Chapter 7
Is Problem Posing a Tool for Identifying and Developing Mathematical Creativity?

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Abstract The mathematical creativity of fourth to sixth graders, high achievers in mathematics, is studied in relation to their problem-posing abilities. The study reveals that in problem-posing situations, mathematically high achievers develop cognitive frames that make them cautious in changing the parameters of their posed problems, even when they make interesting generalizations. These students display a kind of cognitive flexibility that seems mathematically specialized, which emerges from gradual and controlled changes in cognitive framing. More precisely, in a problem-posing context, students' mathematical creativity manifests itself through a process of abstraction-generalization based on small, incremental changes of parameters, in order to achieve synthesis and simplification. This approach results from a tension between the students' tendency to maintain a built-in cognitive frame, and the possibility to overcome it, which is constrained by their need to devise mathematical problems that are coherent and consistent.

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Introduction

Is problem posing a tool for identifying and developing mathematical creativity? This is an intriguing question. Apparently, problem posing refers to generating something new or to revealing something new from a set of data, therefore somehow involving creativity. However, for a more structured answer, we have to investigate the nature of (mathematics) creativity. Consequently, we start by addressing some related issues.

Researchers have used the term "creativity" to characterize quite different behaviors, and this diversity of definitions inevitably led to ambiguity and controversy. Creativity comprises many discrete abilities that often do not correlate very much with each other (Guilford, 1967). Although the general public commonly associates creativity with novelty and surprise, many researchers defined creativity by highlighting two characteristics: originality and appropriateness (e.g., Amabile, 1989; Baer, 1993; Sternberg & Lubart, 1999). Baer and Kaufman (2005) identified some prerequisites that condition the ability to express creative behavior; they are intelligence, motivation, and suitable environments. Creativity can be latent—and, it may not show in the absence of an environment in which it can be nurtured and valued (Csikszentmihalyi, 1996; Gardner, 1993, 2006).

The above-mentioned authors based their definitions of creativity on the conclusions drawn from experiments/observations carried out across a variety of domains, such as arts, genetics, physics, or journalism. The diversity of domains poses the following important question: Is creativity domain specific, or general? This issue is strongly related to the idea of transfer: if creativity is domain general, an adequate training in a domain might be transferable to other domains, helping the trainees to solve any problem more creatively. However, a large body of research suggests that this may not be the case (e.g., Baer, 1993, 1998; Lubart & Guignard, 2004; Nickerson, 1999). In fact, the situation is even worse; the transfer might not work even within the same domain. For example, Baer (1996) investigated the effect of a training course focused on divergent-thinking skills related to poetry writing for middle school students. When after training, these students were asked to write poems, their poems were significantly more creative than those written by their peers from a control group. However, when the same students were asked to write short stories, both groups were nearly as creative. So, as the above researchers and many others (e.g., Dow & Mayer, 2004) have shown, the transfer of creative abilities does not automatically occur even within related sub-domains of the same domain. Therefore, it does make sense to speak about mathematics-specific creativity and to try to understand its specificity as compared to general creativity.

Mathematicians and mathematics educators alike have formulated various solutions to this issue over time. For example, Hadamard (1945/1954) associated creativity with an intuitive mathematical mind and believed that creative expression requires ample time for reflection and incubation of ideas. Hadamard mainly envisaged the creative behavior of the expert mathematician. This option corresponds to the idea that the essence of mathematics is what mathematicians do (Poincaré, 1913). We are equally concerned, however, with what students do when they behave creatively within a mathematical context and what the limits of these behaviors are. This is relevant for learning as we assume that the essence of mathematics is creative thinking, rather than just the identification of the right answer (Dreyfus & Eisenberg, 1996; Ginsburg, 1996).

When comparing students to experts, another dilemma emerges: does mathematical creativity only occur through the discovery of a completely new result, or can it also occur when re-discovering a fact already known by the scientific community? In other words, we can ask what relevance "novelty" has in the students' case. Which aspects are specific to the mathematical creativity of students? How can this be studied? Can creativity be developed?

The answers we arrive at in this study are strictly related to our target population: young students 10–13 years old, proficient in mathematics. We studied their creativity by using problem posing (PP) activities. We further explain the choice of PP as a tool in our research.

Traditionally, the context used to study student's creativity is problem solving (PS). Might it be the case that problem posing is more relevant than problem solving to study creativity? According to Sternberg and Lubart (1991), creative individuals not only solve problems, but also pose the right problems; therefore, the capacity to pose problems might be a sign of creativity. In a series of articles on discovering problems in art contexts, Getzels and Csikszentmihalyi highlighted differences in thinking between the case where the starting point was an already formulated problem, compared to situations in which the problem must be discovered or created (Csikszentmihalyi & Getzels, 1971; Getzels, 1975, 1979; Getzels & Csikszentmihalyi, 1976). In these articles, the ability to "discover" a problem was used as a primary category for analyzing creative processes. Extrapolating these findings to mathematics, we can infer that PP might be more significant than the PS in the study of mathematical creativity, even if a common definition of mathematics refers to it as a problem-solving domain. Other studies confirm this conclusion. For example, Smilansky (1984) showed that there is very low correlation between the abilities of mathematical PS and PP in a group of high school students and undergraduate students. Smilansky's conclusion is that PP is a task more significant than PS for the study of creativity. Starting from Smilansky's findings, we offered our students a context in which they posed and/or modified problems, in order to study their creativity.

If we agree that PP is relevant for the study of creativity, a second question is what taxonomy would be more effective to reveal creative behaviors. Typically, mathematical creativity is studied and assessed through the lenses of: fluency, flexibility, and novelty, the parameters conceptualized by Torrance (1974). We consider that both the study and the development of school creativity should be aligned to new scope and purpose. If in the eighth decade of the twentieth century the creativity focus was on theoretical studies, today, the knowledge society—characterized by complex dynamics and over-information—needs individuals, and especially leaders, able to anticipate changes and to take knowledgeable decisions under varying conditions that are hardly predictable (e.g., European Commission, 2003/2004, 2005; Hargreaves, 2003; Singer, 2006; Singer & Sarivan, 2006).

In other words, more than ever before, today's schools should help students to develop creative approaches as part of leadership qualities, especially in those who are promising high achievers. Previous studies on mathematical creativity in a PP context (Singer, 2012; Singer, Pelczer, & Voica, 2011; Singer & Voica, 2011, 2013; Voica & Singer, 2012, 2013) have concluded that a framework highlighting social integration and leadership could provide better information about students' creativity in a PP context.

We have seen that the transfer of creative abilities from one domain to another is less likely to appear spontaneously. We assume that the study of creativity in a broader, socially-oriented framework that faces opportunities for transfer within the training could offer more relevant data for contemporary research on creativity development.

We support this claim based on the conclusions of a study of Yuan and Sriraman (2011), regarding the achievements in PP activities of groups of students from the USA and China. These students performed several types of tests, including: a mathematics content test; a mathematical problem-posing test; Verbal *Torrance Tests of Creative Thinking* (TTCT) (where students were asked to think with words); and Figural TTCT (where students were asked to express their ideas by drawing pictures).

Yuan and Sriraman (2011) maintained that US students performed much better than Chinese students from the point of view of fluency, flexibility, and originality on the Verbal TTCT. This result is not surprising, if we relate it to the features of the teaching practice in the two countries. US students often work in groups, are involved in projects, and are encouraged to ask questions, to experiment, and to provide explanations (see, for example, National Council of Teachers of Mathematics, 2000). Therefore, US students seem more capable of expressing their ideas in words. Conversely, in the education system in China, where typical lessons are characterized by "order and routine" (Lim, 2007, p. 80), and teachers often maintain control by directly teaching to the whole class (Huang & Leung, 2004), the communication and interaction between students are not important, and the focus is mainly on factual knowledge. In addition, the Chinese language—based on ideograms—offers support for the recourse to drawings and pictures in explaining ideas—a hypothesis taken into account by some psychologists (e.g., Demetriou et al., 2005).

On the other hand, Yuan and Sriraman's study found "no significant correlations" between general Torrance creativity and PP abilities for US students, while for the students from China, PP abilities are "significantly correlated" with Verbal TTCT scores (Yuan & Sriraman, 2011, p. 25). This lack of consistencies between the two groups led us to two major ideas, which we will try to convey in this chapter.

A first claim is that Torrance's criteria do not represent the most suitable framework for the study of creativity in the context of PP. It seems that parameters related to classroom management activities and to students' communication skills are not highlighted enough in such tests. Therefore, we assume that a social-oriented framework is more appropriate for analyzing students' mathematical creativity.

A second claim is that mathematical creativity is of a special nature compared to creativity in general. This is used to explain why there were no significant correlations between TTCT results and PP abilities of the US students in the above-quoted study.

Our research tries to identify this special nature of mathematical creativity in students. Our preliminary studies led us to formulate the following hypothesis: in a PP context, students' mathematical creativity manifest itself through a process of abstraction-generalization based on small, incremental changes of parameters, in order to achieve synthesis and simplification. As a result, students expressed their creativity by making small-scale changes of the mathematical model of a problem, which resulted in maintaining control over the proposed problem.

In this chapter, we try to see if the above hypothesis is confirmed for our sample. More precisely, we seek an answer to the question: How does mathematical creativity manifest in 10- to 12-year-old students? If the hypothesis can be confirmed, it will once again result in the need for a new tool suitable for analyzing mathematical creativity.

Theoretical Background

Given the interdisciplinary nature of this study, we will discuss the theoretical background from four perspectives: mathematical creativity, problem posing, connections between mathematical problem posing and creativity, and cognitive flexibility.

Mathematical Creativity

The topic of mathematical creativity received much attention from researchers who focused on defining it, or on establishing criteria for its evaluation (see, for example, Ervynck, 1991; Freiman & Sriraman, 2007; Silver, 1997; Sriraman, 2004, 2009). The literature contains a variety of definitions and characterizations (e.g., Balka, 1974; Evans, 1964; Getzels & Jackson, 1962; Haylock, 1987; Jensen, 1973; Poincaré, 1948; Prouse, 1967). Earlier references to mathematical creativity came from the work of expert mathematicians like Poincaré and Hadamard (Hadamard, 1945/1954; Poincaré, 1948). Subsequently, various studies have identified certain behaviors that provide evidence of mathematical creativity in students. Haylock (1987) and Singh (1988) assessed mathematical creativity based on the three characteristics defined by Torrance (1974): fluency, flexibility, and novelty. The common interpretation is that these features represent, respectively: the number of identifiable changes in approaching a problem; the number of generated solutions; and the level of their conventionality (e.g., Ervynck, 1991; Leikin & Lev, 2007; Silver, 1997).

Balka (1974) synthesized another line of analysis: he considered convergent thinking—characterized by determining patterns, and divergent thinking—seen as formulating mathematical hypotheses, evaluating unusual mathematical ideas, sensing what is missing from the problem, and splitting general problems into specific sub-problems, as the main components of mathematical creativity. In this context, Haylock (1997) insisted that one of the key elements of creativity is the ability to overcome fixations in mathematical problem-solving (leading, for example, to breaking away from stereotyped solutions).

Problem Posing

There are different terms that are used in reference to problem posing, such as problem finding, problem sensing, problem formulating, creative problem-discovering, problematizing, problem creating, and problem envisaging (Dillon, 1982; Jay & Perkins, 1997). Because of this variety of meanings, different authors use different frameworks for studying PP activities. For example, Brown and Walter (1983/1990) looked at PP within a strategy focused on the phrase "what-if-not." This strategy assumes that, by discussing the significance of the problem components and by trying to modify this, students can come up with a deeper understanding of the problem, rather than just focusing on finding the solution.

Stoyanova and Ellerton defined PP as "the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems" (Stoyanova & Ellerton, 1996, p. 518). In their paper, problem-posing situations were classified into three categories: free, structured, or semistructured (Stoyanova & Ellerton, 1996). In the present study, we adopt Silver's (1994) less restrictive definition, in accordance with which, problem posing refers to both the generation of new problems and the re-formulation of given problems.

Mathematical Problem Posing and Creativity

The literature on PP shows that this activity is important from various perspectives and emphasizes connections between PP and creativity. Some researchers have reported a positive relationship between mathematics achievement and problem-posing abilities (English, 1998; Leung & Silver, 1997). Other researchers (e.g., Cai & Cifarelli, 2005; Singer, Ellerton, Cai, & Leung, 2011; Singer, Pelczer, & Voica, 2011) claimed that instruction that includes problem-posing tasks (problem modification tasks included) can assist students to develop more creative approaches to mathematics.

There are also researchers who have expressed doubts regarding the connection between creativity and PP. For example, Yuan and Sriraman (2011) concluded that "there might not be consistent correlations between creativity and mathematical problem-posing abilities or at least that the correlations between creativity and mathematical problem-posing abilities are complex" (Yuan & Sriraman, 2011, p. 25). However, other studies, for instance Haylock (1997) and Leung (1997), who did not agree that there is a correlation between creativity and problem posing in mathematics, did not consider instruction. From an empirical perspective, Silver (1997) suggested a position that supports our hypothesis: that any relationship between creativity and problem posing might be the product of previous instructional patterns.

Cognitive Flexibility

Cognitive flexibility of a person can be defined as the dynamic activation and modification of cognitive processes in response to changes in task demands, which results in representations and actions that are well adapted to the altered task and context (Deák, 2004). In other words, cognitive flexibility addresses the readiness with which a person's concept system changes selectively in response to appropriate environmental stimuli (Deák, 2004; Scott, 1962). Pragmatically, cognitive flexibility refers to a person's ability to adjust his or her working strategies as task demands are modified (Krems, 1995; Spiro, Feltovich, Jacobson, & Coulson, 1992).

In an organizational context, cognitive flexibility is conceptualized as consisting of three primary constructs: cognitive variety, cognitive novelty, and change in cognitive framing (Furr, 2009). Cognitive variety refers to the diversity of mental templates for problem solving that exists in an organization (Eisenhardt, Furr, & Bingham, 2010), or to the diversity of cognitive pathways or perspectives (Furr, 2009). Cognitive novelty refers to the concepts pertaining to the subject of study and the overall mastery of content (Orion & Hofstein, 1994), or to the addition of external perspectives (Furr, 2009). One's previous experiences, particularly successful experiences, may lead to the phenomenon called cognitive framing: it manifests itself through a person's persistence in trying to solve a new problem by using a certain strategy, previously practiced (Goncalo, Vincent, & Audia, 2010). In certain situations, it denotes an algorithmic fixation (in terms of Haylock, 1997); in these cases, the only possible way to overcome this is to change the thinking frame.

Methodology

Sample

The participants in this research are students in grades 4–6 (10–13 year-olds), winners of a two-round national mathematics competition (the Kangaroo contest). Within this competition, the participants were supposed to choose and solve 30 out of 40 multiple-choice problems (with five possible answers, only one being correct) in 75 minutes. In the Kangaroo contest the problems are graded 3, 4, or 5 points, incorrect answers are penalized with a quarter of the score, and non-responses are ignored. These regulations are publicly available and are reinforced before the test session.

After the first round (involving approximately 60,000 students in grades 4–6, which represents approximately 10% of the Romanian school population for these grades), the top 10% of students attending the first round qualified for the second round (where the competition regulations are the same as in the first round, but the problems are much more difficult). The winners of the second round attended a summer camp.

Due to the selection process, we consider that the participants in the camp (280 students from a total of 60,000) are high achievers or excelling in mathematics. During the camp, the authors of this chapter—as invited professors—launched a call for problems to the students. The 48 students who voluntarily responded to this call represent our sample.

Data Collection

The 53 problems posed by the students of our sample were initially assessed by two reviewers (other than the authors), who worked independently. They graded the problems from 1 to 10, based on the following criteria: the statement completeness, the correctness of the posed solution, and the novelty (expressed as the "distance" between the proposal and the types of "usual" problems of school textbooks and auxiliaries). Subsequently, the two experienced teachers who served as problem reviewers shared the scores they had given and, in each case where they found significant differences, they discussed and reached a consensus.

Following this preliminary assessment, we chose to interview 19 of the students who responded to the call for problems; for this selection, we took into account the scores given by the reviewers, but also some surprising or interesting aspects we had noted in the students' comments and solutions. In some cases, we decided to interview a certain student even if his/her proposal was not highly ranked because a particular aspect of that proposal (e.g., an unusual context for a problem, or unusual comments) suggested that the student showed creative potential. Briefly, we chose the students for interviews either based on the intrinsic qualities of the highly ranked posed problems, or based on the hints that we found in students' proposals that might illuminate the mental mechanisms activated in PP.

We have included the texts of the 20 problems posed by the students invited to the interviews in the Appendix (one of the students suggested two problems). In the following sections, we refer to these problems using the Appendix ordering numbers, but we also quote the problem text when this is needed to enable the reader to follow the line of argument more easily.

Each interview lasted between 10 and 40 minutes. The interviews were video-recorded and subsequently transcribed. Before the interviews, we asked students to re-read the problem they initially posed. We structured the interview protocol around questions such as: What inspired you to pose this problem? How might you change your posed problem? Can you pose a simpler/more complicated problem? What did you change compared to your initially posed problem? How would you proceed to pose new variants of the problem? Therefore, during the interviews, students were given the opportunity to pose new problems, or to modify their initially posed ones. Thus, the interviewed students generated other 26 new problems.

We used the protocol for guidance during the interviews, but we encouraged students to express their ideas as freely as possible. In some cases, the interview departed from the protocol because we sought to identify students' thinking patterns. Thus, we got information about the models students used as starting points in a PP activity (if any), their strategies for generating and correlating problem givens, their perceptions concerning the difficulty and complexity of their posed problems, and finally, the metacognitive processes they activated when posing and solving problems. Based on these, we tried to outline a cognitive profile in problem posing and solving situations for each selected student. We then compared the conclusions formed from the interviews with the participants' behaviors in the Kangaroo national contest. For this, we analyzed students' answers from the contest, obtaining information on: series of correct/incorrect answers, types of wrong answers, types of mistakes, types of preferred/avoided problems. We then compared the results obtained by the students of our sample with the statistical results of all participants in the competition. These comparisons helped us to identify possible correlations between a student's PP-PS cognitive profile and his/her options for posing a certain type of problem, thus validating the identified profile over time.

Therefore, the data analyzed in this chapter come from the following sources: students' posed problems (initially posed problems, problems posed during the interviews, problems obtained by modifying the initial ones), interviews, and statistical databases. Each of the sources was analyzed from several perspectives. Following this multiple-level analysis, we gathered as much information as possible in relation to students' creative behavior by examining students' preferences for particular mathematical domains, their mathematical abilities, students' strategies in PP and PS, and their intra- and inter-personal approaches.

Data Analysis Framework

From our analysis of primary data, we found that, when children posed problems, they involuntarily resorted to their teacher's model. Inevitably, large parts of the students' proposals were tasks that had a specific target audience (of colleagues, friends, competitors, even taking into account different levels of competency of those audiences). As a result, the posed problems did not just represent students' theoretical approaches, but rather encompassed an ensemble of relationships between the poser and potential solvers, expressing a poser's need to feel integrated within a structured social ensemble. This finding, observed in students of different ages and with varying mathematical abilities, emphasizes the fact that, unlike PS, PP activities have an important component of inter-personal interaction which can significantly influence the quality of students' posed problems. In addition, from the perspective of contemporary society, we are interested in those capabilities that enable students to manage their own learning and to be able to identify, pose, and solve problems arising in unpredictable contexts (e.g., Singer, 2006, 2007).

For these reasons, we investigated the relationship between problem posing and mathematical creativity in terms of cognitive flexibility in organizational contexts. In a problem-posing context, we consider that a student exhibits cognitive flexibility when the following three conditions are fulfilled (Pelczer, Singer, & Voica, 2013a): the student poses different new problems starting from a given input (i.e., cognitive variety), generates new proposals that are far from the starting item (i.e., cognitive novelty), and he/she is able to change his/her mental frame related to the proposal, if necessary, in generating and solving problems (i.e., change in cognitive framing).

Criteria for Data Classification

We used the following criteria for classifying students' posed problems: the involved mathematical domain, the coherence, and the consistency of the problem. Further details of these will be provided in the sections which follow.

A first classification concerns the mathematical content of these posed problems. Within this criterion, we used the following categories:

- **Numerical computing**. This includes problems containing instruction(s) that refer to numerical calculations explicitly stated in the text. It may include percentage calculation or computation with fractions.
- **Relations**. Here are problems that use specific properties of sets of numbers, for example: divisibility on **N** or **Z**, or order relation on **Q**.
- **Equations**. Problems where equation solving is essential (even if this is not formalized) are included here. We also included here problems where unknown data can be found using a scheme or a graphical representation.

- **Algebraic computing**. Here are problems involving general features of numbers or abstract schemas for solving, which lead to generalizations that can be expressed by algebraic formulas.
- **Change of patterns**. This includes problems that need to be understood and analyzed in their kinematic development, as they assume successive stages and understanding transitions from one stage to another.
- **Handling data**. This category contains problems in which the analysis of data sets or their distribution is relevant for the solution.
- **Geometry**. Here there are problems in which the students effectively used specific geometric properties (such as parallelism, perpendicularity, and congruency).

The next categories for clustering students' proposals refer to the intrinsic qualities of a posed problem. Since a problem text is expressed in a specific language, we use two criteria—syntax and semantics—that are characteristic of language in a broad sense and used in both natural language and in artificial languages such as computer programming.

To characterize these two attributes, we have adopted the *problem-analysis framework* used by Singer and Voica (2013). According to this framework, the text of a problem contains, in general: a background theme, parameters, (numerical) data, one or more operating schemes (or, simply, operators), constraints over the data and operating schemes, and constraints that involve at least one unknown value of the parameter(s).

Concerning the syntax, we define *the coherence* of a problem, which refers to the rules and principles that govern the structure of a mathematical problem. Essentially, these rules and principles are:

- The following text components—givens, operations, constraints—are present;
- The following text components—givens, operations, constraints—are recognizable or identifiable;
- The givens are not redundant, or missing.

The syntax offers a formal valid shape of a problem, but does not provide any information about the meaning of the problem or the results of its solving. The meaning associated with a combination of text elements belongs to semantics.

Concerning the semantics, we define *the consistency* of a problem. This supposes the existence of meaningful links among the elements of the problem. More specifically:

- The problem data are not contradictory;
- The following text components—givens, operations, constraints are correlated;
- The components of the problem text satisfy a certain assumed mathematical model;
- The information provided leads to at least one solution of the problem (or to the proof that there is no solution).

Within the problems obtained by modifying a given problem, consistency also requires that:

- At least one of the mathematical elements of the starting problem is identifiable in the new problem.

We specify that not all syntactically correct problems are semantically correct. Many syntactically correct problems are nonetheless ill formed and are merely a combination of parts obeying some rules. Such problems may result in error-prone processing. In addition, it may not be possible to assign meaning to a syntactically correct problem, or the wording may be false.

Results

We used the above-mentioned criteria to analyze students' behavior in posing and solving problems based on the students' submitted problems and their answers during the interview sessions.

The Students' Posed Problems

Mathematical content. Table 7.1 presents the distribution of the 20 problems posed by the interviewed students according to the mathematical content criterion. (We remind the reader that the texts of these problems can be found in the Appendix.) For space reasons, the entire classification of the sample consisting of the 53 problems initially posed by all of the students can be found only in Figure 7.1. We relate our classification to the *National Assessment of Educational Progress 2011 Framework* (NAEP, 2011). The NAEP framework describes five mathematics content areas: number properties and operations, measurement, geometry, data analysis

Table 7.1
Distribution of the Problems Posed by the Interviewed Students, According to the Mathematical Content

Math content	Problem number (from the Appendix)	NAEP correspondent
Numerical computing	6, 8, 10	Number properties and operations
Relations	11	
Equations	3, 4, 5, 7, 9,19	Algebra
Algebraic computing	2, 12,15	
Change of patterns	13, 17, 20	
Handling data	1, 16	Data analysis and probability
Geometry	14, 18	Geometry

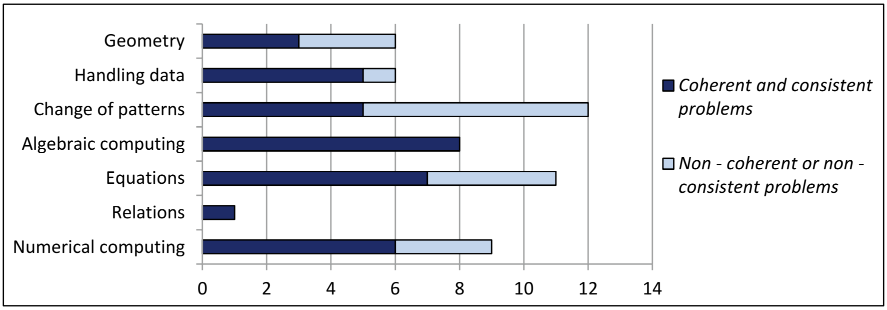

Figure 7.1. Classification of students' posed problems based on the criteria: coherence (mathematical syntax) and consistency (mathematical semantics).

and probability, and algebra. After analyzing students' posed problems, we concluded that the categories listed in Table 7.1, which subdivide some of the NAEP content areas, better describe students' proposals.

Some of the students' posed problems can be included in several content areas. To get a clearer picture of students' preferences for different content subareas, we sought to distribute the posed problems into disjoint classes. Therefore, for each problem, we tried to identify the depth of thought involved, considering both the wording and the problem solution. Subsequently, we checked (when necessary) if our framing matched the student's intention.

To illustrate the way we classified the problems according to their content, we provide below one significant example (problem #16, posed by Mihai, grade 6):

> Because the 6th grade students were the best, they received a prize consisting in one hour free on paintball field. The field has the dimensions 80 m × 120 m, and two people are able (and allowed) to shoot one another if they are at no more than 29 m distance. Prove that howsoever 26 students place themselves on the ground, at least 3 get shot.

Apparently, this problem is one of geometry. At a closer look, we may find that the essential element in its solution is to identify certain regularity in the arbitrary distribution of points inside a rectangle. Indeed, the interview revealed that Mihai has used a grid (he decomposed the rectangle into congruent squares) and made "order in disorder," applying the pigeonhole principle. Therefore, we classified this problem in the category *Handling data*.

Syntax and semantics. In line with recent perspectives on creativity, the outcomes of a creative process should have relevance within a community (scientific, cultural, organizational, etc.). More precisely, not every new product means creativity, unless it is socially valued (e.g., Gardner, 1993; Gardner, Csikszentmihalyi, & Damon, 2001). In particular, in a PP context, a creative mathematical product must be coherent and consistent, since these are minimal conditions for conventionally accepted "correctly formulated" problems.

Earlier in this chapter, we presented a brief description of *coherence* and *consistency* criteria; we now illustrate their application, taking as an example Problem 3 (posed by Malina, grade 4).

> In Princess Rose's jewelry box, there are sapphires, emeralds and rubies. 27 are not rubies, 31 are not emeralds and 32 are not sapphires. In total, there are 45 jewels. How many jewels of each kind does Princess Rose have?

We found that this problem was not coherent, since one of the givens (i.e., the total number of jewels) is redundant. On the other hand, we classified this problem as mathematically consistent because:

- The given data in the posed problem are not contradictory: for example, the numbers 27, 31, 32 are smaller than 45.
- The text elements satisfy a mathematical model: the sum of the numbers 27, 31, 32 is twice the number 45.
- The information given in the text lead to a solution of the problem: Princess Rose has 18 rubies, 14 emeralds, and 13 sapphires.

Figure 7.1 shows the classification of the 53 initially posed problems by coherent-consistent criteria, according to the mathematical content.

Most problems that are both coherent and consistent belong to the categories of Algebraic computing and Handling data, and the fewest are in the categories Change of Patterns and Geometry. (In this discussion, we did not take into account the category Relations, containing only one problem.)

What we hold from this classification is that some content areas seem "safer" in terms of the intrinsic qualities of the posed problems. In other words, coherent and consistent problems occur mainly in the areas of content that require certain formalism, precisely because this formalism provides some stability to a problem statement.

The Interviews

The analysis of the mathematical content and of the text characteristics (syntax and semantics) of the students' posed problems outlined a first overview of the PP *products*. To have a more nuanced understanding of the quality of these problems, we analyzed the interviews to get information on the PP *processes*.

Students' metacognitive strategies in problem posing. We carefully listened to the students' explanations about what they did to elicit a problem. In some cases, they were only able to explain how they had chosen the thematic context of the problem ("My roommates were talking about candies, so I came up with a problem about candies."). In most cases, however, we found that the students had adopted specific strategies for problem posing, which they managed to communicate. Two excerpts from the interviews that exemplify this fact are included below.

When asked how she came up with her problem (#3: "In Princess Rose's jewelry box there are sapphires, emeralds, and rubies. 27 are not rubies, 31 are not emeralds, and 32 are not sapphires. In total, there are 45 jewels. How many jewels of each kind does Princess Rose have?"), Malina explained:

> Malina (grade 4): "You must first establish the answer to the problem, and then you build the wording. You cannot go vice versa; no problem of this type can start otherwise."

Malina explained how she created her posed problem statement: she first decided on the numerical answer, and then she built the givens for the wording. Malina was very categorical in her claim probably because she was aware that the data cannot be random, they should verify certain constraints. In addition, although as a fourth grader she was not previously exposed to PP, she referred not only to her specific problem, but also to an entire class of problems that can be built in that way.

Radu started similarly in posing problem #18 (*Prove that any parallelogram can be divided in 16,384 congruent parallelograms*), but he went deeper into the eliciting process:

> Radu (grade 6): "A problem is made as follows: first we find a purpose: algebra, geometry … an idea … any problem must have a basic idea. After we find the idea, we develop: we add all sorts of tricks, we polish, we re-formulate, and we look for the right numbers. Unfortunately, we got tricks from experience: you cannot do your own problems if, in your turn, you didn't solve problems. We may borrow some ideas; we cannot do something 100% original."

Compared to Malina, Radu had a different approach: he said that the wording of a problem has to be built in successive steps, being modified by a kind of trial and error strategy ("we re-formulate, we look for suitable numbers"). The difference in approach might come from the age difference between the two students. Radu (grade 6) possessed mathematical knowledge certainly more developed than Malina's (grade 4). Radu was confident that, during the problem-solving process, he could anticipate constraints among the data, parameters, and operations, and that he could amend the wording to ensure problem consistency. Indeed, Radu's proposals showed that he had spent a lot of time in formulating and reformulating the problem (compared to other students in our sample). In addition, he engaged himself in qualitative analysis: in his problem, the choice of the number 16,384 was purpose-oriented—it is a perfect square big enough to remove any possibility of reasoning on a geometrical figure. The choice of this number shows that Radu actually developed a generalization (as confirmed during the interview). Radu's behavior in problem posing was of an expert type, as described by Silver and Marshall (1989). This was apparent also in his spontaneous development of explanations and "meta" comments.

As we have seen in the above examples, some students spontaneously shared their visions on what a problem should look like. These comments on the desirable qualities of a problem led us to the conclusion that these students developed metacognitive strategies, which they were able to make explicit. While expressing their opinions concerning the problem-posing process, students revealed a complex philosophy about PP. Some significant examples are included below.

> Radu (grade 6): "[The posed problem] must be original. But it's not worth to be original if it is too easy, so we have to give it difficulty."

Radu asserted that, for posed problems, originality was a necessary condition and, in addition, that a problem must have a degree of difficulty. Radu's claim, in correlation with his other assumptions, can be interpreted as an intuitive understanding of the need for consistency in a problem.

> Malina (grade 4): "Creation usually happens in an artistic composition. You can compose also math problems, but in this case, logic occurs, you have to be very logical and exact in explanations and calculations. It's complicated, because you must keep in mind certain rules of mathematics."

Malina compared PP with an artistic act. Actually, specific literature shows that metacognitive abilities can be related to creative thinking (e.g., Fasko, 2001). Starting from the art comparison, Malina emphasized the differences. She was aware that the wording of a problem must satisfy certain specific constraints that contribute to its mathematical consistency.

These examples serve to illustrate that, in the PP process, students in our sample frequently demonstrated metacognitive behaviors. This allowed them to *look from above* on how to pose a problem; as a result, students can develop strategies for selecting content and constraints to make the new posed problem consistent. Thus, they become able to evolve within the cognitive frames generated by their chosen problem models.

Students' need for social interaction in problem posing. Some students spontaneously included comments for possible collocutors in their posed problems. Others referred to such collocutors in their remarks during the interviews. This observation outlines a need for social interaction of these children through posing and solving problems.

For example, Cristiana (grade 6) seemed to be posing her problem (#13, the "look-and-say sequence") for a friend and displayed a protective role, revealed through her careful reflection on the problem difficulty. Cristiana added in Problem 13 an indication "you must empty your mind of all other mathematical information." When we asked her why she did this, she said:

> I thought that in this way a child would like to read so far.

Already at this step, she entered into the teacher's role and tried to bring both support and motivational elements to the potential solver. In that respect, she summed up how she came to understand the look-and-say sequence herself and tried to translate her own experiences into her proposal.

Cristiana formulated her proposals to provide some support to the solver. This case is not unique: other students also formulated questions keeping in mind the profile of potential solvers. The students who took into account the mathematical skills of their colleagues as potential solvers focused not only on the problem text, but also on how the other person was likely to decode the problem, a fact also noticed in other studies (e.g., Lowrie, 2002).

Unlike the cases described above, other students introduced some elements with the purpose of misleading the solver. Given some situations frequent in the teaching practice in Romania (see, for example, Pelczer, Singer, & Voica, 2013b), we concluded that, through the wording of their posed problems, these students were, in fact, mimicking their teachers' behavior and beliefs. One significant example of this type was Malina's comment about her problem:

> Malina (grade 4, referring to Problem 3): "Instead of saying directly that the yellow and the red ones are 39, I have complicated it, that the children think: we eliminate the blues, and we remain with the yellows and the reds."

In this example, we saw that Malina *intended* complicating this problem to mislead potential solvers, in contrast to those who assumed the role of "protecting" solvers by adding some points of support (as Cristiana did). We have thus highlighted two opposite behaviors exhibited by problem posers, with both emphasizing students' desire for social interaction.

Many educational researchers perceive social interaction as an important factor for stimulating mathematical creativity (e.g., Sfard, 1998; Sriraman, 2004). Most students in our sample spontaneously made connections to social interaction when discussing their posed problems. Their approach in this respect is an additional argument in favor of choosing an organizational framework to study creativity. In this way, we can capture specific aspects, especially related to the field of organizational learning, aspects that are irrelevant for other frameworks of mathematical creativity analysis.

Discussion

Our study focused on students who excel in problem solving, winners of a national contest. Usually, the students proficient in mathematics competitions are specifically trained for this purpose. We were interested to see if these students would be able to manage their own learning, and we provided them with a PP context. We consider that PP is a natural and simple situation where we can separate students' creative behaviors from behaviors learned through systematic practice.

Correlations Between PP and PS: Exploring Cognitive Frames

In posing problems, students showed preferences that influence the types of problems they pose. In this section, we will show that:

1. Students' preferences in PP correlate with their strengths in dealing with a certain mathematical content in problem-solving situations.
2. Students' focus on their strengths suggests that personal strengths are the main elements to build well-defined cognitive frames in PP.

We build the argumentation around three significant examples.

Example 1. Cosma (grade 5) initially posed the following problem (# 10):

> Two boys need £67 to buy a game. The price of the game decreases by 50%. If the first boy pays three times more than the second does, how much money should each pay?

When, during the interview, we asked Cosma where the idea of the problem came from, he said that he likes problems with fractions. To see if this comment is consistent with Cosma's problem-solving capacity, we analyzed his answer sheets from the two rounds of the Kangaroo contest. We noticed that he formulated responses to all 9 problems with fractions (of 80 problems). Cosma was wrong on 15 of the 60 problems he had chosen (so his proportion of mistakes is 25%), but none of these was related to fractions. At least two of the problems with fractions proposed in the second round and correctly solved by Cosma can be considered of high level of difficulty, being correctly solved by less than 20% of the participants. (We should also take into account that more than half of the co-participants were one year older than Cosma.) These observations confirm, on the one hand, the student's real preference for problems with fractions, and on the other hand, his high mathematical capacity in solving problems with this content. Therefore, his preference strongly correlates with the mastery of solving this category of problems.

Cosma proposed a coherent and consistent problem in which operations with fractions appeared as the main working tool and defined his cognitive frame for this problem. The fact that he explicitly claimed a preference for this area strengthens the persistence in this frame. Obviously, in Cosma's case, the cognitive frame correlates with his cognitive strengths.

Example 2. An interesting case is that of Victor (grade 4), who initially posed Problem 8. During the interview, Victor modified it, arriving at the following problem:

> At the "ABC" contest of numbers, each letter has received a number of points. Miss B exceeded Mr. A with 2 points, but D has exceeded Miss B. The Letter D scored so high that only the sum of scores obtained by A and B is equal to D's score. But D wasn't the best! E's score was double of that of D. However, F was the best. He got a score equal to the sum of the scores obtained by E and D. Knowing that if from the score of F we subtract 20 and then divide the result by 7 we get the half of the half of 16, how many points did each participant gain?

For both posed problems, Victor used a graphical method of solving; Figure 7.2, shows the solution he gave to Problem 8.

When we asked him how he came to pose these problems, he said:

> I didn't have a pattern; the idea with graphics that *come one after another* came to me randomly.

Even if he does not seem to be aware of this, Victor developed problems that can be modeled with systems of equations in *row echelon form*, in which the solving can be made "step by step" from the end to the beginning. For both problems, he actually used generating schemes similar to those shown in Figure 7.3. Therefore, Victor acted within a cognitive frame that he systematically used in his posed problems.

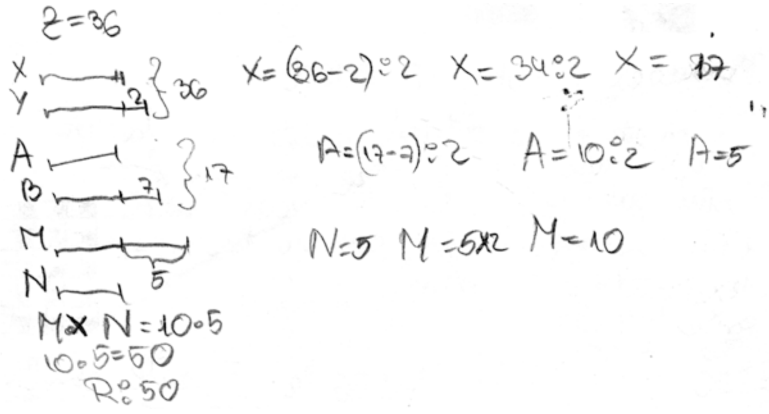

Figure 7.2. Victor's solution to his initially posed problem.

Problem 8, posed initially

$$\begin{array}{ccccccc}
M & \leftrightarrow & N & \square & \square & \square & \square \\
\square & \searrow & \downarrow & \square & \square & \square & \square \\
\square & \square & A & \leftrightarrow & B & \square & \square \\
\square & \square & \square & \searrow & \downarrow & \square & \square \\
\square & \square & \square & \square & X & \leftrightarrow & Y \\
\square & \square & \square & \square & \square & \searrow & \downarrow \\
\square & \square & \square & \square & \square & \square & Z
\end{array}$$

Problem posed during the interview

$$\begin{array}{cccccc}
A & \leftrightarrow & B & \square & \square & \square \\
\square & \searrow & \downarrow & \square & \square & \square \\
\square & \square & D & \leftrightarrow & E & \square \\
\square & \square & \square & \searrow & \downarrow & \square \\
\square & \square & \square & \square & F & \square
\end{array}$$

Figure 7.3. The problem-generating scheme used by Victor (grade 4).

Victor's model for creating problems is one for which a high degree of generalization is possible. However, although just a fourth grader, he was able to control the model for different cases, which demonstrates that this approach was a strength of his.

Example 3. Mihai (grade 6) posed the problems, classified in the category *Handling data*, which we analyzed at the beginning of the Results section of this chapter. We were interested to see if there is any connection between Mihai's preference for this category of content and his response pattern in the Kangaroo competitions. We noticed that the strategy used by Mihai (grade 6) in the competition allowed him to give wrong answers to only 6 problems (10%) of his 60 chosen problems (of the two rounds of the Kangaroo contest). Analysis of his pattern of choices in the competition showed that he jumped over high-complexity problems whose decoding at first sight appeared to be difficult. An example is provided by the following question, difficult to approach at a first glance (and in a relatively short time) by a sixth grader:

> On the number line, the segment between 1 and 100 is divided by points in 2011 equal parts. Find the sum of the coordinates of these points.

A problem of this type, in which many (seemingly unrelated) parameters occur, may generate chaos in a sixth grader's mind because without a culture of solving such problems, the given information could not be structured to minimize the number of independent variables. Mihai avoided this problem, and others of this type, probably because he failed to interpret the wording so as to diminish the number of parameters.

Mihai's preference for structured problems, where the relationship between the different variables of the problem were easily identifiable, is consistent with the model he chose for his posed problem (see Problem 16)—a model in which the identification of regularity in the random distribution of points was essential for solving.

Intuitively, Mihai knew that problems which do not display an immediate structure (or suppose a structure to which he had no access) were to be avoided in competitions. His target was to optimize his actions—a reasoning which is similar to that of a manager who analyzes his/her resources and makes the best knowledgeable decision.

As we have seen above, some students naturally displayed metacognitive abilities. They were able to describe their own approaches to problem posing—they were able to manipulate the constraints and the data, and they were successful in following, both consciously and systematically, a certain strategy in order to get an anticipated result. As in Mihai's case, we see that these students also applied metacognitive strategies in problem solving, in competitions where they had to solve a large number of problems in a short time. The analysis of such metacognitive behaviors of students confirmed our decision to use an organizational framework for analyzing creativity. The above three cases were not isolated examples in our study—for most of the students who posed coherent and consistent problems we found a correlation between their assumed strengths and the cognitive frames within which they built their problems.

The cases presented in this chapter demonstrate that, usually, students posed problems that were associated with their preferred mathematical content areas, and which were connected to their cognitive strengths. Thus, students intuitively felt the need to have a deep understanding of the chosen area in order to keep some control over the quality of their posed problems. This suggests that, when posing problems, students typically work within a well-defined cognitive frame.

Starting Points in Problem Posing: Exploring Cognitive Novelty

In this section, we claim the following:

1. Typically, in PP activities students start from known models, thus limiting cognitive novelty; and

2. When students do not start from a model or when they are not familiar with the model, their posed problems show cognitive novelty, but in most cases, at the expense of problem coherence and consistency.

Initial analyses of the students' posed problems suggest that, in most cases, students seem to use problems previously encountered as support for generating new ones. During the interviews, some students confirmed this assumption, as, for example, in the following excerpt from an interview with Malina:

> Malina (grade 4), with reference to Problem 3: "I had a similar problem in a contest in grade 2. I did not know how to handle it and I was very upset, because at that time I used to get very upset when I was finding something I do not know."

In other situations, we recognized "classical" problems, adapted and transformed. For example, Problem 13, posed by Cristiana (grade 6), is known in the literature as "the look-and-say sequence." Cristiana's contribution was to add some comments, designed to target possible approaches to solving the problem (and a possible collocutor).

The starting point is best visible in the problems generated during the interviews, where the model was clear—the problem originally posed by the student. For example, as shown in the cases presented earlier in this chapter, Mihai managed to generate new problems that did not depart significantly from his original problem, but were coherent and consistent. The same applies to Cristiana. Her Problem 13 was a classical one and Cristiana's contribution was minimal. When asked to generate a new problem, Cristiana proposed the following wording:

> Maria has to solve the following problem: "311311122112, 111312112, 132112, … What is the next term of the string?"

Cristiana did not move away from the assumed model significantly: in the new problem, she just reversed the order of crossing "the look-and-say sequence." During the interview, Cristiana affirmed her belief that every natural number that has an even number of digits may be a term within the look-and-say sequence. Even if this claim is not true (her condition is necessary, but not sufficient), the new posed problem is coherent and consistent, but again, is not far from her model.

Among the posed problems, there were only a few for which we either did not identify a possible model, or uncover it during the interview. One of these exceptions was Problem 17, posed by Nandor (grade 6):

> Dan has a 24 hour-display digital clock that is broken: the first digit of the hours' counter and the last digit of the minutes' one get switched every 5 hours. Example: if a switch occurs at 17:42, the clock will show 24:71. The clock continues to run correctly after that, and stops at 99:99, when it gives an error (1 hour is transformed in 100 minutes). If the clock breaks when the correct time of the day is 10:10, what will be the time before giving the error?

This problem indicates cognitive novelty, because it was far from those in textbooks or school auxiliaries. Nandor's posed problem is, however, neither coherent nor mathematically consistent. The author himself failed to clarify the solution, saying only that "all the numbers should be written—there are about 100." Nandor did not start from a model, and his problem showed cognitive novelty but at the expense of consistency.

We systematically asked the interviewed students to formulate new problems starting from those they originally posed. Consequently, during the interviews, they generated 26 new problems in total (some students posed several new problems, and each offered at least one). For example, Mihai (grade 6) posed the following amendment to his problem (#16):

> Prove that if we have 801 points in a square 60×60, then there exists a triangle with an area smaller than 9, determined by three of the 801 points.

To solve his new problem, Mihai used a technique similar to one he used for his originally posed problem (#16, in the Appendix)—he determined the most dispersed distribution related to a square grid of 3×3 and applied the pigeonhole principle. Later, when he explained his solution, Mihai considerably improved his proposal by finding an optimal version; he replaced 9 with 4.5 without any suggestion or request from the interviewer.

These examples demonstrate that, in general, when a student modified a given problem, she/he changed only some of the elements of that problem. We analyzed those changes based on the problem-analysis framework of Singer and Voica (2013), looking at changes to the following: the background theme, the parameters, (numerical) data, the operating schemes, the constraints over the data and the operating schemes, and the constraints that involve at least one unknown value of the parameter(s). Compared to the problems initially posed, in the 26 new problems, the students in our sample most often changed the givens (in 14 cases), or the background theme (in 8 cases). In only one case was the operating-scheme changed.

Yet, most of the students in our sample posed problems starting from an already known model. The existence of a starting model seemed to prevent the students from showing cognitive novelty. Thus, the vast majority of students in our sample started from a model when they posed problems, and in most cases, the posed problems did not go far from the model. Therefore, in problem posing (and modification) situations, cognitive novelty is limited, probably because of the students' awareness of a predefined cognitive frame.

However, this limitation seems to be relevant beyond the creativity issue, because it seems to ensure coherence and consistency in the new posed problems. Conversely, students who are apparently more creative did not have or have not yet built a cognitive frame, a fact that prevents them, most likely, from offering mathematically consistent problems.

Limits and Challenges of Mathematical Creativity

Within the framework used in this chapter, cognitive flexibility is characterized by cognitive novelty, cognitive variety, and change in cognitive framing. As we saw above, in PP situations, cognitive novelty is limited, and the students feel the need to evolve within a well-defined frame, which corresponds to the outgoing model used for posing a problem. In this section, we focus on how the students made changes to their cognitive frame. More specifically, we present evidence to support the following claims:

1. In PP situations, students are cautious about making major changes to the assumed model;
2. Consequently, students adopt a strategy of "small steps" in changing the starting model; and
3. This strategy of "small steps" seems to characterize mathematical creativity in PP activities.

We will focus this discussion on three concluding examples.

Example 1. Malina (grade 4) originally posed Problem 3. (A discussion on this problem appears in the results section of this chapter.) During the interview, we wanted to see, on the one hand, if Malina could develop new problems starting from her original problem, and on the other hand, whether she understood the mathematical tools she used for solving her problem.

First, Malina modified the numerical data of her problem by proposing the numbers 39, 50, 61, and 75 (=total number of jewels). She later explained us how new wordings can be developed starting from this problem:

> Malina: "If I think about marbles of more colors … So in a box there are black, blue, red and yellow marbles. If I say: a defined number, for example, 13, are not blue, it means that they are yellow, red and black. Of total … it is the same thing, only that there are more numbers; of the total number, I subtract the sum of the three and I got exactly the needed number…"

Malina kept the background theme and the constraints of the original problem, but changed the givens and the number of parameters (she now considered four different objects—i.e., marbles of different colors, instead of the three types of jewelry in the original problem). Malina explained how she generated the new wording: she increased the number of parameters ("I think to marbles of more colors…") and applied the same strategy for solving. Malina actually got to a generalization process for the original problem ("it's the same thing, only that there are more numbers"). We were interested to see if Malina was aware of the constraints on the numerical values of the problem. The interview continued as follows:

Interviewer: So, how do we get the total number?

Malina: Oh, here comes a different kind of problem … if you know that some are not black, some are not blue, some are not yellow and some are not red, you have to add these amounts and you get three times the amount exactly.

I: How is that, 3 times when there are 4 colors?

M: Well, you collect the yellow, the red, the blue [she gestures], then collect the yellow and black and blue, then … and then what's left and every time you notice that each number comes out three times.

I: And if there were 100 colors?

M: Then we would get … uh … uh …
I: Let's not say 100, let's say 6 colors!
M: If there were …we would get 6 colors 6 times … no! 5 times the amount!

It seems that Malina has activated cognitive mechanisms to verify the correctness of this type of problem. These mechanisms allowed her to establish correlations between the data and parameters and to verify the mathematical consistency of the problem. In fact, the mathematical model of the problem described by Malina is a linear system of four equations with four unknowns. As a fourth grader, Malina had no formalized knowledge in solving mathematical systems of equations. Nevertheless, she controlled the system and determined conditions for compatibility. She not only showed the computational strategy to solve the problem, but she was able to generalize this method for other similar conditions, chosen at random. Problem 3, originally posed by Malina, was classified as non-coherent (because redundant data occurred in the wording). The explanations presented above, given by Malina during the interview, convinced us that this redundancy of data seemed to be rather a reassurance that the proposed data were compatible, than an expression of conceptual misunderstandings. Thus, in problem posing, Malina acted within a well-defined cognitive frame set up for her problem.

Further, we wanted to see how far Malina might make changes in her cognitive frame. Consequently, we asked her to pose problems as simple as possible, starting from her initial one. Malina's proposal was:

> In a box there are 75 balls, yellow, red and blue. Of these, 39 are red and yellow, 61 are blue and red and 50 are blue and yellow. How many balls are there in the box?

The interviewer expressed the opinion that this was, in fact, the same problem as one of her previous reformulated problems. Her answer was: "It's the same problem, but told differently, more clearly." The interviewer insisted and asked Malina to pose an even simpler problem. She needed a longer time to think, hesitated, and then posed the following wording:

> In a box there are 75 balls: red, yellow and blue. Of these, 39 are yellow and blue. Find the number of each color.

Malina posed a new problem by reducing the number of constraints and giving up two of the parameters. The change was now more extensive than in the previous cases, but this led to a problem that was neither coherent nor consistent. This was quite surprising, since we thought that Malina had showed deep understanding of her problem's pattern. Because she well understood the relationship between the components of her initial problem, she succeeded in making changes to her cognitive frame, and to keep control over the problems obtained by generalization or by changing the operating scheme, but only as long as the changes were not far. When these changes were wider, she ended up losing control, and posed problems that kept a general pattern, but did not prove consistency and/or coherence. Continuing the analysis, we find that Malina's intuitive attempt to keep control over the new posed problems limited cognitive novelty. Malina intuitively did not go too far from the assumed model, but when she was pushed to do so, her new problems, although simpler, became mathematically inconsistent.

Example 2. Maria (grade 6) initially posed the following problem (#15):

> A number is "special" if it can be written as both a sum of two consecutive integers and a sum of three consecutive integers. Prove that: (a) 2,001 is special, and 3,001 is not special; (b) the product of two special numbers is special; (c) if the product of two numbers is special, then at least one of them is special.

Maria managed to identify equivalent characterization for her so-called "special" numbers: a number is "special," if and only if it is an odd number, divisible by 3. Once she had this general characterization of algebraic nature, Maria could easily pose some new problems:

> Prove that the sum of three "special" numbers is "special."

> A number is "very special" if it is both special and perfect square. Give an example of a very special number.

In posing the first new problem, Maria largely kept the wording and varied only the constraints that involved at least one unknown value of the parameter (she changed the question). For the second problem, Maria included a new constraint (the number must also be a perfect square).

Maria worked in a well-defined cognitive frame: she transposed the problem algebraically and used a general characterization of the "special" numbers to identify new properties of these numbers. Maria did not change her cognitive frame associated with this problem; she always used the same initial properties and did not explore her problem in other directions. The changes she made for her new proposals were minimal, although her posed problems were highly abstract.

Example 3. Radu (grade 6) originally posed Problem 18 (*Prove that any parallelogram can be divided in 16,384 congruent parallelograms*). During the interview, Radu explained that, in posing this problem, he started from the observation that a given parallelogram can be divided into four or nine congruent parallelograms (by dividing each side in two or three equal parts and constructing parallel sides through the points of division). He chose the number 16,384 just to give difficulty to the problem ("There must be a big enough number, perfect square."). Thus, the relatively big distance between the initial model (i.e., for the particular cases 4 and 9) and his final proposal was given by his evolution within a well-internalized cognitive frame. For this proposal, Radu changed only one parameter (the number of congruent parallelograms) and thus obtained a new problem, which was coherent and consistent.

When we asked him to pose another problem of the same type, Radu made the following comment:

> I'd be a bit tempted to say that any triangle can be divided into 16,384 congruent triangles, but I am not sure of the solution. … Yes, I would be tempted to do again with a parallelogram and to apply the same idea, just up here…

In the new posed problem, Radu kept the background theme and numerical data, but modified a parameter (he replaced the parallelogram with a triangle). In fact, Radu formulated a conjecture (“a triangle can be divided into 16,348 congruent triangles”). Radu tried to solve his new problem by completing the triangle to a parallelogram and applying the same idea for solving. In this phase of testing, he remained within the same cognitive frame. Analyzing some particular cases, Radu concluded that the problem required some additional assumptions (such as, for example, that the number of triangles must be even) and that, perhaps, the initial solution method could not be applied. Radu returned later (after 1 day) with new reformulations and attempted to solve this problem. Although his attempts were not entirely correct (probably because the solving of the new problem required knowledge about similarity, to which Radu had no access at that time as a sixth grader), he concluded that it was plausible that the number of triangles must be a perfect square. This showed that Radu was, in fact, able to reframe.

These examples, like others of a similar kind that we found in our sample, led us to conclude that, in problem-posing situations, a student acts within a definite cognitive frame that allows him/her to generate mathematically consistent problems. Further, some students succeed in making changes to those cognitive frames or even to reframe. These changes were not always spectacular because students intuitively tend to maintain coherence and consistency of the posed problems, and changes that are more extensive prevent them from keeping control over the shape of the problem. But when students make small-scale changes (usually by varying a single parameter), they can understand the impact of these changes on the constraints of the problem text and they can choose appropriate numerical data. Therefore, a student’s capacity to generate coherent and consistent problems in the context of problem posing (and modifications) may indicate the existence of a strategy of IN-OUT functional type consisting of small changes followed by checking the outcomes, which seems specific to mathematical creativity. In more general terms, mathematical creativity seems to emerge from changes in cognitive framing, which express a tension between the maintenance within a frame and the possibility of overcoming it for generalizations, possibility constrained by the need for consistency.

Conclusions

This chapter presents an empirical study in which students in grades 4–6, with above-average mathematics abilities, posed problems. We tried to answer the question: How does mathematical creativity manifest in 10–13 year-old high achievers?

The results show that in the PP process, students develop a genuine philosophy, which refers both to practical actions—embodied in their problem-posing strategies—and to the qualitative form of the posed problems. Typically, students start from a model to which they apply certain constraints based on the philosophy they developed, and they then spontaneously try to get a problem that is mathematically consistent and coherent.

We noticed that both the problems posed by the students and the behaviors presented by the students highlighted a social dimension. We have several arguments to support this claim. On the one hand, in most cases, the background theme of the posed problems had a civic connotation: students go to paintball as a prize because they are in an advanced class, or some families do not pay their waste collection fee, and so on. Other posed problems simply involved friends, classmates, or neighbors. On the other hand, some students directly addressed some challenging areas, or provided some support for the solver. Thus, most students took into account possible collocutors within the PP activities. The PP context allowed students to seek ways to distort/alter the magnitude of the problem changes by adding text elements that referred to the author's interaction with a potential recipient of the problem. Students succeeded both in maintaining quality control over the new posed problems and in responding to a need for social interaction.

The social dimension of the PP process revealed by these children's options confirms the meaningfulness of the framework of analysis that we used in this study, in which we discuss the relationship between problem posing and mathematical creativity in terms of cognitive flexibility in an organizational framework.

The study provides evidence that of the three components of cognitive flexibility (i.e., cognitive variety, cognitive novelty, and change in cognitive framing), the last seems to be the most relevant for PP situations. More specifically, the majority of students in our sample started from a model for which they already had a well-defined cognitive frame and posed new problems within this frame.

Students were generally able to make changes to their cognitive frames as they succeeded in posing new problems starting from the initial ones, problems that displayed different approaches compared to the starting point. Yet, among these, it was significant to study the thinking patterns of those students whose proposals, issued either initially or during a modification process, were coherent and consistent.

In modifying a problem, students tried to vary a single parameter; the ones who succeeded to do this could control the consequences of the changes and managed to develop coherent and consistent mathematical problems. Their strategies revealed a kind of cognitive variety that was relatively limited by their desire to control the outcomes of the process. This also limited cognitive novelty.

Therefore, cognitive flexibility seems to be oriented towards finding generalizations and is constrained by the need to maintain the mathematical consistency of the problem. Consequently, students' approaches in the PP process seem to be of an "in-out" functional type, with a careful check of the variations induced over the outcomes.

The study brings evidence that this type of approach in posing/or modifying a problem, which allows for making generalizations, seems to be specific for mathematical creativity, at least for the sample analyzed in this chapter. More specifically, we show that in PP contexts, students tend to make incremental changes to some parameters in order to arrive at simpler and essential forms needed in generalizing sets of data. It follows that mathematical creativity is of a special type—one which requires abstraction and generalization. In addition, students showed awareness of the need for mathematical consistency, which made them persevere as they carefully controlled the changes.

Moreover, the interviews revealed that this awareness meant that most of the students were able to analyze their own proposals critically and their own thinking mechanisms in PP, thus reflecting their metacognitive skills. These metacognitive skills helped them to be aware of their strengths and to use these strengths to reinforce a well-defined cognitive frame for a problem.

On the basis of these conclusions, our study highlights some aspects that have consequences for effective teaching. We briefly present these below.

First, we have seen that students have preferences for some subareas of mathematics, or for some problem-solving strategies, which can be relatively easy identified through PP activities. Students' preferences reveal the strengths on which teachers can focus in order to develop students' mathematical competences.

Second, our data show that students need social interaction. Surprisingly, this need surfaced through problem posing—an individual type of activity. Our conclusion is that social interaction should be part of the teaching-learning process in the class in a consistent way, for example, by means of activities involving posing and solving problems organized in pairs or in teams.

Third, the study shows that PP stimulates metacognitive abilities in students. From this perspective, the use of PP in teaching is beneficial to students' personal development.

Finally, training for the development of mathematical creativity should include features that distinguish it from training for the development of creativity in general. Briefly said, while in the latter more general case, techniques are to be used for stimulating the free development of ideas, in mathematics the variation of parameters should be practiced within a variety of activities where the processes are mindfully controlled and oriented towards abstraction and generalization.

Appendix

Problem 1 (*posed by Diana*, *grade 4*): On the planet Zingo live several types of aliens: with two or three eyes, with two or three ears, and with five or six hands. They are green or red. How many aliens should shake hands with Mimo to be sure that he shook hands with at least two of the same type?

Problem 2 (*posed by Emilia*, *grade 4*): In the Infinite king's castle there are 43 corridors with 18 rooms each. Each room has 52 windows. At every window, there are three princesses. How many princesses are in the Infinite king's castle?

Problem 3 (*posed by Malina*, *grade 4*): In Princess Rose's jewelry boxes there are sapphires, emeralds, and rubies. 27 are not rubies, 31 are not emeralds, and 32 are not sapphires. In total, there are 45 jewels. How many jewels of each kind does Princess Rose have?

Problem 4 (*posed by Paul*, *grade 4*): If a group of students sits by two at their desks, seven students remain standing, and if they are placed by three at the same desks, seven desks remain free. How many students and how many desks are there in the classroom?

Problem 5 (*posed by Sabina*, *grade 4*): In the Fairies' Glade, live 60 unicorns and fairies. They have 160 legs in total. How many beings of each kind are there?

Problem 6 (*posed by Sergiu*, *grade 4*): One day, the plane leaves Cluj [*a city in Romania*] to go to Japan. It departs at sharp hour in the morning, when the hour and the minute hands of a clock form a right angle. The hour hand points to a number bigger than 4. This plane travels at 60 km per hour, and the distance Cluj-Japan is 540 km. In the plane climbed three times more men than women, who are 24,484 people. The cost for a man's tickets is the first odd number greater than 7. Women's tickets cost as double of 3 added with 4 and the result divided by 2. (A) When did the plane leave Cluj? (B) When did the plane arrive in Japan? (C) How many men boarded the plane? (D) How many women boarded the plane? How much money did the pilot receive, if he received all the money, without 3,000 of total?

Problem 7 (*posed by Tudor*, *grade 4*): On a farm, there are two cows, some geese and horses, a total of 86 heads and 328 feet. How many horses are there at the farm?

Problem 8 (*posed by Victor*, *grade 4*): In the world of letters, each letter represents a number. M is two times greater than N, and the difference between these two letters is equal to A. A is less than B by seven, and the sum of A and B is neither bigger nor smaller than X. If we add two to X, we get Y. The sum of X and Y equals Z. Knowing that $Z-(A\colon O+P\colon P+Q\colon Q+A\colon R)=30$, find $M\times N$.

Problem 9 (*posed by Alin*, *grade 5*): (A poem!) If one places three cakes in each box/There'll be three cakes left/If one places five cakes in each box/There'll be an empty box left. (…) How many boxes and how many cakes/Do I put on the shelves?

Problem 10 (*posed by Cosma*, *grade 5*): Two boys need £67 to buy a game. The price of the game decreases by 50%. If the first boy pays three times more than the second does, how much money should each pay?

Problem 11 (*posed by Andrei*, *grade 6*): On the planet Uranus in the T316B2 city, there are less than 101 and more than 49 aliens. 1/2 of them are red, 2/7 are green, 1/14 are yellow, and the rest are blue. How many aliens live in E943S4 city, the capital of the planet, if their number is 149 times greater than the count of blue aliens from T316B2?

Problem 12 (*posed by Cosmin*, *grade 6*): $P\cdot R\cdot I\cdot C\cdot E\cdot P\cdot I\cdot P\cdot R\cdot O\cdot B\cdot L\cdot E\cdot M\cdot A=x$. Knowing that different letters represent different digits, find the last digit of the number x. (*He multiplies the letters meaning YOU UNDERSTAND THE PROBLEM.*)

Problem 13 (*posed by Cristiana*, *grade 6*): Maria has to solve the following problem: "4, 14, 1114, 3114, 132114, 1113122114, … What is the next term of the sequence?" The mathematics teacher gave her some advice: "You must empty your mind of all other mathematical information." Can you help Maria to solve the problem?

Problem 14 (*posed by Cristiana*, *grade 6*): Maya the puppy has six bones. She wants to make four equilateral triangles out of these six bones, but she forgot one essential rule: one has to think out of the box. Can you help her?

Problem 15 (*posed by Maria*, *grade 6*): A number is "special" if it can be written as both a sum of two consecutive integers and a sum of three consecutive integers. Prove that: (a) 2,001 is special, and 3,001 is not special, (b) the product of two special numbers is special, (c) if the product of two numbers is special, then at least one of them is special.

Problem 16 (*posed by Mihai*, *grade 6*): Because the sixth-grade students were the best, they received a prize consisting in 1 h free on paintball field. The field has the dimensions 80 m × 120 m, and two people are able (and allowed) to shoot one another if they are at no more than 29 m distance. Prove that howsoever 26 students place themselves on the ground, at least 3 get shot.

Problem 17 (*posed by Nandor*, *grade 6*): Dan has a 24 hour display digital clock that is broken: the first digit of the hours' counter and the last digit of the minutes' one get switched every 5 hours. Example: if switch occurs at 17:42, the clock will show 24:71. The clock continues to run correctly after that and stops at 99:99, when it gives an error (1 hour is transformed in 100 minutes). If the clock breaks when the correct time of the day is 10:10, what will be the time before giving the error?

Problem 18 (*posed by Radu*, *grade 6*): Prove that any parallelogram can be divided into 16,384 congruent parallelograms.

Problem 19 (*posed by Teofil*, *grade 6*): In 2011, 300 students went on the field trip. Knowing that the percentage of girls was 45%, find the number of boys who participated.

Problem 20 (*posed by Vlad*, *grade 6*): A new quarter was built near a forest. The residents put their garbage in waste containers with a capacity of 750 kg each. At every 10 kg of garbage throw away by the residents, 4 kg disappear, being consumed by bears leaving in the forest. The residents produce 20 kg of garbage per hour. (a) Find out how long it take to fill a waste container; (b) Knowing that in the neighborhood live 500 families that fill 86 containers per month, that each family should pay 7.8 euros garbage fee, but only 400 families are fair and pay, calculate how much money is collected as garbage fees in a month.

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