Chapter 19
Problem-Posing Activities in a Dynamic Geometry Environment: When and How

Ilana Lavy

Abstract In this chapter, results obtained from previous studies on the issue of problem posing in a dynamic software environment using the "what if not?" strategy are presented. These results include outcomes received from prospective teachers' engagement in problem-posing activities both in plane and solid geometry, and outcomes received by the engagement of the researcher in the problem-posing activity. The above-presented results are followed by discussion and a list of implications for instruction. Problem-posing activities should follow activities of problem solving through which the content knowledge of the learnt topic is built. Students should experience problem-posing activities starting at elementary school. In these activities they should be provided with opportunities to develop cognitive processes needed for problem posing such as filtering, comprehending, translating, and editing. When students are exposed to geometrical objects, they should be provided with the option to make sense of the objects via dynamic geometry software.

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Introduction

Many researchers have discussed the importance of incorporating problem-posing activities into mathematics lessons and have emphasized the benefits students might gain from such activities (Cunningham, 2004; English, 1997). However, little attention has been paid to the question of the stage of development at which such activities should be incorporated into mathematics lessons. Relying on Mestre (2002) who asserted that problem posing is intellectually more demanding than problem solving and on my experience in research concerning problem posing both in plane and solid geometry (Lavy & Bershadsky, 2003; Lavy & Shriki, 2010; Shriki & Lavy, 2012), I believe that problem-posing activities are more efficient after students have gained some experience in problem solving. Engagement in problem-posing activities challenges both the meta- and the actual knowledge students have about the learning materials (Lavy & Shriki, 2010). Hence, one should gain first the required knowledge about the learning materials. To be able to think "out of the box" and "produce" meaningful new problems, one has to develop problem-posing skills. Meta-knowledge is essential for the process of problem posing since in this process one has to be able to judge whether the new created problem is mathematically valid. Engagement in problem posing without having the sufficient meta- and actual knowledge of the examined topic may result in poor outcomes (Cemalettin, Tuğrul, Tuğba, & Kıymet, 2011).

Problem posing can be done in an arbitrary or in a structured manner. One of the structured ways to pose new problems is the "what if not" (WIN) strategy (Brown & Walter, 1993). This strategy is based on the idea that each of the attributes of a given problem (the base problem) can be negated and replaced by alternative one—an action that can yield in a new problem situation. The above process can be perceived as a technical one, but in order to yield a valid new problem, students have to think carefully about the alternative suggestion by recalling the attributes of the given, and by considering the relationship between the original problem and the "new" posed problem. Through such considerations, a student's understanding of the problem-posing process may be deepened.

Engagement in problem-posing activities in dynamic geometry environments becomes richer and more useful when technology is involved. The software frees students from the technical work involving computing and graphing, enabling them to invest more efforts in the inquiry process (Ranasinghe & Leisher, 2009).

In order to be able to create meaningful and useful problem-posing activities for their students, prospective and in-service teachers need to develop their own self-confidence regarding their ability to handle such activities successfully. Developing this self-confidence can be achieved by appropriate training in which prospective and in-service teachers can themselves experience various problem-posing activities as students. Studies which discuss problem posing activities for prospective mathematics teachers in dynamic geometry environments include, for example, those by Lavy and Bershadsky (2003), Lavy and Shriki (2010), and Shriki and Lavy (2012); my own experiences on problem posing have been presented in Lavy and Shriki (2009).

Theoretical Background

In this section a brief literature survey is presented in the following related areas: the role of problem posing in students' mathematics education; the role of problem posing in mathematics teacher's education; problem posing activities in a dynamic computerized environment and the "what if not?" strategy.

The Role of Problem Posing in Students' Mathematics Education

In many cases, during their study of mathematics at school, students experience mainly problem solving. Researchers in mathematics education have emphasized the importance of integrating activities of problem posing and have suggested the incorporation of such activities in school mathematics (Brown & Walter, 1983; Ellerton, 1986; Goldenberg, 1993; Leung & Silver, 1997; Mason, 2000; NCTM, 2000; Silver, 1994; Silver, Mamona-Downs, Leung, & Kenney, 1996). The importance of an ability to pose significant problems was recognized by Einstein and Infeld (1938), who wrote:

> The formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skills. To raise new questions, new possibilities, to regard old questions from a new angle, require creative imagination and marks real advance in science. (Quoted in Ellerton and Clarkson, 1996, p. 1010).

Engagement in problem-posing activities can result in students becoming enterprising, creative, and active learners. They have the opportunity to navigate the problems they pose to their domains of interest according to their cognitive abilities (Mason, 2000) and improve their reasoning and reflection skills (Cunningham, 2004).

The importance of incorporating problem-posing activities in mathematics lessons is also supported by the National Council of Teachers of Mathematics (NCTM) in the United States of America (NCTM, 2000) who recommended that students should make and investigate mathematical conjectures and learn how to generalize and extend problems by posing follow-up questions.

Problem posing can also promote a spirit of curiosity and create diverse and flexible thinking (English, 1997). Studies have shown that problem posing might reduce common mathematics fears and anxieties (Brown & Walter, 1993; English, 1997; Moses, Bjork, & Goldenberg, 1990; Silver, 1994). The inclusion of problem-posing activities might improve students' attitudes toward mathematics, reduce erroneous views about the nature of mathematics, and help to encourage students to be more responsible for their own learning. Problem posing can also help to broaden students' perception of mathematics and enrich and consolidate their knowledge of basic concepts (Brown & Walter, 1993; English, 1997; Silver et al., 1996).

Engagement in problem posing may help students to "reason by analogy" when presented with similar questions (English, 1997) and may help them reduce their dependency on their teachers and textbooks and provide them with a sense of responsibility for their own education. By providing students with the opportunity to formulate new problems, the sense of ownership that they need in order to construct their own knowledge is fostered (Cunningham, 2004). This ownership of the problems can result in a high level of engagement and curiosity, as well as enthusiasm towards the process of learning mathematics.

In the process of problem posing, students might end with new problem situations whose mathematical validity has to be checked. For that matter, students need to rethink the mathematical relationships between the concepts involved and, as a result, they might develop and deepen their mathematical and meta-mathematical knowledge. Examining possible links between problem posing and mathematical competence, Mestre (2002) asserted that problem posing can be used to study the transfer of concepts across contexts and to identify students' knowledge, reasoning, and conceptual development.

Researchers have emphasized the inverse process in which the development of problem-solving skills can be helpful in developing problem-posing skills (Brown & Walter, 1993; English, 1997; Skinner, 1990). Research conducted by Philippou, Charalambous, and Christou (2001) have revealed that their study participants realized the importance of developing problem-posing competencies. The participants considered problem posing as harder than problem solving and valued problem posing as the ultimate goal of mathematics learning. However, there are few didactical tools and activities for developing students' skills in problem posing (Yevdokimov, 2005).

Silver (1994) classified problem posing according to whether it takes place before, during, or after problem solving. He argued that problem posing could take place prior to problem solving when problems are being generated as a reaction to a given stimulus such as a picture, a diagram, or a story; during the process of problem solving when students are asked to change the goals and conditions of a problem, or after solving a problem when experiences from the problem-solving context are applied to new situations. Four main cognitive processes are involved in the process of problem posing: filtering (e.g., posing a problem that its answer is 325 sticks); translating (e.g., write a problem based on a given diagram); comprehending (e.g., write an appropriate problem for: $(150-70)+14=x$); and editing (e.g., write an appropriate problem based on a given picture) (Pittalis, Christou, Mousoulides, & Pitta-Pantazi, 2004).

The Role of Problem Posing in Mathematics Education

Teachers have an important role in the incorporation of problem-posing activities into the mathematics lessons (Gonzales, 1996). Nevertheless, although problem posing is recognized as an important teaching method, many students are not provided with the opportunity to engage in problem-posing activities while studying mathematics (Silver et al., 1996). In many cases, teachers tend to emphasize skills, rules, and procedures, which become the essence of learning, instead of focusing on instruments for developing understanding and reasoning (Ernest, 1991). Consequently, mathematics teachers fail to take advantage of the opportunity both to support their students in developing problem-solving skills and to help them build/acquire the required confidence in managing unfamiliar mathematics situations. Teachers rarely use problem posing because they find it difficult to implement in classrooms and because they themselves do not possess the required confidence and skills (Tichá & Hošpesová, 2013; Leung & Silver, 1997). Contreras and Martinez-Cruz (1999) also found that prospective teachers' problem-posing abilities are often underdeveloped, and they should be encouraged to develop their own problem-posing skills (Leung & Silver, 1997; Silver et al., 1996; Southwell, 1998). These skills will enable them to create tasks that include opportunities for their students to be engaged in problem posing (Gonzales, 1996). Southwell (1998) found that posing problems based on given problems could be a useful strategy for developing the problem-solving ability of preservice mathematics teachers. Integrating problem-posing activities in their mathematics lessons enabled preservice teachers to become better acquainted with their own students' mathematical knowledge and understanding. In order to address some of the concerns noted in the literature, it is important that problem-posing activities are included in teacher education programs for prospective teachers.

Problem posing can be integrated in various settings. Crespo and Sinclair (2008) found that engaging prospective teachers in exploratory mathematical activities improved both the range and quality of problems they posed. These authors claimed that such engagement in exploration work enabled the prospective teachers to pose problems that were both interesting and challenging even to them.

Problem-Posing Activities in a Dynamic Computerized Environment

Problem-posing activities become richer and more profound when technology is involved, since the technical work involving computing and graphing is executed by the software more rapidly and efficiently (Ranasinghe & Leisher, 2009). One of the distinctive features of dynamic geometry software (DGS) is the facility to construct geometrical objects and specify relationships between them. Within the DGS, geometrical objects created on the screen can be manipulated, moved, and reshaped interactively (Christou, Mousoulides, Pittalis, & Pitta-Pantazi, 2005). Hence, when working in interactive computerized environments, students can do mathematics differently (Aviram, 2001) and in ways that they could not do with paper and pencil. Their interaction with dynamic geometry software enables students to focus on courses of inquiry without investing time and effort on calculating and drawing, which one cannot avoid while working with paper and pencil. The computerized environment includes tools that mediate students' actions and bridges between the students and the mathematical world (Artigue, 2002). Moreover, dynamic geometry software enables students to represent situations visually and therefore to identify patterns (McKenzie, 2009). Problem posing using computerized environments provides teachers with research-like skills in the development of instructional materials for school mathematics (Abramovich & Cho, 2006).

Problem posing using dynamic geometry software involves unique interactions between the software's interface and the students' actions and understandings. The students are provided with the opportunity to utilize visual reasoning in mathematics, helping them through the dragging facilities, and can help them to generalize problems and relationships, or to examine the validity of a new problem situation (Sinclair, 2004). The exploration techniques—tools, definitions, and visual representations associated with dynamic geometry—contribute to the construction of rich learning environments (Laborde, 1998). Two systems are involved in this interaction between the students and the software: the first system involves the students attempting to pose a problem, and the second system involves the environment, which provides opportunities for students to act and react (Brousseau, 1997).

The "What If Not?" Strategy

Posing new problems can be based on free, semi-structured, and structured situations (Stoyanova, 1998). A *free* problem-posing situation refers to the case in which the student has a free hand in formulating new problems. A *semi-structured* problem-posing activity relates to an open situation in which the student is asked first to explore its structure and complete it, and then to pose new problems. A *structured*

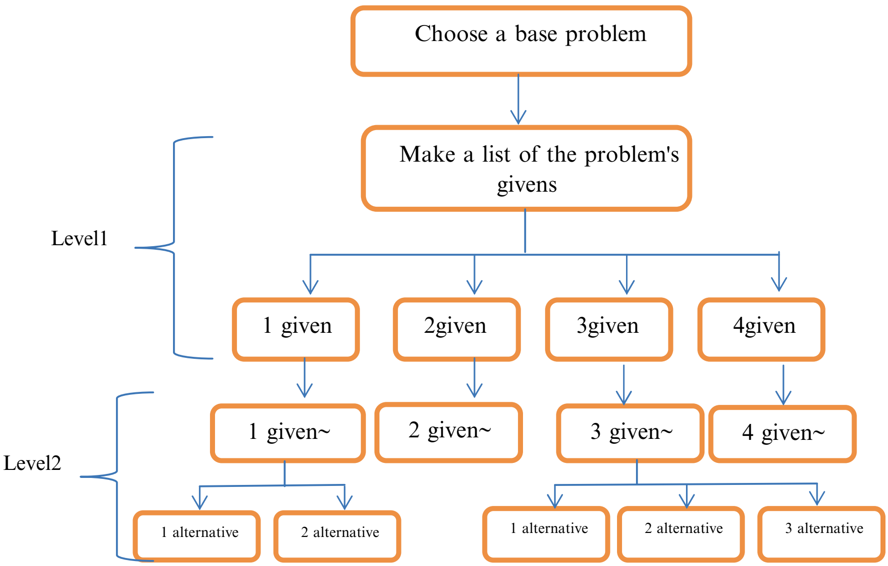


Figure 19.1. Schematic description of the WIN strategy.

problem-posing situation refers to the case in which the learner is asked to suggest new problems that rely on a given base problem.

The “what if not?” (WIN) strategy (Brown & Walter, 1983, 1993) is an example of a structured tool for problem posing. According to WIN strategy, each component of the problem data and the problem question is examined and manipulated through the process of negating one of the base problem’s givens.

In fact, the strategy consisted of two levels: level 1 and level 2 (Figure 19.1). In level 1, one has to list all of the problem’s givens including the question of the problem, and in level 2, one has to negate each of the listed givens by asking “what if not given *k*?” Then she has to make a list of alternatives to the negated given. Part of the offered alternatives results in a new problem situation. Implementing the WIN strategy enables teachers and students to move away from a rigid teaching format that makes students believe that there is only one “right way” to refer to a given problem. Using this problem-posing strategy provides students with the opportunity to discuss a wide range of ideas and to consider the meaning of the problem rather than merely focusing on finding its solution (Brown & Walter, 1993).

Results

In this section, a brief summary of the results obtained from previous studies is presented. These results refer to problem posing done by prospective teachers and to problem posing done by the researcher. The purpose of this comparison is to emphasize that the main difference in the process of problem posing between ones who had not previously experienced problem posing (in this case, the prospective teachers) and those who did (in this case, the researcher) stemmed from a lack of self-confidence in their own mathematical ability. Since the activities in which the prospective teachers were engaged involved mathematical subjects that they were proficient in, it can be assumed that the source of their difficulties stemmed from a lack of confidence in their ability to perform such tasks. Therefore, to enable students to build self-confidence in their ability to perform such tasks, there should be a frequent engagement with problem-posing activities.

Prospective Teachers’ Engagement in Problem Posing

I was involved in several studies in which prospective teachers (PTs) had to pose problems using the WIN strategy (Lavy & Bershadsky, 2003; Lavy & Shriki, 2010; Shriki & Lavy, 2012). In Lavy and Bershadsky (2003) the PTs had to pose problems in solid geometry, while in Lavy and Shriki (2010) they had to pose problems in plane geometry. In Lavy and Bershadsky (2003) the following results were obtained: the majority of the PTs changed one of the givens of the base problem and only a few of them changed the question of the base problem. In the case where the given of the base problem was numerical, most of the PTs suggested another numerical value which was very close to the original one. In the case where the given was a geometrical shape such as a right triangle, most of the PTs suggested to replace it with another shape from the same group of shapes (e.g., from right to isosceles triangle). Although some of the PTs suggested replacing one of the givens of the base problem by a generalization of it, for example, instead of a height of 10 cm they suggested *h* cm, none of them chose to explore this new problem situation. These findings are in line with those of Tichá and Hošpesová (2013) who found that many preservice and inservice teachers tended to regard problem posing as a very unusual activity. Some of them encountered difficulties in coping with such activities, feeling that it was beyond their capabilities.

For the PTs to be able to suggest the above-mentioned alternatives, they applied the cognitive processes of editing, filtering, comprehending, and translating quantitative information. Data obtained from PTs in the Lavy and Bershadsky (2003) study for Problem 1 (Figure 19.2) and Problem 2 (Figure 19.3) are summarized in

What is the distance from the center of the base of a regular triangular pyramid to the pyramid's lateral face given that the pyramid's height is 10 cm and the dihedral angle (the angle between the two faces) is 67^0.

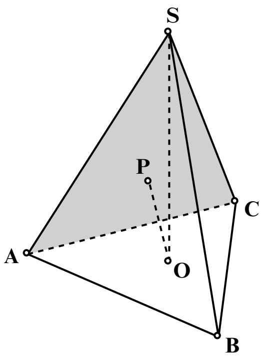
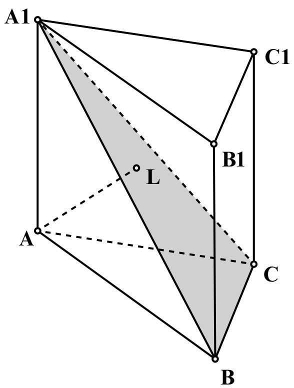


Figure 19.2. Problem 1.



In a regular right prism $ABCA_1B_1C_1$ with isosceles triangular base (AB=BC), AB=5 cm and $\angle ABC=40^0$. The prism height is 8.2 cm. What is the distance between A and the plane A_1BC?

Figure 19.3. Problem 2.

Table 19.1
Distribution of Posed Problems and the Cognitive Processes Involved

Main category				Problem 1 (18 prospective teachers)	Problem 2 (10 prospective teachers)	Cognitive processes
Changing one aspect of the problem's data	Changing of the numerical value of data	Another specific value		6	12	F
		A range of values		4	2	FC
		Negation		2	–	F
		Generalization	Implicit	1	–	FC
			Formal	0	4	FC
	Changing of the data kind	Another specific data kind		15	26	FCE
		Negation		4	1	
		Generalization	Implicit	3	9	FCE
			Formal	1	3	FCE
	Eliminating of one of the problem's data			–	5	F
Changing of the problem question	Another specific question			6	3	FTCE
	Inverting of the given problem into a proof problem			1	–	FTCE
Total				43	65	
Average number of posed problems per one PT				2.4	6.5	

Table 19.1. The PTs' suggestions for changing the problems were categorized according to the four cognitive processes (Table 19.1).

The first five columns in Table 19.1 have been taken from page 377 in Lavy and Bershadsky (2003). A sixth column was added on the right which refers to the cognitive process(es) applied by the PTs while posing new problems. The letters appearing in the sixth column are abbreviations: "E" stands for editing; "F" for filtering; "C" for comprehending; and "T" for translating.

Before discussing the cognitive processes applied by the PTs while posing problems using the WIN strategy, it should be mentioned that there were two sessions of posing problems. In the first session, 18 PTs had to pose new problems while Problem 1 served as a base problem. In this case the PTs were asked not to solve the base problem and only to pose as many problems as they could, based on the given problem. In the second session, ten PTs were asked to solve Problem 2 first, and only then were they asked to pose as many problems as they could, based on the given problem.

Interpretation of Table 19.1 reveals that the average number of posed problems per PT increased from 2.4 to 6.5 (from Problem 1 to Problem 2). This increase may be attributed to the different situations involved in obtaining the two sets of posed problems. The fact that the PTs had to solve the base problem first (in the case of Problem 2) appears to have had a significant impact on the number of problems they were able to pose. While the PTs attempting to solve the base problem, they had to recall the relevant attributes of the geometrical shapes involved, and they had to examine the interrelations between the givens of the base problem. As a result, they could develop some understanding about the various possible modifications that might be applied to the base problem in order to yield new problems.

In what follows I refer to the cognitive processes applied by the PTs in the process of posing problems (sixth column). All of the following examples relate to the first session (Problem 1). Using the WIN strategy, the PTs changed either one of the base problem's givens or the base problem's question. In the case where the base problem included numerical givens, the PTs changed it to another value which was close to the original one. For example: "Change the pyramid height from 10 into 12 cm." In this case it can be said that the cognitive process applied is filtering since they had to choose certain values that would fit the question of the problem which remained the same. However, by changing a numerical given to a range of values, for example: "Change the angle between the lateral faces from 67° into an angle between 67° and 90°," in addition to filtering, the PTs had to think of possible values that could be suggested to replace the given one and yet end with a mathematically valid problem. The dragging facility provided by the dynamic geometry software also enabled the PTs to verify whether their suggestions yield mathematically valid problems or not.

When PTs changed the data type for one of the base problem givens—for example: "Change from a triangular base pyramid to a square base pyramid" (Problem 1), the PTs applied filtering, comprehending, and editing. In this case the PTs had to draw a new sketch of the problem which was completely different from the sketch of the base problem, while at the same time, they did not change the problem's question. To suggest such a given, the PTs had to comprehend the interrelationships between the problem's givens and decide whether such a suggestion could yield a mathematically valid new problem.

All four cognitive processes (filtering, comprehending, editing, and translating) were involved when PTs changed the base problem's question as in this example: "Find the pyramid base area" or: "Prove that sin $\alpha/2 = 5/8$ while the relation between a lateral edge of the regular triangle pyramid to the base edge is 5/9." By leaving the givens of the base problem untouched, they had to filter the possible questions that could be asked to yield a mathematically valid problem. Moreover, to be able to pose a reasonable new question for the given situation, PTs had to demonstrate comprehension of the geometrical shapes involved and their attributes, and they also had to understand the interrelationships between the problem's givens. In changing the question of the base problem, the PTs applied the cognitive process of translating in which they had to write a problem based on a given situation which is composed of certain geometrical shapes (e.g., triangular pyramid) and givens (e.g., pyramid height of 10 cm). Also a process of editing was applied since in this process one has to write an appropriate problem based on a given sketch, and since the givens of the base problem were not changed, the sketch of the base problem remained the same.

Similar results were also reported by Lavy and Shriki (2010). After posing problems using the WIN strategy, the PTs had to choose one of the new posed problems and provide its solution. Most of them chose a problem with a trivial change.

Although the students were familiar with the examined topics, they chose not to challenge themselves with intriguing situations. By making minor change in one of the base problem's givens, the PTs avoided the need to examine the correctness and validity of the new posed problem. This phenomenon can be attributed to their insufficient experience with problem-posing activities. These results are in line with those reported by Cemalettin et al. (2011) who found that prospective teachers' success in problem posing was low. Effective engagement in problem posing necessitates a profound examination of the definitions of the mathematical objects and their interrelationships. To avoid such an engagement, the PTs chose to suggest alternatives which minimized the need to probe the attributes of, and interrelationships between, the mathematical objects involved. Mason (2000) asserted that providing students with the opportunity to pose problems enabled them to navigate the problems they posed to their domains of interest according to their cognitive abilities. However, the results obtained in the above studies revealed that the PTs did not necessarily focus on what they found to be interesting, nor did they always utilize their cognitive abilities in full. Observations of the PTs' initial stages of inquiry (which they soon discarded) suggest that they had the opportunity to develop their mathematical knowledge far beyond what actually occurred. The fact that they overemphasized the need to provide solutions to the new posed problems prevented them from exploring less common shapes and unfamiliar situations.

The Researcher's Engagement in Problem Posing

Before a colleague and I decided to engage our PTs in problem-posing activities, each of us decided to experience this process first. We chose the following to be the base problem for our investigation: *The three medians of a triangle divide it into 6 triangles possessing the same area*. By using the WIN strategy and dynamic geometry software, we experienced a fascinating process and ended with some interesting new insights. Starting from the base problem, we negated the number of divisions of the triangle sides by raising the question: What if each of the triangle sides will be divided into three instead of two segments?

The division of each of the triangle sides into three equal segments created a new posed problem including four triangles and three quadrangles inside the given triangle (Figure 19.4).

Based on measurements taken by means of dynamic geometry software, the following conjectures with respect to the areas and segments were raised:

$$\frac{BK}{KI} = \frac{AL}{LG} = \frac{CJ}{JE} = 6; \quad \frac{S_1}{S_2} = 3$$

$$KJ = JB; \quad LK = KA; \quad JL = LC; \quad S_2 = S_4 = S_6; \quad S_3 = S_5 = S_7$$

In order to investigate and ultimately prove the above conjectures, segments *KD*, *LH,* and *JF* (Figure 19.4) were added to generate triangles *BDK*, *AHL,* and *JCF* and

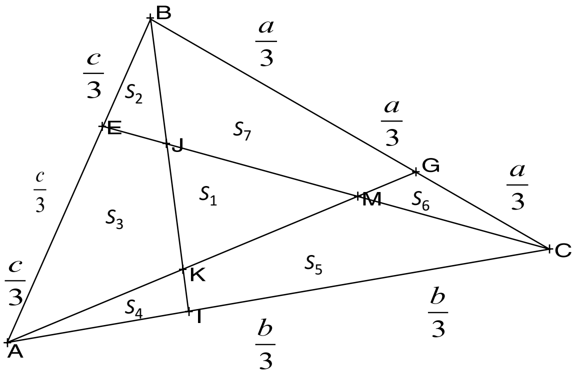


Figure 19.4. Schematic description of the case: $n=3$.

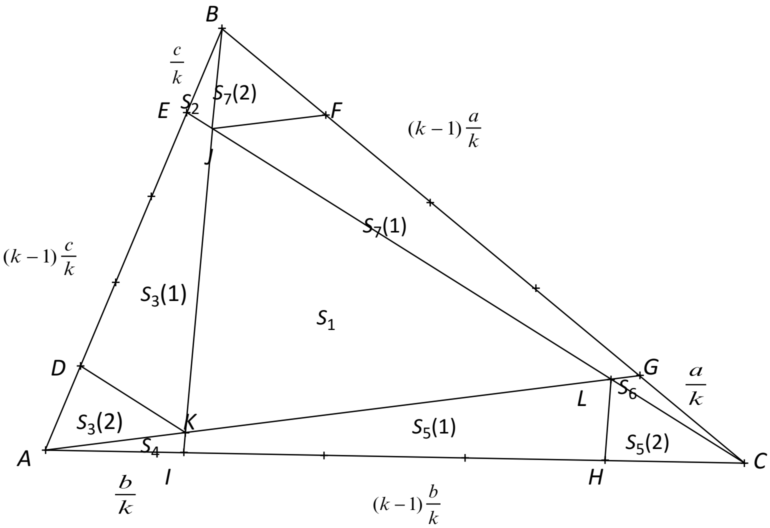


Figure 19.5. Schematic description of the case of $n=k$.

to our surprise we found that: $EJ \parallel DK$; $IK \parallel HL$; $GL \parallel FJ$. Only by using the principles of affine geometry were we able to succeed in proving the parallelism of these segments. Then we examined the general case in which each of the triangle sides is divided into k equal segments (Figure 19.5) and generated the following attributes: $S_2=S_4=S_6$; $S_3=S_5=S_7$; $\frac{S_1}{S_2}=k\left(k-2\right)^2$; $\frac{BK}{KI}=k\left(k-1\right)$; $\frac{JK}{BJ}=k-2$

Finally we examined the case in which each side of the triangle is divided into a different number of equal segments (k-ians) (see Figure 19.6).

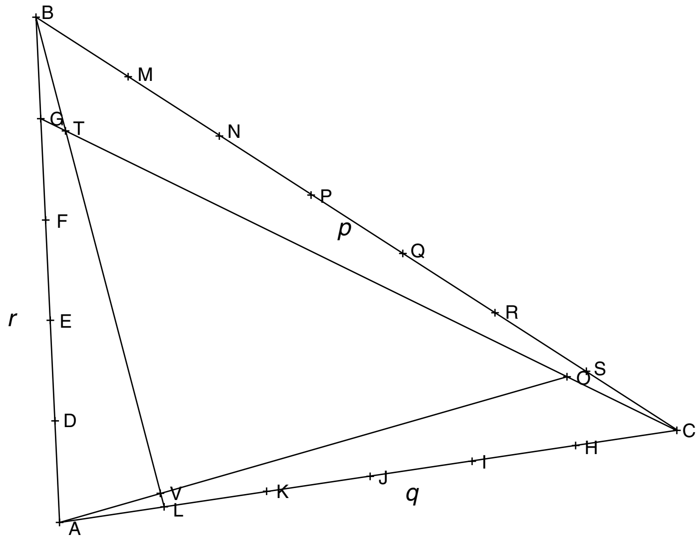


Figure 19.6. Schematic description of the *k*-ians.

In this case we found the following attributes:

$$\frac{BY}{YL}=(p-1)\cdot q;\ \frac{AU}{US}=(r-1)\cdot p;\ \frac{CT}{TG}=(q-1)\cdot r$$

The above-described process demonstrates a sequence of modifications in data from the given base problem that yielded new posed problems with new surprising regularities. Reflection on the above process reveals the potential of applying a sequence of simple modifications to one of the base problem's givens—in this case, yielding supersizing regularities. The question we asked ourselves was: Why do our PTs seem to avoid acting on given problem situations in a similar way?

Discussion and Implications for Instruction

The incorporation of problem-posing activities into the mathematics curriculum is highly recommended by the educational community (NCTM, 2000). Most students, however, are not provided with the opportunity of experiencing problem posing while studying mathematics (Silver et al., 1996; Wilson & Berne, 1999). Hence, when PTs enter teacher education programs, many of them are not yet acquainted with problem-posing activities and when they are exposed to such activities, they refer to them as unusual ones (Tichá & Hošpesová, 2013). PTs should first experience innovative teaching approaches such as problem-posing activities as learners during teacher education programs before they are able to incorporate them effectively in their teaching (Abramovich & Cho, 2006; Crespo & Sinclair, 2008). This is especially true when they encounter unfamiliar approaches which stems from the fact that they were not exposed to them as high school students (Crespo & Sinclair, 2008). If teacher educators wish PTs to implement problem posing in their future classes, they should provide them with opportunities to gain experience in it within various settings. While engaging in various activities associated with problem posing, PTs might become aware of the encompassed cognitive processes involved, discuss and reflect on them, and as a result improve their instruction skills.

My research interest in the issue of problem posing has focused on PTs' engagement in problem-posing activities in geometry. When exposing the PTs to problem-posing activities I noticed that this was often their first exposure to problem posing in mathematics in general and in geometry in particular. Informal conversations with high-school mathematics teachers have confirmed that learners of mathematics in high school rarely engaged in such activities (Lavy & Shriki, 2010). As a result, most university-level students who study to become teachers of mathematics are not familiar with problem-posing activities. My recommendations below relate to PTs who have not had the opportunity to acquire previous experience in such activities. However, in order to create a situation in which PTs will feel most comfortable in acquiring problem-posing knowledge and skills, appropriate activities should be employed earlier—when they are still school students. Considering the results on problem posing by PTs reported in this chapter, I believe that, in order to help students develop problem-posing skills, students should be engaged in problem-posing activities on a regular basis—starting in elementary school.

Problem-posing activities should be planned in a way that they will provide PSTs with the opportunity to apply the cognitive processes of filtering, editing, comprehending, and translating (Pittalis et al., 2004), which are important for the development of problem-posing skills. Teachers should choose various problems relating to the current content topic and initiate problem-posing activities in which the above cognitive processes could be developed. Posing new problems can be based on free, semi-structured, and structured situations (Stoyanova, 1998). Based on my experience, I believe that high school students should be engaged in problem-posing activities in geometry basing on structured situations. Many students find geometry challenging and encounter difficulties when attempting to solve geometrical problems (Gal & Linchevski, 2010; Lin, 2005). Moreover, since many preservice teachers tend to refer to problem posing as a very unusual and complex activity (Tichá & Hošpesová, 2013), I believe that working in a structured situation can make the process of problem posing easier for the PTs. For this reason, I found the WIN strategy (Brown & Walter; 1993) to be useful. Problem posing using the WIN strategy encompasses the four cognitive processes (Pittalis et al., 2004) as was demonstrated in the results section. Changing one of the givens of the base problem or the problem's question can result in a process of filtering, translating, comprehending, or editing. The use of a structured approach to problem posing should provide a gentler transition from problem-solving activities in which students have to cope with valid and solvable problems to problem-posing activities in which new problem situations can be neither mathematically valid nor solvable.

Class discussions on activities involving problem posing are essential for the development of PTs' problem-posing skills (Lavy & Shriki, 2007). Problem-posing activities should be followed by class discussions in which the posed problems can be discussed and guided by the class teacher. In such discussions PTs could reflect on the process they had gone through and ask questions such as: "Does the suggested alternative result in a mathematically valid problem situation?" or "What can be the consequences of changing one of the givens to another geometrical shape?" or "Does the new problem situation include missing/redundant data to solve the new problem?" will be discussed. One of the advantages of class discussions following problem-posing activities is the PTs' exposure to classmates' ideas that they themselves had not thought about. Christou et al. (2005) noted that the discussions which followed a problem-posing activity helped the students to reconsider their generalizations. Before the class discussion the students seemed to over-generalize their solutions, based on particular cases, and they failed to extend the problem to all possible situations. Only after the discussion were the students able to generalize correctly.

One of the important skills PTs have to develop in order to be effectively engaged in problem-posing activities is reflection. Among the means by which reflection skills can be developed are class discussions (McDuffie & Slavit, 2003). Class discussions, in which the participants exchange ideas regarding the attributes and interrelationships of the mathematical objects under examination with other members in class, may stimulate the development of their reflection skills. Each decision students make in the process of problem posing necessitates reflective thoughts regarding the meanings and consequences of such a decision. Cunningham (2004) found that engagement in problem-posing activities improves students' refection skills. Relying on my own experience I believe that a certain degree of reflection skills are needed a priori for engagement in effective problem posing. These skills are essential for probing the attributes and interrelationships of the mathematical object under examination and the possible consequences of replacing any one of these by another.

Frequent engagement in problem-posing activities can contribute to the development of the PTs' self-confidence in their mathematical abilities. This confidence is required especially in cases PTs have to cope with complex situations which may draw upon advanced mathematical topics they have not yet mastered (e.g., affine geometry) in order to investigate a regularity they have discovered. PTs' self-confidence in their mathematical abilities can also help them to develop their ability to think "outside the box" and be free of some traditional constraints. Self-confidence in one's mathematical abilities also applies to meta-knowledge of mathematics. In order to be able to think of possible alternatives to a negated given, one has to be able to probe into the given's attributes and possible interrelationships, as well as understand the possible consequences of suggesting other data with different attributes and different interrelationships. Moreover, PTs' engagement in problem-posing activities can also help them build their self-confidence in their ability to handle problem-posing activities and to manage follow-up class discussions effectively.

Structured and guided activities of problem posing have an important role in shaping PTs' inquiry habits. They need to develop systematic inquiry habits progressing by small steps. Moving forward through a sequence of small changes can help PTs observe whether a mathematical regularity can be unfolded.

From the beginning of students' exposure to geometrical objects, they should be introduced to dynamic geometry environments in which they can create new objects and move and reshape them interactively. The process of problem posing in geometry can be facilitated when using DGS which frees the PTs from technical work and enables them to focus on the inquiry process. The DGS enables the PTs to experiment, observe the stability or instability of phenomena, and state and verify conjectures easily and rapidly (Marrades & Gutiérrez, 2000). The visual aspect provided by the software is also crucially important. By freeing the PTs from technical work which is time-consuming, they are able to invest more efforts into examining interrelationships between the problem's givens, and think of potentially interesting changes. The dragging facilities of the software and the fact that the geometric objects can be easily manipulated and reshaped interactively (Sinclair, 2004) enable the PTs to view on the computer screen a kind of a proof.

Engagement in problem posing necessitates the organization of the PTs' existing knowledge in such a way that they will be able to draw on this knowledge—in not just a technical manner. Problem posing should be implemented in ways that students will be able to make sense of the activity via the cognitive tools already at their disposal. Problem-posing activities should be presented to students in ways that allow them experience a content-related sense of purpose, and that bring them to see the point of extending their existing conceptual knowledge and experiences in fruitful directions.

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