Chapter 2
Problem Posing from a Modelling Perspective

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Abstract In this chapter, we consider how problem posing forms an integral part of mathematical modelling and consider its placement during modelling processes. The problem and its formulation is an essential part of modelling, and a modelling process is usually associated with a continual adjustment and reformulation of the main problem. In addition, one may formulate conjectures, ask monitoring and control questions, and have a critical stance toward the model and its results. We consider how the educational intention of the modelling activity and the placement in the modelling cycle relates to the problems and questions being posed. We briefly consider how problem posing may be implemented in mathematical modelling through the use of students' conjectures and by students acting as consultants and clients.

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Introduction

We see mathematical modelling and problem posing as promoting essential skills necessary for involvement in a democratic society and as integral parts of a balanced mathematics curriculum. The topics of mathematical modelling and problem posing are closely related as modelling is concerned with using mathematics to solve or gain further insight into real-world problems. Our own experience as teacher educators is that posing problems that make good mathematical tasks is no trivial matter, and that student teachers often find it difficult to pose and implement appropriate modelling tasks in their teaching practice. A further layer of difficulty is added when one wishes pupils to take an inquiring stance, where they pose problems related to mathematical modelling. This chapter will look at problem posing from the perspective of the pupil, but much of it will be relevant to teachers' problem posing as well.

In this chapter, we discuss why we see modelling and problem posing as a potentially fruitful combination. This will be followed by short discussions on how problem posing relates to different perspectives on modelling and to the modelling process.[1] We end by sketching some ways to implement problem posing in mathematical modelling. The chapter tries to give some pointers to the many issues present in this under-researched intersection of mathematical modelling and problem posing.

Mathematical Modelling and Problem Posing: Possible Obstacles

An initial example will be presented to illustrate possible obstacles one can meet when trying to combine mathematical modelling and problem posing. In this example, a group of four student teachers were starting a lesson sequence with eighth grade pupils based on mathematical modelling. The student teachers decided to choose the general topic of mathematical modelling themselves. Since one of the student teachers had experience in biology, the topic chosen was plants. In the initial lesson, the student teachers encouraged the pupils to formulate as many questions as possible concerning plant growth. The student teachers planned this as a pure problem-posing lesson. A purpose of this lesson was to provide the student teachers with ideas of authentic mathematical modelling problems to use with the pupils within the context of plant growth, although the student teachers had not considered how to follow-up the problems posed by the pupils. In the continuation of the lesson sequence, the pupils' problems were not, in fact, used. Instead, the pupils were supposed to seed their own plants in small boxes in the classroom and work with modelling and growth prediction in accordance with the schedule made by the

[1] Another relevant theme, which we do not pursue, is how different goals such as decision making, system analysis and design, and trouble shooting (OECD, 2004) affect the type of problem posing relevant for mathematical modelling.

student teachers. The student teachers' main mathematical focus was on the measure and prediction of plant height, using scatter plots and linear functions. Commenting on the problem-posing stage, the student teachers noted that the problems posed by the pupils were to a large degree nonmathematical:

> Student teacher A: "I have been discussing … about ten questions like this, I think: Why a plant is able to grow up through the asphalt, why leaves are yellow in autumn, why some plants are poisonous, why some have thorns, why do flowers need water … it was a lot of that."

And that they had difficulties distinguishing appropriate problems:

> Student teacher B: "Are we likely to ask then what the largest plant in the world may be? Will that be a good enough question?"

Based on this example, we have identified five types of difficulties that the student teachers encountered when they attempted to combine mathematical modelling and problem posing. In particular, we will see the importance of teachers being able to assist in the refinement and reformulation of problems so that they become manageable for the students. For example, a question like "Why do flowers need water?" may be reformulated as a mathematical modelling problem, where one quantifies growth of flowers under the influence of different external conditions.

Five Types of Difficulties

The five types of difficulties faced by the student teachers were:

1. **Posing mathematically relevant problems**. In most cases, the pupils posed problems that were not mathematically relevant, i.e., the problems were stated in such a way that using mathematics to solve them would be contrived. This can in part be explained by the student teachers not being explicit in stating to their pupils that they were mainly interested in problems that could be handled using mathematics. However, we also see an underlying difficulty in posing problems that have mathematical relevance.

   Mathematical modelling is always interdisciplinary. This has several advantages, but also the disadvantage that pupils are not necessarily able to distinguish between problems that are mathematically relevant and problems that are not. Being able to distinguish problems that can be mathematized and being able to reformulate nonmathematical problems so that they can be handled using mathematical tools is part of the learning process. As these are competencies for which the pupils, and the beginning student teachers, were not proficient, this added a level of difficulty to the problem-posing activity.
2. **Posing mathematically suitable problems**. The student teachers had difficulties distinguishing which problems were mathematically suitable for the pupils, i.e., which problems would be neither too easy nor too difficult for the pupils and at the same time would enable the pupils to engage in significant mathematical modelling. Posing problems of an appropriate level of difficulty is, of course, a potential obstacle in any scenario, where one poses nontrivial

problems. However, this obstacle is enhanced in mathematical modelling since it is not always obvious from the initial problem formulation what mathematics will be needed. A needed skill here is to be able to reformulate and adjust problems in appropriate ways so that they attain a reasonable degree of mathematical sophistication.

In mathematical modelling, it is frequently the case that first attempts at problem posing give problems that are unmathematizable or too difficult as stated. It is the norm that repeated adjustments and reformulations of the problem are necessary before one arrives at a problem which is both mathematizable and mathematically manageable.

3. **Posing problems such that the pupils feel ownership of the problems**. In this example, the student teachers encountered pupils just "going through the motions"—that is to say, pupils just spurting out lots of similar looking problems without reflecting on them, or posing pseudo-problems for which they did not really anticipate an answer. Here, one needs to be aware that problem posing is an ongoing process, where reformulations and adjustments of the problem are frequently required. This is especially the case in mathematical modelling, where one continually refers back to the problem situation during the modelling process.
4. **Making problem posing a relevant part of the learning trajectory**. In this case, the pupils' problems were left hanging; they were not reflected upon at the end of the lesson nor were they used in the lessons that followed. If problem posing is to be seen as a mathematically significant activity for the pupils, it needs to be connected to other mathematical activities in the classroom. In particular, if problem posing is to be seen as an integral part of modelling, then the pupils should at times model problems they have posed.
5. **Incorporating the teaching of mathematical content with problem posing and mathematical modelling**. In this example, the student teachers wanted the mathematical content to be connected to scatter plots and linear regression. Two main difficulties can be associated with this: First, this intent had not been communicated to the pupils in the problem-posing lesson, and second, there is an inherent difficulty in posing problems to an unknown or little-known mathematical topic, especially in posing problems where the topic is to be connected to a specific real-world situation.

Why Mathematical Modelling and Problem Posing?

Although some research has been conducted on problem posing in mathematical modelling (e.g., Bonotto, 2010, 2011; English, 2010; English, Fox, & Watters, 2005), in general the topic of problem posing has tended to be peripheral in mathematical modelling research. For example, Goldstein and Pratt (2001) remarked that problem posing "falls outside the classical modelling cycle [of mathematization, transformation, interpretation and validation]" (p. 49). There are several remarks in the literature pointing to the importance of the problem in modelling, to reformulations of problems and to asking appropriate questions throughout the modelling process, although these are mostly incidental. In particular, Ottesen (2002) has drawn attention to the influence of working with mathematical modelling on the question of what makes a problem mathematical. Ottesen wrote:

> [Through working with mathematical modelling] students learn to ask certain types of questions that can only be answered by means of mathematics, as well as types of questions that can only be posed by means of mathematics. (p. 344)

This statement was also used by Swan, Turner, Yoon, and Muller (2007) who saw modelling as promoting "the asking and answering of mathematical questions" (p. 281). Mousoulides, Sriraman, and Christou (2007) drew attention to the potential of ongoing problem-posing activities throughout the modelling process:

> During modeling cycles involved in model eliciting activities students are engaged in problem posing, that is, they are repeatedly revising or refining their conception of the given problem. (p. 35)

Problem posing, in a wide sense, appears in multiple guises in modelling: posing and reformulation of the main problem, making of conjectures, and meta-questions (monitoring and control questions related to the mathematics and/or to the modelling process; or questions taking a critical stance to the model and/or its result). This ongoing problem posing throughout the modelling process makes modelling a natural arena for students' problem posing. According to English et al. (2005):

> Modeling activities promote problem posing as well as problem solving primarily because they evoke repeated asking of questions and posing of conjectures. … Given a rich problem situation, such as mathematical modelling, in which generating problems and questions occurs naturally, numerous opportunities abound for learning by both child and teacher. (p. 156 and p. 158)

In professional modelling, we see problem posing as an essential component that initiates the modelling process as well as defining its parameters and goals. Formulation and reformulation of the problem are necessary throughout the whole modelling process. Being able to pose and adjust a problem appropriately for the data and mathematical tools available is a vital part of using mathematics in real-world situations. In particular, this implies that problem posing is important in the experience of authentic modelling processes.

Several reasons have been given for including problem posing (see other chapters in this book) and mathematical modelling (e.g., Kaiser & Sriraman, 2006; Maaß, 2010) in the mathematics classroom. Potentially, having a focus on problem posing while working on mathematical modelling will give the best from both worlds. Our motives for considering problem posing in conjunction with teaching and learning mathematical modelling is related to problem posing being a vital component of experiencing authentic modelling. Problem posing is helpful in understanding the decisions made during modelling (especially with respect to limitations and possibilities offered by mathematical modelling). Problem posing is also seen as a useful experience to help equip pupils for later engagement in modelling outside the school environment. Finally, problem posing can give students increased ownership of their learning environment, since it is a natural component of inquiry-oriented instruction and is grounded in the belief of giving priority to the question over the answer.

The Priority of the Question

One of the reasons for our interest in problem posing as a topic in mathematics education is the priority of the question over the answer (Hana, 2012). It is questions that drive our search for knowledge, not answers. The problems we engage in determine what knowledge and understanding it is possible to reach. To investigate or explore, there needs to be something to investigate, some kind of problem which lays the groundwork for the investigative and explorative activity. It may be a vague problem of a general nature; maybe one is only somewhat curious about a phenomenon; it may be a specific closed problem. In any case, the problem is there and gives us a goal and a lens through which we make and interpret our inquiries. Popper (1963) expressed this as "It is the problem which challenges us to learn; to advance our knowledge; to experiment; and to observe" (p. 301).

Likewise, several authors have stressed the connection between understanding and the underlying problem which forms our quest for understanding. As Gadamer (2004) wrote: "To understand meaning is to understand it as the answer to a question" (p. 368). This is not to say that answering a question always leads to understanding:

> Understanding starts with a question; not any question, but a real question. … [A] real question expresses a desire to understand. This desire is what moves the questioner to pursue the question until an answer has been made. (Bettencourt, cited in Wells, 2000, p. 64)

The importance of the question for the type of understanding one achieves is well illustrated by Collingwood (1939):

> Experience soon taught me that under these laboratory conditions one found out nothing at all except in answer to a question; and not a vague question either, but a definite one. That when one dug saying merely, 'Let us see what there is here,' one learnt nothing, except casually in so far as casual questions arose in one's mind while digging: 'Is that black stuff peat or occupation soil? Is that a potsherd under your foot? Are those loose stones a ruined wall?' That what one learnt depended not merely on what turned up in one's trenches but also on what questions one was asking: so that a man who was asking questions of one kind learnt one kind of thing from a piece of digging which to another man revealed something different, to a third something illusory, and to a fourth nothing at all. (pp. 24–25)

To take into account the priority of the question over the answer has significant pedagogical consequences. It implies that the goal of education should shift from pupils being able to answer question to pupils also being able to pose questions. To pose real problems is in general at least as difficult as answering them, for to pose one needs to know what one wants to know and, in particular, one needs knowledge of what one does not know (cf. Gadamer, 2004). It is an educational goal to educate citizens who can use and develop mathematics through posing problems which enables them to act and to further their understanding of the world we live in.

Problem Posing and Different Perspectives on Modelling

Within the mathematics education research community, the topic of mathematical modelling has been considered from different perspectives (Barbosa, 2006; Kaiser & Sriraman, 2006). Barbosa (2006), extending Julie (2002), considered three different perspectives: "modelling as content" (modelling competencies and modelling processes are themselves seen to be part of school mathematics); "modelling as vehicle" (modelling is seen as a vehicle for learning and teaching mathematical concepts and procedures); and "modelling as critic" (modelling is seen as essential for critical reflection of mathematics in society). Though rather coarse, we have previously found the classification of Barbosa (2006) to be a useful tool in discussing with student teachers how one's perspective on modelling affects one's implementation of modelling in the classroom and the type of learning one intends to achieve (Hansen & Hana, 2012). In relation to problem posing, we noted that the difficulties observed when student teachers posed modelling tasks were in part due to their perspective on modelling.

Modelling as Content

From the perspective of *modelling as content*, insight into models and the modelling process is seen in itself as a legitimate goal for mathematics teaching (see, for example, the overview of modelling competencies given in Maaß, 2006). This may include the study of mathematical models without the requirement that the models necessarily have to include specific mathematical concepts or techniques. Problem posing within this perspective includes posing problems from a real-world situation (see section "Problem Posing and the Modelling Process" for more details).

Modelling as Vehicle

Another perspective is to consider modelling as a vehicle for learning mathematical content. The aim is not to construct a mathematical model, but rather to use models as a tool to learn about mathematical themes, techniques, procedures, and concepts. Within this perspective modelling is used for both the development and application of mathematical content. If one wants to apply already known, or at least partially known, mathematical content, then it seems possible to ask students to pose problems within a real-world situation, where the specific mathematics content is applicable. If the aim is to develop new mathematical content, there is a serious obstacle in posing problems related to unknown mathematics. This comment is mainly related to posing and reformulation of the main problem. Making conjectures and posing meta-questions should still be manageable in this situation.

Modelling as Critic

Here, one wishes to “create situations in which students are able to identify, interpret, evaluate and critique the mathematics embedded in social and political systems and claims” (Mousoulides et al., 2007, p. 25). A necessary skill then is to be able to pose the questions needed to identify, interpret, evaluate, and critique. In many cases, the mathematical models used are mathematically sophisticated and involve mathematics that would not be accessible to the ordinary citizen. By posing relevant questions, such as questions pertaining to the assumptions and simplifications made in the model, or to the uncertainty of the model, one may be able to engage in meaningful discussion about the models on a meta level.

Problem Posing and the Modelling Process

There have been many descriptions of the modelling process. Here, we follow Galbraith and Stillman (2006). They considered the following transitions as key in the modelling process:

1. From messy real-world situation to real-world problem statement
2. From real-world problem statement to mathematical model
3. From mathematical model to mathematical solution
4. From mathematical solution to real-world meaning of solution
5. From real-world meaning of solution to revising model or accepting solution (p. 144)

It is clear that the first transition is one involving problem posing. Galbraith and Stillman (2006) identified this stage as consisting of clarifying the context of the problem, making simplifying assumptions, identifying strategic entities and specifying the correct elements of strategic entities. In educational modelling contexts, one often starts with the modelling problem, giving little or no attention to its creation. This removes valuable experiences related to modelling assumptions and specifications from students.

The second transition also involves problem solving. This transition hinges on being able to reformulate a real-world problem as a mathematically manageable problem. Problem posing in this transition is then about mathematizing problems through refinement and reformulation. This requires an understanding of the real-world problem and the possible ways it can be mathematized. It also involves control questions about whether the mathematization makes sense from a real-world perspective. Crouch and Haines (2004) pointed out that students often have problems in making transitions between the real world and the mathematical model, indicating that this transition requires more attention in educational research. Sometimes, students and teachers jump directly to the mathematical model, not paying attention to translation processes.

The third transition is within a purely mathematical content area, although it also includes asking control questions about whether the mathematical operations and techniques used are applicable in the real-world situation.

The fourth transition is one of demathematization. If the problem has been revised during the mathematical stages of the modelling process, this includes dematheматizing the problem and comparing it with the original problem.

In the fifth transition, a necessary skill is being able to pose critical questions to analyze the model and solution.

Implementation of Problem Posing in Mathematical Modelling

Modelling Through Conjecturing

A proposed method of problem posing is for students to state conjectures pertaining to a real-world situation that are to be critically examined and refined in attempts to validate or falsify them. To make a conjecture is to move outside the obvious and to test the limits of one's knowledge. As such, conjecturing is a natural way to increase understanding and knowledge of a problem area. To conjecture is to ask "What if…?" and to sharpen one's inquiry toward a concrete statement. In particular, the concreteness of conjectures lessens the chance of one posing vague questions. Furthermore, in trying to validate or falsify a conjecture one necessarily has to pose the types of critical questions which are essential in the verification and examination of mathematical models. When working with conjectures, the focus is automatically moved to reasoning for and against the conjecture, in contrast to the type of problems where students are only interested in finding a numerical answer before moving on to the next question.

As an illustration of using conjectures in modelling, we have provided an example of three student teachers in their practice teaching. The example is related to the "modelling as critic" perspective and is concerned with making dubious conjectures. In general, we see it as beneficial for pupils to engage in making authentic conjectures for which they really wish to determine the validity, but this exercise of making dubious conjectures also seemed to engage the pupils in mathematics in positive ways.

The student teachers wanted to let the eighth grade class experience being critical of the mathematics to which pupils are exposed in society. They decided to implement this by encouraging pupils to make conjectures indicating unusual views or arguments pertaining to real-world situations of the pupils' own choosing. These conjectures were to be presented to the rest of the class, together with some sort of mathematical data and statistical model that supported the claimed conjecture. It was expected that in the ensuing discussion of the conjectures that their fellow pupils would make many critical comments, especially since the pupils were invited to make conjectures that could rather easily be attacked. Through critically evaluating the statistical models it was hoped that the pupils would gain insight into some types of critical questions pertaining to mathematical models and to engage in mathematical reasoning.

One group of students chose the conjecture “The local football team Brann Bergen will beat Barcelona.” To support this conjecture, the students used an argument based on the number of goals the two teams scored. This conjecture resulted in a lively discussion in the classroom, where critical comments played an important role. The pupils were invited by the student teachers to dwell on questions such as “What is it that makes this diagram/argument so convincing/misleading?”

The student teachers’ decision to use obviously dubious conjectures about different real-world scenarios seemed to activate the pupils’ critical engagement in a positive manner. The phases where pupils were “inventing” mathematical models supporting the conjectures and when they presented and compared their conjectures seemed to inspire the pupils to pose critical questions relating to the validity of the conjectures and the mathematical models used.

Pupils as Consultants and Clients

The type of task used in mathematical modelling is one that calls for a mathematical model to be used by an identified client (Mousoulides, 2009). This type of task is intended to give pupils a justification for describing their thinking and considering different possible solutions. In a school–industry partnership where a class collaborated with an oil-valve company, and the pupils were given an authentic consultancy task from the company, we have witnessed firsthand some of the potential inherent in pupils taking on such a role while working on mathematical modelling (Hana, Hansen, Johnsen-Høines, Lilland, & Rangnes, 2011; Lilland, 2012). An important aspect of the activity was that, effectively, the pupils had to define the problems themselves, and that they needed to communicate with the company in order to define the task and gather additional data.

In a similar fashion, Crespo and Sinclair (2008) wrote “there is evidence to suggest that school students are able to generate less narrow and familiar types of problems … when they are invited to pose problems to an audience outside the classroom” (p. 396). In the example above, they were writing about an audience outside the classroom, and it seemed that the essential component was that the students experienced a genuine sense of purpose with the problem-posing task.

We propose combining these two strands of research—mathematical modelling and problem posing. A way to implement this would be to divide a class into groups such that every group has a dual role as a fictional company employing another group as consultants and as consultants to another group’s company. As a fictional company, the groups would set the scene and pose a problem for the group of consultants to work on, within some parameters defined by the teacher. As a consultant, the group would work on and refine the problem given by the company group. During this stage, we would envisage that communication between the groups would be essential so that the problem could be refined and data obtained to help refine conclusions. To conclude the activity, the company groups would critically evaluate the solutions found by the consultancy groups.

Conclusion

Problem posing as a pedagogical tool and as an integral part of mathematical modelling has not yet been systematically investigated. In this chapter, we have sketched some of the different ways problem posing offers opportunities and challenges to mathematical modelling. Further work is needed in this area, especially with respect to implementation in the classroom. All in all, we see some golden opportunities in combining problem posing and mathematical modelling in school contexts.

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