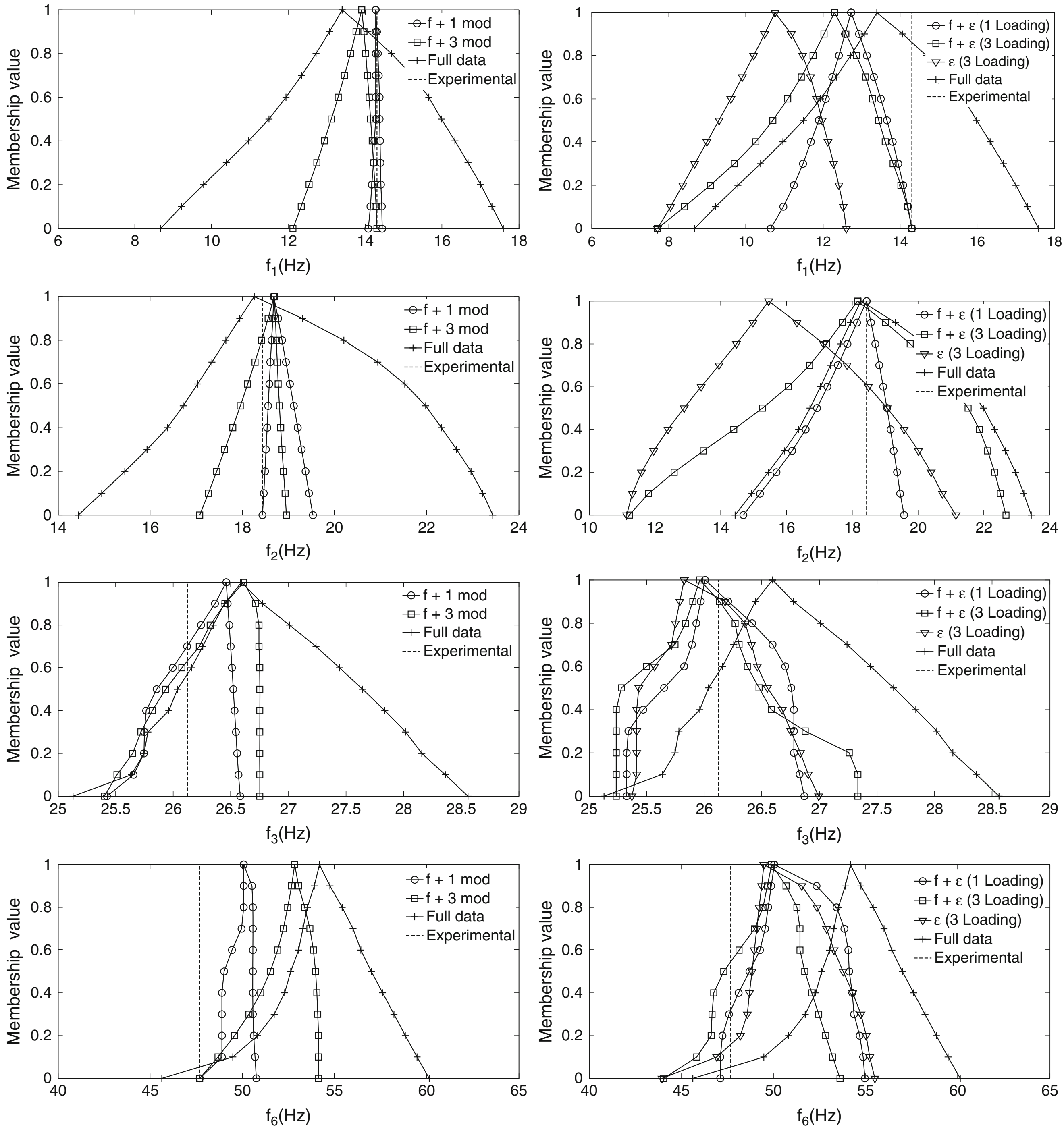

Fig. 7.3 Updated natural frequencies using different SHM data set for the damage case

Table 7.3 Imprecision in the response for different data sets

Response/data set	$f+1$ mode	$f+3$ mode	$f+$ strain (1 loading)	$f+$ strain (3 loading)	Strain (3 loading)	Full data
f_1	0.0101	0.0754	0.1404	0.2422	0.2449	0.3396
f_2	0.0296	0.0471	0.1214	0.3366	0.3724	0.2751
f_3	0.0218	0.0289	0.0380	0.0441	0.0388	0.0595
f_6	0.0254	0.0513	0.1053	0.0852	0.1032	0.0928

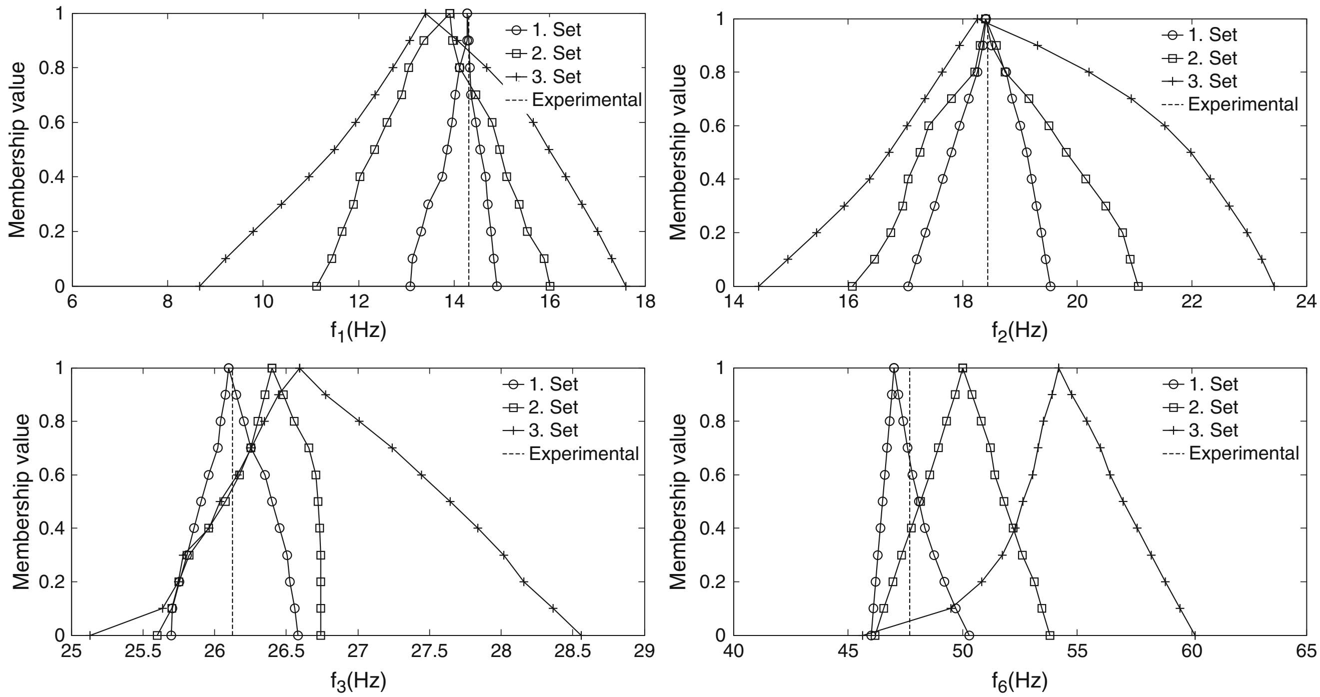

Fig. 7.4 Updated natural frequencies using different updating parameter set

Table 7.4 Imprecision in the response for different updating parameter sets

Response/data set	1. set	2. set	3. set
f_1	0.0511	0.1034	0.3396
f_2	0.0445	0.0814	0.2751
f_3	0.0113	0.0310	0.0595
f_6	0.0333	0.0620	0.0928

experimental values. However, more updating parameters bring some disadvantageous beside low uncertainty and accuracy. The computational complexity increases with higher number of model parameters. In addition, the updated parameters might not provide physically meaningful predictions for different loading conditions. Hence, number of updating parameters should be restricted considering the geometric and material properties of the actual structure.

7.5 Conclusion

Results show that the uncertainty in the response variables increases as the number of experimental data used in FFEMU increases. This can be explained with the fact that the uncertainty in all response variables contributes to total uncertainty in the updated model. This is analogous to applying a high precision curve-fit to very few data points, say two data points. As the number of data points increase, it can be obvious that the imprecision will increase, yet the overall characterization of the entire data set will be better. The imprecision and uncertainty amount in model parameters are observed to increase especially when strain data is used. This increment arises from the difference between the dynamic and the static test setup and the global (modal) and local (strain) responses that these measurements represent. The difference in test setups and responses lead to different uncertainty sources, reducing the reliability of individual parameter predictions. In static tests, additional weights have been used to prevent the loss of contacts in supports while these weights were removed during dynamic tests. Additionally, there is no tension stiffness in boundaries, which introduces nonlinearity in the supports of the test structure. However, similar uncertainties can be expected to exist in real life structures. Hence, special care should be paid for generating appropriate data sets to develop the updated models with the least uncertainty. This can be achieved by ignoring some measurements such as dynamic response at boundaries or strain measurements of "virtually unstressed" members. In addition, low weighting factors can be assigned to the measurements that increase the uncertainty in response predictions. Finally, more complicated models, which are not preferable in most cases, might be another solution in some sense. As demonstrated in the second case, inclusion of more updating parameter lead lower uncertainty amount in the response. However, inclusion of additional model parameters in updating process might not be feasible in all cases since it is always possible to obtain physically inconvenient model parameter values. By this way, the degree of freedom of the updated model is increased which may provide unfeasible response predictions for different loading conditions.

References

1. Catbas FN, Kijewski-Correa TL, Aktan AE (eds) (2012a) Structural identification of constructed systems: approaches, methods and technologies for effective practice of St-Id. ASCE (accepted). ISBN:978-0784411971
2. Xu B, He J, Rovekamp R, Dyke SJ (2012) Structural parameters and dynamic loading identification fromin complete measurements: approach and validation Mech Syst Signal Process 28:244–257
3. Esfandiari A, Bakhtiari-Nejad F, Sanayei M, Rahai A (2010) Structural finite element model updating using transfer function data. Comput Struct 88:54–64
4. Mottershead JE, Michael L, Friswell MI (2011) The sensitivity method in finite element model updating: a tutorial. Mech Syst Signal Process 25(7):2275–2296
5. Bakir PG, Reynders E, De Roeck G (2007) Sensitivity-based finite element model updating using constrained optimization with a trust region algorithm. J Sound Vib 305:211–225
6. Buezas FS, Rosales MB, Filipich CP (2011) Damage detection with genetic algorithms taking into account a crack contact model. Eng Fract Mech 78:695–712
7. Meruane V, Heylen W (2011) An hybrid real genetic algorithm to detect structural damage using modal properties. Mech Syst Signal Process 25:1559–1573
8. Gokce HB, Catbas FN, Gul M, Frangopol DM (2012) Structural identification for performance prediction considering uncertainties: a case study of a movable bridge. J Struct Eng ASCE (accepted). doi:http://dx.doi.org/10.1061/(ASCE)ST.1943-541X.0000601
9. Xiong Y, Chen W, Tsui KL, Apley DW (2009) A better understanding of model updating strategies in validating engineering models. Comput Meth Appl Mech Eng 198:1327–1337
10. Catbas FN, Gokce HB, Frangopol DM (2012b) Predictive analysis by incorporating uncertainty through a family of models calibrated with structural health monitoring data. ASCE J Eng Mech (accepted). doi:http://dx.doi.org/10.1061/(ASCE)EM.1943-7889.0000342
11. Catbas FN, Aktan AE, Brown DL (2004) Parameter estimation for multiple input multiple output analysis of large structures. J Eng Mech ASCE 130(8):921–930
12. Catbas FN, Gul M, Burkett J (2008) Damage assessment using flexibility and flexibility-based curvature for structural health monitoring. Smart Mater Struct 17(1):015–024
13. Catbas FN, Brown DL, Aktan AE (2006) Use of modal flexibility for damage detection and condition assessment: case studies and demonstrations on large structures. J Struct Eng ASCE 132(11):1699–1712
14. Bakir PG (2011) Automation of the stabilization diagrams for subspace based system identification. Expert Syst Appl 38(12):14390–14397
15. Zadeh LA (1965) Fuzzy sets. Inf Control 8(3):338–353
16. Massa F, Ruffin K, Tison T, Lallemand B (2005) A complete method for efficient fuzzy modal analysis. J Sound Vib 309:63–85
17. Nicolai BN, Egea JA, Scheerlinck N, Banga JR, Datta AK (2011) Fuzzy finite element analysis of heat conduction problems with uncertain parameters. J Food Eng 103(1):38–46
18. O'Hagan A (2006) Bayesian analysis of computer code outputs: a tutorial. Reliab Eng Syst Saf 91:1290–1300
19. Box GEP, Wilson KB (1951) On the experimental attainment of optimum conditions (with discussion). J R Stat Soc B 13(1):1–45
20. Mc Farland JM (2008) Uncertainty analysis for computer simulations through validation and calibration. Department of Mechanical Engineering, Vanderbilt University