Chapter 21
Problem Posing in Primary School Teacher Training

Alena Hošpesová and Marie Tichá

Abstract The chapter reports results of a survey whose aim was to contribute to research in the area of problem posing in teacher training. The core of the research project was empirical survey with qualitative design. Preservice and in-service teachers were posing problems in the environment of fractions and reflected on this activity in writing. Analysis of the posed problems and participants' reflections were to answer the following questions: (a) What shortcomings can be identified in the posed problems? (b) How are the posed problems perceived by preservice and in-service teachers? (c) What relations are there between quality of the posed problems and perception of this activity by their authors?

Contents

A. Hošpesová (✉)
University of South Bohemia, České Budějovice, Czech Republic
e-mail: hospes@pf.jcu.cz

M. Tichá
Institute of Mathematics of the Academy of Sciences of the Czech Republic,
University of South Bohemia, Prague, Czech Republic

F.M. Singer et al. (eds.), *Mathematical Problem Posing*,
Research in Mathematics Education, DOI 10.1007/978-1-4614-6258-3_21

Introduction

Development of professional competences of primary school teachers has been one of our focal points for a long time (e.g., Tichá & Hošpesová, 2010). We understand teachers' professional competences as a set of specific knowledge needed for teaching, as an amalgam of skills that teachers should master in order to be able to perform and continuously improve their teaching. We consider that the key to teachers' professional competence is subject didactic competence combining content knowledge, its didactical treatment and application of this knowledge in practice. Teachers, however, often perceive this competence in the narrow sense of simply having knowledge of methodological approaches to teaching a given content, but they are unaware of the need of its didactical analysis (see, e.g., Klafki, 1967). Teachers must not to be satisfied by looking for answers to the question *how*? They must also consider other questions, like *what*? and *why*? Like other researchers, we stress not only theoretical knowledge, but also the capability to act adequately in the development and delivery of mathematics lessons. We are convinced that only with insight and deep knowledge will a teacher be able to assess what can be improved, and how this can be achieved.

Primary school mathematics is the period when foundations for concept, imagery and future understanding of mathematics, and for positive attitudes to the discipline are formed. Teaching mathematics at the primary school level can be understood as a system of propaedeutics[1] of mathematical concepts and solving methods. From this perspective, special demands are made on a teacher's subject didactic competence. Our experience as teacher educators shows that teachers are sometimes unaware of deficits in their competences. This results in inadequate self-efficacy (Bandura, 1997; Gavora, 2010) and in a tendency to overestimate one's own proficiency. This phenomenon is of special importance for in-service teacher training. We came across the following two extremes:

- Teachers who are convinced that they have mastered a sufficient repertoire of methodological approaches which they have tried and tested in their teaching practice; they do not think that any change (let alone improvement) is necessary; and they do not realize that their subject didactic knowledge should be deeper.
- Teachers who are well aware of their weaknesses, who see their knowledge of certain topics are not at a sufficiently high level, and who want to do something about it. There are two categories of teachers who fall under this extreme:
    - Some of them expect that there are ready-made universal recipes they can easily and effortlessly learn by drill, and that will naturally lead to improvement; they fail to see that a change in grasping a problem with

[1] We understand propaedeutics as an introduction to knowledge of preparatory instruction, similarly to Webster's definition "pertaining to or of the nature of preliminary instruction; introductory to some art of science" (Webster's Encyclopedic Unabridged Dictionary of the English Language. New York 1996: Gramercy Books, p. 1152).

understanding is not very likely in these cases; they do not realize that there are no simple recipes leading to understanding.

- Others are well aware of the fact they will have to work hard to gain true insight.

It is often difficult for a teacher educator to discuss any deficiencies with in-service teachers. These teachers may take it very personally and are closed to any new stimuli. Preservice teachers, in general, seem to be more open.

In our practice as teacher educators, we have explored various ways of guiding primary school teachers in their quest to handle mathematical content with deep insight. At first, our focus was on joint reflections of selected teaching episodes (Tichá & Hošpesová, 2006). Later, we turned our attention to problem posing and started exploring the role of joint reflections in the process of assessment of problems posed to meet given criteria (e.g., Tichá and Hošpesová, 2010, 2013; Toluk-Ucar, 2008).

In this chapter, we will describe the rationale, conduct, and outcomes of a study, in which problem posing was used as a teaching method with a group of preservice and a group of in-service teachers. Much attention was paid to posing problems in the environment of fractions. This content area was selected because we are convinced of the importance of the concept of fractions in the curriculum in general, and of the relation between the whole and its parts, in particular. It is vital that teachers realize that they must not only master procedures of calculations with factions, but that they also need to see the importance of concept building and understanding the concept of fractions, and grow aware of the differences between fractions and natural numbers (see, e.g., Toluk-Ucar, 2008). At the end of the course, the participating teachers were asked to reflect on problem posing and on its benefits for teacher training.

Rationale of the Study: Problem Posing in Teacher Training

It is common practice for mathematics to be taught predominantly through problem solving. We understand problem solving as an activity directed towards a certain specific target and that this activity is continuously corrected by this target. The problem-solving process can be perceived as a “dialogue” between the solver and the problem. While solving a problem, solvers ask questions like “How shall I begin?” or How can I carry on at this point? (i.e., they ask themselves questions before starting the solving process, while solving it and having solved it). This can be seen as one of the early forms of problem posing.

We first became involved in the issue of problem posing when doing research on the structure of the process of grasping real-life situations (Koman & Tichá, 1998). By “grasping a real-life situation” we meant the process of thinking, involving especially identification of key objects and phenomena, relationships between them, and identification of problems growing/emerging from the situation, as well as of growing awareness of questions that may be asked—in other words, the process of understanding aspects of a situation that are needed for posing a problem. These stages are followed by solving the posed problem, answering questions, articulating the results, and interpreting and analyzing the results.

Our earlier research established a narrow link between grasping a situation and problem posing. Problem posing is closely interwoven with grasping a situation. If we recall Polya's (2004) characterization of the four heuristic steps in problem solving—getting an insight into the problem, designing a solution plan, implementing the plan, and verifying and critically assessing the solution. Comparing these steps with the above described stages of grasping situations, we can clearly see that some components are characteristic for both activities (i.e., both for grasping a situation and for problem solving).

The interrelatedness of the processes of problem solving and of problem posing has been the focus of a number of studies. A wide range of aspects of these processes have been scrutinized, e.g., their structure, stages, and interaction.

Like a number of mathematics educators (see, e.g., Cai & Brooke, 2006; English, 1997; Freudenthal, 1983; Kilpatrick, 1987; Pittalis, Christou, Mousolides, & Pitta-Pantazi, 2004; Ponte & Henriques, 2013; Silver & Cai, 1996; Singer, Ellerton, Cai, & Leung, 2011), we understand problem posing in teacher training to be a method leading to enhancement of preservice teachers' subject didactic competence. The complex nature of this competence implies that problem posing can have several functions:

- It is an educational tool because a teacher must often pose problems that are, for example, related to a specific situation in the class (Silver & Cai, 2005).
- It is also a diagnostic tool which helps teachers to uncover deficits and obstacles in students' knowledge (English, 1997; Harel, Koichu, & Manaster 2006; Silver & Cai, 1996; Tichá & Hošpesová, 2013).
- On the basis of our experience, we have recently begun stressing that problem posing can be a significant motivational element leading to deeper inquiry into mathematical content areas, resulting in deeper study and effort to improve one's knowledge base for teaching, a deeper understanding of concepts, and in boosting one's repertoire of interpretation.

Goals of the Research Project, Its Conception, and Methodology

The goal of the project reported in this chapter was to explore and describe the role of problem posing in teacher training. Special attention was paid to gaining insight into how preservice and in-service teachers perceive and interpret problem posing because it is these subjective interpretations that play a decisive role in the dynamics of the process of their improvement.

The core of the project was a qualitatively designed empirical survey. Data were collected from an analysis of problems posed by preservice and in-service teachers and from their written reflections on this activity.

Research Questions

The following three research questions were asked at the outset of the project:

1. Which deficits in grasping the concept of fraction hinder teachers' subject didactic competence (and therefore their teaching) can be identified in problems posed by preservice and in-service teachers? Are these shortcomings the same or different in problems posed by preservice and in-service teachers?
2. What are preservice and in-service teachers' beliefs about the importance of problem posing in their own training?
3. What is the relationship between a respondent's view of the role, importance, and benefit of problem posing and quality of problems he/she poses?

When trying to answer the first of these questions, we tried to describe objectively which phenomena were inherent in the process of posing problems involving fractions by preservice and in-service teachers. Other questions were directed to tackling the participants'—preservice and in-service teachers'—perspectives. Data used for objective scrutiny were the posed problems. Participants' subjective perspectives were recorded in the form of their written reflections. Collection of the two different types of data enabled triangulation of the collected data. These data were also the basis for answering Research Question 3.

Participants and Data Collection

The study was carried out with two groups of respondents:

- The first group consisted of 32 preservice teachers, students who posed problems in a compulsory course of Didactics of Mathematics in the second half of their undergraduate studies[2]. This group will be referred to as *students*.
- The other group included 24 participants in the course Didactics of Mathematics, who enrolled to this course to deepen their knowledge. All of these participants were in-service teachers with several years of work experience as primary school teachers. They chose this lifelong learning course for professional development voluntarily. This group will be referred to as *teachers*.

We worked with both respondent groups for approximately 3 months using similar methods.

The course was not originally conceived as a teaching experiment. Participants in both groups studied arithmetical content of primary school mathematics. They solved and posed problems on topics related to the mathematics content taught at

[2] Primary school teachers in the Czech Republic must study 4- or 5-year-long undergraduate courses designed especially for primary school teachers. Their undergraduate teacher education program includes courses in all subjects taught at primary school level.

the primary school level and discussed a variety of questions related to teaching mathematics. In one of the last seminars, the participants' task was *to pose several problems which include fractions* ½ *and* ¾ (similar to that reported in Tichá and Hošpesová (2013) but with a different group of respondents). Deliberately, other constraints were not imposed on the problem-posing task given to the participants, as this was investigated in our earlier studies (e.g., Hošpesová & Tichá, 2010; Tichá & Hošpesová, 2006, 2010). We assumed that this open situation would enable the students and teachers to create such situations deliberately in which fractions were used in different contexts (in the sense of Behr, Lesh, Post, & Silver, 1983).

Transcription of Posed Problems and Approach to Data Analysis

We started by designing a table with three columns in which we matched the posed problems and the written reflections to their authors. Thus were formed triplets [author-problem-reflection]. These were then analyzed. As the design was qualitative, no coding system had been developed in advance and analysis was conducted using open coding. Thus:

- In each of the posed problems, we looked for characteristics showing to which concrete situations the author had linked fractions, how he/she had interpreted them in different situations, and what sub-constructs of fractions had been incorporated into the problems.
- Texts of reflections were classified according to the topics addressed; we tried to find suitable codes for the meaning they connoted (for details about open coding, see Švaříček and Šeďová, 2007); we examined what perspective students and teachers selected when posing word problems, and what opinions they expressed about the process.

We scrutinized word problems from two perspectives: (a) we tried to treat posed problems from an external perspective, indicating their strengths and weaknesses from our perspective; and (b) we tried to determine the respondents' (problem authors') perspective.

Having first analyzed the data individually (each of the authors of this chapter on her own), we then met to discuss our findings, to link the individually created codes, and then to code both types of data again. The subsequent categorization of codes identified substantial topics that were relevant to our study; this was done, as stated above, without any explicit preconceptions or clear ideas. This form of analysis allowed the emergence, for example, of the category "refusal to pose problems," whose occurrence we had not been anticipated and had not been included in our research questions.

Discussion

Shortcomings in the Posed Problems

The analysis of the posed problems showed substantial deficits in respondents' knowledge, and revealed their misconceptions, misunderstandings, and shortcomings. This confirmed the conclusions reached in our previous surveys (Hošpesová & Tichá, 2010; Tichá & Hošpesová, 2013). We are convinced that the reasons for this are to be found in how students and teachers themselves were introduced to the concept of fraction; in other words, in their own evolution of the concept of fraction from its very beginnings. In addition, we believe that any deficits can also be linked to earlier demands there had been on their knowledge of fractions. This assumption is confirmed by the fact that the same or similar shortcomings were characteristic for both groups of respondents. For that matter, the same misconceptions could be seen in pupils and students from different age groups, as our former research has shown (Tichá, 2003). These shortcomings and misconceptions are very pervasive; they can be seen in all of the sets of problems posed by our respondents.

The posed problems indicate that teachers do not realize that:

- It is not sufficient to master arithmetical operations; they lack conceptions, imagery that would enable them to grasp the concept of fraction, solve and pose problems, even application problems (see also Prediger, 2006; Toluk-Ucar, 2008).
- A problem must be carefully formulated and its authors must be very accurate when wording it; Cai & Cifarelli (2005) refer to "ill-structured problems" (p. 47), and cite Kilpatrick (1987), who claims that these problems "lack a clear formulation or a specific procedure that will guarantee a solution, and criteria for determining when a solution has been achieved" (p. 134).
- Problem posing requires knowledge of the curriculum—what knowledge is prerequisite and what the aim of teaching is.

Similarly to our previous research (e.g., Tichá & Hošpesová, 2013), we noted that most of the posed problems were of markedly monotonous nature of situational context (cakes, marbles, etc.), of properties of the environment (either discrete, or continuous, but rarely both), and of interpretation (fraction as operator, quantity (measure), magnitude of physical quantity (in these cases the problems tended to be well formulated, but it was not clear whether their authors were aware of any differences between them, e.g., problems F1 and F2 posed by teacher Filo later in this text), quantity of physical value, and part/whole construct, etc.)

The base was often given ambiguously. Both students and teachers did not realize the need to consider the whole and the part-whole relationship. This can be observed, for example, in the students' and teachers' failure to realize that they must consider the role of the whole (e.g., problems posed by Cecily).

Interference of work with fractions with knowledge of calculations with natural numbers (in which fractions-operators were handled as natural numbers) was another source of mistakes. The core, key deficits spring from the fact that fractions were perceived as quantitative data, as natural numbers and they were handled this way in arithmetical operations. In this context, Streefland (1991) used the term *N*-distractor which warned of the possible interference by knowledge of work with natural numbers, stemming from immersion into the world of natural numbers.

> Dad decorated ½ of the guest-room. Granddad decorated ¾ of the living room. Who decorated more and how much more? (Cecily)

The majority of students and teachers posed problems of an additive nature. Multiplicative problems were very rare. These often showed that their authors did not realize that "whereas multiplication always makes bigger for natural numbers (apart from 0 and 1), this cannot be applied to fractions" (Prediger, 2006, p. 377). Toluk-Ucar (2008) also reported similar findings.

> Honza had ¾ of some dessert. Jana had ½ times less than Honza. How much did they have together? (Vlasta)

When looking for an answer to the first research question, we compared problems posed by students with problems posed by teachers. Problems posed by teachers differed in two aspects: (a) in-service teachers usually asked for specification of pupils of what grade the problems were posed for, and (b) their repertoire of problems was richer. In line with requests that are usually made for sets of problems with fractions (Lamon, 2006) teachers posed more varied *n*-tuples of problems in which different sub-constructs of fractions, various environments (discrete, continuous), various representations, problems related to evidence, construction, etc. were used. Teachers would have been influenced by their experience with problems found in good textbooks and from collections of problems. They may also have been influenced by problems that their pupils had been assigned in various competitions and tests.

For illustration, we can present problems posed by teacher Filo, although her pentad of problems also included the above mentioned misconceptions and confusing formulations, e.g., in problem F1 (what is the whole?) and F5 (are the fractions in the function of an operator or do they give the number of passengers?):

- F1. Children ate cakes. One of them ate ½, the other ¾. How many quarters did they eat?
- F2. In one vessel, there is ½ L of liquid, in another one ¾ L of liquid. How much liquid is in both vessels?
- F3. The sides of a rectangle are ½ cm and ¾ cm. Calculate its area.
- F4. ¾ of a field was seeded with corn but ½ did not germinate. How many quarters germinated?
- F5. There were 20 passengers on a plane. ¾ of the passengers left the plane during the stopover, ½ boarded the plane. How many passengers continued the journey?

What are Preservice and In-service Teachers' Opinions of Problem Posing in Their Own Training?

The majority of comments about the inclusion of activities related to problem posing into preservice and in-service teacher training were positive. In the student group, about two thirds of the participants talked about this question. Mostly, they appreciated inclusion of problem posing into their undergraduate training. There were only two sceptical opinions. In the in-service teacher group, almost everybody answered this question and all answers were positive.

However, we must ask whether the respondents' answers were not mere proclamations. We suspect the respondents may have felt that it was "desirable" to say that problem posing was useful and beneficial as the seminars focused on the issue. Comparison of problems posed and opinions declared by the participants (see below) seems to confirm this suspicion.

When analyzing written reflections on the posed problems, we found that respondents tended to express their opinions on two topics: on subjective feelings when posing problems (codes 1–4) and on the impact of problem posing on a teacher's subject didactic competence (codes 5 and 6). Open coding resulted in the following codes:

1. Problem posing is important
2. Problem posing is surprisingly difficult
3. The teacher finds it easier to work with problems he/she has posed (posing the problem makes it easier to solve)
4. It is not a teacher's task to pose problems
5. Problems posed by a teacher are more appealing for the children and more up-to-date
6. Problems posed by a teacher help children's comprehension

These codes are discussed in more detail in the following section.

Problem posing is important. Students' and teachers' comments showed they were either well aware of the importance and benefit of activities associated with problem posing or at least hint at being aware. Reflections where there were no reasons or justification of this opinion made us ask whether students and teachers had really become convinced about the importance of problem posing in the course, or whether they were just repeating what we had discussed in the course.

> I think it is very important for a teacher to develop this as it enables him/her to understand the structure of already existing problems problem and to be able to carry them out. (Student Soňa)

None of the participants, however, tried to formulate what the prerequisites for successful problem posing are, what knowledge, skills and experiences, etc. a person posing problems for their pupils' needs. The comments, the posed problems, and the following joint discussion showed that these issues were not considered either by the students or by the teachers.

Problem posing is surprisingly difficult. Some of the participants admitted to having had difficulties when they posed the problems. Some of them stressed that the task had been very demanding. Some of their comments showed that the task of posing problems made participants reflect on the adequacy of their knowledge base of mathematics for teaching.

> I think it is very important because when posing problems one often grasps it or starts to understand it but also grows aware of one's deficits. At this moment I feel a bit down as I find it very difficult. The more I think about it and try to come up with something, the more lost I get in it and look for complexities and things that I normally find simple, comprehensible, are now confusing and I have a lot of doubts. (Student Beruška)

> I tried to come up with something but it didn't work too well. (Teacher Filo)

> The "test" today clearly showed how important this is. I've never come across posing problems with fractions and had no idea how difficult it could be. (Student Tereza)

The teacher finds it easier to work with problems he/she has posed (problem posing helps problem solving). This area of comments only confirmed our idea of the closeness of the relationship between problem posing and problem solving.

It is not a teacher's task to pose problems. Some participants expressed their anxiety and refused to do the activity of posing problems entirely. They stated that they did not like problem posing because it was time-consuming; they expected textbooks, i.e., authority, to furnish them with problems. For example, one student (Gábina) spoke of her fear that she would not be able to pose problems:

> Word problems are undoubtedly important for children but no teacher will want to spend their time posing problems when they can find millions of them in textbooks or on the Internet. I will rather be advised. Or I will just modify some existing problems. (Gábina)

Problems posed by a teacher are more appealing for the children and more up-to-date. The students' comments often included the assertion that the competence of and skills in problem posing are an important part of teacher's knowledge. Students often associated problem posing with arousal of pupils' interest, and stressed creativity and inquiry-based mathematics education (see Dagmar's words below). However, we cannot again rule out the possibility that these are not the true beliefs of the participants, but are echoing what they understood to be part of the course.

> Mathematics is an important discipline. Its basis is being able to carry out basic operations such as +, −, *, /. When we go shopping, when we want to solve riddles and puzzles. The teacher should plan his/her lessons playfully and the lessons should be entertaining. Nobody is interested in boring lessons. That is why it is important for the teacher to pay attention to lesson planning, development of skills and competence. (Student Martina)

> I think it is crucial that the teacher understand and be able to pose problems. Teaching could then be more creative and enjoyable. For example, problem posing with pupils and so on. (Student Linn)

> I think it is good to get engaged in problem posing. Problems can correspond to children's hobbies and then they find their solution more attractive because they are more personal. For example, Anička—gymnastics, Pepíček——soldiers, … The sky is the limit. (Student Cecily)

Problems posed by a teacher help children's comprehension. The most common comments in the teachers' group (e.g., teacher Hanka) were related to search for ways of supporting pupils' comprehension. The comments seemed to reflect everyday practical problems teachers that teachers face.

> I based the problem on the math for primary school level. Clarity is crucial. I tried to formulate clear assignment, which the children are able to solve. (Hanka)

What Was the Relationship Between a Respondent's Opinion on Problem Posing and "Quality" of Problems He/She Poses?

We identified several phenomena in our analysis.

Discrepancy: Quality problems vs. opinion of the respondent. Discrepancies between a given participant's beliefs on importance and role of problem posing and the problems he/she actually posed were noteworthy, especially in the student group. We came across proclaimed appreciation of the importance of problem posing accompanied by posed problems of very low quality. We also came across expressions of fear and anxiety of problem posing accompanied by *n*-tuples of problems we evaluated as being of high quality.

For example, above we noted the negative attitude to problem posing expressed by the student Gábina. Paradoxically, this student was very proficient in posing "variegated" problems (Problems G1–G6), in which various interpretations of fractions (quantity, operator, etc.) in the sense presented in Behr et al. (1983). This student also used different contexts. The solutions to her problems required the use of a number of different operations (comparison, addition, multiplication). In spite of the range and form of the problems she posed, some problems included ambiguities and misleading formulations of the definition of a whole (it seems in some cases she did not think of the whole, and posed questions like "How much …?" instead of "What proportion?" or "Which part?"

- G1. How much does an iron rod melded of two rods measure? One rod is ½ m long, the other ¾ m long.
- G2. There is ½ in the swimming pool. We add ¾ more in the swimming pool. Will the swimming pool overflow?
- G3. Mum and Andulka ate together ½ of a cake; Dad and Petřík ate ¾ of the cake. Who ate most?
- G4. Mum and Andulka ate together ½ of a cake; Dad and Petřík ate ¾ of the cake. How much cake is there left?
- G5. Pepíček ate ¾ out of ½ of a cake. How much cake was there left?

However, it was more common for students to reflect on the high demands of posing problems (inquiry-based approach, creativity, interest, etc. (see, for example, the following statements by Dagmar and Hortenzia) and then posed problems similar to those common in textbooks (it seems as if these students wanted "to be safe").

> It is crucially important to develop teachers' abilities and competence because the more interesting, creative and inquiry based the problem is, the more the children enjoy its solution. (Dagmar)

> I think it is very important to develop the ability to pose problems attractive for children. It is not enough just to use textbooks. (Hortenzia)

D1. Mark in the circles such a part that the first represents ½ of the size and the second ¾ of the size.

D2. When you add ½ of 1 m and ¾ of 1 m, how many meters do you get?

D3. What part of a cake is there left if you subtract ½ from ¾?

H1. There was one half of a cake on the table. Petr came and ate ¾ of it. What part of the cake was left on the table?

H2. Mum bought ½ kg of apples. Dad brought ¾ kg of apples. How many apples did they have in total?

H3. Granny baked sponge cake; Tomáš ate ¾ of the sponge cake; Anička later ½ of what was left. What part of the sponge cake was left in the end?

Awareness of the deficit in knowledge. We can say that the participants were unaware of their insufficient knowledge of the content and of their misconceptions. For example, having posed the set of problems (J1–J5), the teacher Jana wrote: "I was guided by my experience from work with children. Most important for me are illustrativeness and understanding. I was focusing on grasping mathematical operations with understanding. The goal was to empathize with the children."

J1. Jane put one half of sweets into one bag and also put there 3/4 from another bag. How many did she have in total?

J2. Compare the following fractions: ½ and ¾.

J3. A dressmaker cut one half from a ¾ long strip of fabrics. How much was she left with?

J4. Two children had their own halves of cake. Can you calculate how much they had together when they got ¾ more?

J5. Calculate the area of an estate whose measurements are ½ (side a) and ¾ km (side b).

Comprehensibility, reality. Even problems posed by the teachers included some inaccurate, ambiguous formulations, despite the fact the teachers often stressed the importance of the comprehensibility of problems. Other teachers' proclaimed demand was that problems be "real" but the posed problems did not meet this criterion. For example, Svatava wrote: "I was guided by my experience, most important: to understand it, to try and use real ideas, the goal comprehensibility," but the problems she posed included the following:

S1. I need ½ m of fabrics for a jacket. I need ¾ m for trousers. How much fabrics will I have to buy? How much will I pay if 1 m costs 200 CZK?

S2. Petr jumped ½, Pavel ¾ m. Who jumped more? By how much?

Discussion of the Role of Problem Posing in Teacher Training

The goal of the study presented in this chapter was to assess the benefit of problem posing in preservice and in-service teacher training and to compare the posed problems with their authors' beliefs. We tried to show that analysis of problems posed in the environment of fractions can lead to identification of misconceptions and major deficits in preservice and in-service teachers' mathematical content knowledge. In the classroom, such misconceptions and deficits could result in the teacher's single-tracked, simplified or even erroneous interpretation of the subject matter, or in his/her failure to react adequately to pupils' valuable contributions.

Problem posing on its own is by no means a sufficient tool for remedy. It must be supplemented by some reaction from others. That is why we used joint reflection, discussed in detail in our preceding studies (Tichá & Hošpesová 2006), as one of the possible ways leading to discovery of deficiencies (especially those of a conceptual nature). This is also called for by other authors. For example, Selter (1997), following Bromme (1994), stated that:

> offering teacher students the necessary background knowledge surely is a precondition for their professionality as teachers. However, teachers actually become professionals while they are teaching and reflecting on their teaching … teacher education … should first and foremost assist prospective teachers in developing their autonomy. This implies to support them in increasing their degree of awareness—about mathematics, about children's mathematical learning, about the quality of teaching material and so forth. (p. 57)

The survey used in our study concluded with a joint reflection. In fact, the participants in both groups were asked for some evaluation of the problems they had posed. Only then, when they were asked to reflect on their own posed problems, were some of the participants willing to admit deficits in their knowledge of mathematics (specifically the concept of fractions), lack of creativity and insufficient knowledge of what their pupils might be interested in.

This survey also confirmed that the teacher educator leading the seminar should have input into the joint reflection, as his/her questions could guide the direction of the discussion. It is possible to proceed in more ways:

- The educator could offer examples of posed (n-tuples) problems, point out flaws, mistakes, misconceptions (e.g., Hošpesová & Tichá, 2010). This approach would stress especially the use of problem posing as an educational tool in teacher training.
- The educator could ask the participants to choose the problems they want to discuss. They should always be able to justify their choice (an interesting problem, open problem, ambiguously formulated problem, etc.). Then problem posing would be used as a diagnostic and re-educational tool.

The question how to persuade in-service and preservice teachers that problem posing should become an integral part of their teaching needs to be answered yet. We came across the belief (especially in case of in-service teachers) that inclusion of "problem posing" distracts from "appropriate, genuine mathematics" oriented on "mastering of craftsmanship—carrying out calculations." Some of the teachers are afraid that their training is not sufficient as to enable them inclusion of these activities in their teaching. Others object that it would require intellectually and time-demanding planning. Many are hindered by the fact they would not know how to evaluate problem posing.

We found out that posing problems in groups ceases to be stimulating. It seems much more convenient for students to work individually or in pairs. However, it is crucial they have a chance to present their problems and discuss them from different points of view (the choice of mathematical topic, continuity, difficulty, symmetry).

The following questions, asked by a number of researchers, still need to be answered:

- What knowledge (mathematical and general) is a prerequisite to successful problem posing?
- How can we assess the benefit of "problem posing" for their authors and the "change" in professional competences of these authors?
- What help or guidance can we offer to teachers who decide to include "problem posing" in their teaching?
- How can teachers and students be persuaded about the potential and benefit of "problem posing" for mathematics education and for the development of mathematical literacy?

Acknowledgement Elaboration of the chapter was supported by RVO 67985840.

References

Bandura, A. (1997). *Self-efficacy. The exercise of control*. New York, NY: W. H. Freeman.

Behr, M., Lesh, R., Post, T., & Silver, E. A. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 91–125). New York, NY: Academic.

Bromme, R. (1994). Beyond subject matter: A psychological topology of teachers' professional knowledge. In R. Biehler et al. (Eds.), *Didactics of mathematics as a scientific discipline* (pp. 73–88). Dordrecht, The Netherlands: Kluwer.

Cai, J., & Brooke, M. (2006). New perspectives of looking back in mathematical problem solving. *Mathematics Teaching, 196*, 42–45.

Cai, J., & Cifarelli, V. (2005). Exploring mathematical exploration: How do two college students formulate and solve their own mathematical problems? *FOCUS on Learning Problems in Mathematics, 27*(3), 43–72.

English, L. D. (1997). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics, 34*, 183–217.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Dodrecht, The Netherlands: Reidel.

Gavora, P. (2010). Slovak preservice teacher self-efficacy: theoretical and research considerations. *The New Educational Review, 21*(2), 17–30.

Harel, G., Koichu, B., & Manaster, A. (2006). Algebra teachers' ways of thinking characterizing the mental act of problem posing. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 241–248). Prague, Czech Republic: Charles University.

Hošpesová, A., & Tichá, M. (2010). Reflexion der aufgabenbildung als weg zu erhöhung der lehrerprofesionalität. In C. Böttinger, K. Bräuning, M. Nührenbörger, R. Schwarzkopf, & E. Söbbeke (Eds.), *Mathematik im Denken der Kinder; Anregung zur mathematik-didaktischen Reflexion* (pp. 122–126). Seelze, Germany: Klett/Kalmeyer.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123–147). Hillsdale, NJ: Erlbaum.

Klafki, W. (1967). *Studie k teorii vzdělání a didaktice (Studien zur Bildungstheorie und Didaktik)*. Praha, Czech Republic: SPN.

Koman, M., & Tichá, M. (1998). On travelling together and sharing expenses. *Teaching Mathematics and its Applications, 17*(3), 117–122.

Lamon, S. (2006). *Teaching fractions and ratios for understanding* (2nd ed.). Mahwah, NJ: Erlbaum.

Pittalis, M., Christou, C., Mousolides, N., & Pitta-Pantazi, D. (2004). A structural model for problem posing. In M. J. Hoines & A. J. Bishop (Eds.), *Proceedings of 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 49–56). Bergen, Norway: PME.

Polya, G. (2004). *How to solve it: A new aspect of mathematical method (with a new foreword by John H. Conway)*. Princeton, NJ: Princeton University Press.

Ponte, J. P., & Henriques, A. (2013). Problem posing based on investigation activities by university students. *Educational Studies in Mathematics, 83*(1), 145–156.

Prediger, S. (2006). Continuities and discontinuities for fractions: a proposal for analysing in different levels. In J. Novotná et al. (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 377–384). Praha, Czech Republic: PME.

Selter, C. (1997). Instructional design for teacher education. In M. Beishuizen, K. P. E. Gravenmeijeer, & E. C. D. M. van Lieshout (Eds.), *The role of contexts and models in the development of mathematical strategies and procedures* (pp. 55–77). Utrecht, The Netherlands: Freudenthal Institute.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*, 521–539.

Silver, E. A., & Cai, J. (2005). Assessing students' mathematical problem posing. *Teaching Children Mathematics, 12*(3), 129–135.

Singer, F. M., Ellerton, N. F., Cai, J., & Leung, E. (2011). Problem posing in mathematics learning and teaching: A research agenda. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 137–166). Ankara, Turkey: PME.

Streefland, L. (1991). *Fractions in realistic mathematics education: A paradigm of developmental research*. Dordrecht, The Netherlands: Kluwer.

Švaříček, R., & Šeďová, K. (2007). Kvalitativní výzkum v pedagogických vědách. (Qualitative research in education.) Praha: Portál.

Tichá, M., & Hošpesová, A. (2006). Qualified pedagogical reflection as a way to improve mathematics education. *Journal for Mathematics Teachers Education, 9*, 129–156. (Special Issue: Inter-relating theory and practice in mathematics teacher education).

Tichá, M., & Hošpesová, A. (2010). Problem posing and development of pedagogical content knowledge in preservice teacher training. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education* (pp. 1941–1950). Lyon, France: Institut National de Recherche Pédagogique.

Tichá, M., & Hošpesová, A. (2013). Developing teachers' subject didactic competence through problem posing. *Educational Studies in Mathematics, 83*(1), 133–143.

Tichá, M. (2003). Following the path of discovering fractions. In J. Novotná (Ed.), *Proceedings of SEMT'03* (pp. 17–27). Praha, Czech Republic: United Kingdom PedF.

Toluk-Ucar, Z. (2008). Developing preservice teachers understanding of fractions through problem posing. *Teaching and Teacher Education, 25*, 166–175.

Webster's Encyclopedic Unabridged Dictionary of the English Language (1996). New York, NY: Gramercy Books.