

# Chapter 6

## Summary and Future Directions

### 6.1 Summary

In this thesis we investigated static decentralized resource allocation problems with strategic users. Unicast service provisioning and multi-rate multicast service provisioning problems arise in wired communication networks; power allocation and spectrum sharing arise in wireless communication networks. The models associated with unicast service provisioning, power allocation and spectrum sharing, and multi-rate multicast service provisioning capture generic issues that arise in market problems, public goods problems, and problems that are a combination of markets and public goods, respectively.

For each of the problems arising in wired networks we developed game forms/mechanisms and analyzed them in equilibrium. We proved that the proposed game forms possess the following properties. (P1) They implement in NE the social welfare maximizing correspondence. (P2) They are budget balanced at the allocations corresponding to all NE of the game induced by the mechanism, as well as at all feasible allocations corresponding to off equilibrium messages. (P3) They are individually rational, that is users voluntarily participate in the allocation process specified by the mechanism. For the problem arising in wireless networks we developed a game form that possesses properties (P2) and (P3), and implements in NE the Pareto correspondence.

Within the context of the above mentioned network problems, the game forms developed in this thesis are the only currently existing mechanisms that possess all the above-stated properties. The results on power allocation and spectrum sharing, as well as the results on multi-rate multicast service provisioning are also a contribution to the state of the art of implementation theory.

There are several problems of paramount importance that remain unsolved and are worthy of investigation. Below we discuss some of these problems.

## 6.2 Future Directions

### 6.2.1 Algorithmic Issues

Currently, we do not have algorithms (tâtonnement processes/iterative processes) for the computation of the NE of the games induced by the game forms we developed. The lack of algorithms for decentralized resource allocation problems where strategic users possess private information is a major open problem in implementation theory. The major difficulty in constructive algorithms that guarantee to converge to NE is the following. Consider an algorithm for a decentralized resource allocation problem where strategic users possess private information. At each stage of the algorithm each user updates his strategy/message. Since the users' utilities are not common knowledge, after each update a strategic user, say user  $i$ , can report any strategy he deems to be advantageous for himself; that is, user  $i$  can misreport/misrepresent his update and the other users can not check whether or not user  $i$  is following the rules of the algorithm. Consequently, the algorithm must provide incentives to the users/agents to follow its rules at each one of its stages. Such a provision of incentives must be based on all the information available at the current stage, and must, in general, take the whole future into account. We have not been able to discover an algorithm with the above feature. To the best of our knowledge, algorithms with the above feature are not currently available.

### 6.2.2 Dynamic Environments

In this thesis we focused on static decentralized resource allocation problems where the system characteristics (e.g. the network topology, the number of users, the users' utilities) do not change with time. The development of mechanisms (that is, situations where the network topology and resources, and/or the number of users, and/or the users' utilities vary with time) is an important open problem. The dynamic mechanisms currently available in the literature [1–3] are direct game forms/direct revelation mechanisms, and the existing results are on truthful implementation, which does not guarantee that for any environment all NE of the game induced by the direct revelation mechanism result in allocations that are in the choice set of the social choice rule/goal correspondence (see [4]). In our opinion, progress in the design of decentralized resource allocation mechanisms for dynamic environments will require a better understanding of the interplay between implementation theory and dynamic game theory. We also believe that resolving the key issues associated with the development of algorithms for static decentralized resource allocation problems (cf. Sect. 6.2.1) will help us understand better the nature of dynamic decentralized resource allocation problems where strategic users possess private information.

### 6.2.3 *Beyond Quasi-Linear Forms*

In this thesis, within the context of unicast and multi-rate multicast service provisioning we addressed resource allocation problems where the users' utilities are quasi-linear. In many real systems the network objective or the users' utilities are not separable in money (tax). Problems with non-quasi-linear objectives are harder to solve as they do not have a general structure or methodology for their solution, and have not received much attention in the mechanism design literature. Developing game forms/mechanisms that implement in some equilibrium concept non-quasi linear network objectives is a problem of fundamental importance. A step in this direction are the results reported in the power allocation and spectrum sharing problem we investigated in Chap. 4.

## References

1. Athey S, Segal I (2007) An efficient dynamic mechanism, preprint
2. Bergemann D, Valimaki J (2010) The dynamic pivot mechanism. *Econometrica* 78(2):771–789
3. Pavan A, Segal I, Toikka J (2011) Dynamic mechanism design: Revenue equivalence, profit maximization and information disclosure, preprint
4. Dasgupta P, Hammond P, Maskin E (1979) The implementation of social choice rules: some general results on incentive compatibility. *Rev Econ Stud* 46(2):185–216