Chapter 12
Problem Posing in the Upper Grades Using Computers

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Abstract Problem-posing activities in mathematics classrooms have been found to have rich outcomes in helping students to develop profound understandings of mathematics and in fostering their problem-solving abilities and creative dispositions. We describe in this chapter practical ways of introducing problem posing using computers, based on our previous studies on problem-posing activities for university students (prospective teachers) and high school students. Studies on problem-posing activities have been rare for students in the upper grades (i.e., high-school and university-level students), and classroom practices involving such activities are less known. We first identify aspects associated with problem posing in the upper grades using computers and introduce practical activities. We report surveys on some of our concrete problem-posing activities and demonstrate their validity. We also present the results concerning the effects of computer use for problem posing in our setting.

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Problem Posing in the Upper Grades Using Computers

One day, in response to an assignment, a university student brought to his teacher a mathematical problem which he thought up by himself. The problem was: "Represent the figure given by the equation $|z-|z-1||=1$ for the complex number z." The student expressed excitement and surprise at how interesting he found the figure. The teacher, one of the authors of this chapter, was also amazed with the student's work since he had not seen such an equation taking twofold magnitudes but with a clear solution. Whereas the student was usually inconspicuous in the class, he seemed to have been intensely stimulated by working out this mathematical problem on his own. He is now a high school mathematics teacher.

Problem posing in mathematics is a discipline in which students create mathematical problems. Problem-posing activities in classrooms appear to offer potential for students' mathematical growth: to help students to deepen their conceptual understanding of mathematical content, to foster their ability to solve problems, and to cultivate students' creativity. Problem-posing activities also serve to build mathematical communication between students mediated by the posed problems. During the past three decades, various studies which support the authenticity of problem posing have been presented (e.g., Brown & Walter, 1983; Ellerton, 1986; Gonzales, 1994; Hashimoto & Sakai, 1983; Lavy & Bershadsky, 2003; Pelzer & Gamboa, 2009; Perrin, 2007; Saito, 1986; Silver, 1994; Silver & Cai, 1996; Silver, Mamona-Downs, Leung, & Kenney, 1996; Singer, Ellerton, & Cai, 2011).

The importance of problem posing in high school mathematics classrooms was recognized by the National Council of Teachers of Mathematics (1989): "Students in grades 9–12 should also have some experience recognizing and formulating their own problems, an activity that is at the heart of doing mathematics" (p. 138). In spite of statements like this, the practice of problem posing by upper-grade students has not been common, and only a few studies about the practical side of problem posing for the upper grades could be found. It may be the case that some consider that inculcation is so necessary for the sufficient understanding of mathematics in the upper grades that self-generated learning like problem posing is unsuitable for effective study. Varying levels or degrees of competency or interest on the part of the students may be another difficulty encountered when implementing the activity. But, it is also conceivable that many upper-grade class teachers do not realize that it is possible to incorporate problem posing effectively into their daily instruction.

In this chapter, we will present some practical ways of introducing problem posing in the upper grades, based on our studies of its use among high school students (Imaoka, 2001; Kanno, Shimomura, & Imaoka, 2007, 2008) and university students (Imaoka, 2001; Shimomura, Imaoka, & Mukaidani, 2002, 2003a, 2004; Shimomura, Imaoka, Mukaidani, & Kanno, 2003b; Shimomura & Imaoka, 2007, 2009, 2011). We believe that problem posing should be closely connected with everyday class activities through appropriate practical activities. Without this, problem posing may be regarded as peripheral to the tight curricula adopted in the upper grades, or as an activity that only expert teachers could manage. Despite such potential deterrents, we believe that problem-posing activities create significant opportunities for upper-grade students to learn from their classmates.

Computer environments provide rich contexts for visualizing objects and for allowing students to experiment on various cases as well as to explore problems developmentally. Although the use of a computer does not always ensure that students can pose problems to their satisfaction, it is, nevertheless a powerful auxiliary for problem-posing activities. Isoda, Okubo, and Ijima (1992) described a method for supporting problem posing using self-developed software and presented data on students' enhanced heuristic learning.

In this chapter we place emphasis on enriching problem-posing activities with the aid of existing computer software. We will present feasible ways for the introduction of problem posing through actual classroom practice, and we aim to demonstrate the effectiveness and significance of problem posing in the upper grades. Our analysis of a survey we conducted with participants will also be presented. We hope that this chapter will encourage upper-grade mathematics teachers to include problem posing in their own mathematics classrooms.

The Study and Framework for Analysis

The problem-posing activities used in the mathematics classrooms involved in this study, and the analysis of data on problem posing which incorporated the use of computers, focused on the following questions:

1. How did teachers organize the problem-posing activity?
2. Which student-created problems were considered adequate?
3. What types of problems were generated by students?
4. In what ways did students utilize computers?
5. How did students evaluate the activity?

We shall describe the organization of the study and criteria for analysis (Questions 1 and 2) in this section; Questions 3 and 4 will be addressed in subsequent sections by referring to our practices; and Question 5 will be discussed in a later section that presents the results of questionnaires.

In designing this study, we applied traditional Japanese methods developed through our practice of using open-ended approaches to mathematics teaching. In Japan, the practice of problem posing was attempted in the 1920s. Teachers of Nara Female Teachers College affiliate elementary school carried out what was described as "arithmetic problem making classroom." This groundbreaking practice was taken up by many teachers, but became controversial at that time, and declined around 1925 because of methodological insufficiency. The basic principle of the practice was to let children freely make "close-at-home" problems, and teachers who adopted the approach were considered to have been influenced by Dewey pragmatism (cf., Hirabayashi, 1958). Inheriting this tradition, the practice of problem posing was re-introduced extensively in the 1970s and was based on studies about the developmental treatment of problems (e.g., Hashimoto, 1997; Nakano, Tsubota, & Takii, 1999; Sawada & Sakai, 1995; Takeuchi & Sawada, 1984). Assessment of students' understanding of mathematics adopted open-ended approaches, and practitioners worked towards establishing a common way of problem posing. Originally devised in elementary schools, the method is better suited for lower grade students. Although we have employed a fundamentally similar approach to problem posing, we have made adjustments appropriate for upper-grade students.

Our approach adopted the following steps:

Step 1: The teacher introduced a problem-posing activity by explaining the process, showing original problems, or by giving remarks as the occasion demanded.

Step 2: Students were assigned the task of posing problems with the aid of computers and were required to give answers; they also needed to show how they contrived their problems and how they used the computers.

Step 3: The teacher checked each student's posed problems individually, and then exhibited them to all students.

Step 4: Each student was assigned several classmate-posed problems to solve and to comment on.

Step 5: As a final step, the teacher chose several posed problems and asked students to solve and develop them in front of the whole class.

In this study, an original problem is defined as a problem prepared by the teacher at the first stage of the activity. It serves as an example for students to refer to later on, and it sets the tone for the later stages. Teachers in Matsubara Elementary School (1984) indicated the following requisites for choosing an original problem: (a) "generalization" for easy consideration; (b) some "analogy" for applicability, and if possible, an "opposite" construction; and (c) "combinations" of these requisites for easy new constructions. Taking into consideration these factors, as well as knowing that the upper-grade students would be asked to add elements for the activity through the use of their computers, we believe that, in addition to the requisites indicated by Matsubara Elementary School, the following additional elements should be present in the original problems.

1. The original problems should have some characteristics that students can target in their problem posing. For instance, the problems should include multiple representations (such as graphs), or should involve measuring of figures, like areas.
2. The original problems should include some elements which would be particularly suitable for involving computer use. For instance, the solutions can be inferred by experimentally examining particular values using computers.

The software used by high school students in this study was the free software *Grapes* (Ver.6.50c) developed in Japan around 2000, and that used by university students was Wolfram Research's *Mathematica*. The high school students used *Grapes* in their school computer room and on their home computers after downloading it from the Internet. The university students installed *Mathematica* on their own computers using the university's group license.

Activities

Activity I (for High-School Students)

One author (Kanno) of this chapter studied 320 second-year high school students in eight classes as they explored various equations through problem posing using their computers. The mathematical focus of the activity was to reinforce the understanding of relationships between equations and graphs of functions. The five steps outlined in the preceding section were followed in the activity. The teacher prepared original problems as summarized below and assigned students the task of posing problems. After completing the task, students were instructed to offer comments on their classmates' problems. Each class invested three class hours to discuss several posed problems, allowing time for the students who posed the problems to explain their work, and for their classmates to solve them.

For the first original problem, the teacher displayed the corresponding graphs for several concrete values of *k*.

Original Problem 1

Find the number of distinct real solutions for the quadratic equation $x^2 - 3x - k = 0$.

Students were familiar with this sort of problem and solved it immediately using the sign of the discriminant $D = 9 + 4k$. Then, the teacher posed the next problem.

Original Problem 2

Find the number of distinct real solutions for the cubic equation $x^3 - 3x - k = 0$.

In this case, many students could not come up with the solution since they had not been acquainted with this sort of problem. The teacher reminded the students that they could get the solution of the first problem by observing the move of the corresponding graph with varying *k*. Students then noticed that the solution of the latter problem could also be detected from the move of the corresponding graph. The teacher gave a further suggestion that it is better to fix the graph $y = x^3 - 3x$ and move the graph $y = k$.

The most popularly posed problems were polynomial equations of higher degrees, such as the following: "What are the distinct real solutions for $x^6 - 14x^4 + 49x^2 - 36 - k = 0$ and $x^6 - 6x^5 + 9x^4 - 10 + k = 0$, respectively?" Figure 12.1 presents two examples of problems (and graphical solutions) posed by students.

It is noteworthy in Figure 12.1 that the students employed the last hint given by the teacher and made use of fixed polynomial graphs. Although Japanese high school textbooks usually treat at most fourth degree polynomial equations, students

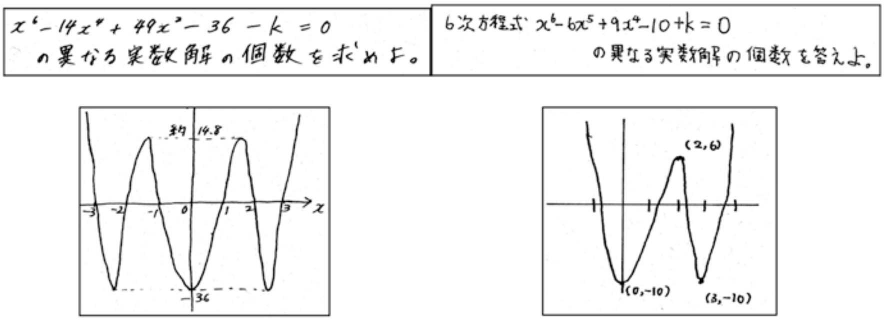

Figure 12.1. Two polynomial problems posed by students.

were free from such a constraint because they were able to use appropriate computer software. In subsequent class discussion, some students noticed that the graphs of polynomial functions associated with posed problems with no odd degree terms were generally symmetric with respect to the *y*-axis, and other students noticed that there exists at least one solution for any equation of an odd degree. Observations such as these were, in fact, not isolated occurrences in which significant mathematical ideas were discussed. Through the problem-posing activity, students were often able to make significant mathematical observations and connections.

Figure 12.2 shows a typical example of a student-posed problem that involved absolute values: "What are the distinct solutions of the equation $|x^3 - 6x^2 + 9x - 2| - k = 0$?".

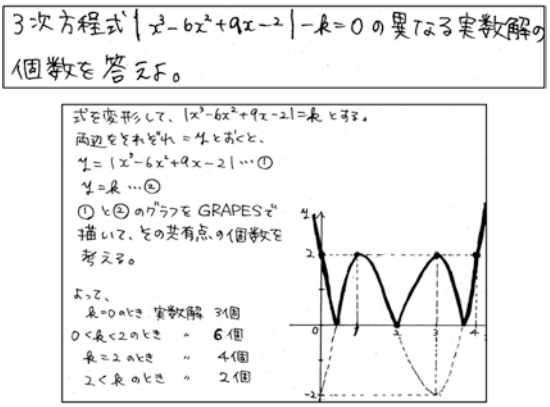

Figure 12.2. An absolute-value problem posed by a student.

Finding solutions to functions which involve absolute values is usually challenging to Japanese high school students. In spite of such challenges, we found students who did create absolute value problems. The ease of finding computer solutions might have made exploring such challenging problems more accessible to students. However, we have observed in other contexts that students are willing to create problems that involve content that they want to master (see, e.g., Imaoka, 2001). This tendency represents a special mentality which appears when students embrace the challenge to think through their work.

Activity II (for Preservice Teacher-Education Students)

Problem-posing activities that incorporate the use of computers have also been studied with preservice teacher-education students (Shimomura & Imaoka, 2007, 2009, 2011, Shimomura et al., 2002, 2003a, 2003b, 2004), and a case will be described here. Although studies on problem posing by preservice or in-service teachers have been presented (e.g., Gonzales, 1994; Lavy & Bershadsky, 2003; Silver et al., 1996), reports of the activity using computers were not found at the time of our research. Our study looks at the practices taking place in preservice teacher-education classes where students are preparing to become secondary school mathematics teachers. The number of students in each class was limited to less than 50. Prior to each time the activity was introduced, both the instructor and the students had learned to solve various problems using *Mathematica*. When activities were introduced, the five steps outlined earlier in this chapter were followed.

The objective of our activities was to enhance students' capacity for devising instruction in their future profession and to let them recognize the possible benefits of using computers for mathematical activities. Being university students, they had considerable competency both in mathematics and in the use of computers, and therefore we could expect them to try to work out quite sophisticated mathematical problems.

We introduced the original problem (referred to as the OPA problem) from Shimomura and Imaoka (2007), as shown in Figure 12.3.

Let P be a point on the curve C: $y=e^x$, and Q the point where the tangent line of C at P intersects with the x-axis. We draw a rectangle whose diagonal is PQ and one edge is on the x-axis. Then, the rectangle is divided into two parts by the curve C. Explore whether or not the ratio of the areas of these two parts is constant for any point P.

Figure 12.3. The OPA problem.

In the class, many students anticipated that the ratio might be constant, and conjectured the ratio as 1:2, 1:3, 2:3, and so on. The teacher urged students to examine the ratio using their computers. The diagram in Figure 12.4 represents the displayed graphs in the cases that the x-coordinates of P's are −.5, .5, and 3, respectively. Students experimented with the graphs and began to determine the ratio using the computer. Surprisingly enough for the students, the ratio became the constant 1:(e−1). After that, the teacher explained the answer using both a computer and the blackboard and assigned the following task as homework: "Referring to OPA, make two developmental problems using the computer effectively. Also, prepare answers for the problems, and describe how you used computers to work out the problems."

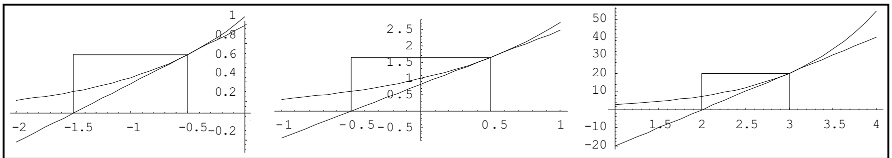

Figure 12.4. Graphs produced for P values of −.5, .5, and 3, respectively, in the OPA problem.

Some students posed problems by making simple changes from e^x to e^{ax}, ae^x or be^{ax}, but many students elaborated their problems by making extensive changes to the OPA problem. In Figures 12.5, 12.6, and 12.7, we present three problems posed by Students 1, 2, and 3, respectively.

Let P be a point on the curve C: $y = \log kx$ ($k > 0$), and Q the point where the tangent line of C at P intersects with the y-axis. We draw a rectangle whose diagonal is PQ and one edge is on the y-axis. Then, the rectangle is divided into two parts by the curve C. Is the ratio of the areas of these two parts constant for any point P?

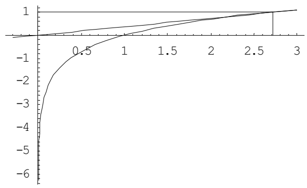

Figure 12.5. Problem posed by Student 1.

Let P be a point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, and Q and R the two intersections of the tangent line at P and the two asymptotes of the hyperbola. Then, is the area of the triangle △ OQR constant for any point P on the hyperbola? Here, O is the origin.

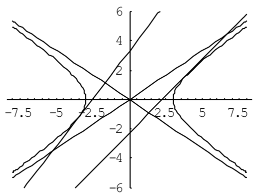

Figure 12.6. Problem posed by Student 2.

Let $z = f(x,y) = x^n + y^n$ and $z = g(x,y) = x + y$ be two surfaces over the region $0 \le x \le 1$ and $0 \le y \le 1$. Does the ratio of the volume of the solid surrounded by the surfaces $z = f(x,y)$ and $z = g(x,y)$ and the volume of the solid surrounded by $z = f(x,y)$ and $z = 0$ have some principle with respect to n?

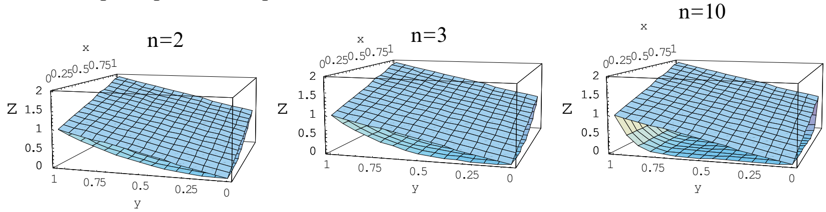

Figure 12.7. Problem posed by Student 3.

Student 1 wrote in his note that he first examined the case of $k = 1$ using a computer, which is a direct analogy to the OPA problem since it changes the original function to its inverse function, and he generalized it with $k > 0$.

Student 2 changed the setting of the OPA problem a little differently, but kept the characteristic concerning invariance of the area. The student reported that she created various quadratic curves using a computer and took a chance on coming upon this problem when she recollected her study on quadratic curves in the class.

Student 3 created the problem shown in Figure 12.7 and changed the ratio of areas in the OPA problem to that of volumes. This student wrote that, when drawing the graphs in the cases of $n=2$, 3, 4, and 10, he found the graphs of $z = f(x, y)$ and $z = g(x, y)$ intersect only at the points (0,0,0), (1,0,1), (0,1,1), (1,1,2). He examined the volumes in each case using the computer, which led him to formulate this problem.

With these examples, we can observe that students tried appropriate experimental methods using computers. In a later class, the teacher chose the problem posed by Student 3 and invited Student 3 to present the problem to the class. After being shown the above computer diagrams, many students arrived at the answer in about 30 minutes.

Although the OPA problem might appear to be advanced, the student opinions summarized in Figure 12.8 suggest that many students were able to cope with the level of mathematics involved. We conclude that the OPA problem satisfies the requisites outlined earlier in this chapter, particularly those concerned with including the use of graphical properties that represent some invariance of an object. Based on the student-posed problems and on students' opinions about the activity, we are convinced that properties associated with graphs are a good fit for problem-posing activities using computers since various explorations about invariance become possible. Students can explore a range of properties of mathematical objects, thus opening creative approaches problem posing.

OPA is adequate to develop the problems. (8 students expressed similar opinions)
OPA was helpful for my problem posing. (3)
OPA is a helpful first step which makes problem posing easy to understand (3)
OPA can also be solved by paper-pencil computation, which is more familiar to me. (2)
Since I have never created a problem like OPA before, it was a significant occasion. (2)
I challenged myself to pose a problem equal to the OPA.
OPA fits the graphical function of the software. (6)
I was impressed by the constant ratio of areas in the OPA. (5)
I stuck to using the ratios in my problem posing and could not devise different kinds of problems. (2)
I associated OPA with problems related to volumes or using the Monte Carlo method.

Figure 12.8. Summary of students' comments about the OPA problem. The number given in parentheses indicates the number of students who expressed a similar opinion.

Survey Results

After Activity I, we asked the following two questions to the high school students who posed mathematics problems based on Original Problem 1 and 2:

1. Was the use of computer valid in your problem posing?
2. Did you find some unintentional findings or interesting results?

In response to the first question, "Was the use of computer valid in your problem posing," 65.5% replied "Yes, very valid" and 31.6% "Yes, rather valid." The responses show that most students believed that the computer had been useful for the activity. We conclude that students appreciated the convenience of the computer which enabled them to treat complicated equations easily through displayed graphs. In response to the second question, "Did you find some unintentional findings or interesting results," 47.7% answered either "Yes, many" or "Yes, some." Nearly half of the students made unexpected discoveries during their problem-posing attempts. Table 12.1 shows a further breakdown of students' responses to Question 2, with responses grouped under one of three levels—high, average, and low—based on the students' terminal mathematics examination results.

Table 12.1
High School Students' Responses to Survey Question 2 Grouped Under Three Performance Levels

	Response to Question 2				
Ranking	Many	Some	Few	None	Total
High	18	32	40	6	96
Average	15	42	54	7	115
Low	12	29	51	7	99
Total	45	103	145	17	310

A χ^2 test of the correlation between the groups and findings was found to be 4.998, and the Cramer V was .090. Hence, we did not find a clear relationship between the responses to Question 2 about whether students made unexpected discoveries in their problem-posing attempts, and their results in their terminal mathematics examination results. We interpret this in the following way: Every student has an equal opportunity to come across some unexpected findings through problem posing, that is, to experience real mathematical activity. Such a low correlation was also reported by Saito (1986) in a different situation of problem posing. In fact, such equality spread across a wide range of student achievement is an excellent feature of problem posing.

Next, we shall present the results of the questionnaires to university students in three studies:

- *Study I*: Our first study on the activity of problem posing using computers.
- *Study II*: Our focus on the reflections of students on their posed problems.
- *Study III*: Our use of the OPA problem, and our subsequent inquiries into the requisites for the original problem.

In Tables 12.2, 12.3, 12.4, 12.5, and 12.6, the notation *a*(*b*) has been used to represent the number of answers, *a*, with *a*'s percentage in each category represented by (*b*).

Since more than 80% of students answered "yes" in every study, we can say that our way of implementing the activity was found to be appropriate and acceptable for the students.

Table 12.2
Preservice Students' Responses to Questionnaire Question 1: Did You Find the Problem Posing Activity Beneficial and Satisfactory?

	Beneficial			Satisfied		
Activity	Study I	Study II	Study III	Study I	Study II	Study III
Yes, absolutely	9 (25.7)	8 (16.3)	16 (38.1)	11 (31.45)	13 (26.5)	14 (33.3)
Yes, mostly	23 (65.7)	36 (73.5)	23 (54.8)	19 (54.3)	27 (55.1)	23 (54.8)
Neither yes or no	3 (8.6)	5 (10.2)	3 (7.1)	4 (11.4)	9 (18.4)	3 (7.1)
Mostly no	0 (0)	0 (0)	0 (0)	1 (2.9)	0 (0)	0 (0)
Absolutely no	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	1 (2.4)

Table 12.3
Preservice Students' Responses to Questionnaire Question 2: How Difficult Was it for You to Think out the Problems?

Difficulty	Study I	Study II	Study III
Very difficult	15 (42.8)	14 (28.6)	11 (26.2)
Somewhat difficult	17 (48.6)	29 (59.2)	24 (57.1)
Difficult	3 (8.6)	6 (12.2)	7 (16.7)
Slightly difficult	0 (0)	0 (0)	0 (0)
Easy	0 (0)	0 (0)	0 (0)

Table 12.4
Preservice Students' Responses to Questionnaire Question 3: What Was the Most Difficult Step in the Problem-Posing Process?

The most difficult process	Study II	Study III
Problem design	41 (83.7)	33 (78.6)
Showing methods, computations, etc.	5 (10.2)	2 (4.8)
Careful examinations	3 (6.1)	7 (16.7)

Table 12.5
Preservice Students' Responses to Questionnaire Question 4: Was the Use of Computers Helpful?

Useful	Study I	Study II	Study III
Yes, very	21 (60.0)	26 (53.1)	26 (61.9)
Yes, somewhat	11 (31.4)	21 (42.9)	11 (26.2)
No opinion	2 (5.7)	2 (4.1)	5 (11.9)
Not so useful	1 (2.9)	0 (0)	0 (0)
No	0 (0)	0 (0)	0 (0)

Table 12.6
Preservice Students' Responses to Questionnaire Question 5: How Was a Computer Useful for Anticipating, Solving Problems, and Acquiring New Knowledge?

Items		Study I	Study II	Study III
A computer is a useful tool in anticipating problems	Yes, very	18 (51.4)	26 (53.1)	24 (53.1)
	Yes, somewhat	13 (37.1)	17 (34.7)	14 (33.3)
	No opinion	1 (2.9)	4 (8.2)	3 (7.1)
	Not so useful	3 (8.6)	2 (4.1)	0 (0)
	No	0 (0)	0 (0)	1 (2.4)
A computer is a useful tool for solving the anticipated problem	Yes, very	24 (68.5)	30 (61.2)	27 (64.3)
	Yes, somewhat	9 (25.7)	13 (26.5)	10 (23.8)
	No opinion	1 (2.9)	3 (6.1)	4 (9.5)
	Not so useful	1 (2.9)	3 (6.1)	1 (2.4)
	No	0 (0)	0 (0)	0 (0)
A computer is a useful tool in acquiring the new knowledge	Yes, very	18 (51.4)	23 (46.9)	25 (59.5)
	Yes, somewhat	16 (45.7)	19 (38.8)	12 (28.6)
	No opinion	1 (2.9)	7 (14.3)	5 (11.9)
	Not so useful	0 (0)	0 (0)	0 (0)
	No	0 (0)	0 (0)	0 (0)

More than 80% of students answered "difficult" ("Very difficult" to "Difficult") to Questionnaire Question 2: "How difficult was it for you to think out the problems?" The results in Table 12.3, along with those for Questionnaire Question 1 in Table 12.2, support our conclusion that problem posing in our setting established sufficient challenge to provide our students with satisfaction after they had completed the activity. This suggests that there may be an important difference between the outcomes of problem-posing activities in elementary schools and in our setting for the upper-grade students.

The types of responses shown in Table 12.4 were anticipated since our target was for students not to make simple modifications to the given problem, but rather for them to engage in more creative problem posing.

Similar rates of approval as in Survey Question 1 in Activity I can be seen in the responses to Questionnaire Question 4 (Table 12.5), but we note that the university students used computers in various ways according to their respective purposes as in Activity II.

Rates of responses for the three items under Questionnaire Question 5 are shown in Table 12.6. Although the number of students was limited in each activity, we observed that students could use computers effectively for every step of the problem-posing process.

Discussion

In the upper grades, giving students sufficient time and opportunity to devise and elaborate mathematics problems is a realistic, and indeed appropriate, mathematical activity. In textbooks or in surveys of elementary or junior high school students in Japan, we find tasks about problem posing related to events or phenomena in the real world (cf., NIER, 2003). Also, in lower grades, problem posing usually took place during class hours. Upper-grade students are generally expected to acquire a certain level of knowledge and skills in mathematics and to enhance their competency through additional mathematical activities. Naturally, problem posing should fit with such circumstances. Activity I (or equivalent activities) would complement students' mathematics classroom experiences and would enable them to explore various equations or functions. Activity II (or equivalent activities) would serve prospective mathematics teachers (and subsequently their students) both by acknowledging the importance of creative thinking and by the effective use of computers. We do not propose that problem posing be the central activity in mathematics learning of upper-grade students, but that the activity is an appropriate supplement for enhancing the study of mathematics.

Our approach to problem posing is also fully applicable without using computers (e.g. Imaoka, 2001; Kanno et al., 2007, 2008). For instance, Kanno, Shimomura, and Imaoka described an activity that took place in senior high school mathematics classrooms, when students were preparing for university entrance examinations. We utilized past problems of entrance examinations as original problems and practiced problem posing without computers. The activity intensified students' attention since it enabled them to experience many kinds of entrance examination problems in a creative way by collaborating with their classmates, exchanging useful information, and discussing solutions with each other.

It is also possible to begin without the stimulus of an original problem. Instead, students can be given only a theme for problem posing and given time to devise their own problems. In particular, problem posing without an original problem seems to be effective in the upper grades when students review what they have been studying in a broader context (cf. Imaoka, 2001; Shimomura et al., 2003a, 2003b).

The exchange of comments, questions, or opinions among students about posed problems increases the effect of problem posing. However, such exchanges might increase teacher involvement since he/she may need to organize the posed problems and may need to facilitate or at minimum initiate discussion about the posed problems. In other words, Step 3 ("The teacher checked each student's posed problems individually, and then exhibited them to all students") needs input from the teacher. In Activity I, the teacher converted all posed problems into PDF files and placed them on his server. Furthermore, he utilized the local area network in the school for distributing posed problems to each student. In this way, if a teacher stores the files of the posed problems in his/her computer, he/she can use them efficiently, for instance, some good posed problems can later be used as original problems.

Figure 12.9 summarizes preservice teacher-education students' comments about Activity II, as a whole, and such opinions have often been observed in other applications of problem posing in our classrooms. Based on student opinions such as those in Figure 12.9, we are convinced that many students feel that our approach to problem posing provided a significant learning activity which supplemented their studies, albeit with some useful suggestions for improvement.

- I found interest and pleasure in examining concrete values first and then making problems by generalizing them while anticipating results.
- I was pleased to be able to make good problems by thinking various patterns and by drawing graphs using computers.
- I tried to make some understandable problems using *Mathematica* , and I was successful.
- I felt that my posed problem was successful, since friends asked me how to solve my problem.
- I almost made problems without a computer. But, I felt that it was very convenient to *Mathematica* to check the result.
- I had only solved given problems before, and I felt strongly that one cannot pose a problem without profound understanding of mathematics.
- It was a good experience, but I felt the shortage of time since I could not use computers freely and quickly.
- It was very difficult, because I could not operate computers well. But, it was a wonderful experience to make problems using *Mathematica*.

Figure 12.9. Opinions expressed by preservice teacher-education students about Activity II.

In conclusion, we would repeat our main findings. Our approach to problem posing was found to encourage our students by inspiring them to create their own work. The experience of problem posing had a positive effect on upper-grade students' overall mathematical experiences. If students are encouraged to exchange comments on posed problems with each other, they were found to gain many ideas through mathematical communication with their peers.

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