Chapter 5
On the Relationship Between Problem Posing, Problem Solving, and Creativity in the Primary School

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Abstract Problem posing is a form of creative activity that can operate within tasks involving semi-structured rich situations, using real-life artefacts and human interactions. Several researchers have linked problem-posing skills with creativity, citing flexibility, fluency, and originality as creativity categories. However, the nature of this relationship still remains unclear. For this reason, the exploratory study presented here sought to begin to investigate the relationship between problem-posing activities (supported by problem-solving activities) and creativity. The study is part of an ongoing research project based on teaching experiments consisting of a series of classroom activities in upper elementary school, using suitable artefacts and interactive teaching methods, in order to create a substantially modified teaching/learning environment. In addition, the study provides a method for analyzing the products of problem posing that teachers could use in the classroom to identify and assess both the activity of problem posing itself and students' creativity in mathematics.

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Introduction

Problems have occupied a central place in the school mathematics curriculum since antiquity. In fact, examples of mathematical and geometrical problems go back to the ancient Egyptians, Chinese, and Greeks. A common belief was that studying mathematics would improve one's ability to think, to reason, and to solve problems that one was likely to confront in the real world. Mathematics problems were a given element of the mathematics curriculum that contributed, like all other elements, to the development of reasoning power (Stanic & Kilpatrick, 1988).

However, traditional school word problems typically focus on the application of operational rules that involve a mapping between the structure of the problem situation and the structure of a symbolic mathematical expression. Often, solving these word problems is not a problem-solving activity for students; rather, it is an exercise that relies on syntactic cues for solution, such as key words or phrases in the problem (for example, "times," "less," "fewer"). While not denying the importance of these types of problems in the curriculum, they do not adequately address the mathematical knowledge, processes, representational fluency, and communication skills that our students need for the twenty-first century (English, 2009).

Furthermore, many researchers have documented that the practice of solving word problems in school mathematics actually promotes in students a suspension of sense-making (Schoenfeld, 1991), and the exclusion of realistic considerations. Primary and secondary school students tend to exclude relevant and plausible familiar aspects of reality from their observation and reasoning.

As a kind of minimal instructional response to this bridging problem, some scholars have made a plea for improving the quality of the word problems by making them resemble somewhat more the real-life problems encountered out-of-school, for example, by making the data, the question, and the contextual constraints more authentic or realistic (see, e.g. Palm, 2006; Verschaffel, Greer, & De Corte, 2000). As an even more radical response, other researchers have argued for the replacement of these word problems by *real* real-life problems that depart from existing (problematic) descriptions of the world (Chen, Van Dooren, Chen, & Verschaffel, 2011).

Our approach falls under the second type of response. If we want to help students to prepare to cope with natural situations they will have to face out of school, we need to rethink the type of problem-solving experiences we present to our students.

Almost all of the mathematical problems a student encounters have been proposed and formulated by another person—the teacher or the textbook author. In real life outside of school, however, many problems, if not most, must be created or discovered by the solver, who gives the problem an initial formulation (Kilpatrick, 1987).

In our opinion, the activities used to create an interplay between mathematics classroom activities and everyday-life experiences must be replaced with more realistic and less stereotyped problem situations, founded on the use of materials, real or reproduced, which children typically meet in real-life situations (Bonotto, 2005). In particular, we deem that classroom activities using suitable artefacts and interactive teaching methods could foster a mindful approach towards realistic mathematical modelling and problem solving, as well as a positive attitude toward problem-posing (Bonotto, 2009). In fact, we maintain that the problem-posing process represents one of the forms of authentic mathematical inquiry which, if suitably implemented in classroom activities, could move well beyond the limitations of word problems, at least as they are typically utilized.

Kilpatrick (1987) maintained that "problem formulation is an important companion to problem solving. It has received little explicit attention, however, in the mathematics curriculum until a few years ago" (p. 123). In the United States, for example, formally and for the first time "the inclusion of activities in which students generate their own problems, in addition to solving pre-formulated examples, has been strongly recommended by the National Council of Teachers of Mathematics" (English, 1998, p. 83). More recently, the Chinese *National Curriculum Standards on Mathematics* (Ministry of Education of Peoples' Republic of China (NCSM), 2001) has emphasized that students must be able to "pose and understand problems mathematically, apply basic knowledge and skills to solve problems and develop application awareness" (p. 7). Also, a document of the Italian Mathematics Union (UMI-CIIM, 2001) and of the Italian Ministry of Education (2007) recognized the importance of problem posing in the mathematics curriculum.

Given the importance of problem-posing activities in school mathematics, some researchers have started to investigate various aspects of the problem-posing process. Several have examined thinking processes related to problem posing (e.g. Brown & Walter, 1990; Christou, Mousoulides, Pittalis, Pitta-Pantazi, & Sriraman, 2005). In particular, Kontorovich, Koichu, Leikin, and Berman (2012) posited that the problem-posing process is constituted by a knowledge base, heuristics and schemes, group dynamics and interactions, individual considerations of aptness, and task organization. Others have underlined the need to incorporate problem-posing activities into mathematics classrooms and have reported approaches that included it in instruction. They have provided evidence that problem posing has a positive influence on students' ability to solve word problems (e.g. Leung, 1996; Silver, 1994). English (1998) asserted that problem posing improves students' thinking, problem-solving skills, attitudes and confidence in mathematics and mathematical problem solving, and contributes to a broader understanding of mathematical concepts.

Furthermore, problem posing is a form of creative activity that can operate within tasks involving structured "rich situations" in the sense of Freudenthal (1991), using real-life artefacts and human interactions (English, 2009). Creativity, understood as the cognitive ability to create and invent, is linked to the activity of mathematical problem posing. In fact, problem posing is a form of mathematical creation: the creation of mathematical problems in a specific context. In particular, Silver and other authors (Cai & Hwang, 2002; Kontorovich, Koichu, Leikin, & Berman, 2011; Silver, 1994; Silver & Cai, 2005; Yuan & Sriraman, 2010) have linked problem-posing skills with creativity, citing flexibility, fluency, and originality as creativity categories. Moreover, some authors have suggested that students' considerations of whether or not the created problems are appropriate could serve as another useful indicator of their creativity (Kontorovich et al., 2011, 2012; Mednick, 1962).

However, the nature of this relationship still remains unclear. For this reason, the exploratory study presented here begins to investigate the relationship between *problem-posing* activities (supported by problem-solving activities) and *creativity*. Also, the study provides a method for analyzing the products of problem posing that teachers could use in the classroom to identify and assess both the activity of problem posing itself and the students' creativity in mathematics.

Problem Posing

Students are usually asked to solve mathematical problems at school that have been presented by teachers or textbooks (Silver, 1994). Therefore, students only have the task of solving problems, while the teachers have to create them.

But, what is a problem? In discussing the nature of problems, Starko stated that "problems come in various shapes, sizes, and forms, some with more potential than others. A 'problem' is not necessarily difficult; it may be a shift in perspective or a perceived opportunity" (Starko, 2010, pp. 30–31). In his studies about problems and creative thinking, Getzels (1979) distinguished between three illustrative types of problems or problem situations: presented problem situations, discovered problem situations, and created problem situations. In the first type of problems, there are three components—a formulation, a method of solution, and a solution known to others if not yet to the problem solver. Most classroom problems are of this type. Problems of the second type "may or may not have a known formulation, known method of solution, or known solution" (Getzels, 1979, p. 169). In the last type of problems, there is no presented problem and someone must invent or create it. As explained by Starko (2010), "Type 1 problems primarily involve memory and retrieval processes. Type 2 problems demand analysis and reasoning. Only Type 3 problems, in which the problem itself becomes a goal, necessitate problem finding" (p. 31). And problem finding is the first step of the problem-posing process.

In mathematics education, after over a decade of studies which have focused on problem solving, researchers have slowly begun to realize that "developing the ability to *pose* mathematics problems is at least as important, educationally, as developing the ability to *solve* them" (Stoyanova & Ellerton, 1996). Problem posing, in fact, is of central importance in the discipline of mathematics and in the nature of mathematical thinking, and it is an important companion to problem solving (Kilpatrick, 1987). Kilpatrick believed that

> Problem formulating should be viewed not only as a *goal* of instruction but also as a *means* of instruction. The experience of discovering and creating one's own mathematics problems ought to be part of every student's education. Instead, it is an experience few students have today—perhaps only if they are candidates for advanced degrees in mathematics. (p. 123)

In recent years, in recommendations for the reform of school mathematics around the world, the results of many studies have supported the central role of problem posing. For example, *The Principles and Standards for School Mathematics* in the United States of America (National Council of Teachers of Mathematics, 2000) called for students to "formulate interesting problems based on a wide variety of situations, both within and outside mathematics" (p. 258) and recommended that students should make and investigate mathematical conjectures and learn how to generalize and extend problems by posing follow-up questions. In *The Interpretation of Mathematics Curriculum* (Mathematics Curriculum Development Group of Basic Education of Education Department, 2002) "it is pointed out that students' abilities in problem solving and problem posing should be emphasized and students should learn to find problems and pose problems in and out of the context of mathematics" (Yuan & Sriraman, 2010, p. 6).

Problem posing has been defined by researchers from different perspectives (Silver & Cai, 1996). The term problem posing has been used to refer both to the generation of new problems and to the reformulation of given problems (e.g. Dunker, 1945; Silver, 1994). Silver (1994) linked problem solving and problem posing and argued that problem posing could occur:

- *Prior* to problem solving when problems were being generated from a particular stimulus (such as a story, a picture, a diagram, a representation, etc.);
- *During* problem solving when an individual intentionally changes the problem's goals and conditions (such as in the cases of using the strategy of "making it simpler"); and
- *After* solving a problem when experiences from the problem-solving context are modified or applied to new situations.

Stoyanova and Ellerton (1996) identified three categories of problem-posing situations: free, semi-structured, or structured. In free situations, students pose problems without restrictions: students are simply asked to make up mathematics problems from a given situation. Semi-structured problem-posing situations refer to ones in which students are "given an open situation and are invited to explore the structure of that situation and to complete it by using knowledge, skills, concepts and relationships from their previous mathematical experiences" (p. 520). Finally, structured problem-posing situations refer to situations where students pose problems by reformulating already solved problems or by varying the conditions or the questions of given problems.

In this chapter, we shall consider mathematical problem posing as suggested by Stoyanova and Ellerton (1996): "the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems" (p. 519). In the study presented here, this process is supported by the use of suitable social or cultural artefacts that, according to this framework, can become a meaningful source for problem-posing activities of the semi-structured type (Bonotto, 2013). A cultural artefact can support a semi-structured problem-posing situation, because it can become a concrete source for types of tasks and activities where the students are invited to explore the mathematical structure, find a problem, and by using knowledge, skills, concepts, and relationships from their previous mathematical experiences, create one or more new mathematical problems.

Problem posing, therefore, becomes an opportunity for interpretation and critical analysis of reality since: (a) the students have to discern significant data from immaterial data; (b) they must discover the relations between the data; (c) they must decide whether the information in their possession is sufficient to solve the problem; and (d) they have to investigate if the numerical data involved is numerically and/or contextually coherent. These activities, quite absent from today's Italian school context, are typical also of mathematical modelling processes and can help students to prepare to cope with natural situations they will have to face out of school (Bonotto, 2009).

A semi-structured situation, as well as an unstructured situation, invites the use of creative thinking inasmuch as it stimulates student sensitivity to a problem—to ideation (the creation of new ideas), originality, the ability to synthesize, and to reorganize the information in a new way, analytical skills, and evaluating ability.

The advancement of mathematics requires creative imagination, which is the result of raising new questions, new possibilities, and viewing old questions from new angles (Ellerton & Clarkson, 1996). Silver (1997) argued that inquiry-oriented mathematics instruction, which includes problem-solving and problem-posing tasks and activities, could assist students to develop more creative approaches to mathematics. It is claimed that through the use of such tasks teachers can increase their students' capacity with respect to the core dimensions of creativity, namely, fluency, flexibility, and originality. We believe in the didactic potential of using suitable artefacts, combined with particular teaching methods, as a source for types of tasks and activities that encourage problem posing and creativity processes—see Bonotto (2005, 2009) for a discussion on the use of artefacts in classroom activities.

Creativity

In the nineteenth and early twentieth centuries, creativity was identified with the genius of a few people of remarkable intelligence who revolutionized their fields. Therefore, early studies on creativity examined the characteristics of these outstanding personalities, such as Mozart and Einstein. These studies were based on three ideas: first, that creativity belonged to exceptional personalities; second, that a creative person was a break with the spirit of the time in which that person lived; and third, that sudden insight was involved. However, it is interesting to note the position of Poincaré (1908), later recaptured by Hadamard (1945), that inventing, at least in mathematics, meant to discern and to choose.

Afterwards, the psychological study of thought addressed aspects of intelligence, and in particular logical mathematical skills. As a result, creativity began to be identified with high intelligence. Beginning in 1950, Guilford dealt with creativity and noted that IQ and creativity could not be overlapped. He, therefore, hypothesized that a person could be creative without exceptional intelligence and vice versa. Then, Guilford hypothesized that there was a different way of thinking, subsequently called divergent thinking, characterized by the ability to imagine a range of solutions to a given problem. Guilford's ideas inspired subsequent research on creativity and the development of tests to measure people's creativity such as the Torrance Tests of Creative Thinking. Thus, creativity began to be recognized as an asset, even if present in different degrees and shapes, for each person. Today, there are many definitions and theories of creativity, each of which considers some aspect of creative thinking.

One of the main lines of research on creativity concerns the distinction between two types of thought proposed by Guilford (1950): productive (divergent thinking) and reproductive (convergent) thinking. Included in the divergent thinking category were the factors of fluency, flexibility, originality, and elaboration. Guilford saw creative thinking as clearly involving what he categorized as divergent production (Yuan & Sriraman, 2010) which he broke down into nine skills: sensitivity to problems, ideational fluency, flexibility of set, originality, the ability to synthesize, analytical skills, the ability to reorganize, span of ideational structure, and evaluation ability. All these skills influence each other and represent the related aspects of a dynamic and unified cognitive system. In particular, sensitivity to problems, flexibility of approach, ability to synthesize, application of analytical skills, and the ability to reorganize are all aspects that characterize mathematical thinking.

It is hardly surprising, therefore, that the main models used to describe the creative process emphasize the importance of sensitivity to the problems (problem finding) and their resolution (problem solving). Problem finding, in particular, may be associated with mathematical problem posing. Problem-posing and problem-solving activities are therefore used by several authors to promote and evaluate creativity (Leung, 1997; Silver, 1997; Silver & Cai, 2005; Siswono, 2010; Sriraman, 2009; Torrance, 1966). For example, in a recent study, Kontorovich et al. (2011) used fluency, flexibility, and originality as indicators of creativity in students' problem posing.

We must not forget that there is a distinction between mathematical creativity at the professional level and at the school level: it is certainly feasible to expect students to offer new insights into a mathematical problem rather than expecting works of extraordinary creativity and innovation (Nadjafikhah, Yaftian, & Bakhashalizadeh, 2012; Sriraman, 2005).

We believe that the creative process in school mathematics may be encouraged by the presence of semi-structured situations (defined by Stoyanova & Ellerton, 1996). These situations are similar to those encountered by professional mathematicians who are frequently engaged in problems which are full of vagueness and uncertainty; the use of appropriate cultural artefacts can help realize these situations.

Through the use of artefacts, children can be encouraged to recognize a great variety of situations as mathematical situations, or more precisely "mathematizable" situations, by asking them: (a) to select other artefacts from their everyday life; (b) to identify the mathematical facts associated with them; (c) to look for analogies and differences (e.g., different number representations); or (d) to generate problems (e.g., discover relationships between quantities) (Bonotto, 2009). These aspects are related to another line of research on creativity that highlights the importance of the process of association of ideas (e.g. Mednick, 1962; Starko, 2010).

In the study presented here, we focused on the analysis of the problem posing and creativity processes. These two processes were studied using a semi-structured situation. We also began to reflect on the relationship between mathematical knowledge and these two processes. Figure 5.1 presents possible relationships between the variables involved in the problem posing and creativity processes at the school level.

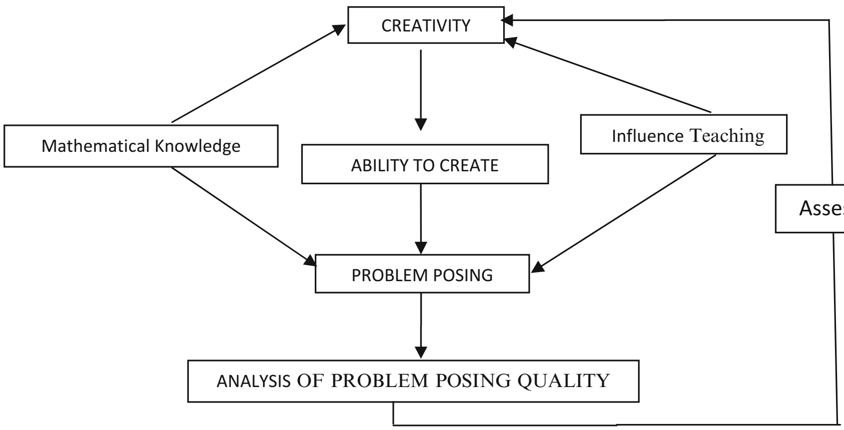

Figure 5.1. Possible relationships between problem posing and creativity.

The Study

The overall aim of this exploratory study, briefly described also in Bonotto (2013), was to examine the relationship between *problem-posing* activities (supported by problem-solving activity) and *creativity*, when these processes are implemented in situations involving the use of real-life artefacts. In particular, the study sought to continue to investigate:

- The role of suitable artefacts as sources of stimulation for the problem-posing process in semi-structured situations; and
- Primary school students' capacity to create and deal with mathematical problems (including open-ended problems).

Furthermore, the study sought to begin to investigate:

- The potential that these problem-posing activities have for identifying creative thinking in mathematics; and
- A method for analyzing the products of problem posing that the teacher could use in the classroom to identify and assess both the activity of problem posing itself and the creativity of the students.

Participants

This exploratory study involved four fifth-grade classes (10–11 years old) from two primary schools in northern Italy. The study was carried out by the second author of this paper in the presence of the official logic-mathematics teacher.

The first primary school was located in an urban area situated within a few miles of the centre of a city. This school participated in the study with two classes, one consisting of 14 students and the other of 16. The children were already familiar with activities using cultural artefacts, group work, and discussions.

The second primary school was located in a mountainous area. This school also participated in the study with two classes, one consisting of 20 students and the other of 21. The children were not already familiar with these types of activities, even though the teacher had once proposed a problem-posing activity where the situation was a drawing of the prices of different products in a shop.

The average marks in mathematics of the students from the two schools were classified into three categories: high, medium, and low. On the basis of this classification the two schools were not uniform; in particular, the second school had more students with averages in the medium-high range in mathematics. These data were obtained in order to make observations concerning the influence that mathematical knowledge can have on the creativity process.

Materials

To perform the problem-posing activity a real-life artefact was used as the initial situation. We wanted to create a semi-structured situation that was as rich and contextualized as possible for the students in order to permit them to use their extra-scholastic experience in the creation and resolution of problems. Thus, the artefact was a page of a brochure containing the special rates for groups visiting the Italian amusement park "Mirabilandia" (Figure 5.2) and shows the menu and applicable discounts, the cost for access to the beach, etc. This artefact was chosen with the belief that all students were already familiar with an amusement park because they had been to one. The page was full of information, including prices (some expressed as decimals), percentages, and constraints on eligibility for the various offers (Figure 5.2 shows part of the artefact). Finally, we gave pupils the individual rates.

Procedure

Assuming, for the reasons discussed previously, that problem posing can be an activity that highlights creativity, we structured a problem-posing activity supported by a problem-solving activity that could be evaluated with regard to creative thinking (in terms of fluency, flexibility, and originality). The experiment was structured

Figure 5.2. Artefact for semi-structured situation.

in three phases: (a) the presentation of the artefact used; (b) a problem-posing activity; and (c) a problem-solving activity. The activities took place on three different days, a few days apart. The students worked individually for part 2. For part 3, they were, at first, divided into groups of two or three students and then participated in a collective discussion. Students could use the artefact and its summary during all three activities.

The first phase, lasting about 2 hours, consisted of the analysis and synthesis of the artefact. This phase was preparatory to the problem-posing activity. After presenting the whole brochure, a copy of one of the pages was given to each student and then he/she was invited to write down everything they could see on that page. Following that, there was a discussion on the observations: the aim was to verify their understanding of the artefact and to create a summary of the mathematical concepts involved.

The second phase, lasting about an hour, consisted of an individual problem-posing activity in which the children had to create the greatest number of solvable mathematics problems (in a maximum time of 45–50 minutes), preferably of various degrees of difficulty, to bring to their partners in the other classroom. The children were not informed of the time limit in order to avoid generating anxiety. Rather, they were told that they would have plenty of time to do this activity and that problems would be collected when the majority of the students had finished. To allow for the pupils' self-assessment, they were given a sheet of paper for their calculations and solutions to the problems they had created.

Then, four problems for the next problem-solving activity were selected from among all the problems that had been created. To facilitate problem solving, problems that would have favoured a discussion among the students were chosen:

- One Multi-step problem, for example "Francesca decided to go to Mirabilandia. There are 15 people including 7 adults and 8 children. Each child spends 26 euro and each adult spend 31 euro. Then, they decide to go to the Mirabilandia beach and they pay an additional 7 euro. What is the total spent?"
- Two Open-ended problems (problems with insufficient information), for example "Luca and his 10 friends go to Mirabilandia to celebrate Luca's birthday. How much did they spend?"
- One Incorrect data problem, for example "A group of 20 people, children and adults, decide to go to Mirabilandia. In total, they spend 480 euro. How much will each person pay to enter?" (The total of 480 was included in the problem by the student. It is incorrect because all of the conditions of the artefact were not taken into consideration—in fact for every 10 entries, 1 entry was free).

For the classes at the second school, the selection criteria of the problems were the same for the first three problems; in the fourth problem, the topic of percentage was included because the students had not yet studied percentage problems. The modified criterion was used since we wanted to study the way in which "anticipatory learning" (Freudenthal, 1991) can be enhanced by the use of an artefact.

The third phase, lasting about 2 hours, consisted of a problem-solving activity by students and ended with a collective discussion. The students were asked to solve problems, to write the procedure that they had used, and to write considerations on the problem itself. Different solutions and ideas that emerged during the discussion were compared and, at the end of the activity, a collective text summarizing the students' conclusions was written.

Methodology and Data Analysis

Data from the teaching experiment included the students' written work, field notes from classroom observations, and audio recordings of the collective discussions.

All of the problems created by the students were analyzed with respect to their quantity and quality. To analyze the types of created problems, the methodology proposed by Leung and Silver (1997) was followed; for the analysis of the text of

Table 5.1
Examples of Each Category of Problem

Category	Example
Non-mathematical problem	Find the name of the following problem.
Implausible mathematical problem	A group of 20 children go to Mirabilandia with the school and each child pays 20 euro. The school children are 130. How much does the school pay to go to Mirabilandia?
Plausible mathematical problem with insufficient data	Giovanni goes to Mirabilandia with his dad. How much does Giovanni spend? How much does his dad spend?
Plausible mathematical problem with sufficient data	A group of 15 people enter in Pizza Time pub and every person pays 7.50 euro. What is total spent?

the problems we referred to the research of Silver and Cai (1996) and Yuan and Sriraman (2010).

Table 5.1 illustrates the first qualitative analysis of the created problems with an example from each category of problem.

In this work, non-mathematical problems are texts which cannot be considered problems or they are not solved through mathematical tools. The mathematical problems were analyzed and divided into implausible mathematical problems and plausible (can apparently be solved, with no discrepant information, and respects the conditions in the artefact) mathematical problems. The plausible mathematical problems were divided further into plausible mathematical problems with insufficient data and plausible mathematical problems with sufficient data.

Plausible mathematical problems with sufficient data were analyzed with respect to their complexity and were assessed by two aspects: the complexity of the text of the problem and the complexity of the solution. With regard to the complexity of the text of the problems, plausible mathematical problems with sufficient data were divided into problems with a question and problems with more than one question. The latter were divided into concatenated questions and non-concatenated questions. With regard to the complexity of the solution, these mathematical problems were divided into multi-step, one-step, and zero-step problems.

Furthermore, only the plausible mathematical problems with sufficient data were re-analyzed to evaluate their creativity. The problems developed by children were grouped taking into account the number and type of details extrapolated from the artefact, the type of questions posed, and the added data included by the students.

To evaluate their creativity in mathematics, three categories were taken into consideration—fluency, flexibility, and originality—as proposed by Guilford (1950) to define creativity, and as used in the tests by Torrance and in other studies such as that by Kontorovich et al. (2011).

When considering the fluency of a problem, the total number of problems created by the pupils of each school in a given time period, as well as the average number of problems created by each student, were taken into account. By contrast, flexibility refers to the number of different and pertinent *ideas* created in a given time period. In order to evaluate the flexibility of the students, the mathematical problems were categorized considering both the number of details present in the brochure (e.g., entrance fee, price of lunch, etc.) which were incorporated into the text of the problem posed, and the additional data introduced by the students (e.g., calculating the change due after a payment). Once the problems had been categorized in the above ways, the various types of problems that occurred in each class were counted.

The originality of the mathematical problems created by the students took into consideration the uniqueness of the problem compared to the others posed in each school. In order to evaluate the originality of a problem, it was considered original if it was posed by less than 10% of the pupils in each school (Yuan & Sriraman, 2010).

Therefore, two different analyses were conducted: one for problem posing and one for creativity. With regard to problem posing, a qualitative analysis was carried out to evaluate students' performance on problem posing and to analyze the structure of the texts of the problems and their solutions. Both quantitative and qualitative analyses were undertaken to evaluate student creativity. The number of problems created per student was counted, and then the texts of the problems associated with each of the problems created by students were analyzed.

Some Results and Comments

A total of 63 students in both schools participated in the problem-posing phase and they created a total of 189 problems. Students from the first school created 58 problems (57 were mathematical problems), while students from the second school created 131 (all mathematical problems).

More than half of the created problems—64% of the problems created by pupils at the first school and about 60% of those created by the pupils at the second school—were solvable mathematical problems (plausible mathematical problems with sufficient data). Table 5.2 shows the main quantitative results for both schools.

After analyzing these solvable mathematical problems we found that:

- 81% of the problems from the first school and 75% of the problems from the second school were multi-step problems; and
- 78% of the problems from the first school and 73% of the problems from the second school were problems with a question.

Table 5.2
Percentage of Problems Created in Each Category

Category	First school (%)	Second school (%)
Non-mathematical problem	1.7	0
Irrelevant mathematical problem[a]	6.9	0
Implausible mathematical problem	19.0	29.0
Plausible mathematical problem with insufficient data	8.6	10.7
Plausible mathematical problem with sufficient data	63.8	60.3

[a] *Note*: Irrelevant mathematical problems did not use any of the information provided in the artefact—the problems, therefore, did not relate to the artefact. The students who posed these problems, in fact, did not understand the presentation of the task.

For problems which involved more than one question, in the first school 62% had concatenated questions, and in the second school, about 43%.

We concluded, from the analysis of the above data, that the first school had better problem-posing performance because the children from first school created fewer implausible problems, more multistep problems, and more problems with concatenated questions than the children from the second school.

Most of the problems created by the pupils were similar to standard problems used in schools (for example: “A father and his son go to Mirabilandia. The adult pays 31 euro, and the child 26. How much do they pay?” And, “How much do they receive in change if they pay with two bills, one of 50 and one of 10 euro?”), although there were some cases (17%, corresponding to 32 out of 189 problems) of creative and open-ended problems.

An example of a creative problem is:

> A group of 50 students go to Mirabilandia. Everyone takes a Ghiotto meal. Then, 50% of this group decide to go to Mirabilandia beach while the other 50% remains in the park area and goes on the rides. The day after, 24 of these students return to the amusement park and 50% of them order a Ghiotto menu while the other 50% takes the Classico menu. 25% of this group wants to return to Mirabilandia. How much does the group pay to go to Mirabilandia beach? And for the food? And for the entrance? And in total?

The text of this problem did not include certain information (for example the entrance fee) because the students who created the problem knew that the other class had the artefact. This consideration also applied to many other problems created by the students.

As far as creativity is concerned, the second school was more successful in all three categories used to assess performance (fluency, flexibility, and originality). With regard to fluency, each student in the first school created two problems on average, while each pupil of the second school created three problems on average. With regard to flexibility, the problems created by the classes of the first school were divided into 11 categories, those of the second school into 16 categories. In evaluating originality, it was found that three original problems were created in the first school and ten original problems in the second school. Original problems included inverse problems and problems involving almost all the information from the artefact. Table 5.3 presents a summary of the creativity results.

In terms of the creativity indicators listed in Table 5.3 (fluency, flexibility, and originality), the students of the second school demonstrated better performance on the parameters used to evaluate fluency and flexibility. It should, however, be noted that the second school had more students with averages in the medium–high range in mathematics, as Table 5.4 shows. The results may suggest that there is a correlation between creativity and performance in mathematics; this aspect deserves to be investigated in a subsequent study.

The results obtained were consistent with those from another study we conducted (see e.g., Bonotto, 2005, 2009) and demonstrate that an extensive use of suitable cultural artefacts, with their associated mathematics, can play a fundamental role in

Table 5.3
Analysis of Problems for Creativity

Category	Method of analysis	Results
Fluency	The total number of problems created by the pupils of each school and the average of the problems created by each student is taken into account	57 mathematical problems were created (two problems per student, on average) in the first school, while 131 problems were created (three problems per student on average) in the second school
Flexibility	The plausible math problems with sufficient data were categorized according to the number and type of information of the artefact present in the text, the type of questions, and the addition of information from the student. Then, the number of categories produced by each school was counted	The problems created by the four classes were divided into 18 total categories, 11 for the first school, and 16 for the second school
Originality	The rarity of the answer was considered: an answer was considered original if it came from less than 10% of pupils in that school	There were three original problems in the first school and ten original problems in the second school. Original problems included inverse problems and involved almost all of the information in the artefact

Table 5.4
Academic Performance in Mathematics of Students in the Two Schools

	Pupils' academic performance in mathematics		
Category	Low (%)	Medium (%)	High (%)
Students of the first school	29	42	29
Students of the second school	12	51	37

bringing students' out-of-school reasoning and experiences into play by creating a new dialectic between school mathematics and the real world. As a paradigmatic example, we have included below some segments from the class discussion concerning the following problem:

> Giovanni decides to celebrate his birthday at Mirabilandia. There are 10 people in total, 6 adults and 4 children. Every 3 children pay 26 euro and each adult pays 31 euro. Giovanni is the birthday boy and he doesn't pay. Also, they decide to make use of the refreshments and they pay 10.50 euro. What is the total spent?

During the discussion, students justified their reasoning using everyday-life experiences and making estimates, as illustrated in this dialogue:

Student 1: This problem isn't written well. The 10.50 euro should be what every person pays for the refreshments, but I realize that the 10.50 euro is the total, because what is written is: Also, they decide to make use of the refreshments and they pay 10.50 euro.

Student 2: But no, because what should be written—and finally all pay 10.50 euro.

Student 3: In the brochure there is written—in the *pacchetto festa* (party package) every person pays 10.50 euro, and not "in total." If you read the brochure carefully, you can understand that the price is per person.

Student 1: But, if you don't have the brochure, how can you solve the problem?

Student 3: It's impossible that ten people pay only 10.50 euro for all the refreshments! It's more likely that the refreshments are more expensive.

[…]

Student 1: It's impossible that all the refreshments cost 105 euro ….

Student 4: If you do the count, 10 people: ten, twenty, thirty, forty [she shows the count with her fingers] fifty, sixty, eighty, ninety, one hundred!

Student 1: For me it's a bit too much.

Student 3: Too much … if you see the table with all the sandwiches, drinks … even the tablecloth has a cost! If there are all the towels, the dishes, the drinks, all these things, all the services cost!

Student 1: But, how can you understand all these things?

Student 5: I think that Martina's considerations about 105 euro are possible. In the brochure there are a lot of things that the children can eat!

Student 3: Then, I think that a drink costs about 3 euro. A drink is enough for two people, because you drink a lot. Then, we image that there are five bottles, therefore five bottles cost already 15 euro. Then there are other things, and each person takes different things; so, it's impossible that all costs 10.50 euro!

[…]

Student 6: Then, here the children are in Mirabilandia; it isn't just any place!

With regard to the problem-solving phase, this appears to be important and helpful in allowing a better understanding of the initial situation, fostering quality control of the problems created by the students themselves, and giving them a starting point for analyzing the structure of problems. We have included below some parts of the class discussion concerning the "incorrect-data problem," reported also in Bonotto (2013). The problem presented incorrect data (480 euro) and the students, during the problem-solving activity, found two different solutions discussed during the collective discussion:

Student 1: We didn't divide by 20. We divided by 18 because the Mirabilandia brochure stated that every 10 entries, 1 entry was free. Therefore, if there were 20 people together, there would be two free entries, and so we divided by 18.

Student 2: I believe that reasoning is wrong because the text of the problem says that they went to Mirabilandia and in total they spent 480 euro, but it doesn't specify if they paid only the entrance or if they went to other places, so the discount is only on the entrance fee and not on the other things.

[…]

Student 3: I think that both solutions are right

[…]

Student 4: One of them must be wrong, because one takes off two people, while the other does not!

[…]

Student 3: Probably, the writer of the problem didn't consider that every 10 entries, 1 entry was free.

Student 2: Practically, the student of the other classroom wrote this problem without realizing that the data was wrong, so we solved it incorrectly.

By solving problems created by their peers, the students became able to analyze them in a more detached and critical way. For example, students reflected on what information was really important and what was not and discovered that numerical information is not always the most important information contained in the text of a problem, as the following problem illustrates:

> It's Giulia's birthday and she invited 9 people to her birthday party, but she didn't benefit from the *pacchetto festa* (party package), how much did she pay for the entrance?

During the discussion, almost all of the students did not read the words of the problem question carefully, because a lot of students calculated the total and not only Giulia's entrance cost. In fact, the total number of people (9) in the problem was superfluous.

Discussion

The specific artefact utilized in this study provided a particularly attractive context inasmuch as it referred to an amusement park known to the children and was desirable because it furnished conditions allowing the students to formulate hypotheses regarding the various possibilities offered. Students were therefore able to create diverse problems with various degrees of difficulty. This activity made it possible to assess problem posing itself and creative thinking in mathematics: children created both original and open-ended problems (in addition to the classic problems), demonstrating that the activity of problem posing can be an environment that fosters creative thinking.

The cultural artefact reflects the complexity of reality and so it offers a rich setting for raising issues, asking questions and formulating hypotheses. It is interesting to reflect on the fact that there were good results for students accustomed to using cultural artefacts (the classes from the first school) as well as those who were using them for the first time (the classes from the second school). In fact, pupils from both schools were able to use the artefact as a context to create problems. This indicates that an artefact provides a useful context for the creation of problems and the mathematization of reality as a result of its accessibility to all students (Bonotto, 2013).

In order to have better performance on the problem-posing task in terms of the greater number of plausible problems, with more complex texts and concatenated questions, it proved to be important to structure, organize, and summarize the information presented in the brochure. In fact, students who had previously performed this type of analysis outperformed the others in the problem-posing activity. With regard to this aspect, students from the first school, who were already familiar with this type of activity, produced fewer implausible problems and therefore appear to have constructed a better analysis and synthesis of the artefact. Instead, about one third of the problems produced by the second school students were implausible problems.

Overall the students involved in the study produced some original problems (13 problems) and open problems (19 open problems). This highlights the fact that pupils were able to deal with open-ended tasks. The problem-solving phase combined with group discussions allowed students to reflect on different types of problems and explore new possibilities (e.g., suggesting that mathematical problems do not always require a numerical answer or a unique solution, and that there are problems which are not solvable). Not only does this confirm the potential of students to create problems, but it also demonstrates the importance of educational action to support students in these kinds of processes.

In fact, almost all of the problems created by the pupils of both schools were classified as mathematically relevant (98% in the case of the first school, 100% in the case of the second school). Of these, more than half of the problems created by the pupils were solvable (about 64% of the problems created by the pupils of the first school and about 60% of those created by the pupils of the second school). This indicates that, at the end of primary school, pupils are not only aware of what mathematical problems are, but they are also able to create appropriate problems.

Furthermore, the results of the discussion in the classroom suggest that asking students to analyze the problems they created facilitated their critical thinking. In this context, students seemed to feel freer to discuss the validity of a given problem, to consider different assumptions, and to decide whether the problem had been solved or not (Bonotto, 2013).

Teachers can assess problem-posing activities and creative thinking several times during the year by applying the proposed method:

- Students are first engaged in problem-posing activities stimulated through a cultural artefact, and this is supported by a problem-solving activity and collective discussions.
- From all of the problems created by the students, plausible mathematical problems with sufficient data are selected for analysis. These can be initially analyzed, from the point of view of problem posing, with respect to complexity of the text and their solutions.
- Then, these same problems can be analyzed from the point of view of creativity with respect to fluency (counting the number of problems created by each student); flexibility (considering both the number of details present in the artefact which were incorporated into the text of the problem posed, and any additional data introduced by the students); and originality (uniqueness of the problem compared to problems created by other pupils).

If these activities are periodically offered to the class, the teacher can then assess changes and improvements over time.

Conclusion and Open Problems

The exploratory study presented here investigated the impact of *problem-posing* activities (supported by problem-solving activities) when these were implemented in meaningful situations involving the use of suitable artefacts. These situations fall under those defined by Stoyanova and Ellerton (1996) as *semi-structured situations*.

Furthermore, this study has allowed us to investigate the potential that problem-posing activities have for identifying critical and creative thinking in mathematics. A method for analyzing the products of problem posing and for assessing both the activity of problem posing itself and the creativity of the students was provided. Furthermore, the study investigated possible relationships between students' knowledge of mathematics, their problem-posing ability, and their creativity.

Two questions arose from the results obtained that require additional research in the future:

1. Does good academic performance in mathematics favour better performance in the three creativity categories (fluency, flexibility, and originality)?
2. How much do teaching practices and classroom experiences influence the creative processes?

Finally, we would like to look more deeply at how children respond over the long term to programs designed to develop their problem-posing skills in the form described here. In agreement with other researchers, we believe that the presence of problem-posing activities should not emanate from a specific part of the curriculum but should permeate the entire curriculum.

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