Chapter 23
A Review on Problem Posing in Teacher Education

Helena P. Osana and Ildiko Pelczer

Abstract Over the last two decades, researchers have shown increased interest in problem posing in mathematics professional development. In the context of teaching mathematics, problem posing can entail asking questions during classroom interactions to assess student understanding, modifying existing problems to adjust the difficulty level of a task, and creating problems to meet instructional objectives. In this chapter, we review the research conducted between 1990 and 2012 on problem posing in mathematics methods courses in elementary teacher education. Despite the range of foci, goals, and theoretical perspectives in the literature, we describe ways in which problem posing has been investigated in the preservice teacher population. Despite the paucity of empirical studies, we were able to group these studies into three distinct categories: (a) problem posing as a skill integral to the practice of teaching mathematics; (b) problem posing as an activity separate from teaching; and (c) problem posing as a tool to assess an outcome variable (for researchers) or as a tool for teaching or assessing the development of preservice teachers' knowledge or beliefs. Implications for mathematics teacher educators that stem from the review of the literature are discussed.

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A Review on Problem Posing in Teacher Education

Contemporary research on problem posing can be traced back to the 1980s and has steadily been gaining interest since that time. Results from cognitive psychology and a reorientation in mathematics education were at the root of this emerging interest, as evidenced by researchers' emphases on identifying the processes underlying mathematical thinking (see Schoenfeld, 1992, for a review on the topic). A growing movement in mathematics education that placed problem solving at the center of school mathematics further contributed to researchers' focus on problem posing, particularly its role in teaching and learning. In this context, problem solving was simultaneously viewed as a means for teaching mathematical reasoning and as a learning objective in and of itself (see, for example, the first *Standards* document published by the National Council of Teachers of Mathematics [NCTM], 1989). Scholars' understanding of the nature and effects of mathematical problem solving over the last several decades, with its roots in cognitive science (e.g., Newell & Simon, 1972), gave rise to the notion that problem posing is a process that is embedded within (and difficult to separate from) problem solving. Indeed, Kilpatrick (1987) claimed that problem posing is an important constituent of mathematical thinking, but some years before, Brown and Walter (1983) had already argued for the central position of problem posing in learning and thinking about mathematics. Similarly, interest in the nature and role of creativity—and its link to other elements of mathematical thinking—further contributed to the study of problem posing. From this perspective, problem posing was seen by many as a way to assess and enhance creativity (Silver, 1997). The history of problem posing, its varied uses in research and teaching, and its inherent cognitive and creative components together attest to its complex nature.

Early research on problem posing was centered on children's thinking and reasoning. In particular, scholars studied the cognitive processes used by children during problem posing (e.g., English, 1997), the types of problems posed (e.g., Gonzales, 1996), and comparisons of the behaviors and attitudes of students from different cultural settings (e.g., Cai & Hwang, 2002). In this line of research, and in parallel with the study of children's problem solving, problem posing was also seen as a way to assess children's mathematical understanding. At a somewhat slower rate, an interest in the problem-posing abilities of teachers was emerging, particularly in mathematics. A series of studies published by Silver and his collaborators in the 1990s looked at relationships between teachers' knowledge, task format, and creativity (e.g., Leung & Silver, 1997). These studies focused on investigating the commonalities and distinguishing features of mathematics teacher knowledge and task-related conditions for posing problems. Although these findings have been informative for the study of teacher knowledge from a theoretical perspective, the data were not, at least in these cases, directly applied to improve professional development initiatives.

Shulman's (1986) well-known article introduced the construct of *pedagogical content knowledge* (*PCK*) as being a critical type of knowledge, along with *subject-matter* and *curricular knowledge*, needed by teachers for effective practice. Shulman's work opened up a new line of research and brought forth efforts by many mathematics educators to specify and clarify the PCK construct further. For example, the construct of *mathematical knowledge for teaching*, and its accompanying conceptual framework, was introduced by Ball, Thames, and Phelps (2008); it describes the nature of knowledge needed for mathematics teachers in their practice. Their framework is useful for our purposes because it provide an analytical framework for our review of problem posing in the context of preservice teacher preparation in mathematics.

The Ball et al. (2008) framework has several components, including common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT). CCK is mathematical knowledge that is not unique to teaching, but is also invoked by professionals in other fields (hence the term, "common"). CCK is required by teachers in a variety of tasks, such as solving the problems that they assign to their students and identifying when textbooks provide inaccurate descriptions of mathematical content. In other words, as stated by Ball et al. (2008), "teachers need to know that material they teach" (p. 399). SCK represents a body of mathematical knowledge beyond what is taught to students and is different from the mathematical knowledge used by research mathematicians or applied in professions where mathematics is used as a means to solve practical problems. It is mathematical knowledge that is specifically needed in teaching, or in the words of Ball and Bass (2001), knowledge that is "pedagogically useful." It is needed in carrying out tasks such as, "recognizing what is involved in a particular representation, finding an example to make a mathematical point, or modifying tasks to be either easier or harder, asking well-chosen follow-up questions" (Ball et al., 2008, p. 400).

KCS centers on student thinking in mathematics: teachers must know the common pitfalls in student thinking about specific topics and problems, the types of thinking afforded by specific tasks, and what will be challenging or trivial for students at any given point in a unit. KCS also entails being able to interpret the ways in which students describe their thinking about mathematical ideas, which, given the frequent incompleteness and complexity of students' explanations, is necessarily based on knowledge of student thinking more generally. Finally, KCT involves making key decisions about instruction, and how specific aspects of instruction will affect student learning. For example, KCT involves knowledge about content sequencing, which means knowing what examples and activities to begin with, and which ones to use to explore the content in more depth.

Problem posing has been viewed as a pedagogical tool for teachers who aim to enhance their students' learning of mathematics (English, 1998; Pirie, 2002), and as a mechanism used by teachers to engage in productive mathematical conversations with their students (Boaler & Brodie, 2004; Franke et al., 2009). With preservice teachers, it has been found to enhance pedagogical content knowledge (Ticha & Hošpesová, 2009) and to have positive influences on their beliefs about and attitudes toward mathematics (Bragg & Nicol, 2008). What is meant by the term *problem*, however, differs in this literature; a problem can be considered a formal word problem written and presented to students to solve (e.g., Crespo, 2003). In this case, problem posing refers to the act of designing such a problem either during planning or in the middle of a lesson. A problem could also be a question that is verbally stated for a specific purpose, such as delving more deeply into the reasoning used by a child or as a way to extend a student's application of a mathematical concept. In this latter conception of the term *problem*, all questions and follow-up questions a teacher asks to test or further the mathematical thinking of her students can be considered problems (e.g., Ball & Forzani, 2009).

Consider the following scenario as an illustration of how problem posing lies at the heart of teacher questioning (the mathematics in this example is offered by Ball & Forzani, 2009). In Mr. Clay's fifth-grade classroom, a student claims that $.2 \times .3 = .6$. It is relatively easy to see what the child did to arrive at her answer (used the same procedure for multiplying decimals as for adding them), but Mr. Clay needs to get at the mathematical reasoning behind the strategy. Given this objective, what would be the best follow-up problem? Giving $.5 \times .2 = \square$ next would likely result in the student answering .10 (the correct answer) for very incorrect reasons, after which the teacher may think that $.2 \times .3 = .6$ was just a calculation error. Instead, giving $.1 \times .5 = \square$ would likely result in the student producing .5 as the answer, confirming that her difficulty lies in understanding that multiplying tenths by tenths produces hundredths. By posing the right follow-up question, Mr. Clay can thereby open key learning opportunities.

Preservice teachers have difficulty engaging in problem posing because it is a skill with which they are not familiar, both as students and as educators (Crespo & Sinclair, 2008). The problems they pose are not cognitively or structurally complex (Stein, Smith, Henningsen, & Silver, 2000; Vacc, 1993) and they often do not align with targeted mathematical concepts (Osana & Royea, 2011). Recent studies on teacher questioning have revealed similar challenges. Franke et al. (2009), for example, found that, after the initial "how did you figure that out?" question, teachers had difficulty asking follow-up questions to delve more deeply into student thinking. Osana et al. (2012) found that, although inservice teachers are quite adept at asking students how they solved problems, they were largely unable to ask specific probing questions that would challenge students' misconceptions about the equal sign. It is clear, therefore, that there is a pressing need to improve teachers' ability to ask purposeful questions and pose useful problems in the context of teaching mathematics.

The newly acknowledged role of problem posing in teaching, coupled with converging findings on prospective teacher's difficulties in problem posing, has prompted teacher educators to include problem posing skills in the elementary mathematics methods curriculum (we shall restrict our discussion here to the elementary level). Although this area of inquiry is still in its infancy, our review of the literature suggests that a synthesis of the existing research on how problem posing has been incorporated into the professional development of prospective teachers would be beneficial for researchers and mathematics teacher educators. Therefore, the purpose of the present review is to analyze and synthesize reported research on problem posing in elementary teacher education. From a theoretical perspective, we aim to interpret the existing research through Ball et al.'s (2008) conceptual framework of mathematical knowledge for teaching. Our interpretations promise to shed light on teacher educators' views of problem posing in mathematics and how problem posing allows them to operationalize mathematics knowledge for teaching in their methods courses. On a practical level, we aim to identify key questions at the heart of using problem posing in teacher education and to identify those practices that have promise for the development of this skill in the preservice teacher population. More generally, therefore, our review can generate useful guidelines for reflection on essential objectives in mathematics teacher education and ways to achieve them.

Below, we describe our selection criteria for the reviewed articles and the ways in which we categorized the selected articles. We also provide a synthesis of the articles using a coding rubric that emerged throughout the reviewing and selection process, and we interpret the selected articles using the framework of mathematical knowledge for teaching presented by Ball et al. (2008). We close by discussing implications of the review for teacher preparation in elementary mathematics.

Method

Selection Criteria

The articles were selected through major indexing databases, available at a large university in Canada. The SCOPUS, Web of Science, Web of Knowledge, Springer, ERIC, JSTOR, OpenDOAR from Sherpa (scopus.com, springer.com, jstor.org, http://www.eric.ed.gov/) repositories were searched using the following keywords: *problem posing*, *preservice/prospective/future teachers*, and *mathematics*. We only included studies published between (1990), which marked a shift in research interest toward teachers' problem posing, to present (2012). From this group of articles, we then selected only those that presented empirical research on problem posing: We excluded those that did not report at least one data set related to the phenomenon, but no specific methodology was excluded.

Finally, we further reduced the sample to include only those articles with elementary preservice teachers as participants; in particular, we were only interested in studies situated in "elementary" teacher education—that is, the professional development of future teachers as generalists who would teach all subjects (e.g., mathematics, language arts, science, social studies) starting in Grade 1 through, in North America, Grade 6. The main characteristic of these teacher education programs is that future teachers receive training for a series of "school subjects." Generally, the majority, if not all, of their mathematics-related courses focus on general guidelines related to the teaching and learning of mathematics (i.e., "methods" courses) and focus on such overarching principles as problem solving and communication in the classroom. Fewer courses are specifically targeted to the content itself, such as the conceptual basis of computational algorithms or the underlying mathematical structure of word problems.

In contrast, secondary teachers receive subject-specific training, which entails enrolling in a number of content courses, such as mathematics and physics. Preservice teachers of elementary mathematics, who do not receive such training, are left with the mathematics they learned when they were students themselves, and the extent to which they engage in additional experiences with mathematics is highly dependent on the teacher education programs they attend. Because the training of preservice teachers differs considerably between elementary and secondary teacher education programs, at least with respect to the emphasis on content, we targeted articles only at the elementary level. Those articles in which it was impossible to determine the type or level of teacher education program from which the participants were selected (e.g., "generalist" vs. "specialist" teacher education) were excluded.

Coding and Analysis

Using the selection criteria, we were left with 8 articles for the review presented in this chapter. Our coding of these articles was guided by our specific focus on teacher educators' practices related to problem posing: in particular, the ways in which they attempted to either foster problem posing among their students or the ways in which they used problem posing as an approach to their instructional practice. The specific codes, based on a careful reading of the articles, were generated using a grounded theory technique (Strauss & Corbin, 1998) in which we engaged in successive rounds of coding, each time followed by discussions that served to resolve any discrepancies and to refine the rubric by generating subcategories for the main codes. The coding process resulted in three major categories of the ways in which problem posing has been incorporated by elementary mathematics teacher educators in their practice. Specifically, these three categories are: (a) fostering problem posing as a skill integral to teaching practice; (b) problem posing as an activity separated from teaching, but conducted by preservice teachers; and (c) problem posing as a tool for researchers (e.g., as the basis for the design of outcome measures) or as a tool for teacher educators to change or enhance preservice teachers' knowledge. The rubric is listed in Table 23.1.

In the first category, problem posing as a skill integral to teaching practice (code: TP), we placed articles in which one of the objectives for the preservice teachers was to generate problems for actual or hypothetical students. In these studies, problem posing was seen as an integral part of mathematics teaching in the sense that the preservice teachers were to use certain criteria to make the problems cognitively appropriate, mathematically suitable, and motivating for their students. The articles we placed in the second category, problem posing as a separate activity (code: SEP), were those in which the process of problem posing itself was the object of study. While the data were collected from preservice teachers, problem posing was not conducted with the mathematical learning of their future students in mind; the focus of the data collected and their analyses was on cognitive aspects of problem posing

Table 23.1
Coding Rubric

Code name	Code	Description
Problem posing as a skill integral to teaching practice	TP	Generating strategically and pedagogically targeted problems to uncover student thinking or mobilize specific mathematical concepts during teaching (e.g., follow-up problems)
Problem posing as an activity separated from teaching	SEP	Cognitive processes involved in problem posing and the factors that are tied to them (e.g., beliefs, epistemology, context)
Problem posing as a tool		
Problem posing as a tool Research tool	T-R	Problem posing used as a way to assess a variable of interest (e.g., conceptual knowledge)
Problem posing as a tool Tool for teacher educators	T-TE	Problem posing used to assess the development of preservice teachers' beliefs, perceptions, knowledge; problem posing used to foster and enhance preservice teachers' pedagogical knowledge

as a form of problem solving, as well as on factors that play a role in posing problems, such as beliefs and context. Finally, the third category was created for studies in which problem posing was used either as a tool for researchers (e.g., as a way to assess a specific variable of interest in preservice teachers, such as conceptual knowledge) or as a tool for teacher educators to assess the development of the preservice teachers in their mathematics methods classes (e.g., learning, changes in beliefs or epistemology) or to foster growth in their pedagogical knowledge. Table 23.2 lists the eight articles included in the present review, including the number of participants in each study, the codes we assigned to the articles according to the rubric in Table 23.1, and the main findings for each.

Analysis and Synthesis of Problem Posing Literature

Problem Posing as Integral to Teaching Practice

In an important article, Ball and Forzani (2009) addressed the role of teachers' follow-up questions during classroom interactions for the purposes of clarifying students' understanding of specific concepts or skills. Problem posing, or the ability to "pose strategically targeted questions," as the authors put it, is central to teaching by offering a way to access and understand students' thinking. This kind of problem posing, along with the ability to "choose tasks, examples, models or analogies, and materials" (p. 501), is particular to the teaching practice. It is used by teachers to mobilize important concepts, test hypotheses about student thinking, and assist students to move through their learning challenges. Moreover, problem posing is considered as a "high leverage practice"—that is, one of the "practices [that is] most likely to equip beginners with capabilities for the fundamental elements of professional work and

Table 23.2
Summary of Study Characteristics

Study	*N*	Code	Main findings
Bragg and Nicol (2008)	33	T-TE	Preservice teachers' views on mathematics teaching and learning changed as a consequence of an open-ended problem-posing task. Curriculum specifications appeared to be both inhibiting and facilitating factors.
Chapman (2012)	40	SEP	Preservice teachers approached problem posing from a variety of perspectives, which in turn influenced the nature of the problems they posed. As a result of engaging in problem posing, preservice teachers became aware of the limitations of their mathematical knowledge for teaching.
Crespo (2003)	20	TP	Preservice teachers' strategies for posing problems changed from posing simple, single-step problems to proposing open-ended, cognitively complex problems. The presence of a "real" audience and discussions with their peers about their problems were possible factors.
Crespo and Sinclair (2008)	22	TP	The quality of problems posed by preservice teachers improved as a consequence of an activity in which a given mathematical situation was explored prior to the posing task. Whole-class discussions on the ways to evaluate problems appeared related to problem posing.
Nicol and Bragg (2009)	33	SEP	Preservice teachers posed open-ended problems, but had difficulties in identifying the intended learning objective targeted by the problem. The specific context (basing problems on digital photographs) was useful in raising awareness of the use of problem posing in mathematics teaching.
Osana and Royea (2011)	8	T-R	The authors assessed improvement of conceptual knowledge in preservice teachers after instruction. A problem-posing task was used as a transfer task of conceptual knowledge. Although there was improvement on one measure of conceptual knowledge, preservice teachers did not improve in their problem-posing abilities.
Ticha and Hošpesová (2009)	24	T-TE	Problem posing was found to be a useful tool for assessing preservice teachers' level of understanding and misconceptions. Personal beliefs about problem posing seem to influence preservice teachers' predisposition when engaging in problem-posing tasks.
Toluk-Uçar (2009)	50	T-TE	Problem posing is a useful tool for assessing preservice teachers' knowledge of specific mathematical concepts (fractions). In addition, it likely had a positive impact on teachers' views about knowing and doing mathematics.

that [is] unlikely to be learned on one's own through experience … [a] teaching [practice] in which the proficient enactment by a teacher is likely to lead to a comparatively large advances in student learning" (p. 460). As such, Ball and Forzani see problem posing as integral to teachers' interactions with students, which, in good teaching, necessitates considering their students' prior knowledge and interests. By viewing problem posing in this way, future teachers must learn, among other things, to take into account feedback received from the student.

Two articles were placed in this category. In the first of these articles, Crespo (2003) examined the processes used by preservice teachers as they posed problems for elementary students in a letter-exchange project. Over the course of a semester-long methods course, Crespo required her students to engage in a pen-pal activity with fourth-grade students at a local elementary school. The letter-writing activity, in the author's view, allowed future teachers to focus on three aspects of mathematics teaching: creating and presenting tasks to students, analyzing pupils' work, and reacting to their ideas outside the context of a busy classroom, thereby affording time for reflection and revision. By excluding the need for classroom management, the prospective teachers were able to concentrate all their efforts on these three aspects of teaching.

Crespo (2003) required her students to pose problems that were modifications of existing ones and to create their own problems "from scratch." She observed that her students used three approaches when they began this activity: (a) making existing problems easier to solve (by simplifying existing problems they had found, for example); (b) posing problems that were structurally similar to familiar ones (such as "typical" textbook word problems); or (c) posing problems "blindly" without reflecting on the mathematics at the heart of the problem or children's thinking about associated concepts. After 11 weeks of letter writing, however, the author reported a significant shift in their strategies: The preservice teachers were more likely to present problems that were less "traditional" to their letter-writing student partners, pose problems that would challenge the children's mathematical thinking, and pose problems designed to gain insight into their thinking. Crespo attributed this change to the authentic nature of the letter-writing activity—the preservice teachers had a "real" audience who received and tackled their problems, and they were confronted with actual responses from children.

Crespo's research illustrates one mathematics teacher educator's view of problem posing: as an activity undergone by teachers as they planned activities in advance of the lessons they conducted in the classroom. As such, she views problem posing as a component that is integral to a teacher's practice. Furthermore, Crespo saw great potential for the letter-writing activity to enhance the student teachers' KCS; the problem posing activity supported their awareness of how problem posing could be tailored specifically to children's needs and interests and increased the preservice teachers' sensitivity to the types of problems that are likely to elicit specific responses. Further, Crespo engaged the preservice teachers in discussions about the characteristics of non-traditional mathematical problems that emerged during the semester. This allowed her students to gain a deeper understanding of what mathematics children can learn in relation to the problems they are given to solve, again emphasizing the critical role of KCS in the preparation of mathematics teachers.

A recurring theme in the research on teachers' problem posing is the challenges they experience posing correct problems (from a mathematical point of view) as well as those that are pedagogically suitable. In another study aimed at enhancing preservice teachers' problem posing, Crespo and Sinclair (2008) set out to investigate the potential effects of holding discussions with preservice teachers on what constitutes good, interesting, appropriate, and even “beautiful” problems. The authors’ hypothesis that mathematical exploration is effective in fostering appropriate problem-posing processes lies in the work of Hawkins (2000) and Dewey (1933), who both claimed that perceiving a certain situation as “problematic” (Hiebert et al., 1997)—one that presents a dilemma—is a necessary condition for proposing alternative solutions. Crespo and Sinclair (2008) suggested that considering exploration as a distinct activity from problem posing leaves the poser unable to recognize potential sources for problems, thereby preventing the generation of suitable and interesting problems. Ultimately, the authors proposed that viewing mathematical exploration and problem posing as separate processes can explain the preponderance of ill-formulated or uninteresting problems in the preservice teacher population.

In this context, Crespo and Sinclair (2008) engaged preservice teachers in explorations by encouraging them to make judgments, using mathematical and pedagogical criteria, about a number of mathematical problems during class discussions. Then, the authors compared the problems posed by preservice teachers before and after the exploration intervention and concluded that the quality and range of the problems they posed increased. Their problems were more cognitively demanding relative to the problems posed by those who did not receive the exploration intervention (a comparison group); their problems would ostensibly require considerable effort and mathematical reasoning to solve. These results provided some evidence that the teachers’ KCS was enhanced considerably: They became more sensitive to characteristics of the problems themselves and how the problems’ features and structure could be modified to make them either more or less challenging for children.

The authors designed a second intervention that was based on classroom discussions about esthetic criteria used by mathematicians in judging problems. Drawing on Dewey’s (1934) interpretation, Sinclair (2004) stated that the term “esthetic” can be understood as related to a “sensibility in combining information and imagination” (p. 262). Crespo and Sinclair argued that esthetic features of problems, such as novelty, surprise, and “fruitfulness,” are important for teachers to consider, even though their objectives are in many ways different than those of mathematicians. Sinclair (2004) argued that teachers can generate rich contexts for mathematical exploration by taking such esthetic criteria into account in their tasks and interactions with students. Crespo and Sinclair (2008) noted the tension that emerged when the preservice teachers grappled with both the pedagogical and mathematical potential of the problems. Seemingly, the prospective teachers generally tended to the pedagogical values of problems and often ignored their mathematical qualities.

By encouraging preservice teachers to engage in open-ended explorations of previously posed problems, Crespo and Sinclair (2008) underscored the importance of teachers reflecting on problems before incorporating them into their mathematics lessons. Thus, their study illustrates that part of problem posing involves the ability to evaluate existing problems on a variety of mathematical and pedagogical dimensions. As such, along the same lines as Crespo (2003), the study places problem posing squarely in the domain of KCS because it can assist teachers to attend to the features of the problems themselves and their affordances for learning. Finally, the conclusions drawn by Crespo and Sinclair underscored the distinct nature, but yet important connection between content knowledge (CCK and SCK) and KCS, an element of pedagogical knowledge. As such, we argue that Crespo and Sinclair view problem posing as a catalyst for the mobilization of both content knowledge and pedagogical knowledge, thereby further promoting the power of problem posing in mathematics methods courses.

The Sense Preservice Teachers Make of Problem Posing

In this category, we placed articles in which problem posing itself is the focus of the research. Although the studies were conducted with preservice teachers, the researchers' emphasis was on the individual cognitive processes of the preservice teachers as they engaged in problem posing activity as well as the sense they made of such activity. In these articles, participants are not tasked with fine-tuning problems to delve at specific aspects of their students' thinking or to further elaborate on a mathematical concept; rather, the researchers' purpose is to reveal how future teachers make sense of a problem posing task, what they find difficult, and how they cope with these difficulties. In terms of mathematical knowledge for teaching, the focus is on the design of instruction, and more specifically, the challenges encountered by preservice teachers in their attempts to engage in various aspects central to the practice of creating mathematics problems for students.

In the first of the two articles we placed in this category, Nicol and Bragg (2009) argued that the types of problems future teachers pose, without any previous training on this skill, act as expressions of the sense they make not only of the task, but also of the beliefs and knowledge they bring to their teacher education programs. The idea that a task, whether attended to or created, is a manifestation of the beliefs and perceptions teachers hold for images about mathematics is reflected in Schoenfeld (1992), who argued that the mathematical tasks given to students can have an impact on the conceptions and beliefs they hold about the discipline.

Nicol and Bragg (2009) presented 33 preservice teachers from a Canadian university with photographs of real life scenes and asked them to pose open-ended problems that incorporated the photograph and its contents. In addition, once they had posed problems, the students were required to connect them to a specific learning objective (using the definition of "learning objective" presented by Martin and Booth, 1997). Using a three-point scale, Nicol and Bragg independently rated the problems according to how closely they actually connected to the learning objectives identified by the preservice teachers. The authors also investigated three specific aspects of the preservice teachers' problem posing: the types of problems posed, the factors that influenced them in the process, and what they found challenging about the task. The analyses of the data revealed, in part, that students experienced difficulty posing open-ended problems, partly also because of their general unfamiliarity with such problems. A more difficult aspect proved to be the correct identification of the intended learning objective—the authors' own ratings demonstrated that only 26% of the problems posed were strongly connected to the preservice teachers' stated learning objective.

A second result pertinent to this discussion was the way in which the preservice teachers incorporated the photographs in the problems they posed. The authors used the term *interactive* for problems in which the photograph was essential for answering them and *illustrative* for the other cases. Although designing interactive problems proved to be challenging for the preservice teachers (59% of the posed problems were of this type), the experience generating interactive problems had an impact on their perceptions of the mathematical potential of their everyday environments, a point to which we return later in the chapter. A second analysis entailed an examination of the strategies that the future teachers employed to generate open-ended problems. The authors classified the strategies as follows: (a) removing information from closed questions (i.e., those problems with only one correct answer); (b) using major curricular areas (e.g., geometry, measurement) as starting points; (c) starting from more specific curricular topics (e.g., patterns) and taking a photo they thought was suitable; (d) imagining being a young child and posing a problem he or she might ask; and (e) focusing on the wording and other linguistic aspects of the problem.

Although this study, like the two previously reviewed, illustrates the authors' view of problem posing as an activity conducted by teachers as they plan their mathematics lessons, Nicol and Bragg (2009) placed the practice of teaching itself in the background. Rather than focusing on problem posing as a teaching task, the authors emphasized the processes undergone by the prospective teachers as they posed problems and described the obstacles encountered during the activity. The authors' analyses of the challenges encountered by the preservice teachers highlight the connection between problem posing and KCT. More specifically, their research points to the ways in which problem posing requires a teacher to consider the types of problems that would be appropriate for specific topics and learning objectives.

Chapman (2012) investigated the sense preservice teachers make of problem posing in the absence of any instruction on it. As part of a larger study, she presented 40 preservice teachers with a variety of tasks, modeled after those found in the problem-posing literature, that entailed posing problems and reformulating given problems. By giving a range of problem posing tasks to the same groups of participants, Chapman was able to make more valid comparisons of problem-posing behavior by task type. The tasks were given to the preservice teachers one at a time, in alternating order of problem posing and problem reformulating.

By analyzing the participants' work, Chapman identified five viewpoints on problem posing held by the preservice teachers. The *paradigmatic* perspective characterizes problem posing as the creation of problems "with universal interpretation, a particular solution and an independent existence from the problem solver" (p. 140). These problems resemble "traditional" word problems and reflect the preservice teachers' own prior experiences as elementary students. The *objectivist* perspective characterizes problem posing as the creation of a problem in a backward fashion, starting with a fact (e.g., a multiplication fact such as "$3 \times 4 = 12$") and then writing a word problem around it. From the *phenomenological* perspective, the creation of a problem emerges from a given situation through the construction of meaning; this involves the individual's point of view and the production of "personalized interpretations and solutions." The *humanistic* perspective is similar to the phenomenological one, but the context in which the problem is situated reflects the individual's experience and interests, including hobbies and personal preferences. Finally, the *utilitarian* perspective characterizes problem posing as a mechanism for getting at students' mathematical thinking, such as problems that target specific representations or that require students to articulate their knowledge.

Chapman focused her discussion on three particular tasks: (a) one in which the students were required to "create a word problem that you think is open ended" (p. 138); (b) another in which the students were asked to pose problems that embody different meanings of multiplication; and (c) a final one that required the creation of a problem based on a given diagram. Similar to the conclusions of other researchers, Chapman highlighted the difficulties experienced by preservice teachers in creating open-ended problems (see Nicol & Bragg, 2009) and creating problems that reflected a different meaning of multiplication (see e.g., Toluk-Uçar, 2009). The specific areas of difficulties she observed revealed weaknesses in both CCK and SCK: a reliance on closed problems, singular interpretations of mathematical concepts, and a lack of awareness of the importance of mathematical structures and representations in problem posing.

Once again taking the perspective that problem posing is an activity conducted in advance of teaching, Chapman nevertheless concluded that the five perspectives on problem posing that emerged from her data could be useful for mathematics teacher educators because they make explicit the affordances and inhibitions in the development of preservice teachers' mathematical thinking. Her focus on disciplinary thinking is a significant departure from the emphasis on the pedagogical aspects of a teacher's knowledge in the studies reviewed earlier in this chapter. Chapman's findings, therefore, allowed her to view problem posing as an activity that is dependent on, and reflective of, the content components of mathematical knowledge for teaching, such as CCK and SCK.

Problem Posing as a Research or Pedagogical Tool

In the articles in this category, problem posing is used either as a research tool or as a pedagogical tool for the teacher educator, whose objective is to foster change in preservice teachers' cognition or affect. When problem posing was used as a research tool, the focus was not on problem posing per se, but rather on a different construct that the researchers believed was correlated with problem posing. For instance, Osana and Royea (2011) used problem posing to measure preservice teachers' conceptual understanding of fractions, which was the focus of their study. When problem posing is used as a pedagogical tool, it is used as a means by which teacher educators either assess preservice teachers' knowledge and beliefs or as a context in which teachers' conceptual understanding of attitudes about mathematics can be fostered. In all the studies in this category, problem posing was not itself the object of investigation, but its role was examined insofar as it could support instructional practice in mathematics methods courses.

Problem posing as a research tool. Osana and Royea (2011) reported the results of a one-on-one intervention with eight preservice teachers on the topic of conceptual and procedural knowledge of fractions. Using a single group pretest–posttest design, the authors gave five individual tutoring sessions to each participant and measured changes in fractions knowledge after the tutoring. During their first session, each participant completed a paper-and-pencil assessment that included items measuring fractions knowledge as well as four problem-posing items, which required the participants to write word problems that corresponded to a given mathematical equation (e.g., $5 \times 1/3$). Given that problem posing was not part of the fractions intervention, the authors used problem posing as the basis for a transfer task to measure conceptual knowledge. During the last session, Osana and Royea administered the same paper and pencil test, which included isomorphically similar problem-posing items.

The authors found significant improvement on the conceptual knowledge scale, but not on the procedural knowledge scale, nor on the problem-posing task. Error analyses highlighted the preservice teachers' difficulty in posing problems for number sentences involving division, and in particular, those in which the divisor is a fraction. The second most frequent error was in cases in which the number sentence involved subtraction: The preservice teachers most often did not attend to the unit when considering fractional parts.

Osana and Royea (2011) viewed problem posing as having a utilitarian function (Chapman, 2012)—that is, although the authors examined in detail the problems posed by the preservice teachers and carefully catalogued their errors, the authors' primary objective was to enhance their fractions knowledge. Problem posing was used as a way to assess whether the preservice teachers were able to apply their conceptual understandings to create problems that accurately reflected specific mathematical operations with fractions. It is clear, therefore, that Osana and Royea (2011) focused their intervention on enriching the CCK and SCK of preservice teachers. In so doing, the authors positioned problem posing as a tool to evaluate their development in these areas, but not as a part of mathematical knowledge for teaching itself. Nevertheless, Osana and Royea's findings prompted them to recommend including explicit instruction on problem posing in mathematics methods courses, which could constitute "a context for highlighting the connection between mathematical concepts and procedures used to solve … problems" (p. 350). The effect of direct instruction on the development of SCK and other aspects of teacher knowledge, however, remains open for further discussion.

Problem posing as a pedagogical tool in mathematics methods courses. Bragg and Nicol (2008), in the same study described earlier, investigated to what extent problem posing could challenge the preservice teachers' views on mathematics as a discipline. At the completion of their methods course that included the problem-pictures assignment, the authors administered a 15-item online questionnaire. Four of the items were designed to measure the participants' perceptions of the assignment. The preservice teachers reported that they found the task challenging, and they indicated that, over the course of the semester, they found it easier to start with a mathematical concept and then find photographs to illustrate it, which was not intended by the original task. Regarding their view of mathematics, the preservice teachers reported that, after engaging in the problem-posing task, they were better able to see the mathematics in their everyday surroundings. In addition, designing open-ended problems gave a quarter of the participants a feeling of "empowerment," which they described as gaining ownership of their created problems and an accompanying sense of confidence in their ability to use them in their future teaching.

In this study, Bragg and Nicol (2008) viewed problem posing as a pedagogical tool for the mathematics teacher educator—that is, they predicted that engaging in problem posing would influence the preservice teachers' attitudes and perceptions about mathematics and their ideas about what is involved in teaching it. Moreoever, the challenges and experiences reported by the preservice teachers bring the authors' view of mathematical knowledge for teaching to the fore. This insight into the broader impact of problem posing on preservice teachers' perceptions demonstrates how Bragg and Nicol viewed teacher preparation as more than "acquiring" specific types of knowledge. Their research on problem posing demonstrates that mathematical knowledge for teaching should also incorporate more affective constructs, such as teachers' personal connections with the mathematics in the world and their confidence in helping children learn mathematics.

In a similar fashion, Ticha and Hošpesová's (2009) main focus was on using problem posing as a means for fostering preservice teachers' pedagogical knowledge in a mathematics methods course. At one point during the course, the authors asked 24 preservice teachers to pose three word problems corresponding to a given symbolic expression involving fraction multiplication (e.g., $\frac{1}{4} \times 2/3$). The problems were collected and stored in a database, and the students then produced ratings of the suitability and the correctness of each other's problems, which were also stored in the database and available for their peers to consult. The authors then selected three problems posed by one preservice teacher and brought them to the class for discussion. The discussions entailed analyses of the problems as well as ways to view them as diagnostic tools in the elementary mathematics classroom. Ticha and Hošpesová observed that once the preservice teachers posed their problems, they did not verify whether the problems reflected the target mathematical expressions, again pointing to preservice teachers' difficulty in invoking KCT, possibly as a result of incomplete mathematical knowledge. The authors were, however, able to correct the problems once any discrepancies were brought to their attention during the class discussions by the authors or their peers.

Ticha and Hošpesová viewed problem posing in a similar way to that demonstrated by Bragg and Nicol (2008)—as a means to foster in preservice teachers specific elements of what they considered to be central to mathematics teaching. In this case, the authors illustrated how problem posing could assist the teacher educator to support prospective teachers in the development of KCT. In particular, through explicit reflection and communication, problem posing can reveal gaps in the teachers'

KCT, which can then be addressed by the teacher educator in a methods course. This said, the use of problem posing as a tool for teacher educators was not conceptualized by Ticha and Hošpesová as part of mathematical knowledge for teaching.

In her mathematics methods course, Toluk-Uçar's (2009) aim was to enhance her students' understanding of fractions, and she hypothesized that this could be achieved by engaging her students in instruction centered on problem posing. The participants were 95 preservice teachers enrolled in a year-long elementary methods course in Turkey. The participants were split into two non-random groups, with 50 preservice teachers in the problem-posing group and 45 in a comparison group. During the first half of the methods course, all participants were given a 2-hour lecture on problem posing that included a description of different types of problem posing as well as general problem posing strategies. After the lecture, participants completed a pretest on fractions that comprised ten items designed to measure their conceptual understanding of fractions concepts and one question requiring the preservice teachers to write about how confident they felt about their knowledge of fractions. At the end of the course in which problem-posing activities were implemented, the same instrument was administered again to both groups. Toluk-Uçar also collected the participants' weekly mathematics journals to gain insight into the development of their fractions knowledge and views of mathematics.

The fractions intervention lasted for 6 weeks and consisted of asking participants to pose problems during class, either individually or in small groups, that would invoke certain fractions concepts, such as equivalence or comparison of fractions and fractions expressions (e.g., $\frac{3}{4} - \frac{1}{2}$). Toluk-Uçar then selected specific problems posed by her students and engaged them in whole-class discussions about these problems. The discussions focused on the appropriateness of the problems to the given situations, the solvability of the problems, and their appropriateness for students at specific developmental levels. The discussions also included possible ways to modify problems that were inappropriate. Throughout the intervention, participants were encouraged to use different representations for fractions and their operations. The comparison group followed the instructional approach that had been traditionally used in the course previously, which entailed developing lesson plans for the same fraction topics as those covered in the problem-posing group.

The results revealed a significant difference in students' conceptual understanding of fractions after the problem-posing intervention, especially with respect to multiplication and division. Furthermore, Toluk-Uçar found that on the pretest, most participants from both groups reported that fraction multiplication and division are characterized by their corresponding algorithms and do not have any connection to real life situations. On the posttest, in contrast, the students in the problem-posing group were better able than those in the comparison group to pose word problems corresponding to those operations using real life contexts. On this measure, the performance of the comparison group did not change.

Along the same lines as the other studies included in this category, the view of problem posing taken by Toluk-Uçar was as a tool to enhance the content knowledge of preservice teachers. In this case, the Toluk-Uçar objective was to use problem posing as a vehicle for the enhancement of their CCK and SCK in the area of fractions. The author's approach illustrates how she chose to mobilize the content component of Ball et al.'s (2008) framework for teacher knowledge. Furthermore, Toluk-Uçar's study is another example of how problem posing can reflect the ways in which teacher educators can target mathematical knowledge for teaching in their methods courses, and at the same time, conceptualize problem posing itself as an activity separate from teaching.

Discussion

It is clear from the present review of the literature that problem posing is a complex endeavor that can, and has been, studied from many angles. One of our observations is that the term "problem posing" has been interpreted in different ways in the literature. This is partly because of the disparate research objectives of the authors, but also because the act of posing a problem (or asking a question or reasoning through an argument) manifests itself in a large number of daily and professional situations. Building on previous definitions of the term, we propose a working definition in the context of teacher education: the act of formulating a new task or situation, or modifying an existing one, with a specific mathematical learning objective and a targeted pedagogical purpose in mind.

The research suggests that efforts to enhance preservice teachers' problem posing must take into account a variety of factors that appear to be related to its development. These factors can be grouped as follows: (a) a focus on the teachers' content and curricular knowledge; (b) the extent to which teachers are required to use a variety of strategies for posing problems; (c) the degree to which teachers are asked to reflect on criteria for the evaluation of problems, which could include mathematical and pedagogical criteria; and (d) a focus on the development of their metacognition, which entails, in part, reflections on personal beliefs and attitudes related to mathematics. The almost ubiquitous reference to pedagogical knowledge (or other conceptualizations of teacher knowledge in line with Ball et al.'s (2008) model), as well as the considerable attention paid to metacognition in preservice teachers, is what makes problem posing in mathematics different from that observed in other contexts. Considering the unique aspects of problem posing in mathematics teaching, the complexity of introducing it into professional development quickly becomes overwhelming, especially with the limited time constraints and multiple goals within any given mathematics methods course (Sierpinska & Osana, 2012).

Most of the authors of the articles reviewed here explicitly stated the need to include problem posing as an objective of teacher education, which entails helping preservice teachers "to build on, reconstruct, and extend their sense-making of it" (Chapman, 2012, p. 144). This conclusion is in line with the general view that professional development is most effective when preservice teachers are actively engaged in the very practices they are expected to carry out when they enter the workforce (Wilson & Berne, 1999). The question remains, however: How can a mathematics teacher educator come to grips with the teaching of problem posing, a construct that is admittedly not well understood, in practical terms? We look for answers by highlighting common themes and key points from the studies reviewed in this chapter and use these to form implications for professional development.

Implications for Professional Development

Often, preservice teachers know that, as practitioners, they will need to select and implement mathematical tasks from textbooks and other sources for their students. Sometimes, they will "use them as they are," but many preservice teachers are unaware that they will often need to assess the instructional value of the tasks and modify them according to specific learning objectives or student needs. It becomes incumbent on teacher educators, therefore, to inform preservice teachers about these and related responsibilities and help them develop the requisite strategies to meet them. Ball and Forzani (2009) introduced the term "high-leverage practice" that "include[s] tasks and activities that are essential for skillful beginning teachers to understand, take responsibility for, and be prepared to carry out in order to enact their core instructional responsibilities" (p. 504). Problem posing, in its most general form of involving the reformulation of existing problems, but also in the generation of new ones, is one of those practices.

A major theme in the literature reviewed here is the impact of the preservice teachers' prior experiences with mathematics on the perceptions, beliefs, attitudes, and skills they bring to problem posing. As a result, we propose that metacognition is an important factor in the development of their skills in this area. For example, as Ticha and Hošpesová (2009) observed, some preservice teachers reject the idea of posing a problem or modifying an existing one. This can often be a consequence of the mathematics instruction they had themselves received when in school—that is, they had been exposed primarily to ready-made problems and rarely, if ever, to teachers who came up with problems themselves. Nicol and Bragg (2009) also identified teachers' unfamiliarity with open-ended problems as contributing to their challenges with problem posing. Such difficulties stem from exposure to problems with only one "right answer" and tasks that require a known procedure for solution (i.e., closed problems), as opposed to problems that are open-ended and that require exploration and inquiry.

Because it has been well established in previous research that the experiences of most preservice teachers are in many ways discrepant with the types of thinking expected from them in their teacher education programs, Ball (1988) mentioned the need for preservice teachers to "unlearn to teach mathematics." Along the same lines, Toluk-Uçar (2009) concluded from her research that, "methods courses can be used as a setting to challenge and revise pre-service teachers' mathematical knowledge and beliefs" (p. 174). To deveop a better understanding of how this might be achieved, we turn to Wilson and Berne (1999), who argued for engaging preservice teachers in the very activities that they will need in their future classrooms. Indeed, researchers have attempted to achieve this objective by engaging preservice teachers in reflections of their own problem posing (through journals, portfolios, and written assessments) and by simulating real audiences (through letter writing, for example). In turn, such activities appear to have honed their problem-posing skills, and the mathematical knowledge and beliefs of preservice teachers changed substantially, even in a relatively short period of time.

At the same time, the initial understandings preservice teachers bring into their methods courses shape their learning. As Chapman (2012) suggested, therefore, it is important for teacher educators to identify how preservice teachers make sense of problem posing and to build on that understanding to develop their students' abilities more fully. She further observed, however, that "…their sense-making of posing 'word problems' often excluded intentional or conscious consideration of mathematical structure or context of the problems or the relationship to the problem situation" (p. 144) and pointed to the importance of mathematical content knowledge in problem posing. Indeed, content knowledge has been shown to be a factor in the quality and quantity of posed problems (e.g., Leung, 1994) and has for some time been viewed as critical in the process. Participants with weak content knowledge, for example, were more likely to pose unsolvable problems or simple ones, while those with stronger content knowledge were able to pose collections of related problems (which required an understanding of the structural relationships between problems) and problems that were structurally more complex (Leung, 1994). Currently, the conception of preservice teachers' problem posing is more targeted: Preservice teachers need to learn how to pose problems that attend to a given learning objective or a certain interpretation of an operation spontaneously during classroom interactions with students. This skill relies heavily on content knowledge (Ball et al., 2008) because it requires teachers to see the mathematical potential of their environment and to build instruction around it.

The literature also points to the fact that content knowledge, although apparently necessary, is not sufficient for teachers' problem posing. As Chapman (2012) argued, "Problem posers have to appropriately combine problem contexts with key concepts and structures in solutions along with constraints and requirements in the task. Thus, both contextual settings and *structural features of problems* are recognized as crucial" (our emphasis, p. 137). Otherwise said, attending to the structure of problems appears to be a necessary element in teacher education, which implies, more broadly, that a focus on appropriate strategies for reasoning about problem posing is necessary. Osana and Royea (2011) made a similar argument in supporting preservice teachers' ability to see the connections between intuitive solutions to fractions problems and formal algorithmic representations of the same solutions. Their data revealed that, although students improved in this respect, they were still unable to transfer their learning to a problem-posing situation. The implication of this finding for teacher educators is that it has been found helpful to engage preservice teachers in class discussions that involve reflecting on the underlying structure of problems, comparing different problems vis-à-vis their structure, and connecting problem structure to targeted learning objectives. Identifying and reflecting on the deep structure of problems can lead preservice teachers to develop schemata for a variety of problem types which, in turn, can be activated under favorable instructional conditions.

Tasks that require preservice teachers to reformulate problems and to assess their modifications on a range of dimensions (solvability, accessibility, solution methods, correctness, contextual features, potential errors, and learning objectives) can result in a greater awareness of the critical aspects of problem posing. Because problem reformulation can occur on a variety of levels (e.g., the same mathematical expression can be worded in different ways), preservice teachers must also learn to consider the relative advantages and disadvantages of each (re)formulation.

A related task is one that requires the modification of problems for specific pedagogical purposes, such as mobilizing a concept for instruction or testing hypotheses about children's mathematical knowledge. These added constraints would focus reformulation on changing specific elements of a problem in response to students' needs (e.g., Crespo, 2003). Elsewhere, such elements have been termed "didactic variables" (Brousseau, 1997). The following excerpt from Brousseau (1997) is helpful in describing the process of problem adaptation:

> A field of problems can be generated from a situation by changing some variables which, in turn, are changing the characteristics of solution strategies (cost, validity, complexity, etc.). … Only changes that affect the hierarchy of strategies should be considered as relevant variables and among the relevant variables, those that can be modified by a teacher are particularly interesting: these are the didactical variables. (p. 208)

Thus, a didactic variable has its values assigned by the teacher, who, by modifying its values, can have an impact on her students' learning. Clearly, problem posing can support preservice teachers in their efforts to identify those variables and modify their values to achieve specific objectives during their teaching (Ball & Forzani, 2009).

Additionally, connecting mathematics to real life has played a major role in the development of problem posing in professional development (Verschaffel, Greer, & De Corte, 2000) because it can function as a criterion for the evaluation of the quality of a problem. As Nicol and Bragg (2009) suggested, a way to foster such connections is by encouraging preservice teachers to "see" the mathematics around them. Within this perspective, teacher educators should design their professional development activities so that preservice teachers are inspired by the environment, art, or science, for example. This could lead naturally to explorations of open-ended situations, known to be notoriously difficult for preservice teachers (Chapman, 2012; Nicol & Bragg, 2009). From activities that are situated in real-life contexts, a variety of different types of mathematical explorations can emerge, such as assessing different solutions according to a variety of criteria and considering situations from different perspectives. Indeed, as Crespo and Sinclair (2008) found, mathematical exploration can be highly beneficial for preservice teachers when it comes to developing criteria for judging the quality of a problem. Open-ended tasks, especially if preceded by exploration, are more conducive to problem posing than tasks that are constrained by specific criteria (such as writing a problem for a number sentence, for example). We argue that mathematical explorations of real-life situations, when combined with both open-ended inquiry and specific pedagogical constraints, could be highly productive for the development of preservice teachers' problem-posing abilities.

In the process of learning how to assess the problems they pose, preservice teachers need to attend to the pedagogical and mathematical fruitfulness of a given problem (Crespo & Sinclair, 2008), and doing so provides opportunities for them to modify problems according to specific criteria or teaching goals. As Crespo and Sinclair argued, however, preservice teachers would also benefit from occasions to appreciate the "mathematical beauty" in a problem. Given that most preservice teachers have not previously been exposed to mathematical esthetics, discussions in methods courses could be effective in this regard. Such explorations could be linked to examinations of non-routine problems, problems with multiple solutions and the relative suitability of the solutions, and extensions of problems to other topics or domains.

Because preservice teachers must acquire a body of professional knowledge specific to their future practice (e.g., Ball et al., 2008; Shulman, 1986), they need to develop an ability to pose and adapt problems with specific pedagogical purposes in mind, such as probing the thinking of their students or extending a specific mathematical concept. For these purposes, preservice teachers need curricular and content knowledge, another factor we identify as critical to the development of their problem posing. Content knowledge supports their understanding of the conceptual underpinnings of the algorithms in the school curriculum (and subsequent explanations of these concepts; e.g., Ball et al., 2008), and as the current review has shown, assists them to produce a larger number of creative and interesting problems for their students. Without understanding the conceptual rationales for topics and procedures in the school curriculum, preservice teachers' problem posing is seriously hindered by the knowledge they bring to their methods courses—knowledge which is usually fragmented and procedural (e.g., Livy & Vale, 2011; Newton, 2008; Simon, 1993; Tirosh & Graeber 1990; Zazkis & Campball, 1996). As such, they rely primarily on reproducing problems they have already seen—a finding that is not uncommon in the problem-posing literature.

We considered it important to highlight in the literature reviewed here the emphasis teacher educators placed on metacognition with their students when engaging in problem posing. From the research designs used in most of the problem-posing literature, however, it is difficult to determine the relative contributions of metacognitive activity and the act of problem posing itself. It may be that by engaging in metacognitive activities such as keeping portfolios, reflecting in written journals, and responding to open-ended questions about their thinking, the preservice teachers learned as much, or perhaps more, about how to pose problems than simply practicing the skill itself. It is clear that more research is needed on the factors that contribute to the development of preservice teachers' problem posing. Whether the objective is problem posing or not, however, we assert that reflective activity is paramount for preservice teachers' professional development, both as teachers-in-training as well as throughout their careers; indeed, much of the literature in teacher professional development would support this assertion (e.g., Van Zoest & Stockero, 2008). Teachers need to remain open to their students' inquiry and allow themselves to capitalize on fruitful comments and questions that arise during classroom interactions. From our analysis of the literature, we see problem posing as an effective vehicle for such growth and a springboard for the further development of curiosity and a continual willingness to learn.

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