Chapter 4
Hybrid MPC: An Application to Semiactive Control of Structures

Wael M. Elhaddad and Erik A. Johnson

Abstract In clipped LQR, a common strategy for semiactive structural control, a primary feedback controller is designed using LQR and a secondary controller clips forces that the semiactive control device cannot realize. However, when the primary controller commands highly non-dissipative forces, the frequent clipping may render a controller far from being optimal. A hybrid system model is better suited for semiactive control as it accurately models the passivity constraints by introducing auxiliary variables into the system model. In this paper, a hybrid model predictive control (MPC) scheme, which uses a system model with both continuous and discrete variables, is used for semiactive control of structures. Optimizing this control results in a mixed integer quadratic programming problem, which can be solved numerically to find the optimal control input. It is shown that hybrid MPC produces nonlinear state feedback control laws that achieve significantly better performance for some control objectives (e.g., the reduction of absolute acceleration). Responses of a typical structure to historical earthquakes, and response statistics from a Monte Carlo simulation with stochastic excitation, are computed. Compared to clipped LQR, hybrid MPC is found to be more consistent in the reduction of the objective functions, although it is more computationally expensive.

Keywords Structural control • Semiactive dampers • Hybrid systems • Model predictive control • Clipped LQR

4.1 Introduction

In the past few decades, extensive research was conducted to study structural control and its application in natural hazards mitigation [1], with the main focus of protecting buildings and bridges against strong earthquakes and extreme winds. Significant efforts were devoted in realizing structural control systems for real-life applications, to protect structural systems, and consequently, human lives; such an application is of great social and economic importance. Today, many buildings and bridges around the world employ structural control systems for vibration reduction [2]. Researchers have investigated the application of different control strategies in structures, such as passive, active, hybrid and semiactive control. Although, passive control strategies [3] are the best established and most well-accepted for structural vibration reduction, they are not the most efficient control technique as they cannot adapt to different loading events and structural conditions. On the other hand, active control strategies are more adaptive and efficient in vibration reduction, though they suffer from reliability issues [1] since they can render a structure unstable and can demand power that might not be available during an extreme loading event such as an earthquake or severe wind exposure.

Recently, semiactive strategies based on controllable passive devices have emerged as an alternative for vibration reduction in structures. Controllable passive devices are ones that exert forces through purely passive means, such as a controllable damper or controllable stiffener, but have controllable properties that affect those forces. Some examples of controllable passive devices are variable orifice dampers, variable friction devices, variable stiffness devices and controllable fluid dampers (*e.g.*, magnetorheological (MR) and electrorheological (ER) fluid dampers) [1, 2, 4–6].

W.M. Elhaddad (✉) • E.A. Johnson
Sonny Astani Department of Civil and Environmental Engineering, University of Southern California,
3620 S Vermont Ave, Los Angeles, CA 90089, USA
e-mail: welhadda@usc.edu; JohnsonE@usc.edu

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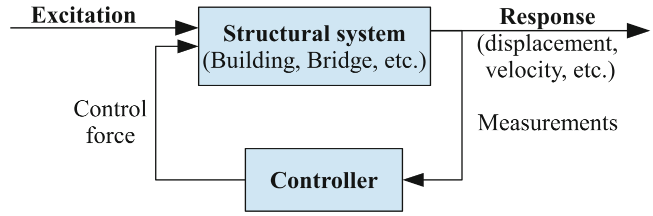


Fig. 4.1 Feedback control for structural systems

Thus, the main benefits of using semiactive control are its inherent stability, as it does not introduce energy into the controlled structure, its ability to focus on multiple (and possibly changing) objectives, exerting a force that can depend on non-local information, as well as the low power requirement that is critical in the case of natural hazards like earthquakes [7]. In addition, it has been shown that semiactive control is capable, in some cases, of achieving performances comparable to that of a fully active system [7]. It is, thus, beneficial to advance the understanding of the behavior of structural systems controlled with such devices, to be able to take full advantage of their capabilities.

Since both active and semiactive control systems have the ability to adapt to different operating conditions, they rely on control algorithms to achieve this adaptability. Both strategies make use of measurements and observations of current structure state, as shown in Fig. 4.1. The measurements are then provided as a feedback to the controllers that are responsible for making a decision about how the device should adapt to the current state of the system in order to efficiently reduce its future vibrations.

4.2 Motivation

The design process of semiactive control for vibration reduction of structures is crucial in order to obtain an effective use of the control device. Different feedback control strategies can be used to minimize vibrations in structures controlled with semiactive devices [8]. Some examples of these control laws are neural/fuzzy control strategies [9], Lyapunov control [8, 10, 11], pseudo-negative stiffness control [12], performance-guarantee approaches [13, 14] and LMI control [15]. Many other applications of semiactive control are based on the clipped-optimal control technique [7], where the desired control input is determined by initially assuming an unconstrained device force model. Then, any non-dissipative force is clipped from the desired control, as shown in Fig. 4.2a, to ensure that the resulting input is dissipative and realizable by the semiactive device. However, when the desired control is highly non-dissipative, the frequent clipping may render the clipped control strategy far from being optimal, as the design methodology does not, and cannot, consider the semiactive control device's inherent passivity constraints — *i.e.*, that it can only exert dissipative forces regardless of the command it is given. For that reason, the design of control with such a method is inconsistent in the minimization of the objective functions. The result of such inconsistency is the use of heuristic approaches that usually involve large scale parametric studies [16, 17] and is not guaranteed optimality.

Model predictive control (MPC) has been employed for control design in different applications. MPC has been widely adopted in industrial engineering in the past three decades [18]. The method is an iterative scheme that seeks to control the future outputs of a dynamical system by using a model to predict those outputs throughout a finite horizon in the future. The MPC scheme attempts to optimize the future outputs of the dynamical system by manipulating the future inputs, given the current state of the system. When the optimal sequence of future inputs is determined, only the first input is implemented and the whole process is repeated again to determine the next optimized input. Today, the MPC method lends itself to different control applications, including applications in automotive, aerospace and process industries.

The main idea of MPC is minimizing a quadratic cost function written in terms of the predicted states and control input in the future time steps, which is why the method is sometimes called in the literature the *receding horizon method*. The number of steps used to predict system states in the future is called the *prediction horizon*, whereas the number of steps over which a control input is considered is called the *control horizon*. The control and prediction horizons for this study are assumed to be equal, which is the case in many applications of MPC. The resulting optimization problem is a quadratic programming problem in terms of the control inputs at each time step of the control horizon. This optimization process is repeated at each time step, which requires that the system analysis be performed fast enough so that the result can be obtained during the sampling time and the control input can be implemented in real time. Although, this can readily be implemented for simple models with a few degrees-of-freedom (DOFs), it can be computationally expensive for higher order or nonlinear systems. Standard MPC has been successfully applied for active control of civil structures, but cannot be easily and directly applied to semiactive strategies because of the nonlinear inequality constraint that arises from the passivity requirements of controllable passive devices.

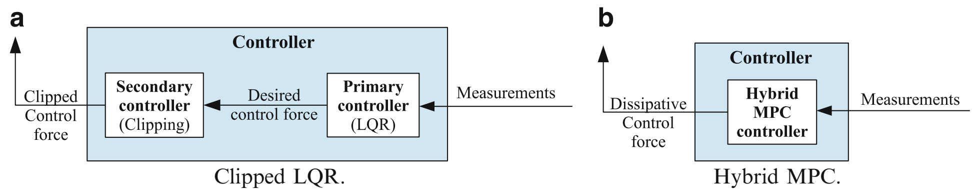


Fig. 4.2 Semiactive control strategies

The application of MPC in structural control has been investigated previously by Mei et al. [19]. In their study, the potential of MPC in structural control was demonstrated on building models with active tendon systems with full state feedback, later extended [20] to use acceleration feedback. Those implementations are, however, not suitable for structures controlled with semiactive devices as they fail to handle the nonlinear passivity constraints that characterize the behavior of such devices.

Hybrid systems are time evolving systems that exhibit mixed dynamics due to interacting physical laws, logical conditions and/or different operating modes [21]. Hybrid systems can be represented in different forms, such as piecewise affine functions [22], discrete hybrid automata [23] and mixed logical dynamical (MLD) system models [24] — all mathematically equivalent [25]. In this paper, MLD systems are used to model structures controlled with a semiactive device as a hybrid system.

A *hybrid system model* is well suited for the design of semiactive control as it can accurately reflect the nonlinear constraints of a semiactive device by introducing *auxiliary variables* into the system model. Hybrid MPC is an MPC scheme that finds optimal control strategies for a hybrid system model. Optimization of this control results in a mixed integer quadratic programming (MIQP) problem, which can be solved numerically to find the optimal control input. The resulting control force satisfies the passivity constraint and no longer needs clipping as shown in Fig. 4.2b. The on-line implementation of hybrid MPC must solve the optimization problem during the sampling time of the system, which is a computationally challenging problem with a complex system model and/or anything but the shortest time horizons. On the other hand, an off-line implementation will carry out most of the computations *a priori*, resulting in less demanding computation, such as a table look-up, to be carried out in real time during the sampling time.

As shown subsequently in the numerical example, hybrid MPC is readily applied to structural systems with a few degrees-of-freedom. In contrast with control laws derived from unconstrained system models (*e.g.*, clipped LQR), hybrid MPC produces nonlinear state feedback control laws that can achieve significantly better performance for some control objectives (*e.g.*, the reduction of absolute accelerations). Despite the nonlinear nature of the control laws, it is found that they satisfy the homogeneity property (*i.e.*, they scale linearly when the state vector is scaled). This property can be exploited to reduce the dimension of table look-ups. The resulting off-line implementation of hybrid MPC is orders of magnitude faster than the on-line implementation, which enables rapid computation of response statistics from a Monte Carlo simulation with stochastic excitation. Compared to clipped LQR, hybrid MPC is found to be more consistent in the reduction of the objective functions, which can help avoid using heuristic approaches and parametric studies in the structural control design. However, it is computationally expensive to design for large structural systems with the current implementations and optimization tools; further research is required to facilitate application to more complex structures.

4.3 Formulation

Consider a structure with n degrees of freedom that is subjected to an external excitation and is controlled by some devices to reduce its response. The equations of motion for such a structure can be written in matrix form as:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \overline{\mathbf{B}}_{\mathbf{w}}\mathbf{w}(t) + \overline{\mathbf{B}}_{\mathbf{u}}\mathbf{u}(t) \tag{4.1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the structural system, respectively; $\mathbf{x}$ is the displacement vector of the structure relative to the ground; $\mathbf{w}$ is the external excitation (*e.g.*, ground acceleration or wind forces) with influence matrix $\overline{\mathbf{B}}_{\mathbf{w}}$; $\mathbf{u}$ is the damping force exerted by the control device with influence matrix $\overline{\mathbf{B}}_{\mathbf{u}}$. The same set of equations can also be written in continuous-time state space form as follows:

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}_{\mathbf{w}}\mathbf{w}(t) + \mathbf{B}_{\mathbf{u}}\mathbf{u}(t) \tag{4.2}$$

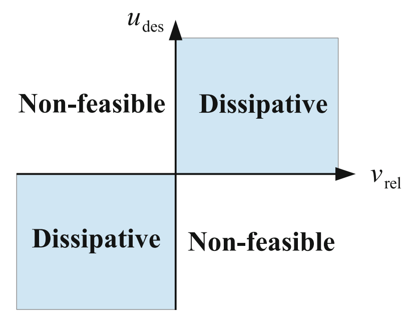


Fig. 4.3 Idealized model of the passivity constraint for controllable damping devices

in which

$$\mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{M}^{-1}\mathbf{K} & \mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{B_w} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\overline{\mathbf{B}}_\mathbf{w} \end{bmatrix} \quad \text{and} \quad \mathbf{B_u} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\overline{\mathbf{B}}_\mathbf{u} \end{bmatrix}$$

where $\mathbf{z}$ is the state vector, $\mathbf{A}$ is the state matrix, $\mathbf{B_w}$ is the influence matrix of the excitation and $\mathbf{B_u}$ is the influence matrix of the control force.

4.3.1 *Clipped-Optimal Control*

Generally, the optimal control design for a linear unconstrained system can be obtained using LQR, where the objective is to minimize an infinite-horizon quadratic performance index of the form:

$$J = \int_0^\infty \left[\mathbf{z}^\mathrm{T}\mathbf{Q}\mathbf{z} + 2\mathbf{z}^\mathrm{T}\mathbf{N}\mathbf{u} + \mathbf{u}^\mathrm{T}\mathbf{R}\mathbf{u}\right] \mathrm{d}t \tag{4.3}$$

in which $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{N}$ are weighting matrices for the different objective terms. If the external excitation is assumed to be white noise with zero mean, the certainty equivalence property [26, 27] of stochastic control theory will apply, and the stochastic optimal control is equivalent to the optimal control obtained from deterministic analysis that assumes an initial condition and ignores the excitation. The LQR control design is a quadratic programming problem, whose optimal solution is a linear state feedback controller [28] with feedback gain $\mathbf{K}_{\mathrm{LQR}}$:

$$\mathbf{u}_{\mathrm{des}}(t) = -\mathbf{K}_{\mathrm{LQR}}\mathbf{z}(t) = -[\mathbf{R}^{-1}(\mathbf{B}^\mathrm{T}\mathbf{P} + \mathbf{N}^\mathrm{T})]\,\mathbf{z}(t) \tag{4.4}$$

where $\mathbf{P}$ is the positive definite solution of the algebraic Riccati equation given by:

$$(\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{N}^\mathrm{T})^\mathrm{T}\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{N}^\mathrm{T}) - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^\mathrm{T}\mathbf{P} + \mathbf{Q} - \mathbf{N}\mathbf{R}^{-1}\mathbf{N}^\mathrm{T} = 0 \tag{4.5}$$

Assuming the states of the structural system (*i.e.*, displacements and velocities) can be measured or estimated, then (4.4) can be used to determine the required control input, and the device can be commanded with the desired damping force. However, semiactive devices can only exert dissipative forces; any non-dissipative forces commanded will not be realized. For that reason, the previous design does not generally result in optimal control forces for controllable passive devices unless, of course, the desired control forces are purely dissipative. This technique is called the *clipped-optimal control* algorithm [8] because a primary controller is obtained assuming unconstrained damping forces (*i.e.*, the passivity constraints of semiactive device is ignored in this stage), and the device will only exert the desired forces that are dissipative. This behavior can be modeled in simulations as if a secondary controller exists that clips the nondissipative control forces.

Different semiactive devices have different constraints on the achievable damping forces; however, they all share the passivity constraint. For the sake of generality, an idealized model is adopted for modeling the passivity constraint, as illustrated graphically in Fig. 4.3. It is assumed that all dissipative forces are realizable by the device and all nondissipative forces are infeasible. The passivity constraint is a nonlinear constraint that can be written as:

$$u_{\mathrm{des}} \cdot v_{\mathrm{rel}} \geq 0 \tag{4.6}$$

where u_{des} is the desired control force and v_{rel} is the relative velocity across the semiactive device (chosen with sign convention such that a positive dissipative force resists motion in the positive velocity direction). The resulting control force, that the semiactive device will realize, can be calculated as follows:

$$u_{\mathrm{sa}} = u_{\mathrm{des}} \cdot H[u_{\mathrm{des}} \cdot v_{\mathrm{rel}}] = \begin{cases} u_{\mathrm{des}}, & u_{\mathrm{des}} \cdot v_{\mathrm{rel}} \geq 0 \\ 0, & u_{\mathrm{des}} \cdot v_{\mathrm{rel}} < 0 \end{cases} \tag{4.7}$$

where $H[\cdot]$ is the Heaviside step function.

4.3.2 Hybrid MPC for the Design of Semiactive Structural Control

Generally, a discrete time form of a mixed logical dynamical system can be written as [21]:

$$\mathbf{z}_{k+1} = \tilde{\mathbf{A}}\mathbf{z}_k + \tilde{\mathbf{B}}_{\mathbf{u}}\mathbf{u}_k + \tilde{\mathbf{B}}_{\delta}\boldsymbol{\delta}_k + \tilde{\mathbf{B}}_{\mathbf{v}}\mathbf{v}_k \tag{4.8a}$$

$$\mathbf{E}_{\delta}\boldsymbol{\delta}_k + \mathbf{E}_{\mathbf{v}}\mathbf{v}_k \leq \mathbf{E}_{\mathbf{u}}\mathbf{u}_k + \mathbf{E}_{\mathbf{z}}\mathbf{z}_k + \mathbf{E}_0 \tag{4.8b}$$

where $\mathbf{z}_k$ is the state vector at time $k\Delta t$; and Δt is the sampling time of the system. $\boldsymbol{\delta}_k$ and $\mathbf{v}_k$ are auxiliary vectors that are binary indicator (*i.e.*, 0 or 1) and real vector functions, respectively, of the states and inputs, and are governed by the inequality in (4.8b). It will be shown later in the numerical example that the nonlinear passivity constraint described in (4.6) can be transformed to linear inequalities as in (4.8b). Note that the certainty equivalence is assumed, similar to the previous section, where the optimal stochastic control can be obtained from deterministic analysis that assumes an initial condition and ignores the excitation.

An MPC scheme based on hybrid model (4.8), can be used to determine the optimal input for semiactive control devices [29]. Such a method is denoted, herein, a *hybrid MPC*. The main idea is to minimize a quadratic cost function similar to the one used in (4.3) for LQR design; however, this scheme should consider the hybrid system model in order to account for the passivity constraint. Since the hybrid system used here is in discrete form, hence the cost function must be written as a summation for p steps, instead of an integral, such as:

$$J(\boldsymbol{\xi}, \mathbf{z}_0) \triangleq \mathbf{z}_p^{\mathrm{T}}\mathbf{Q}_p\mathbf{z}_p + \sum_{k=1}^{p-1}\left[\mathbf{z}_k^{\mathrm{T}}\mathbf{Q}\mathbf{z}_k + 2\mathbf{z}_k^{\mathrm{T}}\mathbf{N}\mathbf{u}_k + \mathbf{u}_k^{\mathrm{T}}\mathbf{R}\mathbf{u}_k\right] \tag{4.9}$$

where $\boldsymbol{\xi} = [\mathbf{u}_0^{\mathrm{T}}, \boldsymbol{\delta}_0^{\mathrm{T}}, \mathbf{v}_0^{\mathrm{T}}, \ldots, \mathbf{u}_{p-1}^{\mathrm{T}}, \boldsymbol{\delta}_{p-1}^{\mathrm{T}}, \mathbf{v}_{p-1}^{\mathrm{T}}]^{\mathrm{T}}$ is a vector that concatenates the optimization parameters together, including both the control inputs and the auxiliary variables at all time steps within the horizon [29]; and $\mathbf{z}_0$ is the current state vector of the system. The first term in (4.9) is the terminal cost term, which is added to emulate an infinite horizon cost function, where $\mathbf{Q}_p$ is the solution of the Riccati equation associated with the discrete LQR problem that minimizes an infinite horizon quadratic cost as in:

$$\min_{\mathbf{K}_{\mathrm{LQR}}} \sum_{k=1}^{\infty}\left[\mathbf{z}_k^{\mathrm{T}}\mathbf{Q}\mathbf{z}_k + 2\mathbf{z}_k^{\mathrm{T}}\mathbf{N}\mathbf{u}_k + \mathbf{u}_k^{\mathrm{T}}\mathbf{R}\mathbf{u}_k\right] \tag{4.10}$$

Note that a horizon of $p = 1$ step results, then, in a control identical to clipped LQR. Cost function (4.9), which is quadratic, must be minimized subject to the linear equality in state equation (4.8a) and to inequality constraints (4.8b), written in terms of both binary and real variables. The resulting optimization problem is a MIQP problem that, when solved, will provide the optimal control input [21].

4.4 Numerical Example

In this section, a numerical application to a single degree of freedom (SDOF) structural model is presented to demonstrate the potential of hybrid MPC in optimization of semiactive control design. Control designs were performed using both clipped

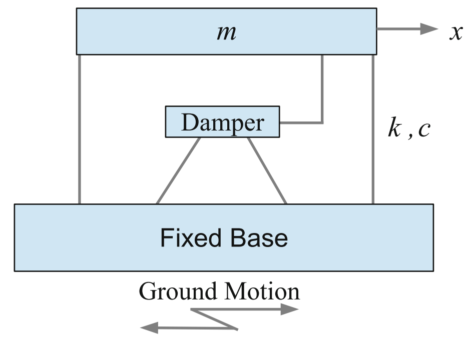


Fig. 4.4 SDOF structure

LQR and hybrid MPC; their respective control performances are compared for various ground motion records and for a Gaussian white noise excitation. The SDOF structural model considered here is shown in Fig. 4.4 with equation of motion:

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{x}_{\mathrm{g}} - u \tag{4.11}$$

where $\ddot{x}_{\mathrm{g}}$ is the ground acceleration.

The model parameters used for this structure are $m = 100\,\mathrm{Mg}$, $k = 3.948\,\mathrm{MN/m}$ and $c = 62.833\,\mathrm{kN{\cdot}s/m}$, which results in natural frequency $\omega = 2\pi$ rad/s and damping ratio $\zeta = 0.05$.

First, semiactive control is designed using the clipped LQR strategy discussed previously in section 4.3.1. The objective of the control design is the minimization of the absolute acceleration of the structure; this is accomplished by using the weighting matrices:

$$\mathbf{Q} = \begin{bmatrix} \omega^4 & 2\zeta\omega^3 \\ 2\zeta\omega^3 & 4\zeta^2\omega^2 \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} \omega^2/m \\ 2\zeta\omega/m \end{bmatrix} \quad \text{and} \quad \mathbf{R} = m^{-2}$$

The dissipativity constraint of the semiactive device can be incorporated into the hybrid MPC formulation (which is not possible for the design of the primary LQR controller for clipped-optimal control). Additionally, without loss of generality, the optimization converges faster with limits on the responses and the control force. Together, these result in three pairs of inequality constraints. **(1)** The nonlinear passivity constraint in this case is $u\dot{x} \geq 0$. Using auxiliary binary variables $\delta_{\mathbf{z}_k} \equiv \delta_{\dot{x}_k} = H[\dot{x}_k]$ and $\delta_{u_k} = H[u_k]$, the passivity constraint can be expressed as a single equality constraint $\delta_{\dot{x}_k} = \delta_{u_k}$ or, to be consistent with the inequality constraints in (4.8b), as the pair of inequalities $\delta_{\dot{x}_k} \leq \delta_{u_k}$ and $\delta_{\dot{x}_k} \geq \delta_{u_k}$. **(2)** A constraint on the velocity can be expressed as a single nonlinear inequality $|\dot{x}_k| \leq v_{\max}$ or, amenable for use in (4.8b), a pair of linear inequalities $\dot{x}_k \leq v_{\max}\delta_{\dot{x}_k}$ (constraining $\dot{x}_k$ when it is positive) and $\dot{x}_k \geq -v_{\max}(1-\delta_{\dot{x}_k})$ (when $\dot{x}_k$ is negative) using the auxiliary binary variables. **(3)** A constraint on the control force can be expressed similar to the velocity: nonlinear $|u_k| \leq u_{\max}$ or the linear pair $u_k \leq u_{\max}\delta_{u_k}$ and $u_k \geq -u_{\max}(1-\delta_{u_k})$. In this example, $v_{\max} = 10$ m/s and $u_{\max}/m = 100$ m/s^2. The resulting MLD system, similar to (4.8), is then:

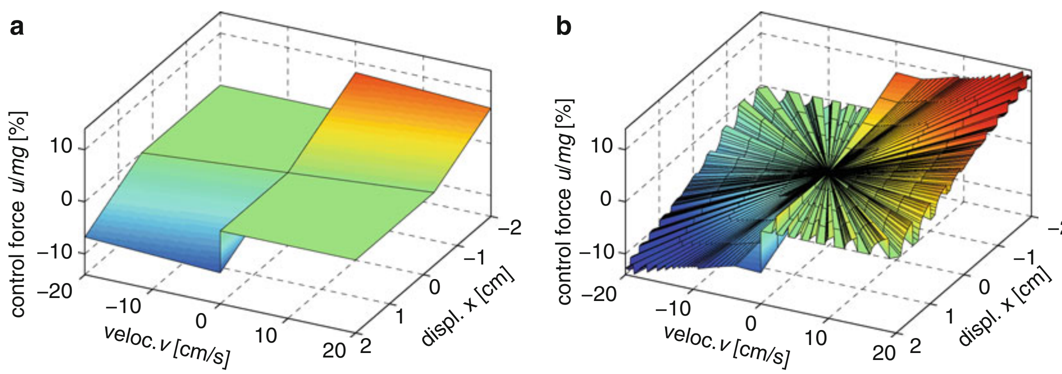
$$\mathbf{z}_{k+1} = \tilde{\mathbf{A}}\mathbf{z}_k + \tilde{\mathbf{B}}_u u_k \tag{4.12a}$$

$$\mathbf{E}_{\boldsymbol{\delta}}\boldsymbol{\delta}_k \leq \mathbf{E}_u u_k + \mathbf{E}_{\mathbf{z}}\mathbf{z}_k + \mathbf{E}_0 \tag{4.12b}$$

where

$$\boldsymbol{\delta}_k = \begin{bmatrix} \delta_{\dot{x}_k} \\ \delta_{u_k} \end{bmatrix} \in \{0,1\}^2, \quad \mathbf{E}_{\boldsymbol{\delta}} = \begin{bmatrix} -v_{\max} & 0 \\ v_{\max} & 0 \\ 0 & -u_{\max} \\ 0 & u_{\max} \\ 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad \mathbf{E}_u = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{E}_{\mathbf{z}} = \begin{bmatrix} 0 & -1 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{E}_0 = \begin{bmatrix} 0 \\ v_{\max} \\ 0 \\ u_{\max} \\ 0 \\ 0 \end{bmatrix}.$$

In order to design the control using hybrid MPC, an optimization problem as in (4.9) must be solved subject to equality constraint (4.12a) and inequality constraint (4.12b). The control for this structural model was designed using both clipped LQR and hybrid MPC; the resulting control forces as functions of the state vector are shown in Fig. 4.5. The hybrid system model of the SDOF structure was generated in MATLAB® using a toolbox called YALMIP [30], which can be used for fast



Clipped LQR. Hybrid MPC.

Fig. 4.5 Comparison of control input for SDOF system

Table 4.1 Minimum cost for various hybrid MPC prediction horizons

p	Cost
1	41.49[a]
5	44.19
10	40.55
15	33.17
20	31.49
25	31.22
30	31.01
40	30.93
50	30.84

[a] =CLQR

prototyping of optimization problems, and is used here to solve the resulting MIQP problems. This toolbox relies on one of several supported external solvers to perform the optimization. In this study, two different solvers were used: IBM CPLEX [31] and Gurobi Optimizer [32]. As a verification of the numerical optimization, both solvers were found to give the same solution for the MIQP problems involved in the presented examples.

For hybrid MPC, a suitable prediction horizon needs to be selected. A parametric study was performed, using the Northridge 1994 (Rinaldi) earthquake record as the excitation, to study the effectiveness of the control obtained from hybrid MPC for different prediction horizons. For a time step of 0.02 secs, the parametric study is shown in Table 4.1. Based on the parametric study, the prediction horizon selected in this study is 30 steps, 0.02 secs each, which provides a good compromise between accuracy and computational expense.

The two control inputs obtained from clipped LQR and hybrid MPC show drastic differences. The control law obtained from LQR is a linear function of the states as in (4.4), and a significant part of it gets clipped when the linear control law commands non-dissipative forces as shown in Fig. 4.5a. On the other hand, the control obtained from hybrid MPC scheme is nonlinear and is generally more dissipative. The differences suggests that the hybrid MPC strategy is using the damping device more efficiently; however, in order to quantitatively compare both strategies, the controlled structure must be simulated under the effect of excitation.

Simulations were performed for the SDOF structure subjected to several historical earthquakes, including the 1940 El Centro and 1995 Kobe records. Simulation results are compared in Table 4.2, including peak responses, control force and values of the objective function. The results show that the hybrid MPC strategy results, consistently, in lower peak responses and cost values. Although, the control was designed to minimize the absolute acceleration, the results show that hybrid MPC can achieve significant reduction, compared to clipped LQR, in the displacements and velocities as well, which suggests that the clipped LQR strategy is not using the damping device effectively.

The previous simulations illustrate the efficacy of the proposed hybrid MPC for control design of structures subjected to earthquakes. It is also beneficial to examine the efficacy of the resulting control when the structure is subjected to wind loading. The loads induced on structures by winds are random and often assumed to be a wide-band stochastic process. Without loss of generality, the wind is approximated here as a Gaussian white noise. In order to quantify the control strategies' efficacy for wind response mitigation, a Monte Carlo simulation is carried out where the controlled SDOF model was subjected to 8 second samples of zero-mean Gaussian white noise excitation, which was a sufficient time for the system

Table 4.2 Comparison of peak responses, control force and cost values using historical earthquakes

	El Centro 1940			Kobe 1995		
	CLQR	HMPC	Δ [%]	CLQR	HMPC	Δ [%]
x_{max} [cm]	5.58	4.29	−23	6.77	4.73	−30
$\dot{x}_{max}$ [cm/s]	39.82	32.80	−18	34.08	24.12	−29
$\ddot{x}^{abs}_{max}$ [cm/s^2]	221.3	189.5	−15	268.5	202.7	−24
u_{max}/mg [%]	22.13	17.21	−22	27.25	18.99	−30
cost [m^2/s^3]	4.761	4.117	−14	3.761	2.804	−25

Note: Δ is the percent change from Clipped LQR to Hybrid MPC; negative numbers are improvements

Table 4.3 Root-mean-square response using Monte Carlo simulation with 1 million realizations

(a) Idealized model I

Response	CLQR	HMPC	Δ [%]
$\ddot{x}_{abs}$ [cm/s^2]	36.88	32.68	−11
u/mg [%]	4.078	3.472	−15
x [cm]	1.374	1.041	−24

(b) Idealized model II

Response	Passive-on	CLQR	HMPC	Δ [%]
$\ddot{x}_{abs}$ [cm/s^2]	37.79	38.95	33.56	−14
u/mg [%]	3.109	2.302	2.325	+1
x [cm]	0.488	1.215	0.849	−30

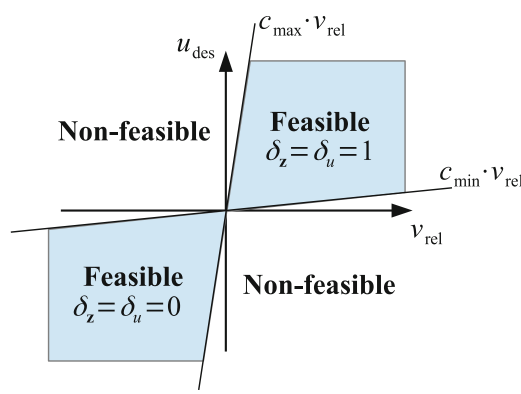


Fig. 4.6 More realistic model for passivity constraint

to reach a stationary response. Responses were obtained for 1 million randomly generated samples and output statistics were computed and compared. Root mean square (RMS) statistics for the stationary response are presented in Table 4.3.

In addition to the idealized model for the semiactive device, described in Fig. 4.3, another idealized model shown in Fig. 4.6 is also considered in the Monte Carlo simulations. This model considers a minimum and maximum energy dissipation in the damping device, which is considered a more realistic representation of a smart damper (*e.g.*, MR damper), and is helpful in this study to illustrate the inconsistency in the clipped LQR design of semiactive control. In this section, the model in Fig. 4.3 is denoted "idealized model I" and the model in Fig. 4.6 is denoted "idealized model II." For idealized model II, it is assumed the maximum and minimum damping coefficients are $c_{max} = 10^4$ kN·s/m and $c_{min} = 1$ kN·s/m which represents a real MR damper.

Despite the fact that hybrid MPC can be implemented in real-time for the SDOF model, the on-line implementation is not computationally practical for carrying out a Monte Carlo simulation with 1 million realizations. In order to make Monte Carlo simulation computationally less demanding, an off-line implementation of hybrid MPC is employed, which speeds up the computation by two orders of magnitude. The implementation suggested here uses table look-up and linear interpolation and exploits the property that the nonlinear control input in Fig. 4.5b is a homogeneous function of order 1 (*i.e.*, it scales linearly when the state vector scales) and, hence, can be written using radial coordinates in the form:

$$u_{HMPC}(x, \dot{x}) = u(r, \theta) = r \cdot \overline{u}(\theta), \quad \text{where } x = r\cos\theta \text{ and } \dot{x} = r\sin\theta \tag{4.13}$$

The nonlinear function $\overline{u}(\theta)$, shown in Fig. 4.7, can be determined *a priori* (before hybrid MPC is implemented), so that most of the computational effort is done off-line and only limited computations are performed during the simulations. The off-line implementation used here is based on a one-dimensional table lookup for the function $\overline{u}(\theta)$.

The results from Monte Carlo simulation for idealized model I (Table 4.3a) shows that hybrid MPC achieves 11% reduction in the stationary RMS absolute acceleration compared to clipped LQR. In addition, hybrid MPC reduces RMS displacement by 24%, compared to clipped LQR. Not only does the hybrid MPC strategy reduce the structural response,

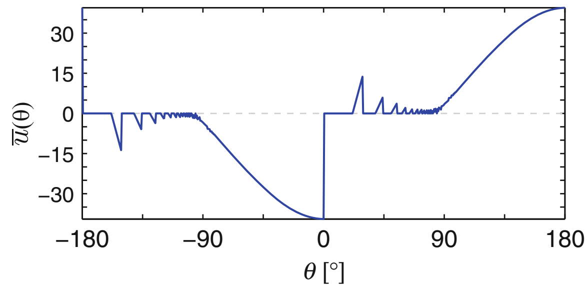


Fig. 4.7 Hybrid MPC control input for unit magnitude state vector

but it does so with a RMS device force that is smaller by 15%, which suggests that hybrid MPC is, in fact, making a more efficient use of the damping device. Results from idealized model II (Table 4.3b) show a similar pattern in the achievable reduction of response. Compared to clipped LQR, hybrid MPC reduces RMS displacements and absolute accelerations by 30% and 14%, respectively. The required damping force, on the other hand, has increased very modestly by 1%. Table 4.3b also shows the results for a passive-on case, for which the it was assumed the damping device is a viscous damper with a damping constant $c_{\max}$; this represents a semiactive device being operated at its maximum dissipative power. Although the passive-on case is more efficient in reducing displacement compared to the two semiactive control strategies, it must be noted that the main objective used in the control design in this example is the reduction of absolute accelerations. One of the most important observations in Table 4.3b is the fact that the passive-on case performs better than the clipped LQR strategy in reducing the absolute acceleration. In fact, this is the main reason why the design of semiactive control using clipped optimal strategy is performed using heuristic approaches involving large parametric studies, because the clipped optimal method is not consistent in minimizing the cost functions and the control optimality is not guaranteed.

4.5 Conclusions

In this paper, the design of semiactive structural control using hybrid MPC method was investigated. The performance of this control design was compared to clipped-optimal control design for a SDOF structural model. The following conclusions can be drawn from the results:

1. Clipped-optimal control strategies are based on unconstrained force models, which might not provide an optimal control performance for semiactive structural control.
2. The nonlinear passivity constraints that characterize semiactive dampers can be taken into account by using a hybrid system model, resulting in a set of linear constraints that makes optimization more tractable, which can be employed by MPC for control design.
3. The optimal control laws for semiactive structural control are generally nonlinear and can achieve higher performance compared to clipped-optimal control strategies for some control objectives.
4. Although better semiactive control designs can be achieved using hybrid MPC, the method is computationally intensive if an on-line implementation is used because MIQP problems have to be solved during the sampling time of the system. Off-line implementations can be more practical for large structural systems, where most of the computations can be carried out *a priori*, resulting in less demanding computations to be carried out during the sampling time.

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