Chapter 17
Problem-Posing/Problem-Solving Dynamics in the Context of a Teaching-Research and Discovery Method

Vrunda Prabhu and Bronislaw Czarnocha

Abstract Problem posing is practiced in the context of an integrated teaching/research methodology which has become known as TR/NYCity methodology (Teaching-Research/New York City methodology) (*Dydaktyka Matematyki,* 2006, 29: 251–272). This approach has been utilized in mathematics classrooms in the New York area for a decade. Problem solving turned out to be an essential teaching strategy for developmental mathematics classrooms of Arithmetic and Algebra, where motivation in learning, interest in mathematics, and the relevance of the subject is unclear to adult learners. Problem posing and problem solving are brought into play together so that moments of understanding occur, and a pattern of these moments of understanding can lead to self-directed discovery, becoming the natural mode of learning. Facilitation of student moments of understanding as manifestations of their creative capacity emerges from classroom teaching-research practice and its relationship with the theory of the act of creation (*The Act of Creation*. 1964. Macmillan) as the integrative element leading to discovery. Discovery returns to the remedial mathematics classroom, jumpstarting reform. This teaching-research report is based on the collaborative teaching experiment (*C3IRG 7 Problem Solving in Remedial Arithmetic: Jumpstart to Reform*. 2010. City University of New York) supported by C^3IRG grant of CUNY.

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Introduction: Posing the General Problem

Enquiry is the path to discovery along which the central problem decomposes into a series of posed questions (see Figure 17.1).

Problem-posing decomposition is the essential link for reaching discovery; its absence derails success by denying access to that discovery. Transformation of the process of enquiry into a series of smaller posed problems generated by the participants allows every student to reach, and to discover, a sought-after solution. Duncker (1945) thought deeply about the psychological processes involved in problem solving, and Silver, Mamona-Downs, Leung, and Kenney (1996) asserted that "problem solving consists of successive reformulations of an initial problem" (p. 294). This view became increasingly common among researchers studying problem solving. Moreover, Brown and Walter (1983), in *The Art of Problem Posing*, posed and answered the question:

> Why, however, would anyone be interested in problem *posing* in the first place? A partial answer is that problem posing can help students to see a standard topic in a sharper light and enable them to acquire a deeper understanding of it as well. It can also encourage the creation of new ideas derived from any given topic—whether a part of the standard curriculum or otherwise (p.169).

The central problem faced by mathematics teachers teaching within an urban community has dimensions that are of both global and local scales. Both ends of the scale can generate the solution of the problem if appropriate questions are posed to reformulate it to the needed precision for the scale at hand. Such a problem is the Achievement Gap. Thus, the central problem addressed in this chapter is how to

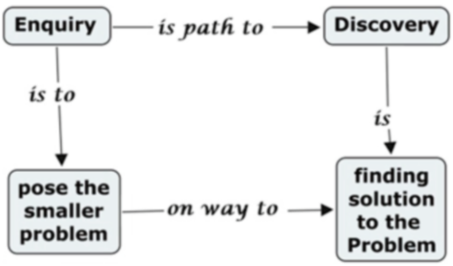

Figure 17.1. Enquiry method of teaching and the decomposition into posed questions/problems.

bridge the Achievement Gap and the role of problem-posing/problem-solving dynamics in this process. Its two scales are, on the one hand, that which drives political machinery: funding initiatives at the National Science Foundation, Department of Education, and other funding agencies, and on the other hand, the situation in a community college mathematics classroom—talent, capacity for deep thinking, yet its clarity disturbed, so grades awarded are not high. The gap for both scales is just a gap; so that the solution to the common posed problem at one end of the scale, of how to fill/bridge/eliminate the gap, can lead to a flow between the local and the national problem, in that the solution at the local scale informs the problem posed at the global national scale. The posed problem has multiple dimensions including:

1. Student voices with the actual classroom difficulties, such as: "what is $-3+5$, why is it not -2," or "why must I take a long answer test, when the final exam is multiple choice," or "why don't you teach, you just make us solve problems";
2. Teachers' voices with the curricular fixes that they think will/has definitely eliminated the gap in their own classroom, of say fractions; and who through that discovery/solved problem, wish to let the secret be available to all students to fix the fraction gap on a broader scale; and
3. Administration obsessed with standardized exams measuring student skills development but not their understanding.

The problems posed by the different constituents are sub-probes to the challenge of closing the Achievement Gap and each of these sub-problems fall into mutually affecting strands. In the classroom, these fall under the categories discussed by Barbatis, Prabhu, and Watson (2012): (a) Cognition; (b) Affect; and (c) Self-Regulated Learning Practices.

In this chapter, we will illustrate our classrooms' problem-posing possibilities. Mathematics is thinking technology through which posing problems, attempting to solve them, and solving them to the extent possible with the available thinking strategies represent the foundational core of the discipline. By repeatedly posing questions to solve the problem in its broad scope, we have discovered that creativity, and in particular, mathematical creativity, can jumpstart remedial reform, thus confirming the assertions of Silver et al. (1996), and Singer, Pelcher, and Voica (2011). Mathematics answers questions—"why?" and "how?" as it uses minimal building blocks on which its edifice is constructed. Thus at any level of the study of mathematics, problem posing and problem solving are inextricable pieces of the endeavor.

TR/NYCity Model is the classroom investigation of students learning conducted simultaneously with teaching by the classroom teacher, whose aim is the improvement of learning in their classroom, and beyond (Czarnocha & Prabhu, 2006). The Teaching-Research, NYCity (TR/NYCity) Model has been used effectively in mathematics classrooms of Bronx Community College and Hostos Community College, the Bronx community colleges of the City University of New York, for more than a decade. The investigation of student learning, as well as related mathematical thinking, necessitates the design of questions and tasks that reveal its nature to the classroom teacher–researcher. That is the original source of problem posing to facilitate student thinking employed by TR/NYCity. This method of teaching naturally connects with the discovery method proposed originally by Dewey and Moore. Utilization of TR/NYCity in conjunction with the discovery method let us, as teacher–researchers, to discover that repeatedly posing questions to students facilitates student creativity, and as such it can jumpstart remedial reform in our classrooms (Czarnocha, Prabhu, Baker, & Dias, 2010). That realization is consistent with the work of Silver et al. (1996), Singer et al. (2011) and others in the field who assert that problem posing is directly related to the facilitation of student creativity.

The *Act of Creation* by Koestler (1964) allows us to extend our understanding of classroom creativity to the methodology of TR/NYCity itself. The *Act of Creation* asserts that bisociation—the moment of creative understanding—is facilitated and can take place only when two or more different frames of discourse or action are present in the activity. Since teaching-research is the integration of two significantly different professional activities, teaching and research, TR/NYCity with its constant probing questions to reveal student thinking presents itself as the natural facilitator of teacher's creativity as well. The TR cycle shown in Figure 17.2 shows the theoretical framework within which problem-posing/problem-solving dynamics as the terrain of student and teacher classroom creativity is being iterated through consecutive semesters. The process of iteration produces new knowledge about learning and problem-posing/problem-solving instructional materials.

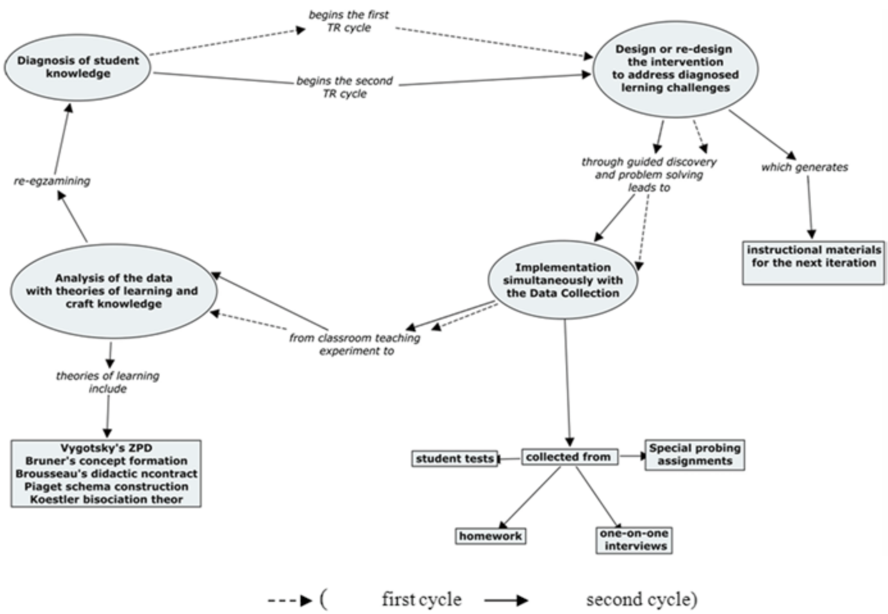

Figure 17.2. Teaching-research cycle with two iterations.

TR cycle iteration is the consecutive run of the investigation or intervention through several subsequent cycles of days, semesters or years. During each semester, student difficulties are cycled over at least twice so that the diagnosed difficulty can be addressed and its success assessed in agreement with the principles of adaptive instruction (Daro, Mosher, & Corcoran, 2011). Over the span of several semesters, the methodology creates an increasing set of materials which are refined over succeeding cycles and acquire characteristics of use to all students studying the mathematical topics under consideration. The learning environment itself develops into a translatable syllabus for the course from several perspectives. Learning environments developed in (TR/NYC) classrooms can be replicated for other instructors facing similar difficulties related to the Achievement Gap in their own classrooms, and for instructors who are interested in becoming teacher–researchers looking for solutions to larger problems in their classrooms.

In classes of Remedial Mathematics (i.e., classes of Arithmetic and Elementary Algebra) at the community college, Teaching-Research Experiments have been carried out since 2006. In the period from 2006 to 2012, success began to be evidenced in 2010 following a broader teaching-research team approach described later in this section.

The initiative in Remedial Mathematics followed the successful use of the methodology in calculus classes under the NSF-ROLE#0126141 award, entitled, Introducing Indivisibles in Calculus Instruction. In the calculus classes (NSF-ROLE#0126141), when the appropriate scaffolding dynamic had been embedded in the Learning Environment, students who were underprepared in, to name the main difficulties, fractions on the line, logic of if-then, algebra of functions and limit (essential for definite integral conception as the limit of the sequence of partial Riemann sums), were nonetheless able to perform at an introductory analysis level (as distinct from the level of standard calculus course). Discovery was the “natural” means of exploration in calculus classes and enquiry leading to discovery through problem-posing/problem-solving dynamics was able to take place without student resistance.

In classes of Remedial Mathematics, however, the situation is markedly different. Student resistance to learning is prompted by years of not succeeding in the subject, and the general attitude is of “just tell me how to do it.” Discovery and enquiry are not welcome means. In the period 2006–2010, development of the mathematical materials was continued, and the learning trajectory for fractions described later in this chapter was also investigated. However, the success was not in student learning. In 2007–2008, as part of a CUNY-funded teaching experiment, Investigating Effectiveness of Fraction Grid, Fraction Domino in mathematics classrooms of community colleges of the Bronx, it was found that a satisfactory student partnership in learning, a didactic contract (Brousseau & Balacheff, 1997) or in classroom language, a mutual “handshake” confirming the commitment to student learning, was essential in confirming the role of problem posing on the affect and self-regulatory learning (Akay & Boz, 2010). In 2010, following a Bronx Community College consultancy to Further Education and Training colleges in

South Africa, a new direction to address the problem was established. The situation in classrooms, whether in South Africa or in the Bronx, needed simultaneous attention to student affect as well as to student learning.

Development of Learning Environments

The relationship between cognitive and affective components of learning has recently received increased attention (see, for example, Araujo et al., 2003; Gomez-Chacon, 2000). According to Goldin (2002) "When individuals are doing mathematics, the affective system is not merely auxiliary to cognition—it is central" (p. 60). Furinghetti and Morselli (2004), in the context of the discussion of mathematical proof, asserted that "the cognitive pathway toward the final proof presents stops, dead ends, impasses, steps forward. The causes of these diversions reside only partially in the domain of cognition; they are also in the domain of the affect" (p. 217). There is a need, in addition to attention being paid to possible cognitive pathways, to consider—and find the impact of—affective pathways. DeBellis and Goldin (1997) described affective pathways as "the sequence of (local) states and feelings, possibly quite complex, that interact with cognitive representation" (p. 211).

A learning environment began to develop under iterative loops of the TR cycle, and the components of this learning environment are captured in the concept map below. At that time, the teaching-research team constituted a counselor (also the Vice President for Student Development), a librarian, and the mathematics instructor.

A brief explanation on how to read the concept map shown in Figure 17.3, with its emphasis on the improvement of classroom performance as a function of motivation, self-regulated learning, and cognitive development, is given in Appendix 2.

In the period 2010–2012, during the process of developing the conducive learning environment, three factors emerged as anchoring the learning environment (Barbatis, Prabhu & Watson, 2012). These authors advocated simultaneous attention to:

1. Cognition (materials and classroom discourse well scaffolded, paying attention to the development of the zone of proximal development via meaningful questioning in the classroom and via instructional materials designed in accordance with Bruner's (1978) theoretical position on concept development with concrete, iconic, and symbolic stages).
2. Affect (classroom discourse and independent learning guided by the development of positive attitudes toward mathematics through instances and moments of understanding of enjoyment of problems at hand, extended by self-directed means of keeping up with students' changing attitudes toward mathematics and its learning).

3. Self-Regulated Learning Practices (learning how to learn, usefulness of careful note-taking, daily attention to homework, asking questions, paying attention to metacognition and independent work).

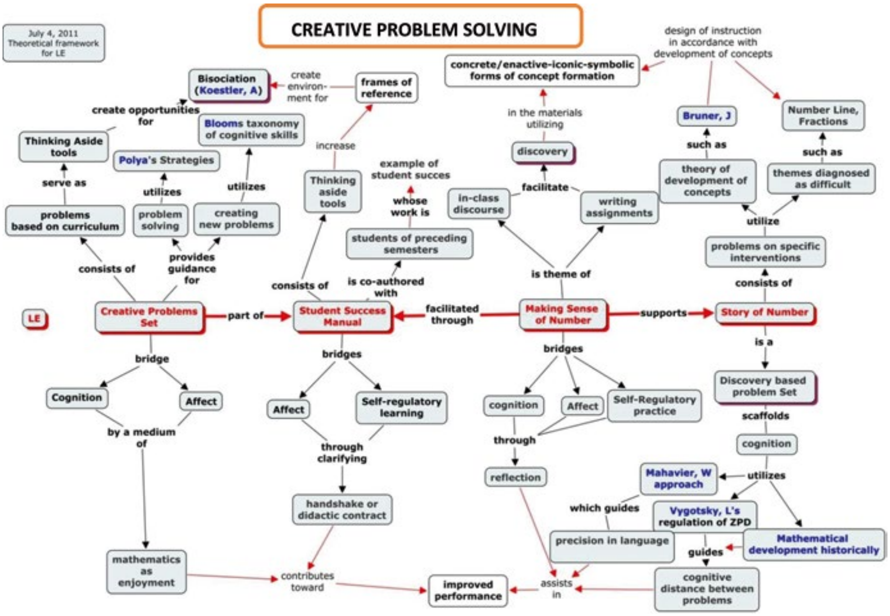

Figure 17.3. The components of a learning environment centered on creative problem solving.

Two simultaneous developments took place during the construction of the learning environment anchored in these three aspects. The craft knowledge of the teaching-research team had a common focus of employment—the development and viewing of the mathematical material on several planes of reference (Koestler, 1964). For example, with a problem such as ½ + 1/3, the counselor of the mathematician–counselor pair would keep the mathematical focus constant while alternating between concrete examples of cookies, pizzas, etc. an approach which exposed students to the process of generalization. This was then extended by the mathematics instructor in removing the monotony of "not remembering" the rules for operations on fractions by using the rules for operations on fractions in more complex problems such as those involving rules of exponents. It was found that the novelty and intrigue of decoding problems that involved exponents made the rules for fractions "easier" to remember or look up. Creativity had emerged as an organic development from the craft knowledge of the instructor. However, it was the support of Arthur Koestler's (1964) *The Act of Creation* that provided a theoretical base in which to anchor thinking and the development of creativity.

Theory of the Act of Creation

Koestler (1964) sketched the theory of the act of creation, or the creative act and coined the term bisociation to indicate the creative act. Bisociation refers to the "flash of insight" resulting from "perceiving reality on several planes at once" and hence, not just associating two familiar frames, but seeing a new one through them, which had not been possible before. This moment of understanding or bisociation is facilitated in the teaching-research classroom through problem posing which can lead to a pattern that changes habit to originality. Mathematics is no longer the "old and boring stuff that needs to be done," but is a source of enjoyment, so that even when the class period ends, students are still interested in continuing to puzzle over problems. Then, when enjoyment translates into performance, the Achievement Gap begins to close, one student at a time.

Koestler's (1964) theory of creativity was based on making connections of the concept in question across three domains or shades of creativity: humor, discovery, and art. Note that our Creative Learning Environment was anchored in Cognition, Affect, and Self-Regulated Learning Practices and assumes overlapping and mutually conducive roles. Humor addresses affect, discovery addresses cognition and learning how to learn when refined so that it is natural, the learner can transform his or her discoveries to deeper levels, or art. A quick glimpse of Koestler's theory is encapsulated in the concept maps shown in Figures 17.4 and 17.5. The habit and originality concept map provides the workings of the transformation involved in the creative process. The Habit + Matrix = Discovery concept map probes more deeply into this transformative process, showing the important role of affect/humor in the creative process. Both become directly usable in the development of the Creative Learning Environment in the classroom.

Mathematics Teaching-Research though the TR cycle clearly lends itself to creating a problem-posing/problem-solving dynamics. How does it do so? In the next section, we provide several classroom instances where problem posing has helped to bring discovery and enquiry "back on track." The concept map shown in Figure 17.5 links creativity with the problem-posing/problem-solving dynamics.

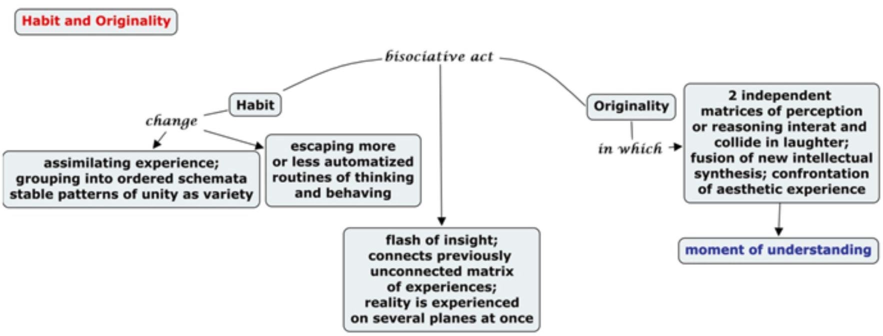

Figure 17.4. The role of the bisociative act in transforming the habit into originality.

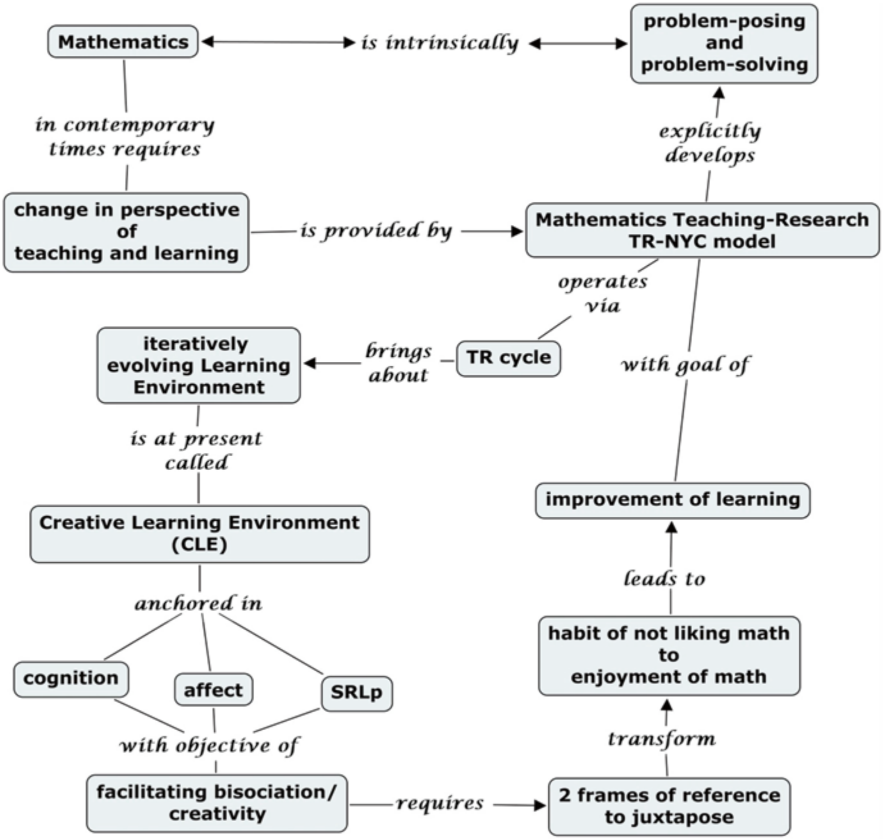

Figure 17.5. The role of mathematical creativity for the improvement of learning.

Problem-Posing/Problem-Solving Dynamics

Problem-Posing Illustration 1

This particular example is from an Elementary Algebra class. The time was just after the first exam, about a month into the semester. Students had had shorter quizzes before. On the day from which this example is taken, almost the entire class staged a rebellion. They stated that the instructor did not teach, that they solved problems, and that since the class is remedial, that means the instructor has to teach. A couple of the students explained what they meant by “teach.” One student stated that her previous instructor did a problem on the board and then students did several like it. Another student adamantly declared that she needed “rules” for how to do every problem. After the uproar subsided, the instructor guided them through the test, assuring them that the student in question is doing the problem—thinking aloud and continually pointing out the rules or the significant places to which to pay attention.

Problem 1 *Compute:*

(a) $36 + (-20) + 50 - (-17) - 10 =$

(b) $2 - (-4 - 10)$

(c) $-18 - (-6 + 2)$

(d) $2 - (-13) + (-7) - 20$

(e) $8 - 5 \times 2 + 9 =$

(f) $6 \times 7(-1) - 3 \times 8(-2)$

(m) $7(-4)(8) - 9 \times 6(-2)$

(n) $15 - 2(-5) - (20 - 4) \div 8$

Each problem was solved/thought out aloud by the student selected by the instructor, and she/he read the problem, and when a symbol was stated, such as parenthesis, the student was asked for the meaning of the symbol (posing a problem). Once the whole problem was read aloud with meaning, the student had to determine the order in which to proceed and why (solving a problem), and then the student actually did the computation in question. It is important to recognize, here, that whether or not a question is a posed problem depends on the state of knowledge of the student. For a student who does not know the meaning of a symbol, the act of asking the question "what does this symbol mean?" is posing a relevant problem. For a student who understands the role of that symbol but has difficulty interpreting this particular case, the question about the symbol is directed toward clarifying that understanding, and hence would not be a posed problem.

At the end of the class, attention was brought back to the work done, how it constituted reading comprehension, paying attention to the structure of the problem and then paying attention to the meaning of individual symbols and thinking of structure and meaning together. There was clarity, satisfaction, and a turnaround in problem solving after this session.

What did this session do in the classroom? First, it debunked the myth that one has to memorize something in order to solve every problem. Second, it took away the authority of the teacher as the knowledgeable one (which the class was reluctant to give up), and finally when each person carefully read and translated/made sense of the problem in terms of symbols and structure, students saw the process of posing and solving working in unison with one of their own classmates carrying out all of the thinking. Hence, for example, when the student who was doing the problem, read "parenthesis," she was questioned about the meaning of "parenthesis," and what role it had to play in the problem (posing problems). The mathematical language with its various hidden symbols, many symbols with one meaning, or one symbol with many meanings are all sources of confusion for students. Situations such as the one narrated here provide for self-reflection, and clarification of the language and of the meaning of the language of mathematics. This approach required many posed questions along the way for clarification. Note, how affect, cognition, and metacognition—all three—enter the dialogic thinking that instructor and students went through together.

Problem-Posing Illustration 2

In this example, the class was Elementary Algebra. Students had trouble determining which rule of exponents was to be applied to the given problem. There was a tendency to use anything arbitrarily without justification. The class problems were followed by a quiz, in which students had much difficulty in determining which rule was applicable for the problem under consideration. Again, it was a matter of not being able to slow down the thinking sufficiently to observe the structure of the problem and the similarity of the structure with one or more rules. Students were asked to work on the following assignment:

Rules of Exponents

1. $a^n \times a^m = a^{n+m}$
2. $\dfrac{a^n}{a^m} = a^{n-m}$
3. $(a^n)^m = a^{nm}$
4. $a^0 = 1$
5. $a^{-n} = 1/a^n$

Make up your own problems using combinations below of the rules of exponents:

- Rules 1 and 2
- Rules 1 and 3
- Rules 1, 2, and 3
- Rules 1 and 4
- Rules 2 and 4
- Rules 1, 2, and 5
- Rules 1 and 5
- Rules 1, 2, 3, 4, and 5

Solve each of the problems you created.

In the work that students submitted, they created problems that had only one term that required the use of say Rule 1 (e.g., x^7y^8) and another term that required the use of Rule 2 (e.g., $\frac{y^5}{y^3}$) but there were no problems that had one term requiring the use of both rules (e.g., $\frac{y^5 \times y^7}{y^{10}}$). This gave the instructor in question a point from which to develop problem solving through deeper problem posing, i.e., through dialogic think-aloud face-to-face sessions, students were asked to observe the structure of the given problem and state the similarity to all rules, where similarity was observed (this led to examples of posed questions which, in turn, led the teacher–researcher to make more complex exercises). This increased students' repertoires in problem solving as evidenced in the quiz and test, described in Problem-Posing Illustration 3.

Problem-Posing Illustration 3

In this illustration, we provide the triptych used in Statistics classes (also used in Arithmetic and Algebra, but not included here), developed through Koestler's work on the development of creativity. A triptych in Koestler's usage is a collection of rows as shown in Figure 17.6, where the columns indicate humor, discovery, and art. In order to get to the discovery of the central concept, the learner can work their way into probing the concept through some word that is known and even funny. Students are provided with the triptych shown in Figure 17.6, with two rows completed. These completed rows were discussed in class as to whether they make sense. Students clarified their understandings in the discussion. It was then expected that students would complete all rows of the triptych and then write a couple of sentences of explanation of the connections between the three words. When all students had submitted their triptychs, the class triptychs were placed on an electronic platform, Blackboard, and students viewed and reflected on each other's work. Students then created a new triptych for the end of the semester and included a few sentences explaining the connections of the concept and its illustration across the row of the triptych.

These classes needed greater scaffolding with the triptych and here the elements of the triptych were introduced "Just-in-Time" as the topic under consideration was being covered in the class. Hence, for example, the triptych Powers ←→ decimal representation ←→ polynomial was discussed during the session on polynomials. Figure 17.7 shows the general strategy that was used to facilitate discovery and understanding from the teacher–researcher's perspective:

Trailblazer ←→ Outlier ←→ Original/ity
←→ Sampling ←→
←→ Probability ←→
←→ Confidence interval ←→
←→ Law of large numbers ←→
Lurking variable ←→ Correlation ←→ Causation

Figure 17.6. The statistics triptych.

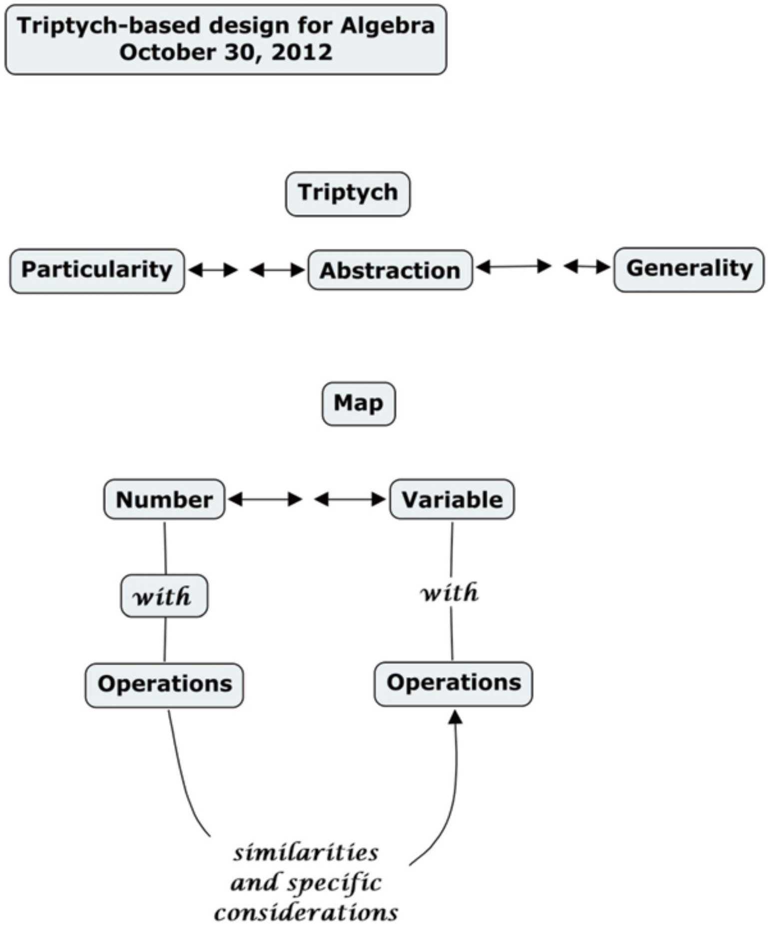

Figure 17.7. Algebra triptychs.

Problem posing was a constant in the discovery-oriented enquiry-based learning environment. Operations on integers and in particular, adding and subtracting with visualizing of the number line, formed the basis for ongoing questioning and posing of problems between students and teacher–researcher.

Algebra as the field of making sense of structure simultaneously with making sense of number provides opportunities for problem posing along the Particularity ← → Abstraction ← → Generality of the Arithmetic–Algebra spectrum. In Algebra classes, it was harder to introduce scaffolding, and problem posing occurred solely on the side of the teaching-research team as they explored ways to include triptychs in the Learning Environment mix. In the process, the triptych rows evolved into "simpler" usable forms.

Results and Discussion

The results discussed in this section were obtained after three teaching-research cycles. Consistent with this model for teaching, the results will be incorporated into the next TR cycle based on the described ideas and practice. We have discussed how our cyclical involvement in TR/NYC Model of teaching-research aims to solve the problem of our classrooms—students' understanding and mastery of mathematics led us to pose to ourselves a general question: *What are the necessary components of student success in mathematics*? Our answer to this problem was investigated in the teaching experiment *Jumpstart to Reform* which directed our attention to student creativity as the motivating factor for their advancement in learning. Quantitative analysis of the data is provided in Appendix 1. In turn, our facilitation of student creativity was scaffolded by a series of posed problems/questions designed either by the teacher or students of the classroom. (Doyle et al., in press) described the quantitative results of the teaching experiment *Problem Solving in Remedial Mathematics—Jumpstarting the Reform* supported by C^3IRG 7 awarded to the team in 2010. These results confirmed the impact of the approach for the improvement of student problem-solving capacity. These authors pointed out that the art of posing series of problems scaffolding student understanding depends strongly on the teacher's judgment concerning the appropriate amount of cognitive challenge.

Solving these problems in practice leads again to the posing of a general question, which, in agreement with the principles of TR/NYCity leads beyond the confines of our classroom: What is a learning trajectory (LT) of, for example, fractions in my classes? We illustrate a learning trajectory for fractions that developed over the period 2006–2012, with some movement at times, none at others, and a lot more when students are active learners. Problem posing has been an active element in that process within the student–teacher mutual understanding. A four-step approach was taken: (a) The meaning of fractions was established and revisited; (b) How should fractions be visualized? A fractions grid was developed as a visual tool (Czarnocha, 2008); (c) Proportional reasoning: Picture in various versions—seeing the interconnectedness of fractions in different representations: decimal, percent, pie chart; and (d) The meaning of fractions was revisited. Over time, the learning trajectory shown in Figure 17.8 was developed and can be summarized through the following six points:

1. Equivalent fractions visualized—operation: scaling—visualize with FG and then scaling
2. Increasing, decreasing order arrangement—prime factorization—common denominator—fraction grid and then reasoning; common denominators are meaningful before any other standard operations
3. Addition and subtraction
4. Multiplication
5. Division
6. Transition to language—what is half of 16?…

This learning trajectory will be refined through subsequent cycles of the course. Developing the LT for fractions is an illustration of how problem-posing works in

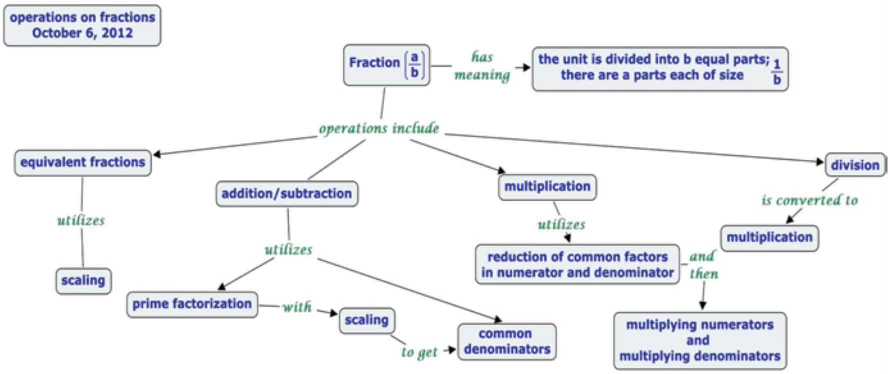

Figure 17.8, Learning trajectory for fractions.

the context of a satisfactory handshake on the part of learners. Problems utilizing exponents is an example of active problem posing leading to its successful integration by learners. The mastery of the language of mathematics through self-directed attention to reading comprehension is an example of how the repertoire needed for problem posing and solving needs to be consistently built up.

The development of several learning trajectories one of which is shown here demonstrate the usability of the methodology and developed materials for a much larger audience of students who fall in the category of self-proclaimed "no good at math," "don't like math," etc. The process of the development of learning trajectories proceeds through the elimination of learning difficulties in the collaborating classrooms.

Repeated problem-posing/problem-solving dynamics increases learners' repertoires for recognizing their own moments of understanding and the emerging patterns of understanding. Writing as the medium utilized for learning to write and writing to learn makes the understanding lasting, concrete, and reusable by learners (Luria & Yudovich, 1968).

The overarching result was that a discovery-based approach to the learning of basic mathematics, coupled with due attention to the cultivation of positive affect, was found to sustain development of learning "how to learn." The learning environment so created was thus a creative learning environment in that it was capable of stimulating creative moments of understanding and extending these to patterns of understanding that could transform learners' habits of doing/learning mathematics to an enquiry-oriented approach that fostered enjoyment and consequently boosted performance. Students' didactic contract/handshake toward their own learning markedly improved once they found mathematics to be enjoyable; their success in tests boosted their confidence; and their desire to achieve. Any fears which students had when the class started, and the accompanying resistance to learning, became nonexistent for the majority of the students. Two students who continued to hold some resistance were in a minority and slowly began to take greater interest. The emphasis on classroom creativity adopted in the teaching experiment outlined a possible pathway across the Achievement Gap.

Conclusion

Mathematics as the creative expression of the human mind is intrinsically questioning/wondering why and how, and through reflection/contemplation, gaining insight through careful justification of the answers to the questions posed. Problem posing and problem solving are thus the core elements of "doing mathematics." In contemporary contexts of teaching and learning of mathematics, this core of mathematics is hidden from sight, and a syllabus, learning objectives, learning outcomes, etc. are more prominent, making mathematics seem like a set of objectives and sometimes even called skills to be mastered by the student who is then considered proficient or competent in those skills. The high failure rate in mathematics starting as early as third grade (funded by MSP-Promyse, 2007), a dislike of mathematics reflected not just among students, but societally, and the low number of students seeking advanced degrees in mathematics are reflective of mathematics not being appreciated for what it is—the quest of the human mind toward knowing, and wanting to know why and how.

In the particular context of community colleges of the Bronx of the City University of New York, and analogously the large percentage of high school students who need remedial/developmental mathematics courses in college, problem posing has to be directly connected and on a regular basis with the classroom curriculum. The objective is urgent: closing the Achievement Gap. The problem as it exists is that an absence of proficiency in mathematics (i.e., scores on placement tests) could well prevent students from college education. The question is how to change this trend?

Knott (2010), in her paper *Problem posing from the foundations of mathematics*, stated:

> Recent developments in mathematics education research have shown that creating active classrooms, posing and solving cognitively challenging problems, promoting reflection, metacognition and facilitating broad ranging discussions, enhances students' understanding of mathematics at all levels. The associated discourse is enabled not only by the teacher's expertise in the content area, but also by what the teacher says, what kind of questions the teacher asks, and what kind of responses and participation the teacher expects and negotiates with the students. Teacher expectations are reflected in the social and socio-mathematical norms established in the classroom (p. 413).

Thus, for classroom environments to be effective, careful integration of simultaneous attention to cognition, affect and self-regulatory learning practices is needed (see also Barbatis et al., 2012). Vygotsky (1978) described the zone of proximal development (ZPD) as "the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or collaboration of more capable peers" (p. 86). The ZPD has to be "characterized from both cognitive and affective perspectives. From the cognitive perspective we say that material should not be too difficult or easy. From the affective perspective we say that the learner should avoid the extremes of being bored and being confused and frustrated" (Murray & Arroyo, 2002, p. 370).

Teaching and learning in a teaching-research environment is necessarily collaborative, as our work has demonstrated. This environment has to take account of the numerous difficulties faced in the classroom. The collaboration creates an open community environment in the classroom, which is beneficial to the problem-posing requirement. Mathematics as enquiry, as enjoyment, and as development of a thinking technology does not remain a collection of terms or unfamiliar notions to learners. In the span of one semester, college readiness has to be achieved so that the regular credit-bearing mathematics courses can be completed satisfactorily. Enquiry facilitating discovery becomes the modus operandi, possible now because of the creative learning environment. It is this environment that can provide learners with the keys to success in the learning and understanding of mathematics.

A problem-posing style of education in general whether it follows Freire's (2000) style of "reading the world," or in the style of Montessori (Montessori & Costelloe, 1972), in the design of the learning environment, all find use and applicability in Remedial Mathematics classrooms. Further, the discovery method, or Moore method (Mahavier, 1999) was applied successfully in calculus classes, and now finds a route into stimulating learners to enjoy and perform well in mathematics in remedial classes, thus paving the way toward closing the Achievement Gap and creating readiness for higher level mathematics classes.

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References

Akay, H., & Boz, N. (2010). The effect of problem posing oriented analyses-II course on the attitudes toward mathematics and mathematics self-efficacy of elementary prospective mathematics teachers. *Australian Journal of Teacher Education, 35*(1), 57–75.

Araujo, C. R., Andrade, F., Hazin, I., Falcao, J. T., do Nascimento, J. C., & Lessa, M. M. L. (2003). Affective aspects on mathematics conceptualization: From dichotomies to integrated approach. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 27th International Group for the Psychology of Mathematics Education, held jointly with the 25th PME-NA* (Vol. 2, pp. 269–276). Honolulu, HI: PME.

Barbatis, P., Leyva, A. Prabhu, V., Sidibe, A., & Watson, J. (2012). Engagement in creative thinking. *Mathematics Teaching Research Journal Online*, *5*(2). www.hostos.cuny.edu/mtrj

Barbatis, P., Prabhu, V., & Watson, J. (2012). *A creative learning environment for making sense of number*. Retrieved from http://catalysis.eprints.iitm.ac.in/2293/1/special_issue.pdf

Brousseau, G., & Balacheff, N. (1997). *Theory of didactical situations in mathematics: Didactique des mathématiques, 1970–1990*. Dordrecht, The Netherlands: Kluwer.

Brown, S. I., & Walter, M. I. (1983). *The art of problem posing*. Philadelphia, PA: Franklin Institute Press.

Bruner, J. S. (1978). *Toward a theory of instruction*. Cambridge, MA: Belknap.

Czarnocha, B. (Ed.). (2008). *Handbook of mathematics teaching-research: Teaching-experiment-a tool for teacher-researchers*. Rzeszow, Poland: University of Rzeszow, Poland.

Czarnocha, B., & Prabhu, V. (2006). Teaching Research NYCity Model. *Dydaktyka Matematyki, 29*, 251–272.

Czarnocha, B., Prabhu, V., Baker, W., & Dias, O. (2010). *C3IRG 7 Problem solving in remedial arithmetic: Jumpstart to reform*. New York, NY: City University of New York.

Daro P., Mosher, F., & Corcoran, T. (2011, January). *Learning trajectories in Mathematics—A Foundation for Standards, Curriculum, Assessment and Instruction*. CPRE Research Report RR-#68. Consortium for Policy Research in Education, Teachers College, Columbia University.

DeBellis, V. A., & Goldin, G. A. (1997). The Affective Domain in Mathematical Problem Solving. In: Pekhonen, E. (Ed.) *Proceedings of the PME 21*, 2, 209–216.

Doyle K., Dias, O., Kennis, J., Czarnocha, B., Baker, W., & Prabhu, V. (in press). The rational number sub-constructs as a foundation for problem solving. *Adult Learning Mathematics*.

Duncker, K. (1945). On problem solving. *Psychological Monographs, 58*(5), 1–113.

Freire, P. (2000). *Pedagogy of the oppressed*. New York, NY: Continuum.

Furinghetti, F., & Morselli, F. (2004). Between affect and cognition: Proving at university level. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 369–376). Bergen, Norway: International Group for the Psychology of Mathematics Education.

Goldin, G. A. (2002). Affect, meta-affect, and mathematical belief structures. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education*? (pp. 59–72). Dordrecht, The Netherlands: Kluwer Academic.

Gomez-Chacon, I. M. (2000). Affective influences in the knowledge of mathematics. *Educational Studies in Mathematics, 43*, 149–161.

Knott, L. (2010). Problem posing from the foundations of mathematics. *The Montana Mathematics Enthusiast, 7*(2–3), 413–432.

Koestler, A. (1964). *The act of creation*. New York, NY: Macmillan.

Luria, A. R., & Yudovich, F. I. (1968). *Speech and the development of mental processes in the child*. London, United Kingdom: Staples Press.

Mahavier, W. (1999). What is the Moore method? *Primus, 9*, 254–339.

Montessori, M., & Costelloe, M. J. (1972). *The secret of childhood*. New York, NY: Ballantine Books.

Murray, T., & Arroyo, I. (2002). Toward measuring and maintaining the zone of proximal development in adaptive instructional systems. In S. Cerri, G. Guarderes, & F. Paraguacu (Eds.), *Intelligent tutoring systems: Lecture notes in computer science, Proceedings of the 6th International Conference ITS* (Vol. 2363, pp. 749–758). Biarritz, France: Intelligent Tutoring Systems.

Silver, E. A., Mamona-Downs, J., Leung, S. S., & Kenny, P. A. (1996). Posing mathematical problems in a complex task environment: An exploratory study. *Journal of Research in Mathematics Education, 27*(3), 293–309.

Singer, M. F., Pelcher, I., & Voica, C. (2011) Problem posing and modification as a criterion of mathematical creativity in M. Pytlak, T. Rowland, & E. Swoboda, (Eds.), *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education*, CERME 7, 9–13 February, Rzeszow, Poland: CERME.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.