Chapter 7
Tuning TMDs to "Fix" Floors in MDOF Shear Buildings

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Abstract Many researchers have examined the optimal design of tuned mass dampers (TMDs) for vibration reduction in single-degree-of-freedom (SDOF) and multiple-degree-of-freedom (MDOF) systems. This work focuses on the design of damped TMDs to "fix" selected floors of a harmonically base-excited building in shear such that the vibrational amplitudes of the selected floors are zero. This method does not require the fixed floors, referred to as "fixed nodes," to coincide with the floors to which the TMDs are attached. This paper presents the proposed tuning method and addresses the feasible arrangements of the fixed and TMD floors. The proposed tuning method is demonstrated with a simulation of a 3-DOF shear building, along with a discussion on the effects of mistuning on the TMD performance.

Keywords tuned-mass dampers • shear buildings • tuning procedure • base excitation • enforcing nodes

7.1 Introduction

Tuned mass dampers (TMDs) are spring-mass-damper systems that are attached to a primary system to divert energy and lessen the vibration of the primary system. Ideally, the primary system is being excited harmonically and the excitation frequency is known exactly. The selection of the TMD parameters, referred to as the "tuning" of the TMDs, can be done in a variety of ways. The most classical tuning method was introduced by Den Hartog [1], who tuned the TMDs to keep the amplitude of the primary mass small. Many other researchers have also used a variety of optimization techniques to determine the optimal TMD parameters for a selected cost function [2–4].

A different method for tuning TMDs selects the TMD parameters so that locations of zero vibrational amplitude, referred to as "fixed nodes," are enforced at desired locations in the system. This method has been investigated by Cha for elastic systems [5], systems with multiple excitation frequencies [6], and damped Euler-Bernoulli beams [7]. The fixed nodes and TMDs need not be collocated, and any number of fixed nodes may be enforced in a continuous system with the same number of attachments. Cha's tuning procedure first determines the steady-state forces the attachments must apply to the system in order to enforce the fixed nodes at the desired locations. Once these required attachment forces are known, it is trivial to determine the steady-state deflection of the system subjected to the attachment forces, check that the required attachment forces are passive, and then solve for the TMD parameters.

The research presented in this paper focuses on modifying this tuning procedure to be used in a discrete system. This modified tuning procedure is derived and presented in Section 7.2, simulation results for an example 3-DOF shear building are presented in Section 7.3, and the work is summarized in Section 7.4.

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7.2 Theory

Consider a discrete primary system with N degrees of freedom that is excited harmonically at N_f locations with a single frequency. It is desired to enforce N_a fixed nodes in the system, which requires tuning N_a attachments. After replacing the attachments with their unknown restoring forces, the equation of motion in generalized coordinates $\boldsymbol{x}$ is

$$[M]\ddot{\boldsymbol{x}} + [C]\dot{\boldsymbol{x}} + [K]\boldsymbol{x} = \boldsymbol{F} \tag{7.1}$$

where $\boldsymbol{F}$ contains contributions from the external forcing and from the attachments.

Assume that the external forcing is harmonic with frequency ω and that the system is in steady-state, in which case

$$\boldsymbol{F} = \bar{\boldsymbol{F}} e^{\mathrm{j}\omega t}, \tag{7.2}$$

$$\boldsymbol{x} = \bar{\boldsymbol{x}} e^{\mathrm{j}\omega t}, \tag{7.3}$$

where $\bar{\cdot}$ indicates phasor notation, and $\mathrm{j} = \sqrt{-1}$. Substituting these equations into Eq. (7.1) and simplifying yields

$$\left(-\omega^2[M] + \mathrm{j}\omega[C] + [K]\right)\bar{\boldsymbol{x}} = \bar{\boldsymbol{F}}. \tag{7.4}$$

Separating the forcing vector into the external and attachment forces yields

$$\bar{\boldsymbol{F}} = \bar{\boldsymbol{F}}_{\boldsymbol{e}} + \bar{\boldsymbol{F}}_{\boldsymbol{a}} \tag{7.5}$$

where $\bar{\boldsymbol{F}}_{\boldsymbol{e}}$ has N_f nonzero entries and $\bar{\boldsymbol{F}}_{\boldsymbol{a}}$ has N_a nonzero entries that correspond to the N_a attachments to be tuned and/or the N_a fixed nodes to be enforced. Defining the impedance matrix as $[Z] = -\omega^2[M] + j\omega[C] + [K]$ and combining Eqs. (7.4) and (7.5) yields

$$\bar{\boldsymbol{x}} = [Z]^{-1}\left(\bar{\boldsymbol{F}}_{\boldsymbol{e}} + \bar{\boldsymbol{F}}_{\boldsymbol{a}}\right). \tag{7.6}$$

Note that $\bar{\boldsymbol{F}}_{\boldsymbol{e}}$ and $[Z]^{-1}$ are known, $\bar{\boldsymbol{F}}_{\boldsymbol{a}}$ is to be determined, and the fixed node enforcement will set the corresponding entries of $\bar{\boldsymbol{x}}$ to zero.

Define two "selection matrices" $[S_n]$ and $[S_a]$ of ones and zeros such that:

$$\bar{\boldsymbol{F}}_{\boldsymbol{a}} = [S_a]\bar{\boldsymbol{f}}_{\boldsymbol{a}}, \qquad \boldsymbol{0} = [S_n]\bar{\boldsymbol{x}}, \tag{7.7}$$

where $\bar{\boldsymbol{f}}_{\boldsymbol{a}}$ is a vector of the N_a attachment phasors that are required to enforce the desired fixed nodes. Note that $[S_n] \in \mathbb{R}^{N_a \times N}$ and the ith row contains a one at the ith fixed node location; $[S_a] \in \mathbb{R}^{N \times N_a}$ and the jth column contains a one at the jth attachment location.

Combining Eqs. (7.6) and (7.7) yields the following expression for the required attachment forces:

$$\bar{\boldsymbol{f}}_{\boldsymbol{a}} = -\left([S_n][Z]^{-1}[S_a]\right)^{-1}[S_n][Z]^{-1}\bar{\boldsymbol{F}}_{\boldsymbol{e}}. \tag{7.8}$$

Once the attachment forces have been found they may be substituted into Eq. (7.6) to determine the steady-state deflection of the system $\bar{\boldsymbol{x}}$.

At this point it is necessary to verify that the required attachment forces can be delivered by a passive system. In particular, the phase of the required restoring force at a particular location must lag behind the steady-state deflection at that point by no more than 180°. In other words, once $\bar{\boldsymbol{f}}_{\boldsymbol{a}}$ has been found, one must verify that

$$0 \leq \angle\bar{x}_i - \angle\bar{f}_{a,i} < \pi \tag{7.9}$$

for all attachment locations i. Note that this matches the result in Cha and Rinker [7].

Because the complex relationship between $\bar{x}_i$ and $\bar{f}_{a,i}$ is known but there are three unknown TMD parameters, the designer must chose one parameter and subsequently solve for the other two. This paper follows the method laid out in Cha and Rinker [7], prescribing the absolute displacement of the TMD $|\bar{x}_i + \bar{d}_i|$, where $\bar{d}_i$ is the relative displacement of the ith TMD. However, one could easily choose any of the TMD parameters (e.g. the stiffness k_i or damper value c_i) and solve for the other TMD parameters. From the referenced paper,

$$m_i = \frac{|\bar{f}_a^i|}{\omega^2 |\bar{x}_i + \bar{d}_i|}, \tag{7.10}$$

$$k_i = \frac{(a_i^2 + b_i^2 - a_i m_i \omega^2) m_i \omega^2}{b_i^2 + (a_i - m_i \omega^2)^2}, \tag{7.11}$$

$$c_i = \frac{-b_i m_i^2 \omega^3}{b_i^2 + (a_i - m_i \omega^2)^2}, \tag{7.12}$$

where $a_i = \Re(\bar{f}_a^i / \bar{x}_i)$ and $b_i = \Im(\bar{f}_a^i / \bar{x}_i)$. Once the TMD parameters have been chosen, the matrix governing equation for the system with the tuned attachments can be assembled and the fixed node enforcement verified.

The tuning procedure may then be summarized as follows:

1. Select a fixed node and attachment arrangement.
2. Use Eq. (7.8) to solve for the required attachment forces.
3. Determine the steady-state deflections with Eq. (7.6) and verify passivity with Eq. (7.9).
4. Choose an amplitude for the TMD.
5. Solve for m_i, k_i, and c_i using Eqs. (7.10)–(7.12).

7.3 Example

Consider a 3-DOF shear building with the parameters shown in Table 7.1. The building is subjected to a harmonic base excitation $x(t) = \bar{x}_b e^{j\omega t}$, where $\bar{x}_b = 0.1$ m and $\omega = 20$ rad/s. Then,

$$[M] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = \begin{bmatrix} 1001 & 0 & 0 \\ 0 & 999 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \text{kg}, \tag{7.13}$$

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} = \begin{bmatrix} 891 & -441 & 0 \\ -441 & 908 & -467 \\ 0 & -467 & 467 \end{bmatrix} \text{kN/m}, \tag{7.14}$$

$$[C] = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} = \begin{bmatrix} 117 & -63 & 0 \\ -63 & 142 & -79 \\ 0 & -79 & 79 \end{bmatrix} \text{kN/m}, \tag{7.15}$$

$$\bar{\boldsymbol{F}}_{\boldsymbol{e}} = \begin{bmatrix} \bar{x}_b (k_1 + j\omega c_1) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 45 + 0.108j \\ 0 \\ 0 \end{bmatrix} \text{kN}. \tag{7.16}$$

Consider the case where a single TMD is attached to the first floor and it is desired to enforce a fixed node at the second floor to remove its vibration. Using the procedure laid out in Section 7.2 with $|d_i| = 0.3$ m, the resulting TMD parameters are $m_a = 375.0$ kg, $k_a = 150.0$ kN/m, $c_a = 0$ kg/s, and the deflected shapes of the building with and without the TMD are shown in Fig. 7.1.

There are two items of note in this example. The first is that the TMD is now undamped, despite the fact that this is a damped system, and the second is that this arrangement of the TMD and desired fixed node leads to vibration suppression in *all three* floors of the building, not just the second. These two facts are not unrelated, and explaining the latter observation will produce an explanation for the former.

Table 7.1 Parameters for 3-DOF shear building example

Parameter	Floor 1	Floor 2	Floor 3
m_i (kg)	1001	999	1000
k_i (kN/m)	450	441	467
c_i (kg/s)	54	63	79

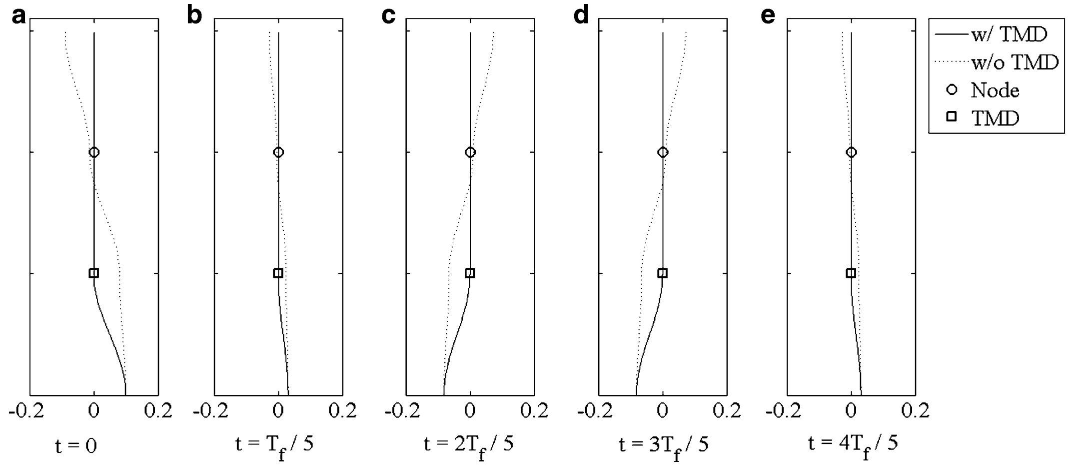

Fig. 7.1 Snapshots of displacement (m) of system with and without TMD

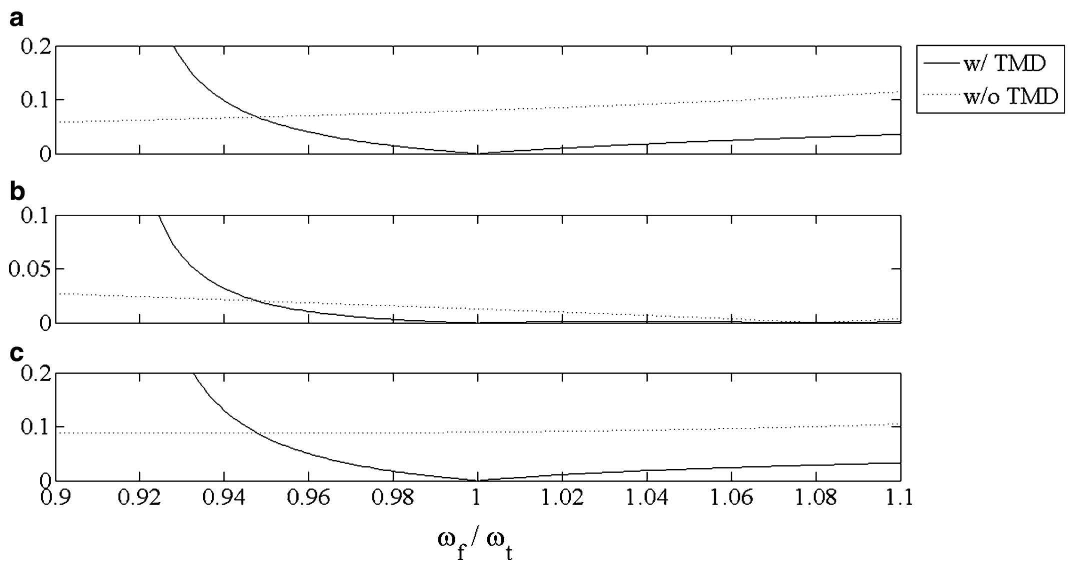

Fig. 7.2 Plots of the amplitude (m) of vibration for (**a**) Floor 1, (**b**) Floor 2, and (**c**) Floor 3 as ω_f varies

This fixed node and TMD arrangement is not the only arrangement that produces these same values for the TMD parameters and the same vibration suppression in all three floors; in fact, any arrangement that places the TMD on the first floor of the building yields these parameters and the vibration suppression in all floors. This is logically intuitive. If the TMD enforces a fixed node at the first floor, then no vibration can be transmitted to the higher floors. Thus, enforcing fixed nodes at the second or third floors is equivalent to enforcing a fixed node at the first floor. In that case, this particular example reduces to the system where the TMD and desired fixed node are collocated on the first floor, in which case the solution features no damper [7]. Note that, by extension, this is true for all cases where the desired fixed node is located on the same floor or above the floor where the TMD is attached.

On the other hand, if the desired fixed node is located below the attachment location, then the required attachment force violates passivity and the TMD/fixed-floor arrangement is infeasible. However, while this was true for all possible arrangements in this particular system, it is not necessarily true for other shear building systems. For example, if $\bar{\boldsymbol{F}}_e = [10\ \ 60\ \ 70]^T$ N, it is possible to place the TMD on the second floor but enforce a fixed node on the first floor. However, this results in an amplification of the third-floor oscillation, so care should always be taken to examine the global system behavior.

Lastly, it is useful to examine what consequences could result from mistuning (i.e., if the actual forcing frequency differs from the tuning frequency). Let ω_f be the true forcing frequency of the system and ω_t be the frequency used in Eqs. (7.4), (7.11) and (7.12). A plot of the amplitudes of vibration of the floors for varying values of ω_f/ω_t is shown in Fig. 7.2. Note that if $\omega_f/\omega_t = 1$ then the system is perfectly tuned and all floors have zero vibration, which matches the earlier result. It is interesting to see that, even with mistuning, if the variation in ω_f is approximately less than 5% of the tuning frequency, the system with the TMD features less vibration than that without the TMD. However, if the variation in ω_f is less than this bound, the system with the TMD has much larger vibration on all three floors.

7.4 Conclusions

This paper develops a procedure to choose the parameters of TMDs in harmonically forced discrete systems to enforce locations of zero amplitude in the system. A 3-DOF shear building with harmonic base excitation and a single TMD is presented as an example. It is shown that fixing floors at or above the TMD location results in zero vibration for all floors at and above the TMD location, whereas placing the TMD above the fixed node is an infeasible arrangement. The paper also includes an examination of the effects of mistuning upon the system with the TMD and shows that if the error in frequencies is less than 5%, then the system with the TMD features less vibration than the system with no TMD, though the floors are no longer perfectly fixed.

References

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