Chapter 20
Developing the Problem-Posing Abilities of Prospective Elementary and Middle School Teachers

Todd A. Grundmeier

Abstract This chapter describes the results of an exploratory study incorporating problem posing in a mathematics content course for prospective elementary and middle school teachers. Problem posing was incorporated as problem generation (posing problems from a set of given information) and problem reformulation (posing problems related to a given problem). The content coverage of the course included problem solving, data analysis and probability, discrete mathematics, and algebraic thinking. Exposure to problem posing had two effects on those who posed the problems. First they began using more sophisticated problem reformulation techniques as the course progressed. Second, with regard to problem generation, participants developed efficient ways of posing problems when time constraints were imposed, and they developed greater aptitude for posing multi-step problems. The development of participants' problem-posing abilities will be described in detail, and qualitative data will be presented to highlight participants' views of the relationship between problem posing and school mathematics.

Contents

Introduction 412
Methodology 413
    Working Definitions 413
    Course Setting 414
    Participants 414
    Instructional Treatment and Data Collection 415
    Problem Generation Coding 416
    Problem Reformulation Coding 416
    Inter-Rater Reliability 417

T.A. Grundmeier (✉)
Department of Mathematics, California Polytechnic State University,
San Luis Obispo, CA, USA
e-mail: tgrundme@calpoly.edu

F.M. Singer et al. (eds.), *Mathematical Problem Posing*,
Research in Mathematics Education, DOI 10.1007/978-1-4614-6258-3_20

Problem-Posing Results ... 418
Pre- and Post-Assessment ... 418
Problem Generation ... 419
Problem Reformulation ... 421
The Relationship Between Problem Posing and School Mathematics ... 424
Discussion ... 428
Conclusion and Implications ... 430
References ... 431

> *If we change the question in the title to 'Where do good mathematics problems come from?', the answer ought to be readily apparent to any competent high school graduate. Mathematics problems obviously come from mathematics teachers and textbooks, so good mathematics problems must come from good mathematics teachers and good mathematics textbooks. The idea that students themselves can be the source of good mathematics problems has probably not occurred to many students or to many of their teachers. (Kilpatrick, 1987, p. 123)*

Introduction

Kilpatrick (1987) suggested that instruction rich in formulating problems that requires students to become problem posers is essential throughout mathematics education. The landscape of mathematics education has encountered much change since Kilpatrick (1987) wrote these words and many educators and authors have considered mathematical problem posing as a skill. However, it can still be argued that students are not "required" to become problem posers.

Through the early 2000s both mathematics educators and professional organizations continued to advocate for the inclusion of problem posing in mathematics classrooms and curricula (Kilpatrick, Swafford, & Findell, 2001; NCTM 1991, 2000; Silver, 1994). Literature within the mathematics education community also focused on the importance of problem posing and research has demonstrated the problem posing capabilities of K-6 students (English, 1997; Silver, 1997; Silver & Cai, 1996; Winograd, 1997). Problem-posing research in the late 1990s likely led to the following suggestion for the incorporation of problem posing in mathematics classrooms and curricula by The National Council of Teachers of Mathematics (NCTM), in *Principles and Standards for School Mathematics* (2000):

> Posing problems comes naturally to young children …Teachers and parents can foster this inclination by helping students make mathematical problems from their worlds … In such supportive environments, students develop confidence in their abilities and a willingness to engage in and explore problems, and they will be more likely to pose problems and persist with challenging problems. (p. 53)

Although much problem-posing research occurred during the 1980s and 1990s, problem posing became prominent again in the mathematics education research literature in the early 2000s (see, e.g., Barlow & Cates, 2006; Stickles, 2010–2011; Whiten, 2004a, 2004b). In order to facilitate the suggestions made by Kilpatrick (1987) and others, to incorporate problem posing at all levels of mathematics instruction, prospective teachers need problem-posing experiences as part of their preservice education (Leung & Silver, 1997).

Gonzales (1994, 1998) examined the incorporation of problem posing in instruction for prospective teachers. Gonzales (1994) suggested engaging prospective elementary and middle-school teachers in problem posing by posing related problems and posing story problems. Gonzales (1994) found that prospective teachers could be guided through a transition from problem solver to problem poser and based on this transition called for the increased use of problem posing with this audience. Gonzales (1998) described a "blueprint" to help teachers and teacher educators include problem posing in their classrooms. The "blueprint" started with posing related problems and after exposure to problem reformulation asked students to generate problems. The project described here incorporated some aspects of this blueprint, as well as subsequent work of Gonzales (1994, 1998), but extended her ideas by formally exploring the outcomes of incorporating problem posing into students' mathematical experiences. This extension of Gonzales' work addresses the need to develop the problem-posing abilities of prospective elementary and middle-school teachers by engaging them in problem posing throughout a mathematics content course designed for prospective teachers. The goal of this work, as suggested by Leung and Silver (1997), was to carry out a careful evaluation of empirical problem posing and to describe changes in the characteristics of participants' posed problems as they gained problem-posing experience.

Methodology

The instructional treatment for this study was the incorporation of problem posing into the expectations of a mathematics content course for prospective teachers ($n = 19$). The main components of the methodology are the working definitions used by the author, the course setting, the participants, the instructional treatment, and data collection. These components will be discussed below.

Working Definitions

In this study problem posing took two forms: (a) the generation of new problems; and (b) the reformulation of given problems (Silver, 1994). It is important to define statement, problem, problem reformulation, and problem generation to give a sense of how the ideas were utilized for the purpose of this research. Definitions are summarized in Table 20.1.

Table 20.1
Working Definitions

Term	Working definition
Statement	A statement will refer to the outcomes of student problem-posing tasks. Statements are all text that is produced as a response to a problem-posing task and is not necessarily a mathematics problem or question
Problem	A mathematical statement for which a valid solution exists
Problem reformulation	The process of posing a problem related to a problem that is or was the focus of problem solving
Problem generation	The process of posing a problem based on a set of given information. Generated problems may include additional information to the original set but must be related to the original set of information

Course Setting

The course was the second in a required sequence of mathematics content courses for prospective elementary and middle-school teachers. The content coverage of the course included problem solving, data analysis and probability, discrete mathematics, and algebraic thinking. The course instructor routinely modeled a mathematics classroom environment that was student-centered, included group work and discussion, and was a safe environment for participants to ask questions and pose conjectures. The class met twice a week for 1 hour and 50 minutes and a typical class would consist of a brief lecture followed by a group activity that asked participants to explore the mathematics content from the lecture in depth. Each class generally concluded with a discussion of the content covered and the goals of the instructional situation. The daily class activities could be described as inquiry-oriented and focused on participant problem solving. Inquiry-oriented in this context refers to the definition offered by Silver (1997) and was "… characterized as one in which some of the responsibility for problem formulation and solution is shared between teacher and students" (p. 77). The inquiry-oriented nature of the class was important because engaging in such problem-solving activities can help students develop more mathematical creativity (Silver, 1997).

Participants

Students enrolled in the course were the participants in this study. Past research has shown that preservice teacher-education students have the ability to pose mathematics problems (Gonzales, 1994). Also, if problem posing is going to become predominant in mathematics classrooms and curricula as suggested by NCTM (2000) and Kilpatrick, Swafford & Findell (2001), it is the author's belief that prospective teachers should not only have experience in posing mathematics problems but also the opportunity to reflect on the role of problem posing in school mathematics. Twenty students were enrolled in the semester-long course and 19 of those students agreed to serve as participants in this study. Participants included 17 undergraduates who were mathematics education or family-studies majors and 2 graduate students working towards their master's degree in education. All participants were prospective teachers and intended to seek certification to teach in the elementary or middle school.

Instructional Treatment and Data Collection

The classroom instructor and the author met before the semester and agreed on incorporating problem posing in the course in the form of a pre- and post-assessment, problem reformulation, problem generation, and journal writing. During the semester the instructor and author met weekly. The goal of these meetings was to examine the course content and plan for the coming week. These meetings led to the development of problem-posing tasks for the instructional treatment and agreement between the instructor and author about how these tasks would be incorporated into the course expectations. All tasks related to problem posing were incorporated into the expectations of the course as part of collected homework assignments, except for the pre- and post-assessments. The only class time directly dedicated to problem-posing tasks was for the pre- and post-assessments, each of which took 25 minutes.

Problem reformulation occurred as an extension of Polya's (1957) four-step problem-solving heuristic. After solving problems using the four step heuristic on the first problem set assigned as homework, participants were asked to use a five-step problem-solving heuristic adding the fifth step—"pose a related problem"—on the remainder of the homework problem sets. Participants were asked to apply this heuristic to a subset of each problem set and in all cases were able to choose the problems to which they applied the heuristic. Problem reformulation occurred on 7 problem sets during the semester and related to a total of 22 problems that were assigned to students to solve.

Problem generation occurred on the pre- and post-assessments, a journal entry, and two problem sets during the semester. The sets of given information provided participants with the context of possible mathematics problems but did not include any questions. The first problem-generation task was presented in a prompted journal entry that was completed as homework and included reflection on the problem-posing process. The final two problem-generation tasks were part of assigned homework problem sets.

The goal of the problem-reformulation and problem-generation activities was to provide participants with opportunities to pose mathematics problems. Therefore, the instructor checked participants' problem-posing work for completeness, but did not grade the assignments or count the assignments in the determination of their course grade. It was also a goal of the project to explore participants' views of the relationship between problem posing and school mathematics. The catalysts for this exploration were journal prompts assigned as homework. The remainder of this chapter will focus on outcomes related to participants' problem posing, while participants' views of the relationship between problem posing and school mathematics will be presented to add context to the problem-posing results.

Problem Generation Coding

An adaptation of Leung and Silver's (1997) scheme was chosen to code problems because they determined that "…the multi-stage analytic scheme … functioned in a reasonable way…" (p. 18). Based on this, a statement determined to be mathematical from a problem-generation activity was coded along three dimensions: plausibility, sufficiency of information, and complexity. An implausible problem is one that contains an invalid assumption and hence is not plausible to solve even with more information. Similar to Leung (1993), implausible problems were not coded further, since the author of this paper was interested in problems that contained a plausible solution. If a posed problem was plausible, the author then determined whether there was sufficient information to solve the problem. Problems with extraneous information were coded as having sufficient information since they were solvable. If a problem was both plausible and contained sufficient information, it was then determined if multiple steps were necessary for solution. Arithmetic steps were not the determining characteristic of a multi-step problem because, as suggested by Silver and Cai (1996), counting steps is easy but could cause simple arithmetic problems to be coded as fairly complex. To solve a multi-step problem, the problem solver must be required to perform at least two mathematical tasks. Problems posed from problem-generation activities were assigned a score as shown in Table 20.2.

Table 20.2
Criteria for Scores from Problem-Generation Coding

Score	Criteria
0 pts	Non-mathematical statement or mathematical statement but not a plausible problem
1 pt	Plausible problem without sufficient information
2 pts	Single-step plausible problem with sufficient information
3 pts	Multi-step plausible problem with sufficient information

Problem Reformulation Coding

Classification of problems began with four posing techniques (switch the given and wanted, change the context, change the given, add information) that describe the relationship between the posed and original problem. Techniques were added until all problems could be described as being posed using at least one technique. It is also important to note that a single problem reformulation could have employed

Table 20.3
Problem Reformulation Techniques

Category	Description
Switch the given and the wanted	A problem in the same context as the original problem with the given and wanted information switched
Change the context	A problem with the same structure but context changed
Change the given	Same problem context and structure but the given information is changed
Change the wanted	Same problem context and structure but what the question asks for is changed
Extension	An extension of the given problem
Add information	Same problem context and structure with added information
Re-word	Same problem with different wording

two or more techniques. For instance, a participant could change the given and change the wanted of the same problem to produce a new related problem. Table 20.3 describes the techniques that exhausted the coding of all problems posed as problem-reformulation.

Inter-Rater Reliability

To check the validity of the coding, two additional raters volunteered to code problems from problem-posing tasks. Raters used the developed schemes to code 90 problems from problem generation tasks and 75 problems from problem reformulation tasks. With regard to problem-generation coding, the author and raters agreed on the plausibility of 96.7% of the problems. Of the 87 problems the raters and author agreed were plausible, all agreed that 87.4% contained sufficient information. Of the 76 problems agreed upon as containing sufficient information, the author and raters agreed that 80.3% required a multi-step solution. All problems not agreed upon were discussed and consensus was reached between the author and raters.

With regard to problem-reformulation tasks, the author asked the raters to code problems based on the seven techniques developed during the initial coding and to report if they believed other techniques were used. Neither rater suggested another technique. Of the 75 problems the author and raters agreed on the coding of 74.7% of the problems. The main discrepancies in coding occurred when the raters coded problems into multiple categories and often considered changing the given as an extension of a problem. The 19 problems coded differently were discussed and coding was agreed upon.

Similar to Leung and Silver (1997), there were high levels of inter-rater agreement on the coding schemes for both problem-reformulation and problem-generation. Based on this the author continued coding all posed problems using the schemes described for problem-generation and problem-reformulation.

Problem-Posing Results

Pre- and Post-Assessment

The same problem generation assessment was administered on the first and then final day of the semester. Participants had 25 minutes in class to pose as many problems as they could. The measure (see Figure 20.1) consisted of a set of information with numeric content and a set of information without numeric content.

The assessment was coded using the described problem generation scheme and each participant received a score for numeric and non-numeric posing based on the scores described in Table 20.2. Aggregate data from the assessments are summarized in Table 20.4.

Directions: Consider the possible combinations of pieces of information given below and pose as many mathematical problems as you can think of.

Numeric Set of Information: You have decided to purchase a computer for college. The new top-of-the-line laptop costs \$2500. You have two options for purchasing the computer, you can use your credit card, which has an annual interest rate of 13.99%, or you can finance it through the university computer store for 48 months at \$70 a month. You have saved \$500, but you need to be able to pay for your books next semester.

Non-Numeric Set of Information: The university has decided to build a parking garage for the use of students and staff. The university has a maximum amount of land that they can use and also a minimum number of faculty/staff spots and a minimum number of student spots needed at certain hours of the day. The university has done research that shows that a fixed number of faculty/staff and a fixed number of students arrive at 8am and 12 noon. The university is also restricted by a fixed budget for paving and general construction.

Figure 20.1. Pre- and post-assessment of problem posing.

Table 20.4
Aggregate Pre- and Post-assessment Problem-Posing Data

	Pre-assessment	Post-assessment
Statements	101	133
Plausible	96 (95%)	122 (92%)
Sufficient information	55 (54%)	87 (65%)
Multi-step solution	16 (16%)	37 (28%)
Numeric average	5.21	8.72
Non-numeric average	3.47	4.89

Results on the pre- and post-assessment were compared using a Tukey–Kramer multiple comparisons matched-pairs test at the alpha equals 0.05 level. The data from the participant who did not complete the post-assessment were not included in the analysis. The statistical analysis showed that the difference in the means of Numeric pre and Numeric post ($q = 0.97$), as well as Numeric post and Non-numeric post ($q = 1.41$), is statistically significant.

With regard to numeric problem-posing ability, the participants' average increased from 5.21 on the pre-assessment to 8.72 on the post-assessment. For non-numeric posing the participants' average changed from 3.47 to 4.89. It is important to consider if these changes are because participants were writing more situations or were generating more plausible problems with sufficient information that required multi-step solutions. The data in Table 20.4 suggest that participants' efficiency in posing problems increased, as they posed 122 plausible problems on the post-assessment compared to 96 on the pre-assessment. It is also clear that participants posed more problems with sufficient information requiring multi-step solutions on the post-assessment. The results of the pre- and post-assessments suggest that, after this course, which included exposure to problem posing, participants became more efficient at posing problems when problems were generated under a time constraint, and they posed more multi-step problems with sufficient information for solution. The remaining results related to problem generation will highlight that the characteristics of participants' problem generation were consistent with the tasks collected during the semester.

Problem Generation

Other than the pre- and post-assessments, participants engaged in problem generation during the fifth, seventh, and tenth weeks of the semester. The first problem generation task was assigned as part of a journal entry and asked students to pose three to five problems. The set of given information and typical problems follow in Table 20.5.

Table 20.5
Typical Week 5 Problem Generation

Given information	Mrs. Smith's and Mr. Jones' fifth-grade classes took the same mathematics test last week. You have been given all the graded exams and the answer key
Not plausible (0 pts)	Do you feel by the overall grades, that it would be fair to scale the grades or should students get the grade they earned?
Plausible without sufficient information (1 pt)	There are 15 students in Mrs. Smith's class and 12 students in Mr. Jones' class. The median of all the tests from both classes is an 82. How many students scored above the median? How many students scored below the median?
Plausible with sufficient information (2 pts)	Mr. Jones' class has an average of 80 and there are 18 students who have taken the exam, but Suzy was absent that day. If she takes the test and gets a 99 what is the new average?
Plausible, sufficient information and multi-step (3 pts)	Does the mean, median, or mode best reflect the class test scores in Mrs. Smith's class [test data was included]? Explain why you feel as you do?

Sixty-seven percent of the problems generated on this task were multi-step problems and only 19% were not plausible or did not contain sufficient information for solution. Participants also added information to 29% of the problems, most likely due to the non-numeric nature of the set of given information.

The week 7 problem generation task was assigned as part of the homework problem set and asked participants to pose at least three problems. The set of given information and typical problems follow in Table 20.6.

Table 20.6
Typical Week 7 Problem Generation

Given information	You arrive at your friend's home and they are sitting at a table with $20, a deck of cards, and red, white, and blue die
Plausible without sufficient information (1 pt)	If there are 4 red chips, 8 white chips, and no blue chips in a pile on the table and the betting is $8 so far what do the red and white chips stand for?
Plausible with sufficient information (2 pts)	If the cards Ace, 2, 3, … up through J, Q, K are each given a value 1–13, in order, what is the probability that a card picked at random will have a value greater than 10?
Plausible, sufficient information and multi-step (3 pts)	Your friend offers to give you $10 if you get a sum of 9, 10, 11, or 12 when all 3 die are rolled. You have four chances. If you do not roll any of these sums in your four chances you owe him $10. Are you going to accept this challenge? Why or why not?

Participants continued the trend of posing multi-step problems on this task, as 56% of the generated problems required a multi-step solution. All problems posed were plausible, only 12% did not contain sufficient information or information was not added to any of the posed problems.

The week 10 problem-generation task was assigned as part of the homework problem set and participants were asked to pose at least two problems. Table 20.7 includes the set of given information and typical posed problems from the task.

Table 20.7
Typical Week 10 Problem Generation

Given information	A roulette wheel has 18 red numbers, 18 black numbers, and 2 green numbers. A person bets on either an individual number or a color. A one dollar bet on an individual number pays $35, on black or red pays $1, and on green pays $12
Not plausible (0 pts)	A roulette wheel has 18 red numbers, 18 black numbers, and 2 green numbers. If Annie puts $5 on one red number and $5 on two black numbers what is the probability that she will win $10 in 2 spins?
Plausible without sufficient information (1 pt)	How many bets would you have to make to win $80?
Plausible with sufficient information (2 pts)	If you bet on black 23 times in a row and win 12 times. Do you have more or less money than when you started?
Plausible, sufficient information and multi-step (3 pts)	Would you bet on an individual number, black, red, or green? Explain your decision using probability

Similar to the first two problem generation tasks, 61% of the posed problems required a multi-step solution and only 13% were not plausible or did not contain sufficient information. As with the second task, information was not added to any of the posed problems. Table 20.8 summarizes the results from the three problem-generation tasks.

Table 20.8
Aggregate Problem Generation Results from the Semester

	Statements	Plausible	Sufficient information	Multi-step solution	Added information
Week 5	42	39 (93%)	34 (81%)	28 (67%)	12 (29%)
Week 7	48	48 (100%)	42 (88%)	27 (56%)	0
Week 10	23	21 (91%)	20 (87%)	14 (61%)	0

Participants were not under a time constraint as they had at least 5 days to complete each of the tasks. The data in Table 20.8 suggest that the characteristics of participants' problem generation during the semester were consistent. These participants were able to pose plausible, multi-step problems from sets of information regardless of whether they contained numeric information. This consistency may have been a pre-cursor to participants' apparent aptitude for posing multi-step problems on the post-assessment.

Problem Reformulation

Participants engaged in problem reformulation on seven homework problem sets during the semester. The participants utilized two distinct types of problem reformulation techniques. "Surface" techniques consisted of adding information, changing the given, changing the wanted, and re-wording. Surface reformulation techniques did not require the problem poser to change the structure of the problem; they required only a change of the surface features of the problem (e.g., numbers, what is asked for). "Structure" techniques included switching the given and wanted, changing the context, and extending the original problem. Structure reformulation techniques required more creativity and a deeper understanding of mathematical content on the part of the problem poser, as they required changing the structure of the problem. The utilization of these two types of problem reformulation techniques will be discussed in this section. Table 20.9 provides an overview of participant problem reformulation.

The mathematical content focus of the problem sets in weeks 4 and 5 was problem solving and data analysis. Reformulation on these problem sets was dominated by changing the given and changing the wanted. Structure reformulation techniques were 22% of the techniques utilized in week 4 and increased to 35% of the techniques utilized in week 5. On both problem sets switching the given and the wanted was the most popular structure technique. The increase in use of structure techniques

Table 20.9
Aggregate Problem Reformulation Data

	Week 4	Week 5	Week 6	Week 7	Week 10	Week 11	Week 15
Total posed problems	49	56	42	40	37	35	32
Total techniques	59	62	45	45	42	37	35
Switch given and wanted	7	11	6	3	3	0	2
Change context	2	5	3	1	4	3	5
Extend original	4	6	0	2	5	13	5
Structure techniques	13 (22%)	22 (35%)	9 (20%)	6 (13%)	12 (29%)	16 (43%)	12 (34%)
Change given	27	26	25	17	16	13	17
Add information	3	3	3	5	2	3	3
Change wanted	15	10	8	15	11	5	3
Re-word original	1	1	0	2	1	0	0
Surface techniques	46 (78%)	40 (65%)	36 (80%)	39 (87%)	30 (71%)	21 (57%)	23 (66%)

Table 20.10
Problem Reformulation from Weeks 4 and 5

Original problem Week 3	A special rubber ball is dropped from the top of a wall that is 16 feet high. Each time the ball hits the ground it bounces back only half as high as the distance it fell. The ball is caught when it bounces back to a high point of 1 foot. How many times does the ball hit the ground?
Switch the given and the wanted	If a special rubber ball is dropped from a wall with an unknown height and bounces four times and is caught at the height of its fourth bounce at 2 feet. If we know that every time the ball bounces it only bounces back half the distance as the distance it fell. How high is the wall the ball dropped off of originally?
Change the given	A special rubber ball is dropped from the top of a wall that is 768 feet tall. Each time the ball hits the ground it bounces back only one-fourth as high as the distance it fell. The ball is caught when it bounces back to a high point of 3 feet. How many times does the ball hit the ground?

in week 5 may be attributed to it being the second problem set related to the mathematical content of problem solving and data analysis. Table 20.10 includes typical examples of problem reformulation on these problem sets.

Data representation and analysis was the mathematical content focus of the problem set in week 6. Participants were still relying heavily on changing the given information and structure techniques were 20% of the techniques utilized. This similarity to reformulation in week 4 may be attributed to this being the only problem set

Table 20.11
Problem Reformulation from Week 6

Original problem Week 5	The average of 7 numbers is 49. If 1 is added to the first number, 2 is added to the second number, 3 is added to the third number, 4 is added to the fourth number, and so on up to the seventh number, what is the new average?
Switch the given and the wanted	The average of 7 numbers is 49. Each of the data points were increased by the same amount. The new average is 53, what value was each data point increased by to raise the mean?
Change the given	The average of 11 numbers is 121. If 1 is added to the first number, 2 to the second number, and so on up to the eleventh number, what is the new average?

Table 20.12
Problem Reformulation from Weeks 7 and 10

Original problem Week 7	In a random drawing of one ticket from a set numbered 1–1,000, you have tickets 8,775–8,785. What is your probability of winning?
Switch the given and the wanted	You have a probability of 3/20 of winning and received the following numbers from a drawing 122–136. What was the total number of tickets distributed for the event?
Change the given and change the wanted	If Beth has 19 tickets for a drawing with 100 total tickets and Veronica has 4 tickets for a drawing with 20 tickets, who has a better probability of winning?
Original problem Week 10	Six people enter a tennis tournament. Each player played each other person one time. How many games were played?
Extension	Three different tournaments, one with four people, one with five people, one with six people. Each player played the other person one time. How many games were played in each tournament? Is there a pattern? Can you find a rule?

on which participants were asked to reformulate problems-related data representation. As with the week 4 and the week 5 problem sets participants favored the structure technique of switching the given and the wanted. Typical examples of posed problems on the week 6 problem set are presented in Table 20.11.

The mathematical content focus of the problem sets during weeks 7 and 10 was counting, chance, and probability. As with previous problem sets, participants relied heavily on the reformulation techniques of changing the given and changing the wanted. During week 7 participants continued to favor switching the given and wanted as a structure reformulation technique, but this gave way to favoring extension during problem reformulation in week 10. Structure techniques were only 13% of the techniques utilized in week 7, but were 28.5% of the techniques utilized in week 10. This increase continued the trend from weeks 4 and 5 of an increased use of structure techniques on the second problem set related to specific course content. Week 7 was the only occurrence of less that 20% structure techniques, and this could be attributed to the difficulty of the material related to probability. Table 20.12 includes typical examples of reformulated problems from weeks 7 to 10.

The mathematical content focus of the problem set in week 11 was discrete mathematics and in week 15 was algebraic thinking. Participants relied on the

Table 20.13
Problem Reformulation from Weeks 11 and 15

Original problem Week 11	Consider networks with 0, 1, 2, 3, and 4 odd vertices. Make a conjecture about the number of odd vertices that are possible in a network. Explain your thinking
Change the given	Consider networks with 0, 1, 2, 3, and 4 even vertices. Make a conjecture about the number of even vertices and the traverse ability of the network. Explain
Extension	Knowing that you can create a network with an even number of odd vertices, is it possible for these types of networks to be traversable?
Original problem Week 15	A whole brick is balanced with ¾ of a pound and ¾ of a brick. What is the weight of the whole brick?
Change the context	If a bottle and a glass balance with a pitcher, a bottle balances with a glass and a plate, and two pitchers balance with three plates, can you figure out how many glasses will balance with a bottle?

reformulation techniques of changing the given and extension in week 11 and structure problem reformulation techniques were 43% of the techniques utilized. Changing the given was the most utilized reformulation technique during week 15 and structure techniques were utilized 34% of the time. Both problem sets were the only problem sets related to the specific mathematical content and there is an increase in the use of structure techniques from previous problem sets. Participant's problem-posing experience on the first five problem sets may have prepared them to use structure techniques when considering new mathematical content. Typical examples of problem reformulation during weeks 11 and 15 can be found in Table 20.13.

In summary, although surface techniques dominated reformulation throughout the semester, changes are evident in participants' problem reformulation. Participants' problem reformulation in weeks 11 and 15 suggest that, as they gained problem-posing experience, they relied more on structure techniques when problems sets were related to course content for the first time. Participants' choice of structure techniques also became more diverse—switching the given and wanted dominated structure reformulation early in the semester, but this gave way to the use of both extension and changing the context later in the semester. These changes in use of structure reformulation techniques suggest that participants developed problem-posing creativity and the ability to generate a more diverse set of problems.

The Relationship Between Problem Posing and School Mathematics

Data related to participants' beliefs about the relationship between problem posing and school mathematics was collected on the pre and post-assessment of beliefs and five journal entries. This data will highlight participants' articulated beliefs that problem posing is a beneficial task for their future students and that they will utilize problem posing in their future classrooms. On the pre-assessment of beliefs

Table 20.14
Participants Pre-assessment Views of Problem Posing

Category	Participants' responses
Problem solving	Help students better understand word problems; students will understand designing problems; create problems that relate to them; develop a better understanding of problem solving; helps students think beyond problem solving
Understanding	Consider information on multiple levels; better understanding of material; help teachers assess student understanding; helps students recognize pertinent information
Feelings	Alleviate student fear of word problems; develop ownership of mathematics; freedom and creativity with numbers and relationships
Negatives	Students may be confused or frustrated at first; may pose unsolvable or non-mathematical questions; questions may take lessons off track; students may take easy way out and ask simple questions; not practicing math directly

instrument, participants were asked to consider problems posed by an elementary-age student and respond to the question, "Do you believe that problem posing from sets of given information is a worthwhile task for elementary school students?" Participants had the prior experience of the pre-assessment of problem posing before completing this task. Participants' descriptions of the possible benefits of problem posing can be organized around three themes, the relationship to problem solving, aiding student understanding, and influencing student feelings about mathematics. Responses from the task are included in Table 20.14, which also includes potential negatives of problem posing suggested by participants.

At the beginning of the semester, participants seemed to believe that although there were some potential drawbacks to problem posing in school mathematics, problem posing had the potential to help students with their problem-solving ability, help students develop understanding, and affect students' creativity and ownership of mathematics. The remainder of this chapter will examine how participants were better able to articulate their beliefs as they gained experience posing problems. This will be highlighted by participants' abilities to discuss possibilities for the utilization of problem posing in school mathematics.

During week 5 of the semester participants responded to the following journal prompt as part of their assigned homework.

> Imagine that you are teaching and someone comes in to observe your classroom and a mathematics lesson that you are teaching. Write a description of your classroom and the lesson from the eyes of the observer. What would they see you doing during the lesson, what would they see the students doing, what would they notice about your classroom?

In response to this prompt only two participants suggested utilizing problem posing in their future classrooms. In the description of her lesson one participant stated that she would have students write word problems for division facts that she had on the chalkboard. Another participant stated that she would give students a journal prompt that asked them to think of a division problem, solve it, and then write in their own words how they would explain the problem to a third grader.

Participants' next journal entry was collected a week later; the prompt asked them to respond to the following:

> Please write a brief reflection on how you think class is going so far this semester, what aspects have you found the most helpful, least helpful and why?, how is the workload?, what aspects would you change?, what additional topics would you like to see covered?

Responses for this journal activity suggested some further reflection on problem posing and its relationship to school mathematics had taken place. Four participants commented that their problem-reformulation and problem-generation experiences have caused them to think beyond the activities and start to relate problem posing to their future classrooms. Other responses related to problem posing included comments that problem posing seemed to be an effective teaching tool and that students should want to pose and solve their own problems in and out of the classroom.

During week 10 participants responded to a journal prompt that specifically asked them to consider problem posing:

> As you are posing related problems or posing problems from a given set of information who is your intended audience? Why? Does the audience change depending on the problem? Would you consider yourself better at posing problems as reformulations or posing problems from sets of given information? Why?

Responses showed evidence that, when prompted, participants were capable of reflecting on the relationship between problem posing and school mathematics. Eleven of the 16 participants who responded stated that they were posing problems for their future students and indicated what they believed an appropriate grade-level range for the problems they created was from second to eighth grade. Ten participants also said that the grade level for which they posed problems was dependent on the original problem or the original set of given information. Participant reflection is highlighted through the following quotes: "When I'm actually teaching, I will need to pose appropriate problems for all children in my class to best facilitate their growth in mathematics," and "What I try to keep in mind most as I am problem posing is whether or not most students at a particular grade level will be able to find a solution with meaning and understanding."

Participants engaged in problem posing for another month before the journal entry collected during week 14 asked them to consider if they would utilize problem posing in their future classrooms through the following prompt. "Do you think you will utilize problem posing in your future classroom? If so, in what ways? Please try to be as specific as possible."

All participants articulated a role for problem posing as a future classroom resource and suggested that they saw potential for student and teacher problem posing. Participants suggested many possibilities to promote student problem posing in their future classrooms including as a whole class, as problem reformulation, as an introduction to new material, on homework, as an extra credit assignment, as a device to give fast students something to do, and by using a "problem-posing box." The most common suggestion was whole-class problem reformulation followed by assigning problem generation tasks when students were more comfortable with problem reformulation. Suggestions also included students posing problems related to a new topic and having the class research answers to these problems to gain introductory knowledge about the topic. Finally, one participant suggested that students could pose problems for homework or during class activities and collect them in a "problem posing box." When time permitted in class, students could choose a problem from the "problem posing box" and attempt to solve it.

Participants' reflection also included possible outcomes and benefits of student problem posing. Participants suggested that problem posing could promote student thinking and could allow for deeper understanding of content. One participant stated "By the problem posing process, students begin to identify key terms and concepts that define a topic, and by structuring problems around these topics, they begin to make connections, which enhances the learning process." Participants also supported their ideas from the pre-assessment of beliefs that problem posing would allow for student control and autonomy and can give students a sense of ownership over a problem. Two statements from participants illustrate these ideas: "I think that when students inquire about topics they are taking learning into their own hands, and that is one of the best things that problem posing can bring to a classroom," and "The questioning can help students determine their level of knowledge and helps students to develop metacognition."

As a tool for teachers, participants suggested using problem posing for assessment, to take advantage of "teachable moments," to accommodate all learning styles more effectively in their classroom, and to help develop activities, problems, tests, and quizzes. One participant described how and why a teacher would utilize problem posing when she wrote, "A teacher must be able to predict what students will find easy and difficult to do, and know her students well enough to be able to pose problems that will be thought provoking and meaningful to them."

In these journal entries participants described similar benefits of problem posing to those identified on the pre-assessment of beliefs, but extended these ideas by articulating specific ways to incorporate problem posing in their classrooms and reasons why problem posing may influence their teaching, student understanding, and student feelings about mathematics. This implies that further problem-posing experience may influence participants' abilities to reflect on and articulate potential roles of problem posing in school mathematics.

Participants' final prompted journal entry was collected in week 15 and participants responded to the following prompt:

> Please write a reflection on your experiences in this course this semester. The following questions might help to guide your reflection: (1) What have I learned about myself as a learner of mathematics? (2) What have I learned about myself as a prospective teacher of mathematics? (3) How has my conception of mathematics or teaching changed? (4) What questions do I still have?

A few participants' quotes stand out to highlight the ideas about problem posing already mentioned in this chapter. Even when they were not specifically prompted to do so, participants still reflected on their problem-posing experiences.

- With problem posing, I as the architect developed the concepts that should be incorporated into the problems and determined the age groups to be assessed, and as the carpenter I wrote the problems, determining what style would suit the students needs best, much like a carpenter must do when building a piece of furniture, or a house.
- I also learned how beneficial it is to having children pose problems, something I didn't like before this class. It is extremely important to give the students a sense of ownership over a problem and a better understanding of the problem.
- Uses in the classroom and importance of problem posing are the biggest thing that I have learned.
- I can also have students pose their own problems to be solved by their classmates. This allows more freedom and power for the students in owning their learning.

In summary, as they gained problem-posing experience, participants articulated detailed beliefs about the relationship between problem posing and school mathematics. It should be noted that the description of these beliefs occurred in both journal entries that specifically prompted for information about problem posing and those that did not. Similar to the results of Akay and Boz (2009), participants saw engaging in problem posing as a beneficial task for their future students and viewed problem posing as a tool that they would utilize in their future teaching. Journal entries collected as homework suggested that participant reflection on problem posing and teaching and learning occurred throughout the semester. This reflection allowed participants to provide detailed descriptions of their new beliefs about problem posing as they developed their own problem-posing skills.

Discussion

The development of participants' abilities and creativity as problem posers was highlighted through quantitative data related to the characteristics of their problem posing. One student summarized class changes with respect to problem posing with clarity in the final journal entry of the semester in the following way:

> However the greatest thing that I will take from this class is my newly discovered talent of problem posing. I remember back to the first class this semester when we were asked to do some problem posing for Dr. G's research project. I was stumped by this task. Posing a problem from the given information was like another language to me. As the problem sets were assigned throughout the semester, I truly dreaded problem posing. But about half way through the semester, it was like a light turned on in my head and I was suddenly able to create problems without all that difficulty. This allowed me to focus on posing valid challenging problems. It was great to have the same packet handed out once again the last day of class for Dr. G's research project, and being asked to pose as many problems as I could. This was such a valuable task for me because I could literally see my growth as a problem

> poser first hand! I sat there and posed problems for minutes without even taking a breather! It was a great feeling to have actually seen how much I grew in this one area of math throughout the course of the semester.

Problem posing was not explicitly discussed in class but as participants' gained problem posing experience they became more efficient problem posers and became more creative at problem reformulation. The changes in participants-posed problems support Leung and Silver's (1997) hypothesis "… that further experience [beyond a single assessment] with problem posing would lead to better, and more sophisticated performance" (p. 17).

Participants' problem generation was consistent when they were not posing problems under a time constraint. On the three problem generation tasks collected, 61% of participants-posed problems required a multi-step solution. As evidenced by the post-assessment of problem posing, participants in this study became more efficient problem posers and were able to pose more complex problems under a time constraint. These results for problem generation support Leung and Silver's (1997) conclusion that prospective elementary school teachers are capable of posing appropriate mathematical problems and are also capable of posing complex mathematics problems.

At the beginning of the semester these prospective teachers relied on surface problem-reformulation techniques the first time they reformulated problems related to specific course content. This reliance gave way to an increased use of structure-reformulation techniques later in the semester. The participants continued to develop their creativity in posing problems as problem reformulation even though the quality of their problems did not influence their grade. Therefore, it seems possible to develop the problem reformulation abilities of prospective teachers through problem-posing experiences that do not have to include explicit instruction in problem posing. Therefore, engaging prospective teachers in problem posing as was done in this study has the potential to help develop both problem generation and reformulation abilities.

Consistent with the work of Leung and Silver (1997) and Stickles (2010–2011), the research reported in this chapter has shown that prospective elementary and middle-school teachers were able to pose more problems and more complex problems on problem generation tasks when the set of information included numeric content. This is evident by the statistically significant difference in their posing from pre-assessment to post-assessment. While there was a positive change in participants' non-numeric problem posing, this change was not statistically significant and after gaining experience posing mathematics problems participants still favored posing problems when numeric content was included. Developing participants' ability to pose problems on tasks that do not include numeric content should be a focus of future problem-posing research.

Participants developed beliefs about the potential benefits of problem posing in school mathematics and developed problem-posing abilities to support the incorporation of problem posing in their future classrooms. As suggested in journal entries, these prospective teachers share Silver's (1997) view that problem posing can be included in mathematics instruction to "… develop in students a more creative disposition towards mathematics" (p. 76). Although problem-posing tasks during the instructional treatment focused on problem reformulation and problem generation, participants' reflection on the role of problem posing in school mathematics went beyond these two techniques and included potential benefits for student learning. The benefits these preservice teachers articulated were consistent with research and writing in mathematics education (Silver, 1994). This understanding of the benefits of problem posing may help these preservice teachers develop practices that mirror their beliefs and incorporate problem posing in their future classrooms.

Conclusion and Implications

Based on this and past work, it is reasonable to assume that prospective teachers have some capability for posing mathematics problems. This research extends previous studies by showing that problem-posing ability and creativity can be developed further by engaging prospective teachers in two forms of problem posing as part of the expectations of a mathematics content course. This incorporation of problem posing does not reduce time in class to discuss mathematics content and does not require class time devoted explicitly to teaching problem posing. Therefore, teacher educators should consider incorporating problem posing in mathematics content courses for prospective elementary and middle-school teachers as problem generation and problem reformulation. Preparing problem-generation tasks and requiring students to pose problems as problem reformulation does not add significant time to the instructor's development of course materials.

The incorporation of problem posing as described in this chapter has the potential to be in the vanguard for the incorporation of problem posing at all levels of mathematics education. Problem-posing experiences may help prepare teacher-education students who are poised to engage their students in problem posing. It may also serve to educate mathematics teacher-educator colleagues about mathematical problem posing. Further research is needed to extend this work. First, due to the study design we cannot directly attribute participants' changes to the problem-posing experience in the class. A study that collects similar data without incorporating problem-posing in the context of the class may be able to shed light on whether the problem-posing experience is key to create change. Second, the prospective teachers in this study were poised to incorporate problem posing in their future classrooms, but did they? Longitudinal studies that incorporate problem posing in classes for prospective teachers are needed. Participants would then be followed into the classroom to determine if and how, as teachers, they implement problem-posing practices.

References

Akay, H., & Boz, N. (2009). Prospective teachers' views about problem-posing activities. *Procedia—Social and Behavioral Sciences, 1*, 1192–1198.

Barlow, A. T., & Cates, J. M. (2006). The impact of problem posing on elementary teachers' beliefs about mathematics and mathematics teaching. *School Science and Mathematics, 106*(2), 64–73.

English, L. D. (1997). Promoting a problem-posing classroom. *Teaching Children Mathematics, 4*(3), 172–179.

Gonzales, N. A. (1994). Problem posing: A neglected component in mathematics courses for prospective elementary and middle school teachers. *School Science and Mathematics, 94*(2), 78–84.

Gonzales, N. A. (1998). A blueprint for problem posing. *School Science and Mathematics, 98*(8), 448–456.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123–147). Hillsdale, NJ: Lawrence Erlbaum.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Leung, S. (1993). *The relation between mathematical knowledge and creative thinking to the mathematical problem posing of prospective elementary school teachers on tasks differing in numerical information content* (Unpublished doctoral dissertation). University of Pittsburgh, Pittsburgh, PA.

Leung, S., & Silver, E. A. (1997). The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers. *Mathematics Education Research Journal, 9*(1), 5–24.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Polya, G. (1957). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19–28.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *International Reviews on Mathematical Education, 29*(3), 75–80.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*(5), 521–539.

Stickles, P. R. (2010–2011). An analysis of secondary and middle school teachers mathematical problem posing. *Investigations in Mathematics Learning, 3*(2), 1–34.

Whiten, D. J. (2004a). Building a mathematical community through problem posing. In R. N. Rubenstein & G. W. Bright (Eds.), *Perspectives on the teaching of mathematics: Sixty-sixth yearbook* (pp. 129–140). Reston, VA: National Council of Teachers of Mathematics.

Whiten, D. J. (2004b). Exploring the strategy of problem posing. In G. W. Bright & R. N. Rubenstein (Eds.), *Professional development guidebook for perspectives on the teaching and learning of mathematics: Companion to the sixty-sixth yearbook* (pp. 1–4). Reston, VA: National Council of Teachers of Mathematics.

Winograd, K. (1997). Ways of sharing student-authored story problems. *Teaching Children Mathematics, 4*(1), 40–47.