Chapter 4
Beyond Deficit Models of Learning Mathematics: Socio-cultural Directions for Change and Research

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Abstract Major paradigmatic changes in mathematics education research are drawing attention to new perspectives on learning. Whereas deficit models were previously in the foreground of research designs, these have been replaced by a wide variety of theoretical directions for studying diverse approaches to learning mathematics. There is now an acceptance of the need for richness and variety in research practices so that approaches can be studied, compared and mutually applied and improved. Psychological and quantitative approaches and methods are now increasingly complemented, or even replaced, by new directions that rely on social and anthropological theories and methods. Rather than reviving ideas about deficit research in mathematics education, the aim of this chapter is to present some socio-cultural perspectives of mathematics learning, and to show how these perspectives go beyond the deficit model of learning. Framing the main traditional markers of discrimination in school mathematics—gender, social class and ethnicity—in a perspective of social justice, the chapter concludes with a reflection on equality in terms of the democratic principle of meritocracy in mathematics education.

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The assumption that people of low socio-economic background, or of different genders or ethnic groups, are intellectually less capable than others has deep implications in society. In educational settings, this assumption is particularly perverse, for it strongly influences the development of policies and practices for dealing with differences as markers of segregation. Under the label of "deficit model"—sometimes labelled as deficit thinking, deficit theory, the deficit paradigm, and the deficit discourse of learning—from the early 1960s, in the USA, for example, the deficit assumption seemed to adopt a definite theoretical perspective in attempts to explain why students "failed." Social and political factors embedded in the educational system, which favoured segregation among groups of students, were ignored. In other countries—such as Australia (with Aborigines), New Zealand (with indigenous peoples), the UK (with immigrants), the Netherlands (with immigrants), South Africa (with Black and poor populations) and Brazil (with poor and indigenous people)—the same debate occurred, not so much under the label of *deficit*, but in relation to the alleged deficit transmitters under investigation: gender, social class, race, culture or familial context.

Richard Valencia (2010) described six main characteristics of the deficit model in the educational context:

- *Victim blaming*. The deficit model of learning links the school failure to a membership community. It attributes the performance of poor students, students of color, students of different genders and ethnic groups to their alleged cognitive and affective deficits.
- *Oppression*. The deficit model holds little possibility of success to these students, privileging some and oppressing others.
- *Pseudoscience*. Deficit research draws on deeply negative bias in relation to persons of color, of different genders, of low socio-economic class and minority culture, "basing their research on flawed assumptions, using psychometrically weak instruments, not controlling for key variables" (p. 95), and communicating their findings in proselytizing ways.
- *Temporal changes*. The deficit discourse varies depending on when they are made. Alleged deficits can be transmitted by low-grade genes, gender, minority culture, social class, familial context, and other related transmitters.
- *Educability*. The deficit model often goes beyond the description, explanation and prediction of elements of poor students, students of color and of different genders, classes and cultures. It is also "a prescriptive model based on educability perceptions" (p. 18) of these students.
- *Heterodoxy*. The deficit model reflects the "dominant, conventional scholarly and ideological climates of the time. Through an evolving discourse, heterodoxy has come to play a major role in the scholarly and ideological spheres in which deficit thinking has been situated" (p. 18).

Taking the USA as the scenario of his critique (which can be fairly extended to other countries), Valencia (2010) carefully scrutinized several North-American documents, dismantling the fallacy of the deficit discourse. One of his main conclusions was that:

> Students are not at risk for academic problems due to their alleged deficits. Rather, schools are organized and run in such oppressive ways (e.g., inequities in the distribution of teacher quality characteristics and inequities in the distribution of economic resources for schooling) that many students are placed at risk for school failure. (p. 125)

At the time deficit models were in the foreground of research designs, the work developed by Klineberg (1935) brought an important psychological contribution in challenging the assumption that certain racial groups are intellectually inferior to others. Klineberg studied Black American children's IQ scores, and showed that they can be directly affected by environmental circumstances. Klineberg's research did not reach to a definite conclusion about the specific role of the environment over these achievements but, as stated by Lieberson (1985), it did "present evidence that such events can occur, that IQ is at least affected by the environment, and that judgments on a rather clear-cut matter can be altered by the influence of a social group" (p. 220).

Similar reactions on intelligence tests among different ethnic groups appeared in the works of Bruner (1990), Canady (1936), Cole (1985), Gould (1995), Long (1925), Menchaca (1997), Thomas (1982), and Van der Veer and Valsiner (1991), among others. All these works share somehow the conclusion that intelligence tests measure the familiarity of certain minority groups with the culture and language proficiency of dominant groups, not intelligence. Bruner (1990), for instance, questioned some ideas concerning the relationships between school learning and development and *intellectual prowess*. Without any reflection about what exactly we want to mean by intellectual prowess, said Bruner, we decided "to use school performance as our measure for assessing 'it' and predicting 'its' development" (p. 26). For Bruner, a definition of intellectual prowess or successful performance intimately depends on which traits a culture selects to honor, reward and cultivate. So, whatever definition of these terms is used, that definition should lead us to issues concerning the use we wish to make of them in "a variety of circumstances—political, social, economic, even scientific" (p. 27). This is to say, the cognitive development of the individuals cannot be evaluated out of the culture they are inserted in, and the operatory power and limits of theoretical models of learning and development adopted by diverse research lines must be analyzed in their emergent political-historical context.

By the 1970s, deficit research designs began to be challenged by a wide variety of theoretical perspectives of learning. Since then, there has been an acceptance of the need for richness and variety in research practices so that approaches can be studied, compared and mutually applied and improved. Psychological and quantitative approaches and methods have been increasingly complemented, or even replaced, by emergent approaches that rely on social and anthropological theories and methods.

Rather than reviving ideas about deficit research in mathematics education, the aim of this present chapter is to present some socio-cultural perspectives of mathematics learning, and to show how these perspectives go beyond the deficit model of learning. The chapter has been structured in four main parts. In the first part, we provide a description on how the research field of mathematics education has been reacting to deficit assumptions. In the second part, we discuss some socio-cultural perspectives of mathematics learning, showing that their mainstream assumptions challenge any deficit discourse. The third part promotes a perspective on social justice. Here some recent research related to the main traditional markers of discrimination in school mathematics—gender, social class and ethnicity—are approached. Finally, in the summary, we present an overview of the main issues and claims discussed.

Socio-cultural Reactions to Deficit Assumptions in Mathematics Education

It is well-documented in the literature that, historically, mathematics education developed as a research field from the late 19th century under the influence of two main disciplines—mathematics itself and psychology (D'Ambrosio, 1993; Kilpatrick, 1992; Lerman, 2000; Schoenfeld, 1992). It is also well-documented that mathematics has had an important role in the intellectual selection, preparation and guidance of students to enter higher education studies. Mathematics has been used to help select those who will occupy different social positions, thereby serving as a *critical filter* (Bishop, 1999; Ernest, 2007a; Gomes, 2008; Sells, 1978). Ernest (2007a) argued that, in Western culture, this "critical social function of mathematics is exacerbated by the preconception that mathematical performance is largely inherited" (p. 2), or, put another way, determined by deficit transmitters like those we have discussed. This discrimination, apparently stronger in mathematics than in other school disciplines, is still supported, in Ernest's view, by a significant corpus of quantitative research that correlates student mathematical achievements with gender, race, class, culture, familial socialization, and other divisors of society such as special needs, disability, sexual orientation, age, creed and religion.

Socio-cultural perspectives that differed from the deficit explicative approach of causality in mathematical performance started to appear by the late 1970s. These perspectives shared the assumption that it is too restrictive to consider merely "the gaps" of a population, and argued that each culture should be examined from tasks or practices that are significant or meaningful to their members. There emerged a number of works within and around mathematics education (e.g., Bernstein, 1996; Bishop, 1988; Bourdieu & Passeron, 1977; Carraher & Schliemann, 2002; D'Ambrosio, 1985; Gardner, 1983; Geertz; 1973; Greenfield & Childs, 1977; Lave, 1977; Scribner & Cole, 1973; Sternberg, 1985) that problematized the deficit approach, in terms of both specificities of the socio-cultural groups under investigation and methods that are pertinent to study these groups. The socio-cultural variable was taken into consideration in these works (Perret-Clermont & Brossard, 1988).

This movement towards new paradigms of learning in academic communities has been described by Lerman (2000) as the *social turn*, having its peak around 1988. Lerman observed that the positive receptivity of new alternative perspectives of learning by the mathematical education community "was due more to political concerns that inequalities in society were reinforced and reproduced by [deficit assumptions] in school mathematics, than social theories of learning" (p. 24). Valero (2004) pointed out that this signalled that some researchers found support, in these socio-cultural perspectives, for their understandings of these inequalities. On the other hand, for other researchers, these perspectives offered an explanatory power for them better to understand the mathematical practices in terms of the interactions, relationships, and discourses that effectively occur in the classroom. Whatever the case, these perspectives became an intellectual commitment for many mathematics education researchers: for some it is more political, and for others more pragmatic or affective.

The social turn in mathematics education was influenced not only by emerging socio-cultural approaches, notably those originating from cultural psychology, anthropology, sociology and philosophy of mathematics, but also by ethnomathematics, issues of gender, social class and ethnicity, history of mathematics, sociolinguistics, semiotics, and other topics in the social sciences (Ernest, Greer, & Sriraman, 2009; Lerman, 2000). In particular, emergent socio-cultural views of intelligence in response to deficit discourses played a special role in the social turn within academic communities in general. Two particular alternative contributions have challenged the deficit paradigm by arguing that intelligence is a social construct that manifests, in many ways and means, different things to different social groups. One contribution came from Gardner's (1983) *Theory of Multiple Intelligence*. Gardner defined intelligence "as the ability to solve problems, or to fashion products, that are valued in one or more cultural or community settings" (p. 7). The other contribution, known as the *Triarchic Theory of Intelligence*, was introduced by Sternberg (1985). It distinguished between three contexts in which intelligence manifests itself: the first relates to successful performance in standardized school norms (e.g., appropriated ways of thinking and reasoning, tests and socio norms); the second is associated with creativity and motivation toward novelty; and the third concerns successful performances in out-of-school activities.

New conceptualizations for intelligence generated new ways of thinking about both cognition and learning, and all of these demanded the development of alternative methods to complement statistical studies, or even replace them. In relation to cognition and learning, Jean Lave's book *Cognition in Practice* (1988) had a very important influence on thinking about mathematics education. Grounded on Vygotsky's ideas, Lave demonstrated that cognition is a phenomenon that emerges in social interactions, and that learning and identity formation occur as a result of participation in social practices. This resulted in a radical shift of paradigm in relation to traditional views of cognition and learning in that meaning, thinking, and reasoning came to be seen as products of social activity (Lerman, 2000).

Alternative methods of empirical research, involving qualitative approaches (see Groulx, 2008), challenged the authority of statistical methods, which came to be seen as being relevant only to events that could be "classified, operationalized and organized" (p. 97; our translation). By contrast, qualitative methods focussed on the particularities, conditions and circumstances of the historical/socio-cultural environments in which the events occurred; the subject-participants become actors in that their voices were heard, revealing a diversity of situations in which they acted in various manners and made use of a varied resource repertoires. Yet, qualitative approaches pushed academic communities to rethink studies concerning the needs of the groups of individuals according to the socio-cultural singularities of their *forms of life,* and not as measurement indicators.

Although the works mentioned so far claimed that intelligence, cognition and learning should not be explained any more from deficit parameters—the issue still remains alive in the agenda of a number of scholars from different Western countries (e.g., Ernest, 2007a; Ford, Harris, Tyson, & Trotman, 2002; Gillborn, 2005; Gomes, 2003; Gorgorió, Planas, & Bishop, 2004; Gutiérrez, 2007; Keitel, 1998; Martin, 2009; Stevens, Clycq, Timmerman, & Van Houtte, 2009; Valencia, 2010; Weiner, 2006).

In the next section some socio-cultural perspectives of mathematics learning are presented, showing that their mainstream assumptions go beyond deficit discourses.

Socio-cultural Perspectives for the Learning of Mathematics

Socio-cultural perspectives of mathematics learning are found under different denominations and within different research foci. Some of these perspectives conform to the main research foci proposed by Bishop (1999)—mathematics learning, mathematics curricula and mathematics teaching. These three foci are described by Bishop in the following words: mathematics learning relates to the ways *cultural learners* learn and use mathematics. This includes “characteristics of learners, types of learning, attitudes, beliefs, motivations, feelings, ways of remembering, imagining, representing” (p. 4). Mathematics curricula deal with *cultural issues* involved in “aspects of content, sequences of ideas, relationship to other topics, other subjects, other contexts, both real and virtual” (p. 4). Mathematics teaching covers all that encompasses *the context of mathematics teaching*, which, at the end, converges to the classrooms in the form of “interactions, explaining, clarifying, linking with other knowledge, inspiring, leading, communicating” (p. 4).

In the analysis which follows we will show that these foci are not disjoint: each overlaps or complements the others.

Cultural Learners, Cognition and Affect

Acknowledging that learning and cognitive processes should not be analyzed outside a learner’s culture led to the development of studies of beyond-school mathematical practices in culturally relevant contexts. Barton (1996) identified four bodies of literature in these studies, one of them focussing on the exploration of relationships between the thinking processes of an individual's cultural group and mathematics education. Thus, for example, Terezinha Nunes, Analúcia Schliemann and David Carraher's studies of street mathematics and school mathematics with some groups of Brazilian children analyzed data on the similarities and differences between different groups of people as they attempted to solve mathematical problems at work and in school. These data, and data from other like studies, constituted strong evidence against deficit models as they showed that, despite failing in school mathematics, children from poor economic backgrounds could understand and apply basic mathematical principles as they solved problems in familiar work contexts.

In their first analysis of the mathematics that people practise in everyday settings, Carraher, Carraher, and Schliemann (1985) found that young street vendors in Brazil correctly solved 99% of the arithmetic problems that emerged during selling transactions. However, when asked to solve similar problems presented to them as school-like computations, the percentage of correct answers dropped to 37%. Nunes, Schliemann, and Carraher's (1993) studies, together with those by other authors (e.g., Lave, 1977, 1988, 1989; Reed & Lave, 1979; Saxe, 1991), demonstrated that specific socio-cultural activities, such as buying and selling, promote the development of mathematical knowledge previously thought of as accessible only through formal instruction. These findings strongly challenged the adequacy of deficit models in relation to mathematical learning: failure to learn mathematics in school cannot be attributed to deficits, given that the same children who failed in school tasks showed mathematical understanding in other contexts. The analysis of school failure needs to focus therefore on the school itself, its values, its assessment procedures, and, above all, the different practices developed in and out-of-school contexts.

Nunes, Schliemann and Carraher, and their students—the so-called *Recife Group*—developed over more than 20 years new contexts of observation in which mathematical activities were not necessarily related to school mathematics patterns (see, e.g., Acioly, 1994; Acioly-Régnier, 1997; Acioly & Schliemann, 1987; Carraher, 1986; Carraher et al., 1985; Da Rocha Falcão, 1995; de Abreu & Carraher, 1989; Nunes, Schliemann, & Carraher, 1993; Schliemann, 1985; Schliemann & Acioly, 1989; Schliemann, Araújo, Cassundé, Macedo, & Nicéas, 1994; Schliemann, & Carraher, 2004; Schliemann & Magalhães, 1990). One of the contexts, discussed by Da Rocha Falcão (2005), referred to a specific community of Brazilian fishermen, the *jangadeiros* from Recife. Although most of these fishermen were illiterate and possessed no conceptual-vectorial schemes at all, Da Rocha Falcão showed how they were able to pilot their sailing boats conforming to vectorial principles of composition of the direction and intensity of the wind and the orientation of the sail and keel.

Refusing to accept deficit models to explain difficulties in learning school mathematics, researchers in the Recife group built upon aspects of Piaget's and Vygotsky's theoretical accounts of cognitive development and followed methods similar to those developed by Cole and Scribner (1974), Luria (1976), and Reed and Lave (1979). Thus, the group developed a conceptual and contextual analysis of empirical data, using methods from anthropology, psychology, and mathematics education, to bring out the different levels of conceptualization and representations of participants in their studies. Vergnaud's (2009) theoretical proposal of *conceptual fields* provided a fruitful background for their analysis of the invariant, symbolic, and situational aspects of concepts developed both in and out-of-school.

Although initially formed with the above-mentioned theoretical and methodological orientations, some members of the Recife group reelaborated them and incorporated others to continue their own investigations. For instance, Da Rocha Falcão pointed out that, although Brazilian fishermen—the *jangadeiros* from Recife—and amateur sailing apprentices displayed clear differences in their psychological competences of sailing, both groups of competences were semiotically and culturally mediated. Supported by Leontiev's (1994) theoretical concept of activity, Da Rocha Falcão argued that the classification of these Brazilian fishermen's sailing competences, proposed by Vergnaud (1991) as being *competences-in-action*, or else *savoir-faire* as proposed by Piaget (1974), suggested the possibility of non-semiotic, strictly practical human actions. Da Rocha Falcão stated that the fact that many people could not explain or discuss their competences should not be taken as evidence that these competences had a purely enactive character.

The systematic research program developed by the Recife group not only drew attention to the weakness of deficit models for learning mathematics to explain the academic failure of children but also demonstrated common aspects of concepts developed out-of-school and in school. In discussing analytical tools for the study of mathematical activity, Araújo et al. (2003) proposed, among other things, to take into account pre-conceptual competences characterized in two ways: First, by their effectiveness in culturally meaningful contexts; and second, by the fact that these competences are, by nature, quite difficult to express using symbolic-explicit representations (see also Frade & Da Rocha Falcão, 2008). For these authors, effectiveness and tacit quality are invariants of mathematical activity, irrespective of whether we are considering school or out-of-school mathematical practices such as those performed by tailors (Lave, 1988), carpenters (Millroy, 1992), *cambistas de jogo do bicho*—Brazilian bookmakers dealing with what is called the "animal lottery" (*jogo do bicho*) (Acioly & Schliemann, 1987), fishermen (Da Rocha Falcão, 2005) and other communities of practice (e.g., Santos & Matos, 2002).

For researchers in the Recife group, the core issue regarding predictors of selective school failure relates to particular characteristics of the semiotic interactions and concepts developed in different practices. But what are those characteristics? Are they linked to the context of learning, to students' identities, or to mathematical concepts involved in the activity? It seems that the *simultaneous* consideration of these three aspects distinguishes this group as researchers of the psychology of mathematics education inspired by the theoretical perspectives of Vygotsky, Piaget, Vergnaud, Leontiev, and Lave, among others.

Within psychology, the role of culture and contexts in the cognitive development of individuals is a fundamental issue. The difficulties of integrating cultural and conceptual aspects within works on mathematical competences can be illustrated in the analysis by Saxe and Posner (1983) of the strengths and weaknesses of transcultural research into the development of the concept of number, associated with the approaches of Piaget and Vygotsky. For Saxe and Posner, each of the theories provides a base to analyze universal or culture-specific processes on the creation of numerical concepts in children.

Piaget described how numerical operations are developed, but he did not analyze the mechanisms through which social factors contribute to the creation of numerical thought. His theory did not give us enough information about the level of conceptualization of the individual in specific domains of knowledge—a fact later recognized by Piaget (1971) himself, at least in relation to what he called formal operations in adolescence and adulthood. The lack of analysis concerning possible differences of conceptualization across contexts or situations led to the abandonment of the theoretical frame of stages of development in the sense Piaget gave to them. By contrast, Vygotsky's approach, as interpreted by a group of American psychologists (Cole, Gay, Glick, & Sharp, 1971; Cole & Scribner, 1974), considered cultural experience as a differentiated theoretical construction. With this approach, concepts are regarded as important, but conceptual development is not analyzed in depth.

Taking advantage of both perspectives, the theoretical perspective of conceptual fields proposed by Vergnaud (1991, 2009) provides a pertinent and operative frame that allows a new type of analysis of different types of conceptualizations occurring in different contexts. The core of Vergnaud's theory lies in the importance attributed to situations for the development of concepts. We recall that this theory defines a concept as a tripolar system constituted by three groups that he called *signifiers, situations, and invariant operatories.* The group of signifiers allows the representation, the communication and the treatment of a concept; the group of situations refers to situations in which the concept operates, and to the idea of reference; the group of invariant operators refers to the idea of meanings.

Using Vergnaud's tripolar system, Acioly-Régnier (2010) identified a distinction between school and non-school contexts in terms of *focus of consciousness*. In this identification, the difficulty an individual is faced with relates to the recognition of whether the concepts or representations are relevant to a given situation, be it a school or a non-school situation, or even to lack of the cultural tools to represent the situation. Acioly-Régnier showed that, within a school frame, the focus of consciousness is essentially directed to the bipolar relation signifier-signify, leaving aside the situations they may refer to. In non-school contexts the stress is mainly on the axis signify-referent. In this case, Acioly-Régnier (2010) noted that the conceptualization becomes somewhat incomplete and the equilibrium of the triple (signifier, referent, signify) is lost.

In terms of the focus of consciousness, this has been justified as everyday concepts are linked to local knowledge as opposed to universal knowledge (Rogoff, 1981). For Acioly-Régnier, this view is controversial. Her study indicated that the same lack of generalization applies to learning that takes place in school contexts. As studies of transfer show (e.g., Boaler, 2002a; Carreira, Evans, Lerman, & Morgan, 2002; Frade, Winbourne, & Braga, 2009; Greeno, Smith, & Moore, 1993; Lerman, 1999), generalization and transfer across contexts are not, in general, without mediation, automatic, or even comfortable.

It is fundamentally important to take explicit account of the contexts in which learning takes place and to study the specificities of the concepts developed by the individual in a given context. That is particularly the case when the specificities of the semiotic interactions cannot be understood by the dichotomy: street mathematics context versus school mathematics absence of context.

In this respect, the perspective taken by Lave and Wenger (1991) illustrated a way of breaking with this dichotomical model. Regarding the appropriation of certain kinds of knowledge, they expanded Lave's initial views, placing the development of learning as a matter of identity development that takes place in social relations within the situations of coparticipation. This participation not only refers to local events that set in motion certain activities with certain people, but to a more global process that integrates the active participants to the practices of social communities and leads them to build their own identities to connect with the community. Lave and Wenger illustrated their theory of situated cognition by considering previous empirical studies of different learning processes among several groups: the midwifes of Yucatec, the tailors of Vai and Gola, the quartermasters of the American marine, the carvers at slaughterhouses, and a group of alcoholics anonymous. At first the individuals who join communities remain mostly at the periphery, where they do their first learning acquisitions. As they become more competent they move to the centre of the community. Therefore, learning is not seen as a simple acquisition of knowledge by individuals, but as a process of social participation in a certain practice or situation.

Acioly-Régnier (2010) adopted the characterization of concept proposed by Vergnaud (2009) in which a concept involves a set of situations, a set of operational invariants, and a set of linguistic and symbolic representations, but took into account the context of the conceptual development. At a more general level, Acioly-Regnier proposed a framework for psychological processes and a conceptualization of reality that includes three poles: culture, cognition and affect, as depicted in Figure 4.1.

This framework considers, simultaneously, the idea that performance in a given context occurs under the triple influence of cognitive, affective and cultural factors. Empirical evaluations of this proposal require multiple methodological approaches aiming at providing different and relevant perspectives regarding the phenomenon

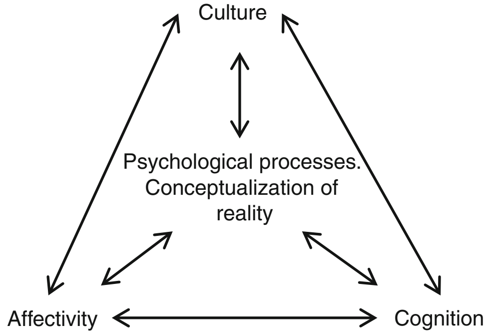

Figure 4.1. Schematization of the frame culture, cognition and affect.

under analysis. For example, one needs to consider the role given to the individual by the researcher—as holder of knowledge, as simply a source of information, or even as someone submissive to a research protocol built by the researcher to test the limits and strengths of her knowledge. The nature of the interaction proposed by the researcher to collect the data, also needs to take into account and be examined from the perspective of, for example, the classic interaction individual-researcher or the binomial interaction within a pair or a larger group (Acioly-Régnier, 1996). Acioly-Régnier (2010) argued that cognitive performance depends on the nature and the context of the question, an individual's familiarity with the situation, the kind of formulations (oral or written) that are required, etc. All these parameters play an important role in the construction of the data, as do classroom-specific factors such as the extent to which students are allowed to display their mathematical understandings.

Numerous studies into cognition and context have clarified important issues regarding individual challenges and strengths as learners develop concepts across different contexts. Questions still remain, however, so far as the relevance of these research findings and theoretical approaches to mathematics education—see Moshkovich and Brenner (2002) for a collection of studies on this issue. Also many questions need to be addressed regarding children who fail to learn mathematics in schools. For instance: What mathematical understandings do these children develop outside of schools, which are relevant to the mathematics curriculum? How are these beyond-school understandings different from the mathematics they are supposed to learn in schools? How can a teacher identify the strengths and limitations of children's previous concepts? How can the teacher create environments that will allow children to learn mathematics that is not merely a set of memorized procedures, but rather a set of meaningful, related concepts—as they seem to be capable of doing when they learn outside of schools? These kinds of questions show that psychology alone is not able to account for socially- and culturally-orientated theoretical perspectives of cognition. Possible responses to these questions can be elaborated from issues outside of psychology, however, and these will be discussed in the following sections.

Culture and Mathematics Curricula

Three particular socio-cultural perspectives of mathematics learning have had a strong influence on mathematics education research on curricula issues: ethnomathematics, Bishop's perspective of mathematical enculturation and acculturation, and situated perspectives originated from Lave's ideas on cognition in social practices.

Ethnomathematics. In his articles *The Name Ethnomathematics: My Personal View* and *Ethnomathematics: My Personal View*, D'Ambrosio (2010a, 2010b), reported on the trajectory of ethnomathematics, presenting his personal view of this already consolidated research field. In the first article, he noted that the word ethnomathematics was always used by him when he was describing the mathematics of other cultures, especially those without writing and those marginalized by the colonial process (D'Ambrosio, 2010a). D'Ambrosio (1997) recognized, however, that this word has been broadened to encompass other ethnomathematics currents, for example, critical approaches to the eurocentric character of mathematical knowledge (Powell & Frankenstein, 1997), research on educational policies and society (Gerdes, 1994), and studies of mathematical ideas of non-literate groups (Ascher & Ascher, 1997). In the second article, D'Ambrosio (2010b) explained what he meant by ethnomathematics and an ethnomathematics program: the former relates to a theoretical framework, and the latter to the empirical dimension of it. In both articles, D'Ambrosio made it clear that ethnomathematics consists of a theory of knowledge of different cultural groups, with special emphasis on both the history and philosophy of mathematics, the aim being to understand, explain, learn about, cope with and manage the natural, social and political environment of processes involving counting, measuring, sorting, ordering and inferring, of well-identified cultural groups (see also D'Ambrosio, 1988, 1997). He linked his theoretical approach to its empirical dimension by saying that his "proposal is a transcultural perception of the nature of mathematical knowledge, which demands a transdisciplinarian approach to knowledge in general" (D'Ambrosio, 2010a). In this sense, an ethnomathematics program focusses on epistemological investigations of mathematical ideas and practices of different cultures—such as those developed or used by indigenous populations, labor and artisan groups, periphery communities in urban environment, farms, and professional groups—using methodological procedures inspired by ethnography.

The paragraph above shows the wide scope of ethnomathematics studies as viewed by its best known scholar. It leads us to reflect, in particular, on some pedagogical implications of these studies as they are directed to societies, communities or groups in which education is structured by any type of formal instruction. The main challenge of ethnomathematics studies, when restricted to any type of formal education, lies with curricular issues, for their assumptions depend on a curriculum planned and developed around the specific socio-cultural needs and life history of these groups. This implies a teaching context design based on the socio-cultural environments of the learners, and a view of cognition in which reasoning reflect cultural roots (D'Ambrosio, 2010c), because meaning-making derives from the learners' socio-cultural needs and life histories. This articulation of *curriculum-context of teaching-cognitio*n was well illustrated in the work carried out by Knijnik (2004) with landless peasant communities in Southern Brazil. In this work, Knijnik clearly showed her careful role as researcher in the construction and planning of such articulation to attend to the social needs of the communities under investigation.

Ethnomathematics has inspired studies into the education of youths and adults (e.g., Fonseca, 2010), indigenous communities (e.g., Barton, 2008; Costa & Silva, 2010), other minority communities (e.g., Knijnik, 1999; Palhares, 2008), professional groups (e.g., Palhares, 2008) and has generated a range of socio-political approaches to research (Gerdes, 1994; Powell & Frankenstein, 1997; Valero &

Zevenbergen, 2004). Yet, ethnomathematics has brought important contributions to mathematics learning and teaching in traditional classrooms, in particular to cultural roots, interactions between mathematics and languages, human interactions and values and beliefs (Bishop, 2002, 2010). However, we do not find substantial ethnomathematics studies in the literature on mathematics education within school environments.

We suspect that this is due to two main reasons. First, the complexity of traditional classrooms in terms of the multiplicity of the students' needs and life histories are much too diverse to permit the students to be considered as well-identified cultural groups in the sense of ethnomathematics. Besides, these classrooms are inserted in a type of educational system in which the mathematics curriculum is basically the same in all countries (Bishop, 2010), generally elaborated and developed by pre-established contents, guided by national policies and mechanisms of assessment performances, and with little opening for changes. The second reason, which can be viewed as a consequence of the first, concerns a probable lack of interest in researching mathematics classrooms because ethnomathematical thinking, as Knijnik (2004) observed, emphasizes "other mathematics, usually silenced in school, as the cultural production of non-hegemonic groups" (p. 136). What we find in the literature about ethnomathematics and formal mathematics learning and teaching is a set of proposals concerning pedagogical lines of actions to incorporate the cultural diversity in the educational context (Borba, 1997; Gerdes, 1996; Palhares, 2008; Shirley, 1995).

Mathematical enculturation and acculturation. Bishop's perspective of mathematical enculturation (Bishop, 1988) and acculturation (Bishop, 2002) fills the space left open by ethnomathematics concerning school mathematics in some important aspects. It is a perspective that helps us to understand affective imbalances within mathematics classrooms, especially multi-ethnic classrooms, between students and teachers, *culturally*. Borrowing from the literature on anthropology, Bishop introduced the concepts of enculturation and acculturation into mathematics education. These concepts primarily address curricular issues in that they are strongly linked to the culture the student brings from home and the teachers' cultures, values, beliefs and choices in relation to mathematics, mathematics education and education in general (Bishop, 1988, 2002; Bishop, FitzSimons, Seah, & Clarkson, 1999; Seah & Bishop, 2000).

Frade and Faria (2008) noted that Bishop's educational analysis was initially grounded on the perspective of enculturation, where enculturation was taken to mean the induction, by a particular cultural group, of young people into their culture. This perspective presupposes the existence of a cultural consonance between school mathematics and the culture the student brings from home. Frade and Faria (2008) observed, however, that Bishop (1994) re-evaluated his premises for the purpose of reaching an understanding of cultural conflicts within multi-ethnic classrooms—moving from the assumption that mathematics education may not be a process of enculturation, but rather a process of *acculturation*, the induction into an outside culture by an outside agent. Often one of the contact cultures is dominant, irrespective of whether such dominance is intended. At this stage, according to Frade and Faria, Bishop's studies began to focus not so much on individual students, but on the acculturation process per se, and on the role of the so-called *acculturators*. After observing apprentices (in general) during their experiences with cultural conflicts, Bishop (2002) proposed a more radical hypothesis: "all mathematics education is a process of acculturation … every learner experiences cultural conflicts in that process. However, cultural conflicts need not be conceptualized exclusively in a negative way" (p. 192).

Frade (2006) reported that for Bishop (2002) mathematics teachers are the main agents of mathematics acculturation. He considered two types of acculturator-teachers: the teacher who does not make any reference to any out-of-school mathematical knowledge; and the teacher who imposes what she wants through her privileged position and power. In both cases, Bishop claimed, although the resulting cultural conflicts contain a cognitive component, they are infused with emotional and affective traces or nuances indicating deeper and more fundamental aspects than can be accounted for from a cognitive perspective. These affective traces clearly appeared in the works of Frade and Machado (2008), and Frade and Faria (2008), who reported on two studies into teachers' mathematical culture and values, and the corresponding affective reactions of the students to both their learning and their teachers' practices.

In an attempt to humanize the imbalanced relationship between the culture of the teachers and the culture of the students, Frade and Faria (2008) suggested that Bishop (2002) proposed to reconceptualize mathematics learning environments based, to a great extent, on Gee's (1996) theoretical construct of *borderland discourse*. This would correspond to the area of intersection between the students' primary and secondary discourses. The primary discourse refers to the discourse learned and used within the family, at home or with surrounding groups. The secondary discourse, more institutional or formal than the primary one, is related to traditions passed forward to us by various generations through time, aiming at learning conducted in external environments. According to Frade and Faria, the potential oppressive character of an acculturation process led Bishop to state that the intentional mathematics acculturation of a young person is turned into some type of cultural production while schools should be the place where the primary discourse of the students' families and communities meet the secondary discourse of the mathematics community. This *turning* was explained by Bishop as he explored the idea of *transition* (see de Abreu, Bishop, & Presmeg, 2002).

More recently, Bishop (2010) revisited the evolution of his works and discussed the universality of mathematics curriculum—that is to say, the fact that school mathematics curricula are almost the same in every country, apparently disregarding the cultural diversity that characterizes the population in general. Bishop urged that "mathematics curricula be designed which deal with numeracy/ethnomathematics practices as one strand together with Mathematical theory as a separate but related strand" (p. 339). By Mathematical theory for a school's curriculum, Bishop meant "an approach which focusses on the many 'Why' questions provoked by numeracy/ethnomathematics practices" (p. 339). In this way, Bishop suggested that we would have "a balanced mathematics curriculum for all—one which respects local ethnomaths/numeracy practices, 'legitimizes' them, and accepts them within the school context, and investigates their rationales by using appropriate Mathematical theory" (p. 340). In relation to values, Bishop restated their importance in educational research. For him, although we can perceive an increased interest in culture-based research constructs, they are still insufficiently addressed, given that "shared values are a significant part of any culture" (p. 341).

Bishop's perspective has influenced not only studies focussing on school mathematics. His ideas have been taken up by those conducting general studies into the development of curricula in conflictive political and/or multicultural contexts (de Abreu et al, 2002; Civil, 2007; Gorgorió & Planas, 2001; Powell & Frankenstein, 1997; Valero & Zevenbergen, 2004), and also on teachers' values and students' affect (Clarkson, FitzSimons, & Seah, 1999; Frade & Faria, 2008; Frade & Machado, 2008).

It is clear that both ethomathematics and Bishop's perspective are unquestionably rooted in humanistic views such as respect for cultural diversity, equality and social justice, and human rights. Mathematics education researchers and teachers who have a commitment to these approaches do not accept assumptions and claims based on the cultivation of deficit practices or discrimination of any type. In particular, ethomathematics and Bishop's perspective are very aligned with Bruner's (1990) position concerning the socio-cultural character of intellectual prowess. In the case of ethnomathematics, it is common to refer to those groups which, for some reason, are excluded from the cultural production of Western hegemonic education. In Bishop's perspective, teachers and students are the main perpetrators of cultural conflicts in which students' "primary" cultures are often oppressed by the "secondary" cultures of the teachers.

Situated perspectives of learning. With a different emphasis on culture from that given by ethnomathematics and by Bishop's perspective, but still relying on anthropology, we find the situated approaches to mathematics learning originating from Lave's (1988) perspective on cognition in practice (see also Lave and Wenger, 1991).

Lave's core idea is that cognition is a product of semiotic interactions between the individuals and the social practices in which they participate. According to Frade et al. (2009), this implies that cognition is "a phenomenon that emerges from the practice, from the fact that an essential feature of the practice is making resources available for … involving and encouraging the individuals" (p. 16) to interact semiotically within it. For these authors, this is what it can be understood by learning as a process which does not depend on an individual only, but notably on the potential of the appeals of practice to bring individuals to participate in it (for other situated approaches, see, for example, Boaler, 2000; Brown, Collins, & Duguid, 1989; Cobb & Bowers, 1999; Engeström, 1999; Greeno, 1997; Kirshner & Whitson, 1997; Watson & Winbourne, 2008). By focussing on the practices in which individuals are expected to learn to participate, learning is then seen as a process of changing participation and identity formation within these practices (Lave & Wenger, 1991).

Lave and Wenger's (1991) social-practice perspective of learning had profound implications not only for those studying mathematics practices in and out-of-school settings but also for the interdisciplinary character of research in mathematics education. The emphasis on practices as the emergent locus for the production of meanings indicated that mathematics learning and its use by common people was no longer a matter for discussion by psychologists only. Psychology alone was not able to account for the processes involved in learning and using mathematics from a situated point of view (Boaler, 2000).

In the context of school mathematics, learning as a result of participation and identity formation is particularly challenging in two special aspects. First, it claims a refocussing of the teachers' attention away from students' cognitive differences/performances (conveyed by expressions such as "good student" and "weak student") towards the students' semiotic interactions within mathematical practices or activities, which are situated in a broader historical/socio-cultural context. This does not imply that students' individual needs are ignored. On the contrary, cognition viewed in terms of symbolic mediation means that different individuals will interact in semiotically different ways. In producing different meanings, teachers should be aware that students will have distinct needs that must be considered by the practice.

Second, and consequently, it demands a rethinking of the idea that the mathematics curriculum should be centred on a "universal" or taken-for-granted list of pre-established subjects. What is being called for, now, is a re-direction to propositions of mathematical practices within the space of signification/meaning of the students, to allow them to interact semiotically. Participation, in Lave and Wenger's sense, is not merely an act of engagement in a certain practice or activity; changing forms of participation are part of a process that shape identity formation. That is to say, by beginning to participate in new ways, participants come to see and deal with concepts or situations that either they have not seen before or, if they have, they now see and deal with them in different ways. It might be said that individuals have learned or become different persons in relation to a certain domain. Indeed, according to Lave and Wenger (1991), the concept of participation involves, above all, a contribution of the individual to the development of the practice and the contribution of the practice to the development of the individual as well.

This refocussing of the teachers' attention and the rethinking of the universal character of mathematics curricula are key aspects of situated approaches that challenge deficit assumptions. Cognition is now to be seen as a process which does not depend on an individual's "natural" attributes. And, by focussing mathematics curricula on mathematical practices that include diversity considerations as earlier proposed by Bishop (2010), for instance, while at the same time providing access to globalized knowledge, educators foster student participation and avoiding differentiation, division, qualification and disqualification among students (Popkewitz, 2004).

Some researchers (e.g., Walkerdine, 1997) have pointed out that Lave's situated perspective does not clarify how subjectivities are produced in social practices. Similarly to the discussion developed by Frade and Meira (2010) about the social nature of affective behaviours and the constitution of identity, the production of one's subjectivity can be also seen as results of internalizations that occur from the interactions between individuals and those practices in which they are involved. How an individual reacts to and internalizes what she has learnt as a result of her engagement in social practices depends not only on her previous life experiences, choices and judgments, but also—and perhaps mainly—on a combination of the contingencies, circumstances and social norms to which she is subjected at the moment. Thus, according to Frade and Meira (2010), one's subjectivity is subjected to continuous changes, and depends on the historical/socio-cultural circumstances which the individual has experienced and is experiencing at that particular moment of life.

Studies on ethnomathematics, Bishop's perspective, and situated learning perspectives clarify important issues concerning the learning and the use of mathematics by different cultural groups. However, there are many unanswered questions that need to be investigated: What effects do mathematical content and its use have on the processes of participation and identity formation? How can teachers effectively evaluate their students' mathematical developments in terms of participation and identity formation? What needs to be done to create curricula based on mathematical practices, rather than merely on content? Which kind of curricular materials would teachers need to support work with a curriculum based on mathematical practices? How can cultural specificities be incorporated into the curriculum of different cultural groups without avoiding the mere reproduction of their cultures?

The Classroom Dynamic: The End Point of Mathematics Teaching

As indicated earlier, the context of any formal teaching is configured under the influence of a number of factors—like institutional issues, curricular policies, pedagogical organization, teacher qualifications, values and beliefs, power relationships, and people's expectations and needs. In classrooms, all of these are manifested, somehow, in the form of interactions, explanations, clarifications, linking with other knowledge, inspiration, leadership, and communication. The unique combination of these in a classroom produces a classroom dynamic in terms of norms, negotiations, designs and modes of teaching. Below we provide illustrations of some socio-cultural approaches, the intention being to explore how these factors combine in classrooms to affect teaching and learning.

Social negotiations. A classic example of a proposal for the negotiation of meanings and conduct between teacher and students comes from Guy Brousseau, one of the pioneer scholars of the well-known *didactique Française*. Brousseau (1986, 2006) introduced the notion of *didactic contract* as a theoretical framework aimed at understanding certain didactic situations involving the triple relationship "teacher–students–*savoir* (knowing)." Chevallard, Bosch, and Gascón (1997) described a didactical contract as a set of generally tacit norms or clauses which regulates the reciprocal duties of teacher and students concerning a common project of study. For Chevallard et al. (1997), this set of norms or clauses is not static, for it evolves as long as the didactical process goes forward. A didactical contract is therefore a construct to illuminate studies whose objective is to understand and support the work developed in the classroom (e.g., Galligan, 2005; Novotná & Hošpesová, 2008; Passos & Teixeira, 2011; Sierpinska, 2007).

Another example of a framework for analyzing interactions in classrooms was offered by Cobb and his colleagues (Cobb, 2000; Cobb, Stephan, McClain, & Gravemeijer, 2001; Yackel & Cobb, 1996). Based on a situated approach, these authors developed a framework that was intended to link the "social" and the "individual" dimensions of classroom interactions. In doing so, they saw these interactions as a coordination between the establishment of common mathematical practices (a social perspective) and the individuals' reorganization of mathematical reasoning during the evolution of these practices (a psychological perspective). The social perspective concerns the regulation of the classroom microculture regarding three main features: classroom social norms (established jointly by the teacher and students), socio-mathematical norms (normative aspects of classroom discourse and interaction that are specific to mathematics), and classroom mathematical practices (normative ways of reasoning mathematically during specific tasks). The psychological perspective focusses on the individual students' particular ways of participating in such common mathematical practices, more precisely on the individuals' mathematical beliefs (about their own role, the role of others, and the general nature of mathematical activity in school), values, interpretation and reasoning. For Cobb (2000), "each perspective constitutes the background against which mathematical activity is interpreted from the other perspective" (p. 64). Recent explorations of this interpretative framework in classrooms can be found in the works of Levenson, Tirosh, and Tsamir (2009), Lopez and Allal (2007), Tatsis and Kolezab (2008), Yackel (2001), and Yackel, Rasmussen, and King (2000).

Classroom designs. From a critical mathematics perspective, the construct of *landscapes of investigation,* introduced by Skovsmose (2001), refers to a dialogical environment in which mathematics is discussed through thematic projects that lead the students to develop a critical position about the role of the discipline in society concerning social, political and economical interests. In proposing these landscapes of investigation, Skovsmose observed that the teacher will probably have the experience of transposing a risk zone marked by the unpredictability of some events. On the other hand, he argued that not only may the students' mathematical abilities be developed in relation to certain contents but also their competence to interpret critically and act in a social and political situation structured by mathematics. Skovsmose stated that landscapes of investigation can be set no matter how the learning processes are organized.

Complementing the illustrations provided by Skovsmose, the work of Araújo (2009) offered an example of the processes of negotiation, production and development needed to build landscapes of investigation in the classroom. Acting as teacher-researcher, Araújo reported on a whole-year term course for undergraduate geography students designed from mathematical modelling projects (see also Barbosa, 2006, for a discussion on mathematical modelling in classroom from a socio-critical and discursive perspective, and the work of Pontes, 2003, about classroom design based on activities in mathematical investigations).

Another classroom design that has been taken up by researchers and teachers came from the core ideas surrounding Lave and Wenger's concept of *community of practice*. Winbourne and Watson (1998) proposed an adaptation of such ideas so that they would become applicable to certain school settings. This adaptation was intended to account for a design for teaching, and provide an analytical tool to evaluate relationships and student participation in mathematics classrooms. Winbourne and Watson suggested that, in some school settings, we can have, or not have, what they called a *local community of practice.* Frade et al. (2009) synthesized this construct as follows:

> A local community of practice in a school setting is, amongst other things, continuing activity where the participants—teacher and students—work purposefully together towards the achievement of a common goal. In doing so they share … ways of behaving, language, habits, values and tool-use, and can see themselves as an essential part of the regulation of their activity and progress towards the common goal. (p. 15)

Based on these ideas, Frade et al. (2009) examined the students' crossing of boundaries between some specific, apparently insulated school practices. This crossing of boundaries was the focus of research carried out by secondary mathematics and science teachers who planned and developed an interdisciplinary collaboration aimed at creating a local community of practice. The authors concluded that it was mainly the activity of these teachers that enabled the students to cross the boundaries between their disciplines: the teachers translated their specific discipline language codes, worked together to prepare and organize their collaborative work, and shared their goals and purposes with the students.

Other studies—for instance, those by Watson and Winbourne (2008) and Graven (2004)—have applied the concept of communities of practices to educational and professional mathematical settings. Thus, Graven (2004) used Wenger's (1998) concept of communities of practice to investigate the teacher learning which occurred within a mathematics senior-phase inservice program fostered by a change in the curriculum in South Africa.

Modes of teaching. Here, we offer some illustrations that share the assumption that the enhancement of "the competencies and identities of all learners, to a large extent, rests with how teachers operationalize the core dimensions of pedagogy" (Walshaw & Anthony, 2008, p. 518)

The social turn translated into new pedagogies and classroom organization demands considerable effort and commitment from teachers. They are expected to move their pedagogical actions from traditional modes and conducts of teaching, as well as their ways to organize the classroom, toward the production of a new pedagogy featuring classroom dynamics which foster mathematical and social interactions. From a socio-cultural perspective committed to equity to the students' access to globalized knowledge and to educational change, Walshaw and Anthony (2008), in discussing the teacher's role in discursive practices within the classroom, have provided a rich and comprehensive review of the literature about connections between teachers' pedagogies and desirable mathematical and social outcomes for students. Walshaw and Anthony showed how engagement in mathematical discourse can successfully develop students' understanding while at the same time fostering a respectful exchange of ideas (between the teacher and the students and among students) as well as teacher listening, attentiveness, and reflection-interaction. These authors also discussed the effectiveness of the teacher's role in building bridges between students' everyday ideas and their mathematical ideas. It is through language—in particular by acknowledging students' difficulties when attempting to use mathematical language—that teachers can build these bridges. Walshaw and Anthony argued that by teaching and involving students in mathematical language, teachers contribute to students' development of mathematical clarity through argumentations, critiques, and justification of assertions.

For David (2004), teachers have a decisive influence over the interactions that occur in the classroom. Her assumption was that all enunciations made by the teacher directly act on how the students internalize mathematics (see Blanton, Stylianou, & David (2003) for a development of patterns of these enunciations). Based on the work of Vygotsky and his colleagues (Luria, 1976; Vygotsky, 1962, 1978), David's analysis of a number of lessons taught by a group of elementary and secondary mathematics teachers, revealed how much the mediation role of the teacher's language and discourse can contribute to the development of aspects of the students' mathematical thinking and actions.

Meira and Lerman (2010) focussed on the role of language and discourse in conceptual development. They employed Vygotsky's notion of the *zone of proximal development* (ZPD) as a semiotic space when analyzing interactions between a preschool teacher and her students. When investigating the communicative moves of the teacher and a 2.5 year-old child around a plantation of beans set-up on cotton wool, Meira and Lerman demonstrated how the teacher positioned herself to be receptive to the pupil's attempts to use new words in idiosyncratic ways. In this way they sustained a shared field of attention that enhanced communication by allowing both the teacher and the child to recognize ambiguities in their own discursive contributions.

Numerous other studies grounded on socio-cultural perspectives have directly or indirectly drawn attention to the roles of teachers in classrooms. These studies addressed distinct factors such as the relationships that both the students and teachers developed with mathematics and mathematical practices as a result of *participation* (e.g., Back & Pratt, 2007; David & Watson, 2008; Frade & Tatsis, 2009; Goos, Galbraith, & Renshaw, 2002; Jaworski, 2008; Martin, Towers, & Pirie, 2006; McVittie, 2004; Williams & Clarke, 2003) and *identity* (e.g., Boaler, 2002b; Boaler & Greeno, 2000; Brown & McNamara, 2011; Frade, Roesken, & Hannula, 2010; Ingram, 2008; Sfard & Pursak, 2005). Other matters addressed by researchers have been the effects of possibilities for communication between the teacher and students (e.g., Chronaki & Christiansen, 2005; Setati & Adler, 2000; Silver & Smith, 1996), classroom culture (e.g., Seeger, Voigt, & Waschescio, 1998), teachers' mathematical knowledge for teaching (e.g., Ball, 2003; Moreira & David, 2008; Tardif, Lessard, & Lahaye, 1991), and teacher's professional development (e.g., Fiorentini, 2003; Nicol, 2002). Although these studies and those discussed above clarify many aspects concerning how socio-cultural perspectives can reach the classrooms, there is an important question which needs more attention—To what extent do findings of research on classroom interactions and teacher communication and behaviour offer guidance on how students can be *guaranteed* reasonable participation?

In completing the present section, we suggest that there is a fourth emergent focus of research in mathematics learning represented by a substantial and increasing corpus of research whose main concern is specifically with socio-political dimensions of mathematics education (Baldino, 1998; Knijnik, 2010; Mattos & Batarce, 2010; Restivo & Sloan, 2007; Valero & Zevenbergen, 2004). Although methods and approaches in this area of research need to be consolidated (Valero, 2004), the socio-political perspectives neatly go across the three foci or areas we have been discussing so far. They are intimately associated with the notion of *power* and *inclusion* in their different manifestations (social, economic, political, educational and cultural) and include critical mathematics approaches (e.g., Chronaki, 2004; Mellin-Olsen, 1987, Powell & Frankenstein, 1997; Skovsmose, 2001; Skovsmose & Borba, 2004), and equality issues concerning learner gender, ethnicity, social class, language, and other divisors of society (e.g., Barton, 1996, 2008; D'Ambrosio, 2001; Ernest, 2007a, 2007b; Ernest et al., 2009; Frankenstein, 1995; Gerdes, 1996; Gorgorió & Planas, 2001; Keitel, 1998).

The next section is dedicated to a discussion on social justice and equality in mathematics education. For equality, we selected issues of gender, ethnicity and social class.

Social Justice: An Emergent Discourse in Mathematics Education

Social justice refers to the realization of a common good to be applicable in a certain practice of a group, community or society; it is a concept associated with *praxis*, with human action. The discussion in this chapter will make it clear that social justice is an emergent discourse in mathematics education: the common good is *equality* (and all that it subsumes, like diversity, inclusion, accessibility, dignity, respect, assistance, opportunity)—applicable to *all mathematics learners and users as well* (not withstanding ethnicity, gender, social class, age, …), and to *mathematical practice in its several manifestations* (educational, social, cultural, professional, economic, political and technological).

There are many ways in which a discussion on social justice in mathematics education can be organized (see, e.g., Atweh & Keitel, 2007; Dowling, 2007). We will consider the *meritocratic model* to structuring our discussion for two main reasons: (a) it has a direct impact on the students' lives and identities inside and outside the school; and (b) mathematics teachers are very familiar with it and its mode of functioning.

In his analysis of equal opportunities and its limitations in formal systems of education, François Dubet (2004), the sociologist of education, discussed why social justice, in the sense described above, is not really found. Dubet reported that, contrary to the aristocratic societies which prioritized the "well born," democratic societies have chosen merit as the essential principle of justice in education: by giving equality of access to all, the school becomes fair because everyone can become successful as a result of their efforts and qualities. This principle, said Dubet, was progressively implemented in the modern and rich countries with the expansion of common compulsory schooling, and the opening of tertiary and secondary education. Then, gradually, the formal frame of both the equal opportunity and merit principles was globally installed in a great number of countries. For Dubet, this school, however, did not become fairer for reducing performance differences between social classes, even though all students were allowed to enter into a supposedly balanced competition—see Kariya (2011) for a discussion of this remark in the context of Japanese education.

This purely meritocratic aspect of justice in schools brought a number of difficulties in that it reinforced markers of segregation between various groups of students (Valencia, 2010). In particular:

- The accessibility of the meritocratic model to all did not eliminate the inequalities of social classes, gender and social groups. The more favored students still had decisive advantages.
- The meritocratic school especially did not adequately address the needs of the most disadvantaged students. The barriers are more rigid for the poor, and teachers' expectations are less favorable to children from disadvantaged families. In competing with others, disadvantaged students more than often lose and become the despair of their teachers. They are left aside, marginalized within a differentiated curriculum, and become increasingly weak.
- The "losers," that is to say, the students who fail, are seen as solely responsible for their failure—because, it is argued, the school gave them the same opportunities to succeed as it gave the other students. As a consequence, these students tend to lose their self-esteem and motivation, refuse to attend school, or particular classes within school, and, in many cases, become violent individuals: after all, the meritocratic school placed them in a competitive environment without giving them the support they needed to succeed. From the point of view of the teachers, the meritocratic school is also cruel as they become the major agent of social and educational selection.

Dubet (2004) stated that he doubted whether the model of justice based on merit would be abandoned because within a society that, in principle, demands equality between all, individual merit is seen, by many, as the only way of producing fair or legitimate inequalities—other inequalities, based on birth and biological attributes, for example, are recognized as unacceptable.

Yet, as Dubet noted, we cannot ignore the fact that inequalities within schools can cause social and economic inequalities. As Cole (1985), a US scholar, stated over 25 years ago:

> Our society, founded upon the principle that all men are created equal, has never lived easily with the recognition of enormous *de facto* social inequality. We need a rationale for such inequality and our traditions strongly bias us to seek the causes of inequality, in properties of individuals, not society. At the same time, we realize that social and economic inequality can be the causes of individual intellectual inequalities, as well as their consequences. (p. 218)

This issue poses a challenging question to educators: Can social justice ever be achieved in schools in which, in the name of democracy, the meritocratic model is adopted?

For Dubet, social justice in schools (and other formal educational institutions) should consist, on the one hand, of assuring education accessibility to all, and on the other hand, of using this accessibility to suppress obvious privileges and complicity between the school and certain social groups. This understanding of social justice would be measured by *the way the school treats the disadvantaged students; by recognizing them as individuals in evolution, rather than students engaged in a competition*. According to Dubet, a school committed to social justice does not humiliate and hurt the students usually identified as "losers." Instead, it values and works on those students' interests and needs, assists them in their evolution preserving their dignity and the equality of principles in relation to the others and in the fair sharing of human and material intellectual resources available.

As already suggested in this chapter, when applied to the teaching and learning of mathematics, the meritocratic principle seems to play a special role in the provision—or obstruction—of social justice. It is through it that mathematics acts as a *critical filter* in schools. And the results of several recent research studies, mentioned earlier in this chapter in the context of a discussion on deficit models have confirmed that *the fair treatment to all in schools is still far from being achieved.* Those studies mostly reacted to deficit discourses involving issues of gender, ethnicity and social class (though other markers are also claimed by social justice). They shared the conclusion that inequalities in mathematics education concerning these traditional deficit markers were consequences of socially-constructed discourses to meeting the political, social and economic interests of some groups within the society. In this sense, the meritocratic model can be seen as an efficient, but a perverse mechanism of implementation of these interests in schools.

Next, we will briefly approach issues of gender, ethnicity and social class, the aim being to present current views of researchers in mathematics education on these issues.

Gender

Recent researchers on gender have seemed to agree that inequalities between females and males in mathematics education emerged from a traditional discourse which relied on the premise that females have a fragile emotional nature in comparison to the strong rational nature of males (Walkerdine, 1998). This premise had other associated beliefs and assumptions: females tend to do better in affective matters demanding care, assistance and sensitive support, as well as in the humanities and professional areas. This explains in part why it is not surprising that many young and adult women do not hesitate to comment on their supposed incompetence in mathematics and related subjects. Males, on the other hand, are supposed to do better in objective and rational matters, in the hard sciences, and in competitive professions. Although this is a simplistic description for the presence of inequality between genders, there is a sense in which it is true: the public image *is* that mathematics is a male domain (e.g., Burton, 1986; Forgasz, 1998; Hyde, Fennema, Ryan, Frost, & Hopp, 1990; Keitel, 1998).

In terms of the meritocratic model, females and males are the actors of the competition. Females always start with a disadvantage in comparison to males, for, the public perception is that women are competing in a territory essentially meant for men. This competition appears more or less explicit in several studies. Willis (1998) reported that the Head of the Mathematics Department of a school in Australia believed that girls did not enrol in mathematics classes at the senior levels as much as boys do, not because they lack mathematical skills, but because of emotional insecurities. The school authorities, and often the girls themselves, thought that girls could not cope as well as boys with social pressures (see Sukthankar, 1998, for comments on familial pressures with respect to the career choice of females). And, if for any reason, the rules of the competition are changed so that females perform better than males, then this is blamed on "feminist initiatives" (Zevenberger, 1998). Skelton (2010), in discussing the repositioning of girls from "victims" to "victors" at school, showed how the Australian media dealt with the fact that in 1996, for the first time, girls from one State performed higher than boys in the end-of-secondary-school mathematics examinations. A similar study focussing on community reactions to this fact was reported by Coupland and Wood (1998).

The situation with respect to females' participation and performance in mathematics is showing significant signs of changing, although these signs vary according to time, nation, ethnicity, school level, and socio-economic status (Ernest, 2007b; Fennema, 1995; Forgasz, Leder, & Kloosterman, 2010; Grevholm, 2007; Nkhwalume, 2007; Rossi-Becker, 1998). In relation to the image of mathematics as a male domain, the study by Forgasz et al. (2010) indicated that most North American and Australian research student-participants see mathematics as relatively gender neutral. Seliktar and Malik's (1998) study showed that, in the USA, differences between males and females in academic choices are reducing, resulting in a fairer competition. Seliktar and Malik attributed this to a socio-economic need, typical of modern countries: the traditional occupations of women are sufficiently low in prestige, autonomy and financial compensation that they do not enable a modern and autonomous woman to support a family. So, more women are wanting to secure higher paying jobs which often require technical qualifications. Without mathematics qualifications and expertise, these jobs may not be achievable. Rossi-Becker (1998) attributed the change to extensive and diverse intervention programs aiming at increasing the participation of women in mathematics and related professions. Ernest (2007b) reported that in Latin-American countries, the Caribbean and Scandinavia, a higher proportion (at least 50%) of those taking up mathematics and science studies, and occupations, are women. Regarding imbalances in mathematical performance, Ernest analyzed data from different countries and concluded that "no unambiguous differences in achievement levels can be identified" (p. 5).

In many communities the meritocratic model has been replaced by a model based on everyday competence. For instance, studies conducted by Knijnik (1998), with landless peasant communities in Brazil, and McMurchy-Pilkington (1998), with the Maori community in New Zealand, reported data suggesting that for females, school mathematical subjects were not useful or related to their everyday lives or to their roles in their communities. In the case of the Maori community, McMurchy-Pilkington showed that although the women regarded themselves as not mathematically competent (considering the school parameters), they were able to think in mathematically complex ways, especially in family-related situations. Thus, what was valued by the Maori women was their ethnomathematical competence in everyday life. Singh (1998), who studied a group of South African Indian females, reported a strong tension between the females' aspirations for gaining better qualifications in order to gain control over their lives and be able to enter the labor market, and barriers imposed by historical, economic and social hegemonic forces.

Ethnicity

Although studies relating gender and participation in mathematics seem to point to real signs of changes, even in countries in which women have been oppressed, the same cannot be said in relation to ethnic and social class differences. Over the past four decades, much research on race and minority ethnic culture in mathematics education has continually shown the cruel effects for many students of meritocratic competition in mathematics. Here, the actors of the competition are often the socially and politically constructed individuals identified as the "non-whites" and the "whites." As mentioned previously, many teachers continue to maintain low expectations and negative images of some under-achieving groups of students, especially the "non-white" ones. Consequently, these students are often subjected to a differentiated treatment and curriculum, reinforcing inequalities and obstructing their access to quality education. In addition, in most developed and developing countries, issues of race and minority ethnic cultures are strictly linked to issues of class. This is the case, for instance, of Brazil (e.g. Costa & Silva, 2010), South Africa (e.g., Dowling, 2007), and the USA (e.g., Livingston, 2007), where the color of the skin clearly reveals separate social positions in society. In the Netherlands, research carried out by Stevens et al. (2009) called attention to the importance of considering family processes and characteristics as an essential aspect of understanding the relationship between race/ethnicity and educational inequality.

The meritocratic model relies on the democratic principle of equality and is based on the logic of a universal curriculum for mathematics: everybody should learn the same set of contents and achieve a fixed set of abilities or skills (Bishop, 2010). By effectively ignoring the cultural diversities of groups of students, those supporting this model align themselves with the continuation of educational deficit practices (Ford et al., 2002; Gillborn, 2005; Glevey, 2007; Martin, 2009; Powell, 2002; Valencia, 2010), cultural conflicts (e.g., Gorgorió et al., 2004), and conflictive communications and affective relationships between students and their teachers (e.g., Gates, 2002; Gillborn, 1990; Sewell, 1997; Wright, Weeks, & McGlaughlin, 2000). In relation to the maintenance of deficit practices in schools, the studies of Glevey (2007) and Martin (2009) are particularly noteworthy in that they not only provide complementary explanations for the persistent inequality between some groups of students, but they also propose actions to remove these inequalities within classrooms.

Taking the classroom in England as the context of his study, Glevey (2007) discussed the persistent underachievement of Black students (that is to say, students of any African heritage). Some of his conclusions were

- The lack of care, attention, teacher expectation, and consequently the non-access to education quality can be considered the major factors in the mathematical underachievement of pupils of color, minority ethnic cultures and low social classes. These pupils often develop unhealthy identities marked by painful feelings of poor self-esteem and low self-expectations in life.
- How schools succeed in providing social justice to the disadvantaged pupils depend on their appreciation of the ideological positions and tensions within which they function. We have noted that competencies, abilities, skills and motivation to participate are not innate—they result from learning which develops in healthy affective environments.
- The persistent underachievement of these pupils (all over the world) is a challenge that must be confronted and defeated. According to Glevey (2007), "while legislations are useful in persuading teachers to treat all pupils with dignity, the crucial importance of genuine care and compassion cannot be overlooked if real progress is to be made in supporting all pupils" (p. 12).

Similar illuminations are also found in the works of Martin (2006, 2009), who has discussed the learning of mathematics by Afro-American pupils in the USA. Martin (2009) called for teachers and schools to implement mathematics classroom practices that "promote the development of positive racial and mathematical identities and situate the learning of mathematics in the social (and racial) realities confronting students" (p. 299).

Regarding cultural conflicts, research carried out by Gorgorió et al. (2004) with a group of immigrant youngsters in a Catalonian school district clearly revealed the major tensions to which immigrant students are subjected. Although Gorgorió et al.'s (2004) research focussed on identifying social and political circumstances which generated conflicts in the context of the research, it indirectly revealed the impact of such conflicts on the communication and relationship between students and teachers. For instance, Gorgorió et al. showed that a teacher-participant in the research attributed these conflicts to the immigrant students' rejection of the Catalonian school culture. As a consequence, this teacher-participant had, as would be predicted by Dubet (2004), Glevey (2007), Martin (2009) and many other researchers, low learning expectations for those students, as well as a negative image of them, and a lack of awareness of the need to understand the particularities of their cultural roots. Gorgorió et al.'s proposal is "to spread the idea [in schools] that cultural and social diversity, far from being a problem, can be a source of richness if the teachers can take the advantage of it" (p. 121).

Social Class

Most of the studies on social class in the mathematics education literature frame their questionings, arguments and claims in the terms of theories and models put forward by scholars, like Freire, Bourdieu, Bernstein, Foucault, and others. Relatively few modern scholars have reported empirical studies in which social class and mathematics learning are variables (Cooper & Dunne, 2000).

Nonetheless, three empirical studies deserve our attention due to the inequalities with which they are often associated: differentiated curriculum for low-class students, poor performance of low-class children on national curriculum tests, and difficult school and life conditions for poor children. The first study, reported by Dowling (1998, 2007), provided a critical investigation of school mathematics textbooks in the UK. Dowling analyzed a series of popular mathematics textbooks for school years 7 and 8—namely, the SMP 11–16 textbooks. According to Dowling, these textbooks consisted of a large number of booklets organized for levels and topics which could be used flexibly by all students. However, at the beginning of school year 9, the format of the SMP 11–16 changed, presenting three series of textbooks for use in year 9 and the subsequent 2 years. In a careful analysis of two samples of these series (series *Y* and series *G*), Dowling concluded that the *Y* series and the *G* series were clearly distinguished in terms of the "ability" of the proposed student audience. His main findings pointed out to a strong bias concerning perceived ability and social class: the *Y* series was specifically directed at high-ability students, and the *G* series at lower-ability students. The result was that student groups reflected social class and, among other things, these were marked by differentiated content and classroom discourse.

Dowling's study demonstrated that the meritocratic model, assumed by the use of the SMP series, orientated teachers and students to the belief that ability and social class positioning walked hand in hand, that mathematical ability is an attribute somehow encapsulated in social positioning, and that ability is not changeable or achieved during school years. This same belief clearly was in evidence in the following statement of a mathematics teacher during a conversation with Gates (2006):

> You know, a lot of my bottom group really struggle with maths—and I've noticed they all come from the same part of town, and they have got similar family backgrounds. Surely that can't be a coincidence? (p. 367)

A second study, reported by Cooper and Dunne (2000), addressed the relationships between mathematics success or failure, and social class. Taking the context of the UK National Curriculum and assessment in mathematics, these authors compared a large number of test and interview data. Cooper and Dunne showed that many children failed these tests because they got confused when interpreting items that were concerned with supposedly "realistic" situations, and not because they lacked related mathematical knowledge and understanding. Drawing on Bernstein's and Bourdieu's accounts of social-class differences and cultural orientation, Cooper and Dunne explored whether the same patterns of responses occurred with male and female children and with children from different social classes. They concluded that performance on National Curriculum items in general, and what they called "esoteric" and "realistic" items—referring to Dowling's (1998) introduction of these terms—in particular, varied by both gender and social class.

For instance, in relation to the primary school context, Cooper and Dunne's (2000) results indicated that middle-class children tended to move flexibly and appropriately between and across the boundaries of the "esoteric" and the "realistic" items but working-class children did not. Cooper and Dunne also showed that the tendency of working-class children to solve esoteric items was marked by bringing to their responses considerations of their everyday lives, which were not always appropriate from the point of view of the *language games* that were being played. This indicated at least two things: (a) working-class students seemed to be subjected to a differentiated curriculum, which somehow prioritized mathematical contents drawn on the public domain, rather than on the esoteric domain; and (b) National Curriculum items seemed to be designed for middle-class students. Whatever the case, both teachers and designers of these item tests needed to be aware that inequalities between social classes were being reproduced through National Curriculum test data.

The third, study was developed by Vithal (2003, 2004). Vithal reported on the painful life and school experiences of two Black adolescents—a boy, Wiseman, and a girl, Nellie—who were identified as living in the margins of society. Wiseman and Nellie were *street children* in the city of Durban, South Africa. Both had lived in and attended shelter (usually called "home") schools. Nellie had moved on to a "normal" school. In both cases, Vithal noted that the physical and intellectual conditions of the schools were very poor, insofar as they needed more adequate physical and pedagogical resources. Nellie had attended three different schools and had had a disrupted primary schooling. Like many street children, Nellie had faced experiences of abuse, neglect and poor health while trying to cope with schooling.

The extent of the discrimination suffered by Nellie from both her teacher and her classmates was revealed in an interview, when she commented that because the other students did not understand her situation, they laughed at her, and teased her. Nellie said she liked mathematics, but her test results were very low (Vithal, 2004).

The experience of Wiseman was quite different. He was recognized not only by his teacher, but also by his peers as one of the best students, someone who would definitely be placed into one of the public schools. He was proud and confident of his mathematical ability and assisted his classmates when participating in his mathematics classes.

Nellie's and Wiseman's appreciation for mathematics, despite their harsh conditions of life and schooling, point to, as indicated by Vithal, the need for future research in mathematics education to consider more seriously why and how learners in poverty and in potentially violent situations continue to learn, and want to learn mathematics.

Results of the studies summarized above strengthen the claim that social justice is an urgent matter that needs to be more carefully considered by education policy makers, mathematics teachers, and mathematics education researchers. Social justice in mathematics education cannot be achieved without a political and affective commitment from those responsible for creating mathematics education learning environments.

Appropriate and culturally-sensitive policies, based on modern research findings, need to be devised and implemented. There has been too much rhetoric and too much deficit thinking. In terms of research, how can studies on mathematics learning, mathematics curricula and mathematics teaching effectively lead to fair treatment for disadvantaged students? Can issues like affect, education policies and actions that divide society, and the need for social justice, become central issues in mathematics teacher development courses? Can the powerful meritocratic model be tweaked, so that it becomes a mechanism for equality? If it can, then how?

We close this section with a message to all Nellies and Wisemans, adapted from a reflection of Richard Rorty (1989): "To fail as a human being is to accept somebody else's description of oneself" (p. 28).

Summary

This chapter offers a view of how various socio-cultural perspectives of learning mathematics go beyond the deficit model of learning. The chapter was not intended to revive ideas or discuss data from the deficit research. Instead, it attempted to address the issue in a broad sense, showing a variety of perspectives and reporting on a number of relevant studies, within and around mathematics education. In reporting these studies, we chose to highlight the main conclusions, rather than discuss methods, arguments and evidence used to reach conclusions.

By contrasting deficit models and socio-cultural perspectives of mathematics learning, the chapter displays an uncomfortable reality: despite all academic advances and efforts to emphasize the fundamental role of culture in any individual's learning and development, deficit thinking is still a cloud hanging over the educational context, particularly in relation to mathematics education. By looking at the results of several current research studies that generated results that challenged deficit discourses, and by providing a brief overview of recent research concerning the three traditional deficit markers in mathematics education—gender, ethnicity and social class—the chapter has shown that inequality does persist within the walls of many schools, manifesting itself in different ways and varying across time and within and between nations.

The perspective from which we addressed issues of gender matches what Ernest (2007b) called as "The Public Educator" view, which is that "the gender and mathematics problem is a product of the distorted social construction of gender roles and differences and of mathematics itself" (p. 7). The result of this distortion, said Ernest, can be explained in terms of a vicious cycle: Gender-stereotyped cultural views (mathematics = male, mathematics ≠ feminine) → Lack of equal opportunities to learn mathematics, plus the stereotyped self perceptions of mathematics and mathematical abilities by women → Women's lower participation rate in mathematics → Unequal opportunities to study and work: "critical filter" → Women in lower paid jobs → Reproduction of gender inequality in society → Confirmation of gender stereotyping → Gender-stereotyped cultural views (closing the cycle). In his conclusion, Ernest stated that "only if every link in the cycle is attacked can the reproductive cycle of gender inequality in mathematics education be broken" (p. 8). It is clear that the reproduction of this cycle involves distinct factors that are associated with economic and political conditions, theories and research methodologies, and education practice. The challenge to mathematics educators might be formulated as follows: "What can mathematics educators, teachers and policy makers effectively do to reduce, or even break this cycle?"

Regarding ethnic issues, a very strong argument about the biological non-existence of human races has been provided by Birchal and Pena (2010), who stated:

> The notion of "race" was imported from the common sense to science … Recently, however, the advances of the molecular genetics and the sequencing of the human genome … showed that the labels previously used to distinguish races do not have biological importance. It may seem easy to distinguish phenotypically a European from an African or an Asian, but such ease disappears completely when we look for evidence of these racial differences in genomes. In spite of that, the concept of race persists, qua social and cultural construction, as a way of favouring cultures, languages, beliefs and emphasizing the differences between groups with different economic interests. (p. 24)

These authors analyzed some aspects of the tension between the social and the biological views of race (in connection with the philosophical question of the relation between science and ethics).

Birchal and Pena (2010) cited Relethford (1994, 2002), and Jablonski and Chaplin (2000, 2002)—to support the assumption that, from the biological point of view, human races do not exist. This evidence strongly indicated that there is an excellent correlation between levels of UV radiation and levels of skin pigmentation worldwide: "The degree of skin pigmentation is determined by the amount and the type of melanin in the skin, and these in turn are apparently determined by a small number of genes (4–6) of which the melanotropic hormone receptor appears to be the most important" (p. 24). Birchal and Pena added that external phenotypic features (e.g., nose format, thickness, hair colour and texture) most likely indicate adaptation to environmental conditions and are influenced by sexual selection. And these phenotypical features also depend on relatively few genes. For these authors, these iconic "race" features correlate well with the continent of origin, but depend on variation in an insignificantly small portion of the human genome. In this sense they argued that "race is skin deep. Yet, human societies have constructed elaborated systems of privilege and oppression based on these insignificant genetic differences" (p. 24).

Birchal and Pena illustrated their assumption by analyzing the broad admixture of genes within the three founding continental groups forming the Brazilian population—the Amerindians, Europeans and Africans. The evidence produced a weak correlation between colour (a race correlate) and ancestrality. Consequently, they concluded, that "in Brazil, the colour, as socially perceived, has little or no biological consequence" (p. 24), and raised the question: "Since race does not exist from a biological point of view, would it lead to the moral consequence that the social use of the concept of race should be banned?" (p. 25) This question offers a strong challenge not only for society in general but also for those who support, consciously or subconsciously, the deficit model of learning concerning ethnicity.

Socio-cultural perspectives of mathematics learning cannot by themselves guarantee equality in mathematics education. But they can guide and help policy makers and mathematics teachers to improve their understandings of the diversity of identities and *forms of life* that are encountered in classrooms. It is a matter of being sensitive and dealing with differences not as deficit qualities, but instead as evidence of varieties of singular human beings and familial realities, who need different levels of assistance and care. It is in this sense that this chapter makes claims for social justice.

Osler and Starkey (2010), in proposing to discuss educational inequality and discrimination in terms of human rights, stated

> These standards provide a common point of reference for teachers and educators as they engage with students from a wide diversity of cultural, [economic], ethnic and religious backgrounds. Schools can help to ensure that human rights are known and understood, not simply as normative standards for encouraging pro-social behaviour, but also as a set of principles for critically engaging with social and political realities. (p. 43)

Osler and Starkey argued that the realization of justice is at the heart of the human rights project.

The approach taken in this chapter is consistent with the view expressed by Osler and Starkey (2010). Our discussion of inequality and discrimination in terms of social justice can be viewed as a claim for human rights concerning the specific case of deficit thinking in mathematics education. Our decision to address the main issues in terms of social justice allowed us to develop the critique of the meritocratic model of justice as presented by Dubet (2004). As previously suggested, it is important to question this model since, on the one hand, it is based on the democratic principle of equality, and on the other hand, it has been used as a mechanism of discrimination and exclusion, especially in relation to inequality between social classes. Of course, our approach also has economic and political implications.

Further research on the effects of the meritocratic model on practices, and therefore on people, is needed. How can socio-cultural perspectives guide and support mathematics education researchers, policy makers and teachers to implement these perspectives in educational systems based on the meritocratic model more effectively? If mathematics education is to become a domain that features justice and equality, then responses to this question must incorporate ways of rethinking the model and its use. This is a challenge that this chapter leaves to both practice and to future research in mathematics education.

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