

# The Changing Role of Optimization in Urban Planning

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**Abstract** Most world cities are now planned in one way or another. Through the deliberate positioning of activity and transportation facilities, urban authorities hope to ensure the success of their cities in economic, social and environmental terms. Urban planning models are an important tool to help them in this task, and in this chapter, we examine the use of optimization techniques in urban planning modelling. Through a broad review of the field, we highlight the distinction between single-goal urban-environment models and multi-objective land use and transportation models. While it is shown that optimization no longer plays a stand-alone role in land use and transportation modelling, it does contribute to the overall modelling workflow. Furthermore, optimization forms the basis of two niche applications: excess commuting and sketch modelling. This last field holds the most promise for the future, enabling planners to establish minimum resource consumption benchmarks for their city as a means of comparison with other cities and to evaluate the ambition and feasibility of new plans.

**Key words** Cities • Urban planning • Mixed integer linear programming • Review

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## 1 Introduction

A city's character is greatly shaped by the organization of space and activities within its boundaries. This sense of place is partly an aesthetic attribute: by invoking the name of major world cities, we can quickly picture their structure and form such as the grid-iron streets and high-rises of New York or the hillside favelas of Rio de Janeiro. However, the city's social, economic and environmental performance is arguably the more important consequence of urban form.

For much of human history, cities evolved in an organic fashion “without preconceived planned intervention” [39, p. 10]. Natural determinants such as topography, climate and the availability of construction materials were major driving forces, shaping both architectural styles and activity location. The requirements of religion, politics, defence and logistics also played a role [39]. While the resultant forms may look random and uncoordinated, research has demonstrated a number of possible organic growth mechanisms including “preferential attachment” to existing settlements and transport networks [1, 14], economic processes (such as von Thünen's 1826 model of land rents) or analogies with physical processes such as diffusion-limited aggregation and dielectric breakdown [5].

The shift towards a more active form of urban planning often arose in response to the limitations imposed by haphazard urban growth. In Renaissance Rome for example, the tightly woven medieval structure of the city began to place significant constraints on the health and mobility of citizens and visitors. One of a number of planning popes, Sixtus V (1585–1590) located four major obelisks throughout the city to guide future planners in the construction of major thoroughfares that could connect prominent piazzas and churches. Similarly Haussmann's boulevards were an intentional effort to reshape Paris in response to changing defence requirements, inadequate sanitation and other factors [39]. While the specific constraints may vary over time and by location, the planning departments of modern cities essentially fulfil the same function: to create vibrant thriving urban areas subject to limitations of land, resources, finance and time. It is worth noting however that planned urban forms cannot be divorced from organic growth processes. Planned activity and transportation developments create opportunities for new patterns of urban living, which in turn need to be accommodated by new plans and construction. This cycle can be seen as the feedback loop which drives the growth of urban systems [52].

For the purpose of this chapter, we can broadly define urban planning as the policies that configure patterns of land use, associated activities and transportation. Urban planning is an interdisciplinary field, incorporating the expertise of architects, engineers, economists, sociologists and others. The planning process is necessarily a compromise between competing interests and multiple stakeholders, each of which may hold very different views about what constitutes a liveable neighbourhood or an effective strategic plan. In this context, it may seem that a technique as deterministic as optimization (or mathematical programming) has little to offer. However, this chapter will demonstrate that optimization techniques have been widely used in urban planning, although their precise contributions have shifted as other modelling techniques and the needs of analysts have changed.

This chapter is organized as follows. In Sect. 2, we provide an overview of the major uses of optimization modelling in urban planning over the past 50 years. The aim is not to provide a comprehensive review but to illustrate the range of applications, the specific techniques used, and to identify the reasons why mainstream urban planning analyses tend not to use these techniques now (or rather, do so indirectly). Section 3 then considers two current urban planning applications where optimization models are more commonly used, the fields of “excess commuting” and sketch planning. We review the structure of these models and offer comment on their formulation and applicability in various circumstances. Section 4 concludes by considering how optimization techniques for urban planning might evolve in future, concentrating on their use in the design of eco-cities.

## 2 Past Applications of Optimization in Urban Planning

The modern history of optimization might be said to begin with Dantzig’s 1947 simplex algorithm for linear programming. Since then, advances in algorithms and computing technology have helped the field to expand, and mathematical programming models are now used in a variety of disciplines and formulations. This section provides an overview of the use of optimization in urban planning. First, we present a top-down review to identify major categories of practice and the types of optimization techniques employed. We then narrow the scope and work from key review articles to describe the major trends in the specific area of urban land use and transportation (LUT) planning.

### 2.1 Top-Down Review

We began our review by considering which general urban planning fields employ optimization techniques. To do this, we searched the ISI Web of Knowledge index<sup>1</sup> for the terms “(optimization OR optimisation) AND (urban OR cities OR city) AND (planning)” in both the topic and title fields. This led to 581 results, broken down into the subject areas shown in Table 1. While there is clearly a bias towards the more numerate subjects, the list of disciplines is very diverse. As noted in the introduction, the urban context attracts researchers from a wide variety of fields, and it is interesting to see that optimization offers at least some insight within all of these disciplines.

After inspecting the results, the query was further limited by adding “AND land use” to the search terms. This avoids a large number of papers that focus on topics not directly related to the question of urban land use. This includes work on the

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<sup>1</sup>[www.isiknowledge.com](http://www.isiknowledge.com).

**Table 1** Top ten subject areas for urban planning and optimization papers, as found by an ISI Web of Knowledge query

Subject area	Number of papers
Engineering	229
Environmental sciences and ecology	222
Computer science	123
Business and economics	110
Water resources	105
Mathematics	101
Public administration	69
Transportation	56
Public, environmental and occupational health	52
Agriculture	46
Total	581

See text for query phrasing

**Table 2** Cross-tabulated categorization of selected urban planning and optimization papers; values indicate number of papers

Topic	Optimization method					Total
	LP	NLP	MOO	Other	Unknown	
Land use	4	2	13	4	6	29
Transport	2	0	2	1	2	7
Ecology	4	2	1	0	4	11
Water	0	2	3	2	1	8
Total	10	6	19	7	13	55

*LP* linear programming (including mixed-integer variants), *NLP* non-linear programming (including mixed-integer variants), *MOO* multi-objective optimization, *Other* hybrid methods such as cellular automata or agent-based simulation with an optimization component, as well as random and grey optimization, *Unknown* studies which do not specify the technique used

planning of large infrastructure systems without an explicit land use component (e.g. ant colony optimization applied to electricity networks, [16] or long-term water supply portfolio planning, [26]) and detailed operational optimization and control problems (e.g. for traffic light timing, [44]). The narrower search terms resulted in a more manageable 61 unique records. These papers were then categorized according to the optimization techniques used and the field of application. The cross-tabulated results shown in Table 2 therefore provide an indication of practice in this area. Six studies from the original sample were removed as they did not actually apply mathematical programming techniques, but rather referred to “optimization” in a non-technical manner (often as a synonym for improvement). From a methodological perspective, the table shows a mix of linear and non-linear, single and multi-objective formulations. The formulations are often, but not always, of mixed-integer form where integer variables are typically used to represent the classification of a discrete land use parcel [e.g. 10, 15]. Multi-objective approaches are commonly used as well and are typically solved by genetic algorithm [e.g. 3, 43], simulated annealing [e.g. 11] and to a lesser extent single aggregate objectives [e.g. 29].

Although the data set is diverse, a rough split in problem type can be seen. First, there are those studies which apply optimization techniques to examine the ecological impacts of urban development (the “ecology” and “water” categories above). In these cases, the natural environment imposes constraints on a city’s growth and so might be called *urban-environment* planning models. For example, [10] describe how the growth of urban areas and pressures from agriculture can lead to fragmentation and degradation of nearby habitats, with negative impacts on biodiversity. They therefore use spatial optimization, based on a mixed-integer linear programming formulation, to allocate land function so that the minimum number of locations need to be actively restored after development. Constraints on the problem include overall areal targets for intact habitats and restrictions imposed by the local geography (e.g. soil types, vegetation). A similar approach is adopted by [38], who use non-linear programming to minimize impacts on water resources from urban growth policies.

The second set of problems focuses on urban planning in a more traditional economic or social context (the “land use” and “transportation” (LUT) categories); [2] is a typical example. Here a genetic algorithm is applied to determine future land-use and transportation provision for a growing city, subject to constraints on housing provision. The problem is one with multiple objectives such as minimizing cost, disruption, and traffic congestion. Multi-objective frameworks are significantly more common in this category, compared to urban-environment models that may have a narrower focus (42% of LUT models, 21% of urban-environment models,  $\chi^2 = 5.55, p = 0.018$ ).

Two final points from this brief survey. First, researchers often combine optimization with other techniques, particularly for spatial analyses with cellular automata (e.g. for forestry planning, [37]) or agent-based models (e.g. for biodiversity planning involving multiple stakeholders, [27]). In these cases, the optimization routines can be embedded within heterogeneous agents to simulate the behaviour of individuals within a more complex interactive system. The second issue is that the selected papers span from 1979 to 2010 (a limitation of the ISI data set) and are therefore likely to be missing some of the early applications of these techniques to urban planning. The second part of this review will therefore provide more detailed perspective on the evolution of the field.

## 2.2 Optimization in LUT Models

For this second review, we limit our definition of urban planning to incorporate only the LUT sectors. Although environmental motivations are increasingly important, urban planning is still primarily concerned with creating economic and social opportunities [for an overview, see 41]. Quantitative work on the relationship between urban form and function typically falls under the general title of LUT modelling.

Review studies identify four major urban modelling approaches—regression, optimization, aggregate spatial models, and disaggregate individual models (including both random utility frameworks and activity-based models) [9, 20, 35]. Over time, the field has trended towards increasing behavioural realism and disaggregation. That is, whereas earlier studies could only simulate a few districts and had to aggregate all activity supply and demand within that zone, advances in computing power and model formulations mean that behaviours at the level of the individual or household can now be simulated (e.g. by modelling individual choices within an econometric random utility choice framework). The motivation for this shift can be explained by the operational use of these models, i.e. their deployment in real cities to answer policy questions such as how a city might expand over time [51], where people choose to live and work within cities [50], and which modes of transportation will be used to facilitate urban travel [20]. Greater behavioural fidelity allows these models to test sophisticated policy interventions and increases confidence that the salient processes have been effectively represented: “the value of more complex, behaviourally valid, microscopic models is not that one obtains microscopic forecasts, but that one obtains macroscopic forecasts based on microscopic principles” [48, p. 239].

However, this trend has meant that optimization has, over time, taken on a secondary role within urban LUT modelling. Several early studies used optimization as the primary technique to determine activity location and traffic flows within a city. For example, the earliest operational LUT optimization model appears to be TOPAZ from Australia [12, 46]. This model sought to allocate activities to discrete zones minimizing total cost from construction and travel. However, even in its early stages, limitations on data inputs and computational ability made these models impractical for everyday use [18, 32, 35]. Yet while optimization models have “all but disappeared” as stand-alone tools [52, p. 7], the techniques are still used in conjunction with more mainstream LUT modelling approaches, as so-called “combined” models [e.g. 9, 33, 42]. These tools use optimization to determine transportation costs endogenously, capturing spatial interactions and user behaviour in a more realistic fashion. More generally, optimization remains a useful technique within LUT models, for example to perform mean square error fitting of econometric models as a preliminary step in random utility choice modelling or to calculate market clearing equilibrium prices for land and transportation [52].

### **3 Present Applications: Excess Commuting and Sketch Planning**

There are urban modelling niches within which optimization remains a valuable primary modelling methodology. It has been acknowledged that optimization techniques have the potential to find “extreme solutions” and to meet a specific objective during the “preparation of plans” (i.e. to be used in a normative fashion)

rather than forecasting detailed descriptive behavioural patterns over time [35, p. 330-331]. The use of optimization models for designing hypothetical optimal configurations, but not necessarily in assessments of existing cities, is also supported by [20]. Two such applications which will be discussed here are excess commuting and sketch layout modelling.

### 3.1 Excess Commuting

Excess commuting can be defined as commuting longer or further than suggested by the actual spatial layout of a city and assumptions about rational commuter behaviour [19]. Metrics based on this concept help analysts to compare the efficiency of urban layouts and suggest strategies for re-development and rationalization of spatial activity patterns. To estimate the level of excess commuting, original work in this field assumed a stylized monocentric city and found that, for a range of US and Japanese cities, average commuting journey distances were approximately 8 times greater than might be expected if commuters tried to minimize their average commute [17]. However, [53] reinterpreted this issue as a linear programming problem showing the level of excess commuting to be on the order of 11%. Research since 2000 has resulted in average estimates of excess commuting in the range of 50–70%, and the optimization framework is now the most common method of calculating benchmark minimum and maximum commutes for a given spatial layout [34].

The basic formulation of the problem, based on White’s paper [53], is as follows:

$$\begin{aligned}
 &\text{Minimize} && \frac{1}{N} \sum_i \sum_j c_{i,j} n_{i,j}^* \\
 &\text{subject to} && \sum_i n_{i,j} = \sum_i n_{i,j}^* = D_j \\
 &&& \sum_j n_{i,j} = \sum_j n_{i,j}^* = O_i,
 \end{aligned}$$

where  $c_{i,j}$  is the cost of commuting from zone  $i$  to  $j$  (e.g. distance or time),  $n_{i,j}$  is the existing (exogenous) number of commuters travelling between zones  $i$  and  $j$ ,  $n_{i,j}^*$  is the optimized (endogenous) number of commuters,  $N$  is the total number of commuters,  $O_i$  is the number of workers living at zone  $i$ , and  $D_j$  is the number of workers employed in zone  $j$ . Readers familiar with operations research will recognize this as an example of the *transportation problem*, wherein the goal is to connect suppliers and customers with a network at minimum cost, subject to constraints on customer demand and supplier capacity [see 55]. It assumed that network capacities and transport requirements are specified at the outset. In the

urban context, this is analogous to knowing where people live and work and trying to determine the lowest cost routing for customers, i.e. the minimum flow of visits from home to each activity over the course of a year. This therefore results in an idealized measure of commuting flows that can be compared with observed flows and normalized to calculate the excess commuting statistic.

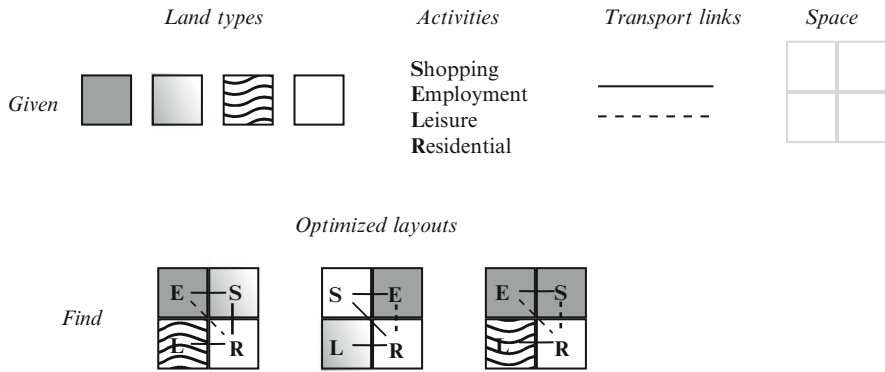
Optimization-based excess commuting has been applied in a variety of locations, but [34] highlight some common themes and outstanding questions. First there are methodological questions regarding biases in the calculation of the excess commute statistic. For example, measured data may need to be aggregated at different scales depending on availability and tractability. However the value of excess commute statistic depends on the chosen scale, with more aggregated spatial data resulting in a smaller estimate of excess commuting. This suggests that for accurate estimates, highly disaggregated data is preferred; however, this may create computational difficulties. A second issue is contextual, i.e. is the estimate a meaningful indicator of possible commuter flows? The technique described here relies upon an aggregate approach, i.e. considering travel to and from given urban areas for a particular activity class as a single homogeneous flow. However, more recent studies have looked to disaggregate individuals and households within these calculations, for example, to account for households with two working persons who need to live together but work in separate locations [7, 40]. Finally there are questions about the suitability of the measure for policy decisions, for example, on zoning regulations, housing policy, and road pricing.

The use of optimization modelling for the calculation of excess commuting is therefore an important application of the technique, even if theoretical and practical questions remain about its validity. However, a key point is that excess commuting measures are focused primarily on explaining observed behaviour in *existing* cities, where the locations of work, residence and other activities are exogenous inputs to the benchmark calculation. When considering new cities, these activity locations can be endogenous to the model so that transportation requirements are jointly determined with the location of individuals and activities. For this problem, we turn to a second current application of optimization: sketch models.

### 3.2 *Sketch Models*

Sketch models assess both activity location and transportation flows within an optimization framework; Fig. 1 provides a schematic overview of the technique. Clearly when compared with most operational urban planning models, sketch models provide an incomplete representation of LUT dynamics. When used in the context of master planning and benchmarking processes however, their reduced data requirements enables models to be tested quickly against multiple scenarios. The results can be then tested within a more rigorous LUT modelling framework as required.





**Fig. 1** A schematic description of a sketch model, based on [30]. The various combinations of land uses and transportation flows are assessed within an optimization framework according to objective functions which vary with the chosen application

Although [4] was not the first to use these techniques, his paper provides an excellent overview of the approach and illustrates a typical application wherein three goals are pursued in a multi-objective framework: minimization of land development costs, maximization of residential accessibility, and minimization of transportation energy costs. This work has been extended and revised by others, most notably in the sketch layout model [13, 30]. In its various incarnations, this model seeks to generate alternative master plan sketches as an input to participatory planning processes. Again a multi-objective approach is adopted and the model considers “harmony” (the similarity of adjacent land uses), “relevance” (the compatibility of adjacent land uses), and “traffic accessibility” (the shortest path between two cells). Related work in this area has examined the positioning of individual facilities within a city [e.g. shopping malls, 56], layout of space within buildings [47], the planning of development densities around transit stations [31], and the use of multi-objective optimization to generate a range of Pareto optimal layouts for discussion with planners [22].

The general formulation for sketch models is essentially a hybrid of two canonical operations research problems [see 55]. The first is the *transportation problem* discussed above. The second problem is the *assignment problem*, i.e. finding an optimal combination of tasks and agents where each pairing incurs a given cost. This problem can be formulated as follows:

$$\begin{aligned}
 &\text{Minimize} && \sum_{i,j} t_{i,j} x_{i,j} \\
 &\text{subject to} && \sum_i x_{i,j} = 1 \quad \forall j \\
 &&& \sum_j x_{i,j} = 1 \quad \forall i,
 \end{aligned}$$

where  $x_{i,j}$  equals 1 if person  $i$  is assigned to job  $j$ , else 0, and  $t_{i,j}$  is the cost of the assignment. Although it appears to be a mixed-integer problem, it can in fact be solved as an LP owing to the “integer-in-integer-out” properties of a network model [55]. In an urban context, the assignment problem can be seen as the task of allocating activity provision to different land areas, where “activity” might be interpreted variously as work, schools, residential housing and so on. The model can also be split so that building and activity types are assigned separately. In the case below for example, which focuses on minimizing urban energy consumption, there might be two building types that can support a single activity category.

The hybrid formulation is known as the *facility layout* problem, and while there are multiple forms of the problem’s definition, the general aim is to determine the position of processes within a factory so that the combined costs of performing a task at a given work station and moving materials between each work station are minimized. Recent examples of this literature include [8, 28, 49]. As noted above, when planning a new city, there is no a priori reason for taking the location of housing and activities as fixed. Therefore a joint problem can be constructed which seeks to minimize the cost of assignment both function to land plots, and the travel required to move between those two locations.

The general formulation can be adapted with many application specific constraints. For example, in a factory layout problem, each piece of equipment might have a certain footprint and therefore require a certain amount of space. An analogous situation exists in the sketch planning case, where certain activities might require a minimum site area (e.g. for a school with a playground) or minimum total area (e.g. sufficient green space is provided for the whole city). An optimization-based sketch model can also handle constraints such as the capacity of transportation network links, housing requirements for the population, or prohibiting certain kinds of development on particular land plots.

### ***3.3 An Example: Calculating a Minimum Energy Urban Layout***

In our own work we have applied sketch models to the question of eco-cities, focusing particularly on urban energy consumption. Cities are major energy consumers, accounting for an estimated 67 % of global primary energy demand and 71 % of energy-related greenhouse gas emissions [21]. While there is some dispute about the precise allocation [45], it is clear that urban energy efficiency must be improved if economic and cultural opportunities are to be maintained while avoiding the worst environmental effects.

Urban energy consumption is the consequence of decisions taken at a variety of spatial and temporal scales. Using domestic energy consumption as an example, the temporal scale spans from short-term decisions such as when to use appliances (seconds to days), medium-term choices about which appliances to purchase (months

to years), and long-term decisions about the built fabric of the home (decades to centuries). For example, [6] note that, at current rates, the UK's housing stock could take approximately 1,300 years to be replaced completely. Variations in spatial scale also have a strong influence on energy consumption. End-use energy conversion technologies such as household gas boilers may be relatively easy to reposition or replace for improved efficiency but large infrastructure systems, such as resource distribution and transportation networks or the location of buildings and activities, are more persistent [e.g. 39]. Consequently the layout of urban environments is perhaps the most difficult aspect of improving urban energy efficiency.

For existing cities, urban expansion and retrofit projects can lead to improvements in energy consumption as seen in London's Canary Wharf and La Défense in Paris [54]. However, working within the constraints of existing infrastructures is expensive and difficult and so new construction arguably offers the greatest opportunities to create energy-efficient cities. In recent years, the eco-cities movement, both in the UK and abroad, has created visions of new sustainable urban areas [23], such as Masdar, the world's first "carbon-neutral zero-waste city" near Abu Dhabi in the United Arab Emirates [36]. However, these ambitions raise serious questions about the limits of low-energy urban forms, boundaries which must be identified if new developments are to set realistic goals and existing cities are to understand their improvement potential.

Not all of the energy issues listed above can be dealt with in an optimization model, or if they can be addressed, they may impose significant computational costs. However, a sketch model can be used to examine some key trade-offs such as the balance between transport capacity (i.e. the maximum number of trips that can occur on a route) and the maximum capacity of an activity site (i.e. the number of visitors that can be satisfied at a given location).

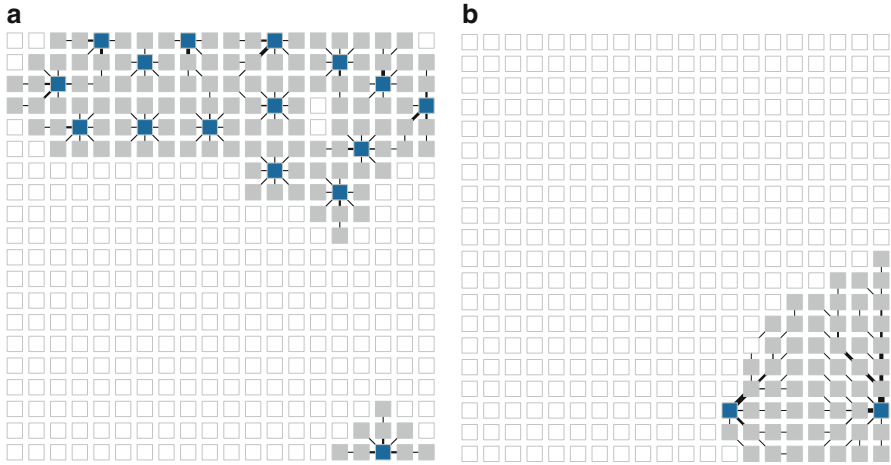
Our sketch model is described in full in [24] and contains both the core elements of an optimization-based sketch model, as described above, as well as some energy-specific features. As input, users provide information about:

- The population of the city
- The generic housing and transportation types that are available
- The spatial layout of the city (i.e. the location and size of the empty zones to be populated)
- The activities to be performed by the population

With the objective of minimizing "cost" (in this case, annual energy consumption), the model will then determine:

- The location of buildings and activities
- The location of network connections
- The number of daily trips from zone  $z$  to  $z'$  by mode  $m$
- Other summary information (e.g. passenger km by mode)

Constraints on the problem generally fall into two categories: feasibility constraints (i.e. those that are necessary to obtain a valid solution, for example, that all



**Fig. 2** Minimum energy layouts for the provision of work and housing under different assumptions. *Light grey cells* represent domestic housing, blue work locations. Transport links are shown in *black arrows*, with width proportional to the flows. Population of 100,000 in all cases. (a) Residential housing 60 dw/ha, each work site offers 3,200 jobs (b) Residential housing 130 dw/ha, each work site offers 48,000 jobs

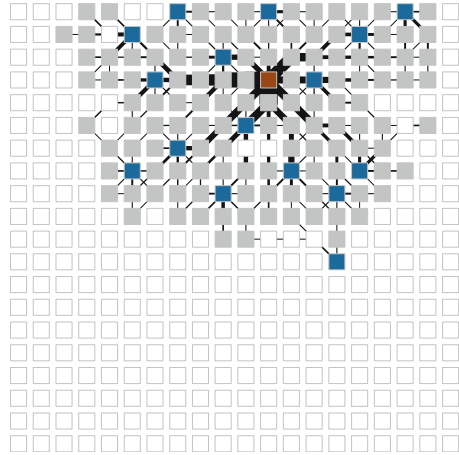
citizens must be housed) and context constraints (i.e. additional restrictions to reflect planning laws or other user-specified concerns).

In this particular problem, the goal is to house 100,000 citizens and provide them with sufficient work. Each cell is 16 hectares and from UK statistics, it is assumed that 48% of the population works. Figure 2a shows the results when we assume that the housing density is low (approximately 60 dwellings per hectare) and each work site can provide only 3,200 jobs (equivalent to a small office). However, in Figure 2b, higher-density housing is used (130 dwellings per hectare), and a single work site can employ 48,000 people (roughly equivalent to a dense central business district). In both cases, we have not added any binding constraints on transportation flows. The high-density case delivers an energy saving of approximately 15%, accounting for both building and transport demands.

A closer inspection of these figures reveals the stylized nature of the analysis. In the low-density case, each work cluster is completely isolated; there are no traffic flows between centres as the model is able to satisfy work demand through these local “village” offices. Even in the high-density case, a clear divide between the two halves of the city can be seen. However, if one adds further activities besides work, the structure begins to change. In Fig. 3, a shopping activity is added and, when a single site is large enough to satisfy the city’s resultant demand (e.g. a shopping mall), it is centrally located by the model so that all citizens can access it with minimal travel requirements.

The distributed structure of these results matches well with the predictions of Christaller’s 1933 central place theory, which suggests that, for an unobstructed

**Fig. 3** A low-density layout for a hypothetical city with a central shop



landscape with an evenly distributed population, activity clusters should emerge at the centre of population areas subject to constraints on minimum market (threshold) and maximum distance (range) [41]. However, central place theory also predicts that individual local activity clusters feed into the demands for similar clusters at larger scales, in a hierarchical fashion. For example, a city may have distinct entertainment and business districts, but within each of these larger clusters, there will also be smaller local provision of these activities. Such a comparison therefore highlights the lack of multiple spatial scales within a simplified sketch model. To date, sketch models have tended to focus on a single spatial scale as the initial problem specification is built around discrete plots with an intended homogeneous purpose. This limitation is one reason why the results of such models are best used to inform planning discussion of smaller developments, where each zone contains a discrete activity, rather than to provide definitive plans or for the analysis of larger systems. Finally, a note about the optimization process itself. As a combinatorial optimization, the tractability of a sketch layout model is limited by the number of cells and activities to be positioned. Table 3 compares the capabilities of historic sketch models but note that these figures do not represent the limits of performance but the application size used for each study. While the table does show improvements over time, the problem remains fundamentally difficult. Looking at the solution in Fig. 2a for example, it can clearly be seen that a number of rotational and translational symmetries exist within the resulting structure. Therefore to improve performance and reduce degeneracy, it is useful for the user to provide some sensible constraints on the problem. This may include fixing an “anchor” activity at a given location within the model, in addition to using standard termination criteria for mixed-integer models such as a timeout or optimality gap.

**Table 3** Comparison of previous urban layout optimization studies

Study	Case study	Computer set-up	Formulation	$n_c$	$n_a$
[4]	Germantown, WI, USA (7,000 people)	–	LP	9	11
[13]	Tanhai, Taiwan (300k people)	IBM/PC 486–33 with Turbo Pascal 6.0	MINLP	13	10
[30]	–	IBM Pentium II 300 MHz with Turbo Pascal 6.0	MINLP	13	7
[31]	Taipei central business district	–	LP	–	–
[56]	Dalian, China (2.23 million)	8 CPU cluster with C++	NLP	300	2
This paper	Generic UK	500 CPU cluster with GAMS/CPLEX 9.0 (though run on one core)	MILP	400	3

Other variables are used in these models, but only the core assignment problem variables, i.e. the number of cells (i.e. discrete zones within the model,  $n_c$ ) and number of activities (i.e. land use categories,  $n_a$ ) are shown here for an indicative comparison

Model formulations: *LP* linear programming, *NLP* non-linear programming, *MILP* mixed-integer linear programming, *MINLP* mixed-integer non-linear programming

## 4 Future Applications and Conclusions

This chapter has shown that optimization is a widely used technique in the urban modelling community. While in the past it was used on its own to determine LUT patterns, it has now fallen out of favour as a stand-alone technique with more behaviourally realistic models based on a disaggregated view of urban activities dominating current practice. As [9, p. 345] notes in his review, mathematical programming models have the advantage of “a simple mathematical form linked to system efficiency; however, the aggregate nature of the model means that there are inherent difficulties in representing the systematic properties of locations and the behavioural context of decision-makers.” However, even in these problems, optimization is used to fit statistical models and in hybrid modelling applications, e.g. in conjunction with agent-based modelling.

Chang also observes that LUT modellers have focused “too much detail of the issues rather than the refinement of the foundational relationship.” (p. 346). This suggests that a new look at the role of optimization modelling in urban planning might be in order. In Sect. 3, we showed that optimization remains a popular

technique in two niche applications: the determination of minimum commuting configurations and the rapid creation of sketch layouts early in the planning process. With the excess commuting literature pushing towards increased disaggregation and behavioural realism, sketch modelling seems like the most promising area for continued work on a purely optimization-based form of aggregate urban planning.

Early sketch modelling applications sought to contribute to the planning process by generating alternative plan ideas early in the planning process. This is still the general goal of such models, but the specific objective should be reworked slightly. Instead of focusing on realistic looking alternative plans that “rationally” balance multi-objectives, we would argue that there is significant scope for using sketch models to develop extreme scenarios with a specific goal in mind. In particular, the eco-cities movement has grand visions of low-impact urban settlements, driven by concerns over specific issues like carbon emissions. A resource-based sketch model could therefore be used to establish minimum benchmark values, i.e. patterns of development that meet basic goals of activity provision and housing with the lowest possible resource consumption. It is not envisioned that such plans would be built directly, but that by establishing a minimum benchmark, stakeholders could evaluate the ambition and difficulty of their actual designs in a more quantitative manner. In many ways, this application is similar to the urban-environment models highlighted in the review above. The emphasis is not on necessarily on multi-objective optimization, but on the pursuit of a single goal with the aim of identifying the limits of practice (although multi-objective optimization might still have a role as a goal like a “low-energy” city might have multiple energy-specific objectives such as carbon emissions and security of supply).

However, there appears to be at least three major obstacles or challenges in this field. First there is a question of scale, both spatial and temporal. Our analysis to date has focused on snapshot optimizations, as would be required to inform a single planning decision. However, resource infrastructure systems take decades to develop and must continually adapt to the needs of an evolving city. Multi-period optimization to look at minimum resource development pathways over time is therefore a promising area of research. The appropriate spatial scale, is also an issue. As identified above, most sketch models have tended to focus on a single spatial scale whereas the structure of cities consists of nested spatial scales. Hierarchical optimization methods might offer valuable insights here. This could offer performance improvements as well, for example, by solving a simplified relaxed version of the problem at a coarse spatial scale and then introducing integer variables to allocate homogeneous land functions at a local level.

The second major question is model fidelity. If sketch modelling is to be used to estimate the minimum resource consumption layouts for a city, what level of detail is needed by decision makers and can the models provide this? Taking the decision-makers perspective first of all, we can imagine a scenario where the goal is to establish a minimum energy baseline for a city. In such a scenario, building energy demands might be parameterized by means of normalized benchmarks (e.g. in kWh per square metre). However, the decision-maker may want to know if these demands can be reduced through demand side measures, such as flexible pricing,

or if higher per square demands are even relevant if the primary fuel source is low carbon. Clearly the more factors that become endogenous to the model, the more difficult it will be to construct and validate. The corollary to this problem is one of computational ability. Ideally, because sketch models represent a single optimized solution, they should be run multiple times to capture the uncertainty of input parameters and the range of possible outcomes; the goal should be to deliver a distribution of minimum resource benchmarks, not a single value [as in 29]. However, such a goal is in conflict with the use of a sketch model as a tool to quickly inform planning at an early stage. Multiple model runs will need to be solved and the mixed-integer formulation used here can result in slow solve times if not properly formulated. This can be partly resolved by parallel computing, but improved knowledge of the problem description and relevant heuristics will be valuable.

The third question is whether optimization is indeed the most effective technique for identifying the limits of feasible urban performance. Certainly its use in excess commuting and the simple examples shown here demonstrate that the basic idea is feasible, but it is uncertain what other techniques might be used to the same purpose. A specific question is whether or not the method works for existing cities. In new cities, a relatively unconstrained optimization makes sense; however in a city with substantial existing infrastructure, it is not clear whether the city has sufficient degrees of freedom to make a sketch model optimization meaningful. A more sensible approach in this case might be data envelopment analysis (which of course is also based on optimization) to identify the relative efficiency and performance of other cities.

Future research in this area should develop the concept of resource-based minimum benchmark urban plans and critically assess them in a variety of contexts (both locations and resource categories). One example is to combine these benchmark models with optimization-based models of resource supply systems. In [25], we examined the design of an eco-town by considering the layout and energy supply systems as separate optimization problems; however, the models could be combined to offer minimum energy benchmarks that consider both the supply and demand sides. While ultimately it may be found that a pure optimization-based approach is insufficient to capture the complexities of the urban environment, the history of the field suggests that optimization will continue to have an important role to play within the implementation of other techniques.

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