Chapter 49
An Kalman Filter-Based Method for BeiDou/GPS Integrated Navigation System

Xu Yang, Qing Xu, and Haijiang Wang

Abstract To improve precision of double-star positioning system (DSPS) in China and overcome the defects due to the shadiness of DSPS in the war, an indirect Kalman filter-based loosely coupled method for tri-star passive BeiDou/Global Positioning System (BeiDou/GPS) integrated navigation system is presented. This chapter uses tri-star passive positioning technique to solve the shadiness of DSPS in the war. An indirect Kalman filter-based method is proposed to combine BeiDou with GPS to fuse into an integrated navigation system to enhance precision of BeiDou when GPS is available. Experimental results demonstrate that the proposed Kalman filter-Based method can effectively reduce the positioning error, and precision of the BeiDou/GPS integrated navigation is better than GPS-only navigation.

Keywords BeiDou • GPS • Kalman filter • Tri-star passive positioning • Integrated navigation

49.1   Introduction

Currently, a sole satellite positioning system cannot meet the increasing demand for high-precision positioning. DSPS using two geostationary satellites providing radio determination satellite service (RDSS) has been established in China since 2000. The third navigation and positioning geostationary satellite was launched in May 2003 which is a backup satellite for DSPS. That is milestone that China owns a regional RDSS system which combines rapid positioning with short message communication. But, unlike GPS and GLONASS, DSPS works in an active mode

X. Yang (✉) • Q. Xu • H. Wang
College of Electronic Engineering, Chengdu University of Information Technology, Chengdu 610225, China
e-mail: joyodsp@163.com; whj@cuit.edu.cn

[1], which means the positioning function is provided by the center station and transferred to users by satellite. The fatal defect of this mode is that the users are easily exposed in the war. Another defect is that the capacity of user is subject to the control center's ability of calculation and communication.

GPS is a satellite-based high-precision positioning system which provides positioning and time information in all weather, anywhere on or near the earth, where there is an unobstructed line of sight to four or more GPS satellites. It is widely applied to a variety of areas such as aerospace, aeronautics, missile guidance etc. However GPS is maintained by the United States government and it is not available for our troops in the war. BeiDou/GPS integration can combine reliability with high-precision.

There have been many literatures over last decades on the improvement of the DSPS performance. Xiaofeng He et al. [2] employ fuzzy inference-based Kalman filtering SINS/RDSS integrated algorithm to improve DSPS performance. Benlei Su et al. [3] use BeiDou/INS integration navigation system to avoid the limitation of the DSPS. Erhu Wei et al. [4] propose the protective system with the integration of GPS and BeiDou navigation system to overcome the security of using GPS-only positioning. Little attention is paid to BeiDou with GPS integration navigation system.

Herein, a new technique for BeiDou/GPS integration based on an indirect Kalman filter is proposed to overcome defects of the easily exposed in the war and the limited capacity of users of the DSPS. Above all, this method can improve precision of navigation system. The experiential results demonstrate that precision of this integrated navigation system is better than GPS-only navigation or BeiDou-only navigation.

49.2 Principle of Tri-Star Passive Positioning

In July 2007, the first receiver of BeiDou positioning system has been made successfully [5] which overcome the limited capacity of users, the easily exposed, and the low precision of positioning etc.

As the principle of positioning of GPS, the technique of BeiDou positioning system is based on measurement of pseudo-range [6]. Together with the backup satellite of DSPS, there are three satellites to form tri-star positioning system. When all of the three satellites are available, a receiver of BeiDou positioning system can reckon the location (x_i, y_i, z_i) in the Earth Centered Earth Fixed (ECEF) frame of satellite i in terms of ephemeris data received from signals of BeiDou positioning system transmitted. By means of measuring time difference between instant of time arrived at receiver and instant of time transmitted from satellites, it can calculate the range ρ_i from the receiver to satellite i, the formula of range is given by

$$\rho_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} + c\Delta t, i = 1, 2, 3 \qquad (49.1)$$

where (x, y, z) is the location of receiver of BeiDou positioning system in the ECEF frame, $c = 3 \times 10^8 m/s$ and Δt is time difference. Because the time of users is asynchronous with the time of BeiDou, it needs the fourth equation to solve more accurate time difference. By using an elevation device, it can obtain the height h of the receiver of BeiDou. At the same time, the earth is ellipsis on the *xoz* plane. The firth equation is given by

$$\frac{x^2}{(a+h)^2} + \frac{z^2}{(b+h)^2} = 1, \tag{49.2}$$

where a and b is the long and short axle of earth respectively. Therefore we obtain the location (x, y, z) of receiver of BeiDou and the time difference by combining (49.1) with (49.2).

49.3   System Design

Kalman filter has been widely used for optimal estimation. The block diagram of the integrated BeiDou/GPS navigation system is illustrated in Fig. 49.1. Figuer 49.1 shows that the integrated navigation system consists of a BeiDou receiver, an elevation device, a GPS receiver, the computer of tri-star passive BeiDou solution, and a Kalman filter. The scheme of the integrated BeiDou/GPS navigation system uses an indirect Kalman filter-based loosely coupled technique to improve the precision of BeiDou-only navigation. Where (x_b, y_b, z_b) is position error of a BeiDou receiver, h is an elevation error, and $\left(x_g, y_g, z_g\right)$ is position error of a GPS receiver. In this system, (x_b, y_b, z_b) and h are inputs to the computer of tri-star passive algorithm, the output of computer of tri-star passive algorithm and $\left(x_g, y_g, z_g\right)$ are inputs to Kalman filter. Then, the output of Kalman filter input to BeiDou to correct position error.

49.3.1   Error Model of BeiDou and GPS

According to practical experience, the model of tracking filter of BeiDou with GPS is based on the Langevin equation [7]

$$\frac{d}{dt}v(t) = \underbrace{-\frac{1}{\tau_{vel}}v(t)}_{F(t)} + \omega(t), \tag{49.3}$$

where $v(t)$ is a velocity component, τ_{vel} is a correlate time constant, and $\omega(t)$ is a zero-mean white-noise process in continuous time. A position error model of BeiDou or GPS in terms of (49.3) is given by

$$
\frac{d}{dt}\begin{bmatrix} \delta_{pGNSSN} \\ \delta_{pGNSSE} \\ \delta_{pGNSSD} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{-1}{\tau_{hor}} & 0 & 0 \\ 0 & \frac{-1}{\tau_{hor}} & 0 \\ 0 & 0 & \frac{-1}{\tau_{vert}} \end{bmatrix}}_{F(t)} \begin{bmatrix} \delta_{pGNSSN} \\ \delta_{pGNSSE} \\ \delta_{pGNSSD} \end{bmatrix} + \begin{bmatrix} \omega_{hor}(t) \\ \omega_{hor}(t) \\ \omega_{vert}(t) \end{bmatrix}, \qquad (49.4)
$$

where τ_{hor} and τ_{vert} is a horizontal correlate time constant and a vertical correlate time constant of BeiDou or GPS respectively, $\delta_{pGNSSN}, \delta_{pGNSSE}, \delta_{pGNSSD}$ is a north position error component, a east position error component, and a down position error component of BeiDou or GPS respectively, $\omega_{hor}(t), \omega_{vert}(t)$ is a horizontal, a vertical zero-mean white-noise process in continuous time of BeiDou or GPS respectively.

Let $\left(\delta_{pGNSSN}, \delta_{pGNSSE}, \delta_{pGNSSD}\right)^T$ and $\left(\omega_{hor}(t), \omega_{hor}(t), \omega_{vert}(t)\right)^T$ be δ and ω respectively. $\delta_0(t)$ is assumed to be δ_0 and the solution to (49.4) is given by

$$
\delta(t) = \Phi(t, t_0)\delta(t_0) + \int_{t_0}^{t} \Phi(t, \tau)\omega(\tau)d\tau, \qquad (49.5)
$$

where $\Phi(t, t_0)$ is a 3-order matrix of state transition and the solution of equation given by

$$
\left\{ \dot{\Phi}(t, t_0) = F(t)\Phi(t, t_0)\Phi(t_0, t_0) = I_n, \right. (49.6)
$$

where the solution of (49.6) is given by

$$
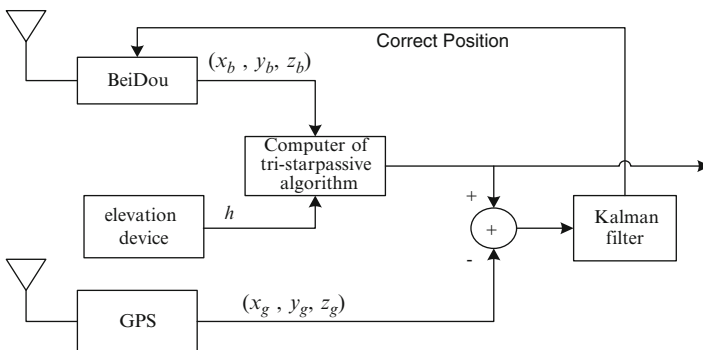\Phi(t, t_0) = e^{\int_{t_0}^{t} F(t)dt}. \qquad (49.7)
$$

Fig. 49.1 Block diagram of an indirect Kalman filtering-based loosely-coupled BeiDou/GPS integrated navigation system

49.3.2 Vehicle Dynamic Model for GNSS Receiver

In Kalman filter, dynamic models are completely specified by two matrix parameters:

1. The dynamic coefficient matrix F(or equivalent state transition matrix Φ)
2. The dynamic disturbance covariance matrix Q

In this chapter, we use a model called DAMP3 [7].This type of filter is designed for vehicles with limited but nonzero position variation such as land vehicle. The model parameters are given by

$$\underbrace{\begin{bmatrix} -1/\tau_{pos} & 1 & 0 \\ 0 & -1/\tau_{vel} & 1 \\ 0 & 0 & -1/\tau_{acc} \end{bmatrix}}_{F} \text{ and } \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{jerk}^2 \Delta t^2 \end{bmatrix}}_{Q}$$

Because Kalman filter is based on discrete state, so we need to discretize continuous state. When do equally spaced sampling on continuous state and the sampling interval $\Delta t = t_{k+1} - t_k (k = 0, 1, 2, \cdots)$ is very small, $F(t)$ is seen as a constant, i.e.,

$$F(t) \approx F(t_k) \, t_k \leq t \leq t_{k+1}. \tag{49.8}$$

therefore $\Phi(t_{k+1}, t_k) = e^{\Delta t F(t_k)}$. The discrete state equation is given by

$$\delta(k) = \Phi(k, k-1)\delta(k-1) + w(k-1), \tag{49.9}$$

where $w(k-1)$ is a zero-mean white-noise process in discrete time. The discrete observation equation is given by

$$Y(k) = C_k \delta(k) + v(k), \tag{49.10}$$

where $v(k)$ and C_k is a zero-mean white-noise process in discrete time and a coefficient matrix of the observation equation.

49.3.3 Kalman Fiter

In this chapter, the six measurements include three position outputs from the GPS receiver and three position outputs from the BeiDou receiver. By combining (49.9) with (49.10), we can obtain a set of equations [8] given by

$$\begin{cases} \hat{\delta}(k) = \Phi(k, k-1)\hat{\delta}_{k-1} + H_k(Y_k - C_k\Phi(k, k-1)\hat{\delta}_{k-1}) \\ H_k = P'_k C_k^T (C_k P'_k C_k^T + R_k)^{-1} \\ P'_k = \Phi(k, k-1)P_{k-1}(\Phi(k, k-1))^T + Q_{k-1} \\ P_k = (I - H_k C_k)P'_k, \end{cases} \quad (49.11)$$

where the coefficient matrix C_k of observation equation is given by

$$C_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (49.12)$$

49.4 Experimental Results

When BeiDou receiver and GPS receiver work normally, the state space of Kalman filter consists of 3 position components from tri-star passive BeiDou receiver, 3 position components from GPS receiver, and 9 state variables of dynamic vehicle i.e., position, velocity, and accelerate component in the horizontal level, the vertical level and the down direction. The simulation parameters of integrated navigation system are given by Tables 49.1 and 49.2. The interval of Kalman filter is 1 s, the length of simulation time is 2 h, and R is 0.1 m.

As shown in Figs. 49.2 and 49.3, the results of simulation demonstrate that an indirect Kalman filter-based loosely coupled BeiDou/GPS integration navigation system can improve the positioning accuracy greatly in contrast to GPS-only or BeiDou-only in the horizontal level and the vertical level.

Table 49.1 Parameters for simulation of GPS and BeiDou

Navigation system	Correlate time/s	Position errors/m		
—	—	North	East	Down
BeiDou	39	20.1024	15.1103	2.2930
GPS	39	10.0715	9.1029	1.5347

Table 49.2 Parameters for simulation of DAMP3

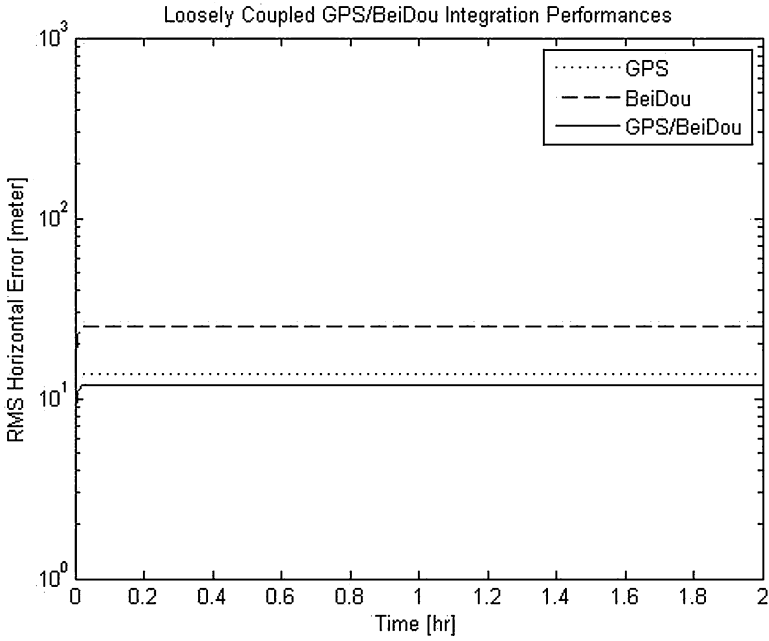| Components	North	East	Down
Position(m)	212.934	70.9768	3.5361
Velocity(m/s)	22.3017	14.8678	0.3724
Accelerate(m/s/s)	2.335	3.1134	0.0377
Accelerate correlate time(s)	13.4097	7.6696	9.6786

Fig. 49.2 Curve: performance of indirect Kalman filtering-based loosely coupled BeiDou/GPS integrated navigation system on horizontal level
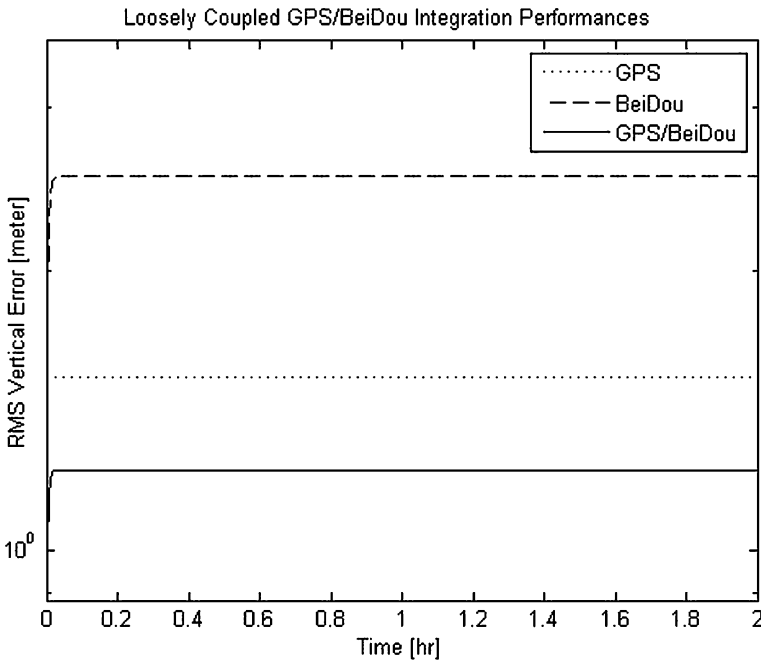

Fig. 49.3 Curve: performance of indirect Kalman filtering-based loosely coupled BeiDou/GPS integrated navigation system on vertical level

49.5 Discussions and Conclusions

An indirect Kalman filter-based loosely coupled BeiDou/GPS integrated navigation system is presented here not only to improve accuracy of positioning of DSPS but also to overcome the fatal defects of shadiness of DSPS in the war. The experimental results verify that the proposed algorithm is effective and feasible. Although the precision of DSPS is less than GPS, it provides our country with a way to explore BeiDou-II in future. In addition, these contributions can be applied not only to land vehicles, but also to the fields of ships and airplanes by necessary modifications.

References

1. Kimura K, Morikawa E, Kozono S, Obara N, Wakana H (1996) Communication and radio determination system using two geostationary satellites. II. Analysis of positioning accuracy. Aerospace Electron Syst IEEE Trans 32:314–325. doi:10.1109/7.481271
2. He X, Xiaoping H, Wenqi W et al (2009) Intelligent SINS/RDSS integrated algorithms for land vehicle navigation. Aerospace Electron Syst Mag IEEE 24:4–11. doi:10.1109/MAES.2009.4811083
3. Benlei S, Qing X, Yao Z (2008) The study on low cost passive-BD/INS integration navigation system. Microelectron Comput 25:169–171
4. Wei E, Zhang X, An Z, Feng X (2008) On the design of the protective system with the integration of GPS and Beidou navigation systems. Bull Surv Mapp 7:4–6
5. Xiaodong W, Siliang W, Li J (2009) Application of Kalman filter in BeiDou passive position. Microcomput Inf 25:247–249
6. Liu Y (2006) Passive location technique with three BeiDou geostationary satellites. Radio Eng China 36:36–39
7. Grewal MS, Weill LR, Andrews AP (2007) Global positioning systems, inertial navigation, and integration. Wiley-Interscience, Hoboken
8. Grewal MS, Andrews AP (2008) Kalman filtering: theory and practice using MATLAB. Wiley-IEEE Press, Hoboken