Chapter 24
Technology-Driven Developments and Policy Implications for Mathematics Education

L. Trouche, P. Drijvers, G. Gueudet, and A. I. Sacristán

Abstract The advent of technology has done more than merely increase the range of resources available for mathematics teaching and learning: it represents the emergence of a new culture—a virtual culture with new paradigms—which differs crucially from preceding cultural forms. In this chapter, the implications of this paradigm shift for policies concerning learning, curriculum design, and teacher education will be discussed. Also, the ubiquitous possibility of emergence of ever-new forms of technology brings about both new opportunities for learning and collaborative work (involving students and teachers), as well as potential dangers. Policy measures may give priority to technological access and developments, over the intellectual growth of learners and the professional development of teachers—which should be more demanding goals of mathematics education. Such policy issues will be discussed.

Introduction

The previous chapters in this section of the *Third Handbook* suggest that the emergence and dissemination of digital technology provides opportunities for mathematics education and affects teaching and learning practices in different ways.

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The influence on different mathematical fields has been discussed: for example geometry (Chapter 19), algebra and calculus (Chapter 20), statistics (Chapter 21); as well as on different aspects of mathematics education, such as curriculum design (Chapter 17), modelling (Chapter 18), proving (Chapter 19), the use of interactive resources (Chapter 22), and assessment (Chapter 23).

The impact on mathematics education, however, is not just a matter of individual teachers and students finding their ways to use and benefit from the affordances offered by technological means; the integration of technology in mathematics education involves setting standards (International Society for Technology in Education, 2011) and is also a matter of institutional and national policies with regard to educational reform (UNESCO, 2008). Therefore, this final chapter of this section in the *Handbook* addresses technology-driven developments and policy implications for mathematics education.

Let us begin by clarifying how we understand the expressions used in the chapter's title. By *technology-driven developments* we refer to two levels of developments. At a first level, we consider the developments of digital technology that can be used in mathematics education. For example, interactive whiteboards have been integrated to many mathematics classrooms nowadays. Students have hand-held technological devices at their disposal such as calculators, netbook or laptop computers, in the classroom as well as at home. Through the Internet, both students and teachers have access to online content and resources, to communication facilities and to student management systems which monitor student progress. These first-level developments foster second-level developments, namely individual students and teachers learning to work in new technological contexts. For example, students may change the way they work on tasks and in preparing for tests. Teachers may be tempted to develop new teaching and/or assessment practices. The availability of technology confronts both teachers and students with questions on the relation between paper-and-pencil work and work with technological tools, and on the approach to mathematics—as an experimental science or as a more structural, formal science.

These types of technology-driven developments have repercussions initially at local and individual scales. However, they also have an impact on more global, institutional and national policy levels. Therefore, *policy implications* need to be considered. For example, a school, a group of schools, or a regional school board may decide to abandon textbooks and to use—and eventually co-design—online resources that cover the curriculum. Also, national authorities may decide to allow specific types of technology in centralized assessments. As a third example, teachers may benefit from online collaboration with their colleagues, so as to share, and collectively develop, resources and practices.

Two dimensions seem to be of particular interest in describing policies related to the development of educational technology, namely the top-down/bottom-up dimension and the access/support dimension. The top-down/bottom-up dimension refers to the differences between policies that, on the one hand, may emerge from the needs expressed by students, teachers, parents and other persons involved in mathematics education, and on the other hand may be imposed on the mathematics education community as a result of political choices made by top-level administrations and, thus, at a distance from educational reality. For example, a top-down policy could be a national directive to impose access to graphing calculators during national examinations; whereas support for teachers who start to design their own online resources can be seen as a bottom-up policy.

The access/support dimension refers to the difference between, on the one hand, policies which focus primarily on providing teachers and students with access to technology, and leave the implementation up to the educational field itself; and, on the other hand, policies that focus on supporting teachers and students in the process of integrating technology. For example, providing schools with high-speed Internet connections is typically an access policy, whereas measures for professional development and guidelines for implementation may be more supportive. This access/support dimension is manifest in different statements on the integration of technology in mathematics education. For example, in the USA, the National Council of Teachers of Mathematics (NCTM), in a 2008 Position Statement, claimed that "all schools must ensure that all their students have access to technology" and that "programs in teacher education and professional development must continually update practitioners' knowledge of technology and its classroom applications" (NCTM, 2008).

The two policy dimensions are depicted in the left part of Figure 24.1. We believe that policies are more effective if they emerge from, and respond to, bottom-up developments rather than resulting from top-down initiatives, as will be illustrated in this chapter.

Merely providing access to technology is not enough for promoting educational change; support for teachers' professional development is a necessary precondition for a thoughtful and fruitful integration of technology. In line with this position, the right part of Figure 24.1 shows a potential trajectory towards effective policies, and

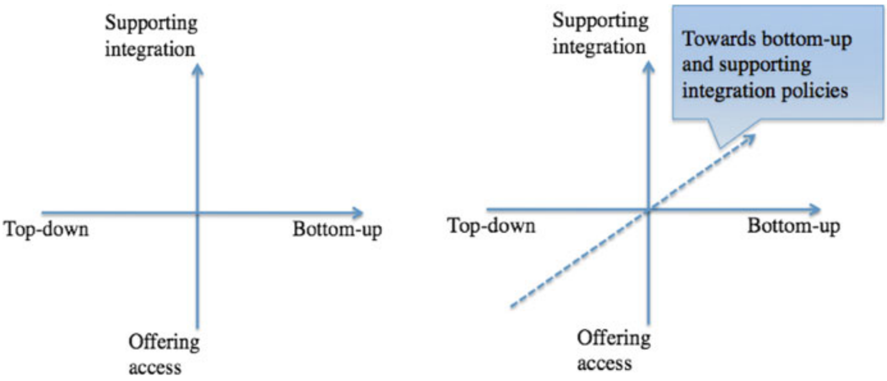

Figure 24.1. The two policy dimensions (*left*), with potential orientation towards bottom-up and supporting policies (*right*).

as such represents a policy shift. Whether these types of shifts can be observed in national developments will be discussed in this chapter. Policy shifts do not fall out of the blue, but reflect or intend to support underlying views on learning, and are mediated by new paradigms of teaching and learning. Therefore, we cannot address policy shifts without discussing, as well, shifting paradigms of learning.

The issue of educational policies and learning paradigms related to technological developments is addressed through the next four sections of this chapter, each offering a different view and illustrated through related national experiments or *windows* on experiences. Part 2 addresses some challenges of policy, curricula and assessment implementation. The shifting learning paradigm that underpins policy changes is addressed in Part 3, and illustrates how new spaces for learning can be opened. Part 4 then describes the role of digital resources in policy making, questioning the two articulated issues of design and quality. Then, since teachers are of crucial importance in mathematics education policies, Part 5 delves more deeply into teacher education, and highlights the new opportunities—such as through networking possibilities— that technology may offer for this. Finally, in the conclusion, we propose an extension to the two-dimensional top-down/bottom-up and access/support model.

Part 2: Policy, Curricula and Assessment Implementations: Evolution and Challenges

In this section we discuss how policies and curricula have tended to integrate technologies for mathematical education and their evolution linked to developments in technologies. We present some cases that illustrate the two-dimensional model discussed above; the policy tendencies in different regions; and how different policies (even within a same region) have different emphases. Finally we address the issue of technological assessment policies.

Historical Evolution of Technology Integration and the Shift Away from Technologies for New Educational Paradigms

The incredibly rapid development and dissemination of technology in society has led to a demand for policies for incorporating technologies into education—such as was proposed in UNESCO's (2005) World Report or in the *Bento Gonçalves Declaration for Action* (Carvalho, Kendall, & Cornu, 2009)—and of setting standards at national and international levels (International Society for Technology in Education, 2011; UNESCO, 2008). However, though there is a generalized political discourse that emphasizes the need to incorporate technology, there seems to be limited visions on *how* to carry this out (Fonseca, 2005). A comprehensive meta-review on research on the integration of technologies (ICT) into education in general (LeBaron & McDonough, 2009) pointed to a gap between educational practice and policies with background theory and research; and calls for research strategies that will support educators to make the best use of the resources that are emerging. It also calls for policies that will help teachers go beyond a technical focus and think of technologies as a means for improving teaching and learning. In fact, as we will illustrate here, it would seem that many policies focus on *digital power*, rather than contemplating a rethinking of educational paradigms in the light of what technologies can bring and change.

Historically, mathematics education was one of the first fields to glimpse the potential of digital technologies, and consider them for mathematics education curricula: For instance, in the 1980s, following the publication of *Mindstorms* (Papert, 1980), the Logo programming language was introduced into mainstream schools and programs, particularly for developing mathematical thinking, in many countries, including the USA and UK (Agalianos, Noss, & Whitty, 2001). Other technologies, such as calculators, spreadsheets and dynamic geometry, also were seen early on as having great potential (as evident in the first ICMI Study—Churchhouse et al., 1986).

Pimm and Johnston-Wilder (2004) provided an interesting historical account, from a UK perspective, of the evolution of the inclusion, policies and relationship in and with school mathematics of technology—from the first calculators and computer programming, to the recent interactive whiteboards. They narrated that, even before the advent of microcomputers, computer programming was part of the UK's mathematics syllabus because of the *special relationship of computers and mathematics*. In fact, technologies and computer programming (e.g., with Logo) were used as a means to develop mathematical thinking (e.g., through *construction and expression*) and to seek deep educational transformations (as inspired by the Logo philosophy). But as computer science evolved, school mathematics distanced itself from it, as explained by Ruthven (2008):

> The rise of Logo … was facilitated by an educational climate receptive to progressive educational ideas … the majority of classrooms took up Logo as part of an incremental view of educational change and were quick to absorb it into existing modes of work … In terms of *disciplinary congruence*, during the period of Logo's rise the *algorithmic thinking* associated with computer programming was being proposed as a modern equivalent of Klein's *functional thinking* … However, this position … lost ground as a wider range of software became available with new types of user interface which pushed programming into the background … In terms of *adoptive facility*, … the lack of a viable platform suited to conventional classroom use was an important barrier … Finally, in terms of *educational advantage*, the perceived value of Logo diminished as the place of more open and extended work in school mathematics was downplayed. (p. 99)

Thus, with the evolution of the nature of the technologies involved, and mathematics increasingly hidden in the software used (Pimm & Johnston-Wilder, 2004), there has been a shift in the past 15 years in how technology and its role is conceived in policy and curricula. In many cases, rather than harnessing the potential of technologies for creating new paradigms of thinking about mathematics and/or of school mathematical practices, technologies are often used to assist in existing traditional mathematical practices (used as tools for visualization, presentation, or for their computational power—see Julie et al., 2010).

Also with the increasing availability of hardware and the development of online resources, Web sites, and the possibilities of networking, there is a focus—at least at top or national levels—to *access,* seeking to provide schools and pupils with technologies (both in terms of equipment and resources). In the case of many developing countries, as discussed in the next section, access seems to be the priority, together with developing computer "literacy," which in some countries implies developing technical competencies for the use of pervading software (e.g., office suites). In fact, as some of the general research reviewed by LeBaron and McDonough (2009) pointed out, there has been a lack of sufficient technological resources in classrooms, as well as of professional development. We will now discuss some cases of national policies with regard to the incorporation of technologies in mathematics education.

Some National Curricula Recommendations and Policy Implementations

In developed countries, technology has been part of national mathematics education policies for several decades. For example, in the USA, as far back as 1980, the NCTM had as one its main recommendations that "mathematics programs must take full advantage of the power of calculators and computers at all grade levels," and that access to those tools should be provided in classrooms (NCTM, 1980); in 2000 it claimed, boldly: "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 24; see also Ferrini-Mundy & Breaux, 2008).

In France, mastering common information and communication technologies is considered one of the major seven competencies of the curriculum (Ministère de l'Éducation Nationale, de l'Enseignement Supérieur et de la Recherche, 2006). At the end of 2009, reforms were announced proposing to offer two weekly hours of computer science in the last-year of high school (*Terminale S*) to science and mathematics students (Ministère de l'Éducation Nationale, 2009).

Julie et al. (2010) described some developments of access to and implementation of technologies in mathematics education in various countries or regions—for example, government initiatives in Hong Kong and South Africa were described, as well as three types of integration in Latin America—the first two, bottom-up, and the third top-down: (a) due to the initiative of individual teachers and/or schools; (b) privately-funded projects (IBM, Microsoft, Intel, etc.); and (c) government-sponsored projects. The paper offered a vision of large-scale projects in several countries (such as those expanded below in Windows 1 and 2, for the case of

Mexico), highlighting the difficulty of such projects, and the problem of the digital divide. It concluded:

> The outstanding similarity is the acceptance at political and bureaucratic level of the use of digital technologies for mathematics teaching and learning in all the countries. However, the translation of policy into practice is a much more daunting task. … Even under massive government implementation, there remain unequal access, unequal resources, and sporadic use of the digital technologies in schools. Political decisions and administrative issues also affect the implementations, the quality of the training of teachers as well as its continuity and that of the projects themselves. (Julie et al., 2010, p. 380)

More recently, many developing countries have ordered hundreds of thousands of *One Laptop per Child* (OLPC) computers, particularly Peru, Uruguay, Argentina and Rwanda (OLPC Foundation, 2011). Though some early reports (Australian Council for Educational Research, 2010) pointed to some positive results, careful evaluations of the effects of activities with these machines—on teacher training, and on mathematics teaching and learning in schools—still need to be carried out.

It is worthwhile taking up the case of Mexico in terms of its national top-down policies for the integration of technologies for mathematical teaching and learning. Between 1997 and 2007, the Mexican Ministry of Education (SEP) launched, in this respect, two very different initiatives with opposite pedagogical and implementation strategies (as explained below): The *Teaching Mathematics with Technology* (EMAT) program (Window 1) and *Enciclomedia* (Window 2). These examples offer insight into the dimensions discussed at the beginning of the chapter, with *Enciclomedia* having a top-down and access nature, whereas *EMAT* conceived as a bottom-up implementation, supporting integration. With government changes in 2007, federal support for both *EMAT* and *Enciclomedia* was discontinued, though *EMAT* continues at regional levels. In 2003, there were 731 schools officially participating in the *EMAT* program.

The availability of *Enciclomedia* resources is limited nowadays (and is no longer available from *Enciclomedia's* official Web site, http://www.enciclomedia.edu.mx/). However, some teachers still use them. The government has now conceived a program called *Habilidades Digitales para Todos* (Digital Abilities for All) with very different aims from those of past projects: this program aims to provide all

Window 1: A First Case of Mexico's National Implementations: The EMAT Project

EMAT, which began in 1997 (together with parallel sciences programs—*ECIT-ECAMM*) aimed to incorporate technologies in middle schools (for students from 12 to 15 years) in order to transform educational practices from the traditional teacher-to-student, top-down approach towards student-centred, exploratory, bottom-up practices. An international team of mathematics education researchers designed a constructivist, pedagogical model

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Window 1: (continued)

and activities. Universal open tools (that allowed different objectives) were preferred, such as spreadsheets, dynamic geometry (*Cabri-Géomètre*), the TI-92 algebraic calculator and, later, Logo. Emphasis was put on changes in the classroom structure, on collaborative exploratory work, and on a teaching model based on mediation and guidance (Ursini & Rojano, 2000). *EMAT* was designed to be implemented gradually—beginning with eight schools in 1997, and gradually expanding over the course of several years—so that adjustments and support could be provided, and the quality of teacher education and implementation in classrooms would be optimized.

Though the implementation in schools was not as straightforward as planned (for example, preservice and inservice education were limited in scope— see Trigueros & Sacristán, 2008)—*EMAT* was groundbreaking in the ways it opened doors to integrate technologies in schools. Its use was recommended in the official national mathematics curriculum, and has extended beyond the originally-conceived policies. Some teachers who have been working with *EMAT* over many years, have been able to integrate the use of diverse tools and develop their own long-term projects—like, for example, the series of long-term *Painless Trigonometry* projects (Jiménez-Molotla & Sacristán, 2010), which was developed by a couple of teachers on the basis of EMAT's triangle activities. In *Painless Trigonometry* projects, students participated in activities which helped develop their trigonometric concepts and ideas through complementary explorations and constructions with the *EMAT* tools and other software (Figure 24.2). This led, in one case, to the construction, by the students themselves, of 3D computer models of triangle-based figures (such as pyramids).

In some regions, local officials still coordinate and support teachers' communities of practice for *EMAT*, hold monthly workshops, develop new materials, and have developed anthologies of *EMAT* activities for different tools—see Figure 24.3 (Sacristán & Rojano, 2009).

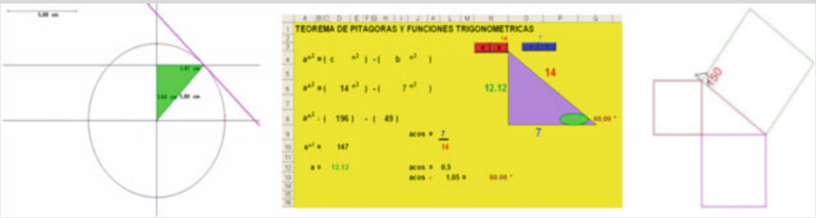

Figure 24.2. Complementary trigonometrical explorations with Cabri, Excel and Logo.

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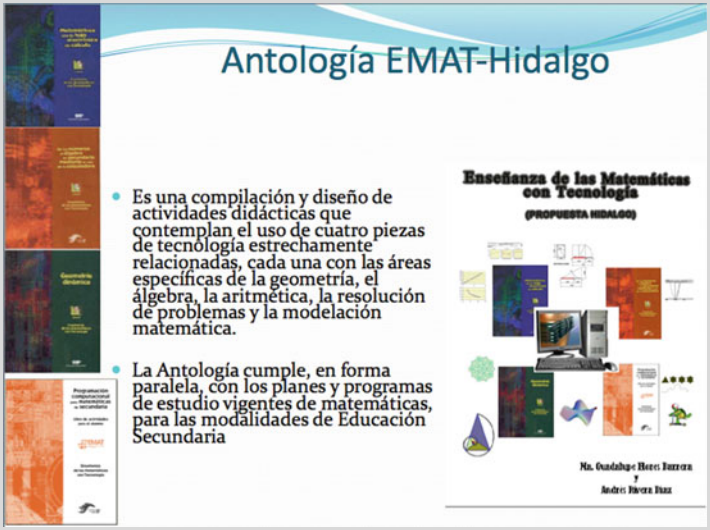

Antología EMAT-Hidalgo

- Es una compilación y diseño de actividades didácticas que contemplan el uso de cuatro piezas de tecnología estrechamente relacionadas, cada una con las áreas específicas de la geometría, el álgebra, la aritmética, la resolución de problemas y la modelación matemática.
- La Antología cumple, en forma paralela, con los planes y programas de estudio vigentes de matemáticas, para las modalidades de Educación Secundaria

Enseñanza de las Matemáticas con Tecnología

(PROPUESTA HIDALGO)

Ma. Guadalupe Flores Herrera
y
Andrés Rivera Díaz

Figure 24.3. *EMAT* activities from the state of Hidalgo.

Window 2: A Different National Implementation in Mexico: Enciclomedia

Unlike *EMAT*, *Enciclomedia* was the result of an ambitious political decision, made in 2004, to implement digitalized versions of official Grade 5 and Grade 6 textbooks in all subjects in all primary schools in Mexico. It included accompanying digital resources and interactive whiteboards. For this project, a huge number of ad hoc interactive resources (applets) were produced in a very short time. However, the use of open universal tools, such as those used in EMAT, did not occur (Rojano, 2011). A view of this production of interactive resources for the mathematics curriculum (such as the one illustrated Figure 24.4) has been presented by Trigueros and Lozano (2007).

One of the most successful (and popular) mathematics resources from *Enciclomedia* was *La Balanza* (“The Scale,” see Figure 24.4), for which users input numbers (e.g., fractions, decimals) and, using the scale metaphor, investigate notions such as equivalent fractions. Trigueros and Lozano (2007) found that this applet gave students and teachers freedom to explore mathematical situations through interesting mathematical activities and challenges.

Despite some successes, the haste with which *Enciclomedia* was implemented resulted in shortcomings. Rojano (2011) explained that there was an

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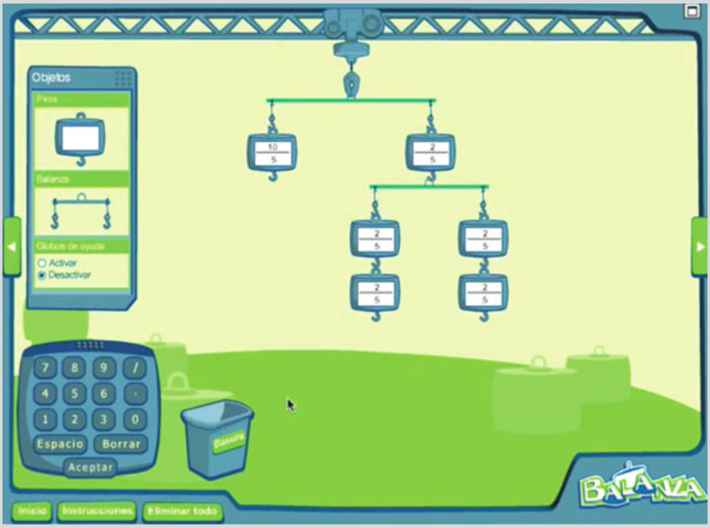

Figure 24.4. Screen capture of Enciclomedia's *La Balanza*.

obvious jump from resource availability to its use in the classroom, with many teachers not ready to experiment and appropriate the tools in ways suggested by Artigue's (2002) *instrumental genesis* theory. Artigue (2002) had called for a *gradual implementation* that allowed for feedback from research as well as the inclusion and linking with other types of resources, such as those from EMAT.

students with laptops (the access dimension) and promotes the view that all teachers should have competencies in basic software, specifically in MS Office (Bernáldez, 2011).

The case of Mexico draws attention to an issue that arose in many implementations—specifically, a lack of continuity in policies. Mexico is an example of a country where policies increasingly shifted towards *access*, and away from meaningful and supportive *integration* for mathematical learning. It also points to how social, adoptive, practical and other factors can affect policy implementation with respect to technology. Many other factors come into play (for examples, see Julie et al., 2010) and these can create gaps between political will and school and teacher implementation (Ruthven, 2007). Assude, Buteau, and Forgasz's (2010) classification into levels of factors influencing this contradiction included the social and political levels, the mathematical and epistemological level, the school and institutional level, and the classroom and didactical level. Difficulties arising from the need to develop technological competencies among teachers, and associated pedagogical difficulties, proved to be especially important (Trigueros & Sacristán, 2008).

A further consideration, related to policy and curricular changes, is that of the role of technologies for assessment. This is now discussed.

Assessment Policies

Assessment is an important and widely debated aspect of national policies, with respect to the use of technology in mathematics education. It is beyond any doubt that assessment drives teaching and affects educational reform. This particularly holds in countries where national, externally-set final examinations are used as a main form of assessment. Meanwhile, research findings on this topic are limited. In the frame of the ICMI Study 17, Sangwin, Cazes, Lee, and Wong (2010) focussed on computer use for automatic feedback during online assessment, but did not discuss policy aspects of the use of technology in assessment—issues related to the kinds of tasks that might be appropriate, and implications for pedagogy, were not considered in depth.

Leigh-Lancaster (2010), by studying how CAS technology has been incorporated into upper secondary mathematics curriculum and examinations since the year 2000 in Victoria (Australia), offered a broad perspective of the challenges and experiences of assessment that is congruent with technology integration in mathematics programs. One issue is that standard models of assessment seem to be incompatible with new educational paradigms that are promoted by the use of technologies (Stroup & Wilensky, 2000). The rationale for assessment related to these new paradigms perhaps needs further elaboration which takes into account the learner's development (Lesh, Hoover, Hole, Kelly, & Post, 2000). Some research (e.g., Hernandez-Sánchez, 2009) has delved into the issue of how to evaluate students' work and learning in classrooms in which contemporary technology tools are being used (Window 3).

As mentioned in the last chapter (Chapter 23) of this *Handbook*, concerning the role of technology in national mathematics examinations, Drijvers (2009) distinguished between four assessment policies:

1. Technology is (partially) not allowed;
2. Technology is allowed, but offers no advantage;

Window 3: A Search for Developing Assessment Methodology for Work with Technology

Hernandez-Sánchez (2009) identified three areas to assess: (a) development of abilities and mathematical content knowledge, (b) use of resources, and (c) collaboration and participation. In order to observe the work in progress

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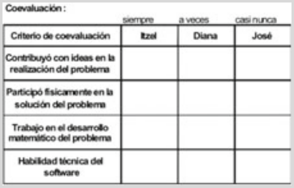
Window 3: (continued)

with technology in a classroom, she developed a series of instruments for her own assessment and for students' self-assessment and co-assessment (with student in teams evaluating each other—see Figure 24.5).



Coevaluación :

	siempre	a veces	casi nunca
Criterio de coevaluación	Itzel	Diana	José
Contribuyó con ideas en la realización del problema			
Participó físicamente en la solución del problema			
Trabajo en el desarrollo matemático del problema			
Habilidad técnica del software			

Figure 24.5. A co-assessment form to be used by a team of students.

3. Technology is recommended and useful, but its use is not rewarded; and
4. Technology is required and its use is rewarded.

With the fourth of these policies, conceptual skills, such as interpretation, reasoning, mathematization, justification and modelling, are examined. However, designing appropriate examination tasks for such goals is not trivial. Brown (2010) developed a similar scheme of analysis that identified four categories for technology in assessment: namely active required, active optional, active neutral and active excluded.

Drijvers (2009) investigated policies in some countries in Western Europe, and concluded that although many countries have Type 3 and Type 4 policies, they nevertheless concentrate on assessing paper-and-pencil skills, either through a non-technology part of the examination (consistent with a Type 1 policy) or through the use of specific vocabulary in the wording of items that indicates that paper-and-pencil methods are required. If technology is allowed during the assessment, a common limitation concerns communication facilities. An exception to this can be found in experimental examinations in Denmark, in which students have Internet access during the session. However, in France, after attempting to organize an "experimental test" for the *baccalauréat* in mathematics, the national authorities finally decided it was too difficult to organize both the assessment itself and the class preparation (Sur l'épreuve pratique, 2007).

Some Closing Remarks to Part 2

In this section we have presented part of the evolution of technology integration into mathematics education and related policies, which shows a shift-away from the early tendencies where technologies and computer programming were viewed as means to innovate education towards constructivist—and *constructionist* (Papert, 1991)—educational paradigms. Social and implementation difficulties, as well as the profusion of technological resources (to be discussed in Part 3 of this chapter) have brought about a change in these tendencies (Agalianos et al., 2001; Ruthven, 2008; Pimm & Johnston-Wilder, 2004).

We have also presented some examples of national policies. The contrast between the Mexican *EMAT* and *Enciclomedia* policies, not only illustrated some of the support/access, and bottom-up/top-down dimensions but also highlighted the contrast between focus on individual learning versus a collective approach, a dimension which we will discuss in the concluding section of this chapter.

Finally, concerning assessment, we claim that this is an important, but underestimated, aspect of policies on the integration of technology in mathematics education. As Kaye Stacey and Dylan Wiliam have pointed out in Chapter 23, issues relating to technology and assessment deserve more attention from the research community.

Part 3: Mathematics Learning and Teaching Spaces

The impact of national policies and strategies finally come down to teachers, either individually or collaboratively, getting involved in the design of digital resources, and facing the challenge of how to turn the available resources into effective education. Such design and integration processes, however, are not neutral, in the sense that they reflect views on learning and teaching. These views may be affected by the new opportunities technology offers. In the present section, therefore, we elaborate on this by considering relationships between the integration of technology in mathematics education, and the paradigms of its learning and teaching.

Let us first focus on learning. Technology offers opportunities to enlarge students' learning spaces. As such, it potentially extends the scope of learning, the repertory of forms of learning, and offers opportunities for new paradigms for learning. But what do we mean when we speak about "enlarging learning spaces" for mathematics? We now address some aspects of this multi-faceted concept.

Mathematical Learning Spaces

What are potential dimensions of an enlarged technology-supported learning space? A first obvious, but non-trivial, dimension that technology may bring about, concerns the learning space, in the literal sense of distance and time: technology offers new means for *ubiquitous learning*, in which students can access resources at every moment, in every place, and in a variety of synchronous as well as asynchronous modes. As an anecdotal example, it is not uncommon, these days, to see students sitting in the bus to the university campus watching video recordings of last week's class on their smart phones. Learning becomes independent from time and location, becomes *mobile*, and this is indeed an extension of the learning space. Thanks to technology, and to online resources in particular, distant learning has become quite common. The learner decides on what, where and when to learn.

A second, related aspect of the enlarged learning space concerns the opportunities for organized forms of *out-of-the-classroom* or *out-of-school learning*. Students equipped with handheld devices can go outside classrooms to gather real-life data that inform their biology or chemistry lessons. More specifically for mathematics, students can use GPS technology for a mobile geometry game in the school-yard (Window 4).

A third and more subtle aspect of the extended learning space brought about by technology, concerns what we would like to call the student's *mental learning space*. The use of technology may, on the one hand, invite mental activity, and on the other, free students from basic mental activities that may distract them from

Window 4: MobileMath Game with Handheld GPS Technology

In this example, taken from Wijers, Jonker, and Drijvers (2010), teams of Grades 7 and 8 students used handheld GPS devices to play an outdoor game in which they had to construct parallelograms and try to destroy other groups' geometrical shapes. The aims were to make students experience properties of geometrical figures in a lively, embodied game context.

Student actions while playing the game include looking at the map to imagine where they want to make a shape, walking to the location for the first vertex to enter this location in the mobile device, which generates a dot on the map, walking again to the location of the second vertex of their imagined shape which provides a line on the screen connecting the first vertex with the current (moving) location, etc.

The map in Figure 24.6 illustrates some student constructions. The results of the pilot experiments suggest high student engagement and motivation. Students learned how to use the GPS, to read a map, and to construct quadrilaterals. The study suggested mathematical learning opportunities that need further investigation.

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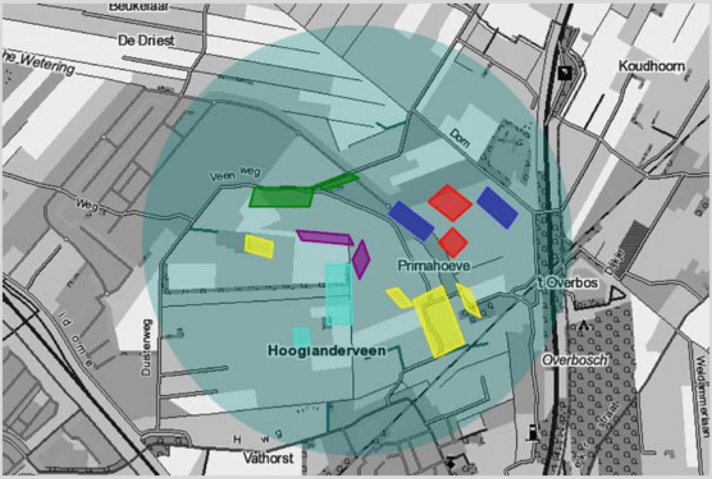

Figure 24.6. Map of students' parallelogram constructions using GPS.

higher goals. Depending on the task, technology may provide space for exploration, for discoveries in microworlds, for dynamical investigation of variance and invariance, for design of—and links between—representations; in short, for knowledge construction. Through technology, students can have early access to advanced mathematical ideas in a non-structured or nonlinear way (see Sacristán et al., 2010), as expressed by the *webbing* idea proposed by Noss and Hoyles (1996). A point of concern here, however, is that these challenging potentials are not easy to exploit in every-day mathematics teaching. For example, the seemingly trivial techniques for using technological tools are often interrelated to conceptual aspects (Lagrange, 2000).

A fourth, interesting aspect of how technology can enlarge the learning space, concerns the opportunities technology offers for *collaborative learning*. Thanks to online connectivity and social media, communication, exchange, and collaborative work are not limited to face-to-face meetings but can take place at a distance. This affects the paradigm of learning as an individual activity and widens the horizon to more intensive online collaborations (Hoyles et al., 2010).

A fifth and final aspect is that technology also enlarges the *learning space for teachers*, who are confronted with challenging questions on how to exploit the opportunities technology offers, how to organize the learning, and how to learn to organize the learning. This aspect is addressed in more detail later in the chapter.

To summarize all of the above, a new paradigm for learning has emerged, one which is influenced by the seemingly unlimited learning spaces generated by new technologies. The more classical view on learning as an individual, in-school, linear process has been challenged. Learning is now being seen: as ubiquitous, rather than in-school; as involving active construction, rather than passive reproduction; as a Web-like, rather than a linear process; as bottom-up, rather than top-down; as self-dependent, rather than teacher-dependent; as collaborative, rather than individual; and, finally, as aiming at conceptual, rather than procedural knowledge.

Even if this new paradigm for learning may sound very appropriate for the 21st century, as well as appealing in the light of new demands for workers and citizens, its realization in classroom practice—within its institutional constraints—turns out to be far from a trivial matter (Ruthven & Hennessy, 2002). Therefore, we now consider the exploitation of the teaching space as it is opened up by the availability of educational technology.

Mathematical Teaching Spaces

If technology has the potential to enlarge students' learning spaces, how does this affect teaching practice? How can teachers manage the learning spaces and *orchestrate* classroom situations to exploit them? What are the consequences of new paradigms for learning and for educational formats, classroom organization, pedagogical approaches and teaching strategies?

As a means to address these questions, Trouche and colleagues developed the notion of *instrumental orchestration* (Drijvers & Trouche, 2008; Trouche, 2004). An instrumental orchestration is a teacher's intentional and systematic organization and use of the various artefacts available in a—in this case computerized—learning environment for a given mathematical task; it includes setting up the scene, exploiting it and taking ad hoc decisions. Other models are available. For example, Ruthven and Hennessy (2002) designed a *practitioner model* for the use of technology in mathematics teaching. Pierce and Stacey (2010) offered a *pedagogical map*, which may guide teachers in their articulation of tools, task and teaching techniques. Finally, the notion of *Technological Pedagogical and Content Knowledge* (TPACK, Koehler, Mishra, & Yahya, 2007) identifies different types of knowledge that teachers need, as well as their interactions, and as such may help teachers to position their knowledge and identify possible weaknesses. Whether these models really can help teachers in their professional development on the issue of teaching with technology, is still to be investigated.

Earlier, we claimed that technology offers opportunities for ubiquitous and out-of-school learning, for widening students' learning spaces and for collaborative learning. How can these opportunities be dealt with in teaching? The idea of *ubiquitous and out-of-school learning* challenges the traditional teaching formats, as it is difficult for the teacher to know what students do and learn. Learning trajectories may take different directions at different speeds. However, technology also offers solutions to this through the availability of student monitoring systems, which allow teachers to access online students' computers or devices. This allows for the preparation of face-to-face teaching that takes into account the students' proceedings and benefits from the different approaches they could have developed during their out-of-class work. Window 5 sketches such an approach, in what was called a *Spot-and-Show* orchestration (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010). It illustrates the way in which the availability of technology can enlarge the mathematical teaching space, by offering the opportunity to access students' work and monitor students' progress through digital means, and fine-tune the face-to-face teaching to that.

Window 5: The "Spot-and-Show" Orchestration

In this example, taken from Drijvers et al. (2010), we imagine a teaching situation in which ICT allows a teacher to access digital student work while preparing his lesson. As he does that, he notices something special in the work of one of the students—such as a remarkable mistake, a misconception, or a surprisingly original solution. The teacher decides to exploit this during the lesson and shows the student's work to the whole class by means of a projection. Next, he may ask the student to explain his approach or reasoning. Peers can comment and the teacher can explain why he considered that this particular solution was worthy of special attention.

As an example of Spot-and-show, Grade 6 students had compared dot graphs of the square and the square root function (Figure 24.7). One pair of students typed in the digital environment: "And the square of a number is

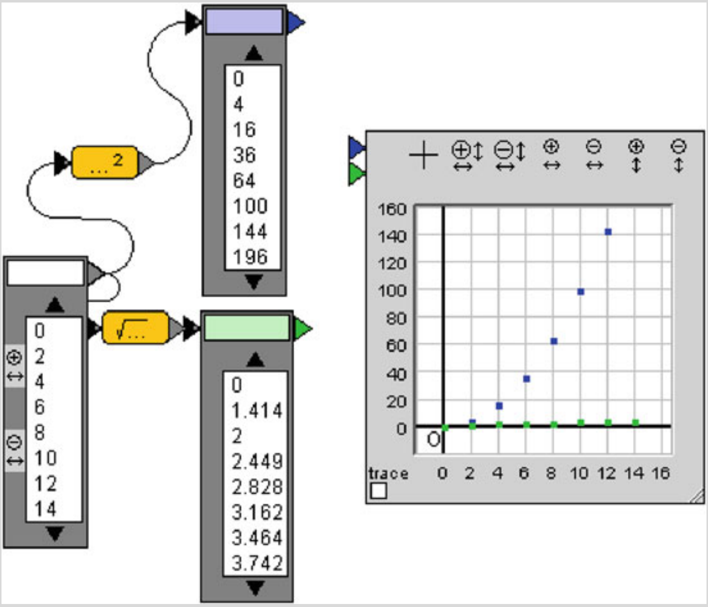

Figure 24.7. Comparing the square and the square root.

(continued)

Window 5: (continued)

always right above the root." The teacher wanted to draw attention to the fact that the value of the dependent variable is always positioned vertically above the value of the independent variable, and that this has nothing to do with the type of function involved. Therefore, she projected this answer to the classroom. After a whole-class discussion, one of the students said: "That's because the line underneath, that's got a number on it, which you take the square root of and square, so it's on the same line anyway."

Concerning the widening of *students' mental learning space*, the question of how best to exploit this is not an easy one to answer. Of course, students' mental activity is not stimulated by the availability of technology in itself, but largely depends on the task, the affordances and constraints of the tool, and orchestration of all this by the teacher. As a teacher, one needs to be aware of the subtle interaction between techniques for using the tool and mental activity, as it is reflected in the notion of instrumental genesis (Artigue, 2002). To enhance this, new organizational forms of teaching might be designed. Some studies suggest that teachers are less drawn to whole-class teaching in technology-rich education than they are in regular lessons (Drijvers, 2012). We strongly believe, however, that interactive forms of whole-class teaching are crucial for exploiting, making explicit and reflecting on students' individual hands-on experiences. For enhancing such whole-class interactive teaching formats, classroom connectivity tools are available, such as the *TI-Navigator*, voting boxes or different types of digital pen technology (Hoyles et al., 2010).

Technology opens new horizons for addressing *collaborative learning* in teaching. Collaborative work can be part of assessment and students could be encouraged to use online chat while working on their mathematical tasks at home or to have other types of online peer interaction. The teacher himself may be engaged in these types of collaboration. An online consultation hour for students might increase student–teacher interaction. As will be explained later in this chapter, collaborative learning also applies to teachers' collaborative work and their professional development. Technology may support teacher education through the sharing of experiences and the collaborative design and use of online resources. In this sense, technology also enlarges the teachers' own learning space. Results from the nationwide evaluation of *EMAT*, discussed earlier, showed that teachers' learning is enhanced (Trigueros & Sacristán, 2008).

To summarize this section, we claim that, on the one hand, the availability of technology enlarges students' learning spaces in several aspects and leads to new paradigms of learning. On the other hand, ways by which teachers can fully exploit the potential of these resources are not yet evident. Nevertheless, the design and diffusion of teaching resources is a major issue within educational policies. This relationship between the dissemination of resources and educational policy is the main theme of the next section.

Part 4: A Profusion of Resources, Opportunities and Questions

We now turn to a central issue for educational policy: as in the case of the examples in Mexico, provision of resources has often been seen as a way to influence what happens in the classroom (see, e.g., Ball & Cohen, 1996; Pepin, 2009). Although traditional textbooks remain central, digital textbooks are becoming much more prevalent, and there is a profusion of other available digital resources: Web sites, interactive applications, online videos, forum discussions, etc. The devisers of these resources and participants in these online exchanges may be professional designers, teachers, educators and educational researchers.

This situation raises new policy questions, such as the following:

- What are the key design modes of these new resources? Who designs and what do the design processes look like?
- How to assess the quality of the resources? Which criteria are set for linking quality and design mode, and by which assessing authority?

In the course of discussing these questions, we draw, in particular, on two examples of innovative projects in Europe: *Sesamath* and *Intergeo*.

Towards New Design Modes

From a technical point of view, designing and broadcasting online resources is within the scope of most teachers. The networking possibilities foster the development of online communities, designing resources. For example, the Geogebra community [http://www.geogebra.org/] (Lavicza, Hohenwarter, Jones, Lu, & Dawes, 2010) gathers teachers and researchers all over the world, designing resources, organizing training sessions, and conferences around this educational software. In France, an example of such an online community is the *Sesamath* association (see Window 6), whose Web site records more than 1.3 million visitors each month.

Window 6: From Drill-and-Practice to Virtual Environment: Sesamath

Sesamath [http://www.sesamath.net/], a French online association of mathematics teachers (most of them teaching in Grades 6–9), started in 2001. Its spirit is summarized on its Web site as “Mathematics for all.” It offers several kinds of free resources: online exercises, dynamic geometry software, online textbooks, etc.

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Window 6: (continued)

Sesamath started with a gathering of some 20 mathematics teachers, who shared their personal Web sites and then designed together a drill-and-practice piece of software called *Mathenpoche* (Gueudet & Trouche, 2012a). *Mathenpoche* was immediately very successful, in the sense that it was used by many teachers and students. In some regions, the local educational or political authorities supported its development by offering dedicated servers.

Several changes took place between 2005 and 2006. The association started to collaborate with researchers (Kuntz, Clerc, & Hache, 2009) and the designed resources integrated results of these collaborations. For example, a virtual abacus [http://cii.sesamath.net/lille/exos_boulier/boulier.swf] was developed for primary school, and new exercises, with several solutions, were added in *Mathenpoche*. At the same time, *Sesamath* decided to develop textbooks and, through the use of an online platform, involved others teachers—outside of the association—as authors. The resulting textbooks, freely available online, were also published on paper, and sold for half of the price of regular textbooks. Some commercial publishers attempted legal action. Due to the importance acquired by *Sesamath* resources, some educational authorities started to question their quality.

The development of the association's activities continued with a Web site, *Sesaprof*, allowing users to contribute to the design of resources (Sabra, 2009). The main current *Sesamath* product is *LaboMEP* (see Figure 24.8), a virtual environment where teachers can choose various kinds of activities: online exercises, dynamic figures, extracts of textbooks. They can, among a range of possibilities, combine some of them, or assign them to specific pupils.

Explaining the reasons for the success of *Sesamath* requires specific research. The existence in France of the IREMs (Institutes for Research on Mathematics Education), a national network that involves many mathematics teachers, has played an important role. A similar project could perhaps not succeed in countries were such a network, linked with mathematics education, did not exist.

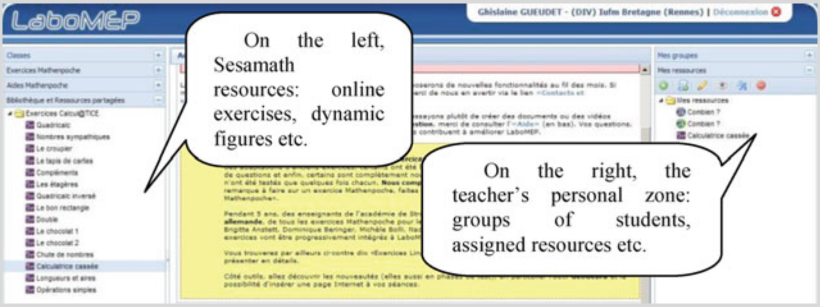


Figure 24.8. LaboMEP, a virtual environment for the teacher.

In France, no "official" online resources exist. Though files can be downloaded from several institutional Web sites (such as those of the Ministry of Education or regional academic authorities), they only concern specific topics. This is different from the *Enciclomedia* project in Mexico (Window 2), directed by the government, and providing ad hoc resources to support the official textbook; or the *Enlaces* project [http://www.enlaces.cl] in Chile that has similar features to *Enciclomedia*. Although the *Sesamath* example shows how new, bottom-up modes of design and collaboration can emerge, the examples in Latin America show that traditional centralized modes of expert production for system-wide dissemination also exist.

The availability of free resources is of economic importance, as it raises the issue of competition with commercial resources (in countries where commercial teaching resources are allowed). In some countries, governmental institutions themselves design resources, or offer opportunities for teachers to engage in the creation of resources, competing with the commercial productions (for example Wikiwijs in the Netherlands [see http://www.wikiwijs.nl/sector]).

Design issues should not be seen merely as a simple bottom-up versus top-down, or private versus public confrontation; they are more complex, involving a variety of agents. Communities of designers and users of resources include members with different positions: including regular teachers, expert teachers (with the status of teacher trainers, in some countries), and researchers.

The collaborative design of online resources is important for educational research. That is not only because research is needed to enlighten the new design modes, but also because many researchers are actively involved in the design process. This involvement is rooted in a long tradition, both in the field of research on technologies and in the field of task design (Watson & De Geest, 2005). Digital networks offer new possibilities for large projects associating teachers and researchers. Below we discuss the case of the *Intergeo* project (Window 7). Another important example of such collaboration is the UK's National Centre for Excellence in the Teaching of Mathematics (NCETM—see Chapter 16). Joint work for the design of online resources can enhance relations between researchers and teachers.

Window 7: Quality of Dynamic Geometry Resources: The Intergeo Project

Intergeo [see http://i2geo.net/] (Kortenkamp et al., 2009) is a European project that began in 2007. It has three aims: (a) inter-operability of the main existing DGS (Dynamic Geometry Systems); (b) sharing pedagogical resources; (c) quality assessment of resources (Trgalovà, Jahn, & Soury-Lavergne, 2009, p. 1162).

Any user logged on the *Intergeo* platform can propose a resource, which will be immediately published online (more than 3,500 resources were published

(continued)

Window 7: (continued)

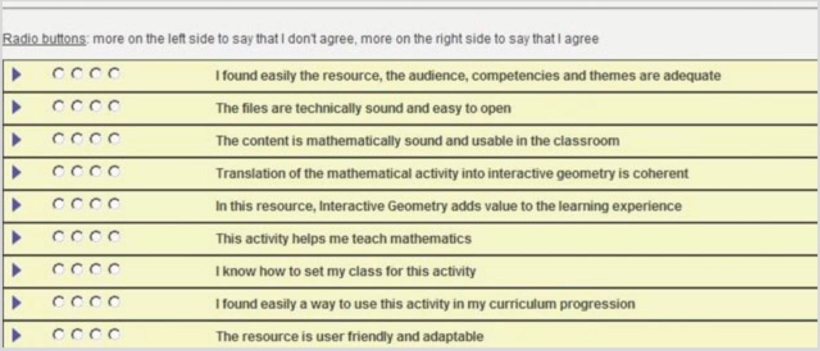
in January 2011). This feature makes the resource quality assessment essential. This quality assessment in *Intergeo* draws on the users' opinion, considering that the quality of a resource can only be defined in relation with a given teaching context.

The main assessment tool is a questionnaire (Figure 24.9) proposed on the user's platform (Trgalovà et al., 2009). This questionnaire takes into account nine different dimensions: metadata, technical aspect, mathematical content, instrumental content, added-value of dynamic geometry, didactical implementation, pedagogical implementation, integration in a teaching sequence, ergonomic aspects.

A user can choose to answer only a simple version of the questionnaire (giving an opinion on each dimension) or to give more details. For each dimension there are several precise statements. For example: "The activities are appropriate, given curricular and institutional constraints" (mathematical content); "The DG provides an experimental field for the learner's activity" (added-value of DG); and, "The resource describes possible students' strategies and answers" (didactical implementation). The answers are automatically collected and treated, and this treatment leads to a label (a number of *stars*) associated to the resource on the Web site.

The authors can freely modify their resources. If a participant, who is not the original author, wants to modify a resource, he/she has to copy it. The system allows following and connecting all the versions. Modifications can help improve the resource's quality; moreover, the questionnaire itself also contributes to this improvement, by raising the awareness of designers (who completed the questionnaire as users) on important dimensions of the resources.

In June 2011, *Intergeo* gathered 1,200 registered members. It contains around 3,500 resources; and altogether 700 evaluations have been proposed. This amount might seem to be limited; but the evaluation process only started in 2009.



Radio buttons: more on the left side to say that I don't agree, more on the right side to say that I agree

▶ ○ ○ ○ ○ I found easily the resource, the audience, competencies and themes are adequate

▶ ○ ○ ○ ○ The files are technically sound and easy to open

▶ ○ ○ ○ ○ The content is mathematically sound and usable in the classroom

▶ ○ ○ ○ ○ Translation of the mathematical activity into interactive geometry is coherent

▶ ○ ○ ○ ○ In this resource, Interactive Geometry adds value to the learning experience

▶ ○ ○ ○ ○ This activity helps me teach mathematics

▶ ○ ○ ○ ○ I know how to set my class for this activity

▶ ○ ○ ○ ○ I found easily a way to use this activity in my curriculum progression

▶ ○ ○ ○ ○ The resource is user friendly and adaptable

Figure 24.9. *Intergeo* questionnaire on the platform, short version.

The evolution of design modes also has an impact on the articulation of design/use as well as on the very notion of authorship. Users send their comments and suggestions; and designers modify the resources according to these contributions. A given initial resource can lead to many different versions, and identifying the contributors of one of these versions is often impossible. Moreover, teachers naturally adapt resources to their own use. This process is not new: teachers have always selected parts of textbooks, extracts from students' productions, etc. Nevertheless, the technical possibilities foster this process: teachers download files, and can easily copy and paste parts of these to produce their own files. This *documentation work* (Gueudet & Trouche, 2009) views teachers as designers of their own resources; and generally points to a need to reconsider borders between design and use.

These evolutions introduce a paradigm shift for the design of resources: the resources are never complete, but always involved in design processes. Directing this permanent move towards an increased *quality* is an essential policy issue that we discuss in the next section.

Assessing and Improving Resources Quality

Choosing a resource, for a given teaching or learning objective, is a difficult task. It is, firstly, linked to the issue of *indexation*, investigated by many computer scientists and also educational researchers (Lee, Tsai, & Wang, 2008). But the choice problem is not restricted to indexation; the metadata cannot certify the resource's *quality* that is considered both in terms of *intrinsic* quality and for its *adequacy* with respect to a user's expectations.

Defining the intrinsic quality of an online resource, for the teaching of mathematics, is not straightforward. Which criteria can guarantee this quality? Naturally, such criteria have to take into account three dimensions: *mathematical*, *didactical*, and *ergonomic* (ease of use). But even these dimensions do not fully take into account the *appropriation* by a user. Quality also encompasses the *potential* of a resource: potential for uses in class, for further design, and even for teacher professional development (see later in this chapter). In fact this question cannot have a general, unique answer. With the *Intergeo* project (Window 7) quality criteria were defined, with a focus on the added-value of dynamic geometry, particularly in terms of investigation possibilities for the students. Other criteria could be used for other foci.

Beyond the choice of criteria, the issue of *who assesses the quality* can also be delicate. In some countries educational authorities have developed certifications (in France, a national label, attributed by the Ministry of Education, indicates a resource of "Recognized Pedagogical Interest"). Different kinds of agents can intervene in the assessment process: like, for example, stakeholders such as teachers (expert or not) and researchers. In some cases, the Ministry of Education calls for researchers to intervene as experts in quality assessment tasks (as in the *Pairform@nce* program—see Window 10).

Answering the "who assesses the quality?" question drives us back to the bottom-up versus top-down confrontation and to all the intermediate possibilities.

In the *Intergeo* project, the quality assessment is grounded on the users' opinions (as these opinions are expressed by a carefully designed questionnaire). Quality and design issues are intertwined. The involvement of users in the design of a resource and the organization of *design loops* (design-use-feedback-new design) are presented by several authors (see, e.g., Hegedus & Lesh, 2008) as likely to contribute to quality, in particular by fostering the resource's appropriation potential.

Resources, Policies and Practices

We developed, in the previous sections, two important—and articulated—aspects of educational policies, concerning digital resources: their design and their quality (assessment, and improvement of quality).

These aspects can help to situate a given policy in our 2D system of axes. Indeed the design of resources can be more top-down, linked with official resources, designed by experts; or bottom-up, with a support for communities of teachers designing resources. Web sites (whoever the designers are) can propose ready-made resources, expecting the users' alignment, or can take into account the complexity of the appropriation processes, offering possibilities of adaptation. The quality assessment can be in the hands of experts; it can also be entrusted to the resources users (as in *Intergeo*).

A new important dimension appears here, concerning the production paradigm: the design of resources seems to be an increasingly collective process. We could thus complement the initial two axes displayed in Figure 24.1 with a third one, representing an individual/collective evolution, and could figure the paradigm's shift, concerning the production of resources for teaching, as a move in this 3D system of axes.

This third axis, individual/collective, is also very important for characterizing the teacher education aspects of a policy, an issue that will now be discussed.

As a final remark on designing and integrating resources, we notice that, whereas at the present time, students can be considered *digital natives*, most teachers are learning to speak *technological language* as their second, third, fourth, …, language. This brings us to the issue of teacher education and pre- and inservice professional development.

Part 5: Teacher Education Strategies, Policies and Practices

Technology opens the horizon for new forms of orchestrations, but "the process of orchestrating technology-integrated mathematics learning is neither a spontaneous nor a rapid one" (Healy & Lagrange, 2010, p. 288). This certainly requires new resources and new competencies for teachers. To what extent do the resources for such a development exist? To what extent do new teacher education programs help teachers build such competencies? In this final section of the chapter we shall examine these questions, drawing special attention to two examples of innovative programs.

Teacher Education: Back to the Future

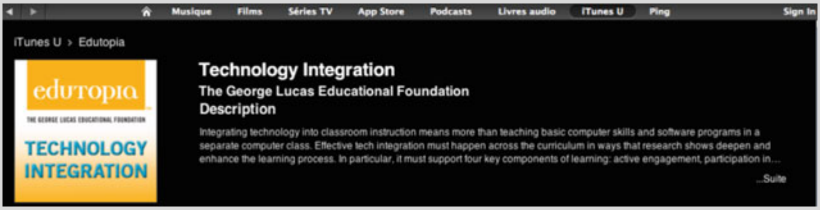
In the *Second International Handbook of Mathematics Education*, Mousley, Lambdin and Koc (2003) anticipated some major features of the present situation:

> There are many ways of using technology in teacher education. Generally, these meet three different purposes: … the creation and use of videotape, videodisc and multimedia resources …; varied facilities such as the Internet and communication software packages, …; the use of computers, calculators and other electronic resources for doing mathematics. … It is now not difficult to foresee a time when today's tools for meeting all three of the purposes outlined above will be able to be attended to in one apparently Internet-based seamless, interactive technological environment. (p. 396)

The time, mentioned by these authors, has apparently come (Window 8), providing resources freely, guaranteed … or not.

Window 8: Video Resources for Helping Teachers to Integrate Technology

Figure 24.10, below, shows iTunes U, a guaranteed repository of videos linked to the results of research (videos from Universities, well-known institutions, etc.). Figure 24.11, on the other hand, shows a video obtained from the Google "jungle," via a search using as keywords "teacher education for mathematics with technology." One resource is *supporting integration* (cf., the introduction to this chapter), and the other is *offering* (magic) *access*…



Technology Integration
The George Lucas Educational Foundation
Description

Integrating technology into classroom instruction means more than teaching basic computer skills and software programs in a separate computer class. Effective tech integration must happen across the curriculum in ways that research shows deepen and enhance the learning process. In particular, it must support four key components of learning: active engagement, participation in…

…Suite

Figure 24.10. Screen capture of iTunes U.

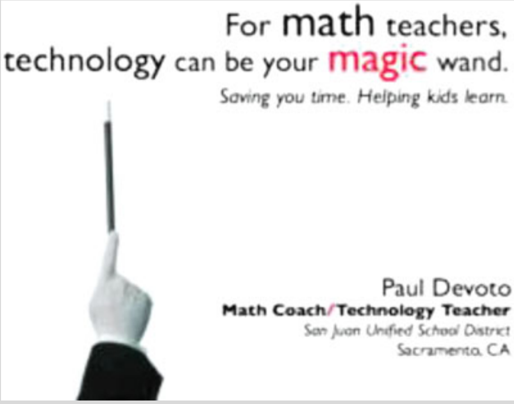


Figure 24.11. Video capture from a source obtained via Google.

More generally, looking at the mathematics teacher education landscape, we can now observe a wide range of resources, situations and devices: individual versus collective, associative (*Sesamath*, see Window 6) versus institutional (*Enciclomedia*, see Window 2), with various content–strategy privileging. Grugeon, Lagrange, and Jarvis (2010, p. 344) pointed out different strategies focussing on: mathematical knowledge, teaching skills, technology potentialities, virtual communication or dialectic old/new tasks. Throughout this diversity, some new trends appear:

- After a time of *institutional injunctions* ("teachers *have to* integrate technologies, to change their way of teaching"), there emerges a *consciousness of the complexity* of the technology integration into mathematics teaching. The perpetual and rapid technological and social changes impose the idea of *lifelong learning* by which teacher education becomes an *ongoing process.* These evolutions push a metamorphosis of *teacher training* to *teacher supporting* along deep evolutions of mathematics teacher work.
- The question is no more to privilege content, or pedagogy, or technology, but to articulate these three components: "Good teaching with technology requires understanding the mutually reinforcing relationships between all three elements taken together to develop appropriate, context specific strategies and representations" (Koehler et al., 2007, p. 741)
- The *Second Handbook* underlined a dominant point of view on teacher education as *introducing*, in a relevant way, *resources to* teachers:

> How technological resources are introduced to teachers and used in teacher education is just as important as what they are designed to do and how well they are constructed … Most authors stress the need to use the resources in the same way as one would expect teachers to use them with children (Mousley et al., 2003, p. 401).

- The idea of *supporting* teacher work implies, not only *providing* resources, but *helping them to design* their own resources. This is in line with the tendency towards *supportive policies* discussed in the introduction to this chapter.
- Helping teachers, as "instructional designers" (Visnovska, Cobb, & Dean, 2012), to design their own resources, is in line with the tendency towards bottom-up approaches presented in our introduction. It leads to conceive new devices for continuous exchanges (via Web sites or platforms) and to take into account different agents of resource design: existing resources available, particularly via the Web; student and classroom interactions, as well as teachers' interactions.

It is this new landscape that we want to illustrate now, through two contexts, one about preservice teacher education; the other, concerning inservice teacher education. Even if the border between both, in the context of lifelong learning, is vanishing, there remain some specificities: entering, and moving within, a profession, are not the same "thing."

Preservice Teacher Education: Towards New Modes of Articulating Classroom Practice and Training

In this section we want to draw attention to the role of technology for supporting teachers at the beginning of their career. This theme raises important questions that need to be faced at this time when, for economic reasons, in some countries (in France, since 2010) persons intending to be teachers are "dropped" into classrooms at the end of their academic studies, before completing their education in the field. In these conditions, new forms of training emerge, often driven by researchers, where video can have a major place, in forms of training that aim collectively to work on *cases* and to develop a reflective stance (see Window 9).

There is therefore a move "from videotape to interactive multimedia," as anticipated by Mousley et al. (2003, p. 398). The use of video is combined with the potentialities of an interactive platform, and carefully orchestrated by teacher educators. As Santagata, Zannoni, and Stigler (2007) emphasized: "The responses pre-service teachers gave to the analysis task prior to the course confirm the need for a framework to guide their observations" (p. 138). The use of video can be found in both preservice and inservice teacher education. In this case it seems to be efficient for supporting discussions, through excerpts of video, on each other's practice—see, for example, the experience of *video clubs* related by Van Es and Sherin (2010).

Window 9: Teacher Education Through Online Discussions

Llinares and Valls (2010) relate an experiment of integrating video-clips from videotaped mathematics lessons, and asynchronous, computer-mediated discussion groups (online discussions and workshops) for prospective primary teachers.

By using resources of an interactive environment (Figure 24.12), video cases and excerpts of interviews with the teacher who was "in the video," these teachers—prospective or already practising—have to: (a) notice aspects of teaching that might influence the development of primary pupils' mathematical competence; and (b) design a mathematical task to foster mathematical understanding by taking into account primary pupils' thinking. The task is realized through online discussions and online workshops, with the help of a tutor, providing the young teachers with questions and theoretical information on demand.

The authors underline the efficiency of this program, enabling the prospective teachers to reflect on, and integrate, multiple aspects of teaching. For them, this success results from the structure of the learning environment, articulating video-clips of actual mathematics lessons, providing structured guidance (task and discussion questions), participating in online debates, collaborating for designing a task; and providing theoretical background.

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Window 9: (continued)

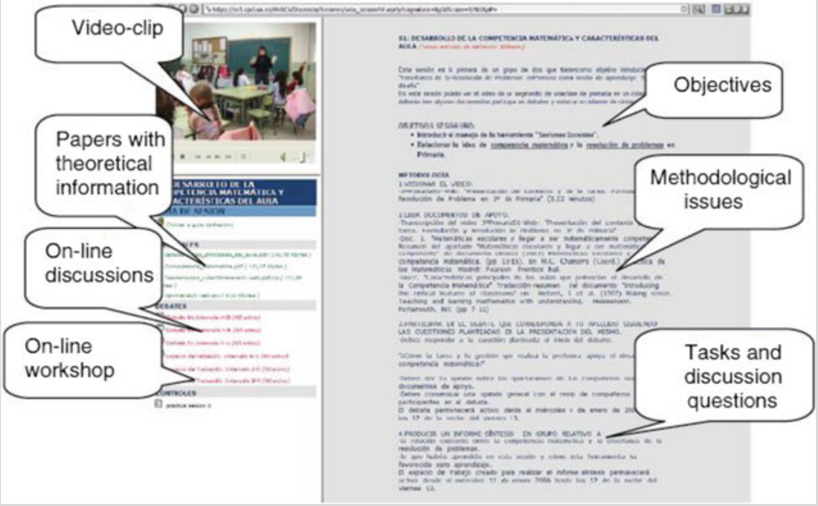

Figure 24.12. An online environment, including videos, for teacher training.

Inservice Teacher Education: Teachers as *Actors* of Their Own Development

After examining the use of digital resources for teacher education, in this section we study *teacher education for technology integration*. To illustrate this, we choose the French Pairform@nce program (Window 10), because it relies on two principles, characteristic of what we consider as new trends in teacher education:

1. *Collaboration* among teachers: Professional development, especially related to technology; results from collective activity and experience with peers, that is in line with the importance of teams; communities and networks as participants in mathematics teacher education (Krainer & Woods, 2008).
2. *Resources design and implementation in class*: A development program for teachers necessarily implies experimentation of resources in the field and, afterwards, a shared reflection that is in line with the strategy. As emphasized by Fugelstadt, Healy, Kynigos, and Monaghan (2010), “centre activities around the process of elaborating and experimenting with new instruments aimed to support new mediations of mathematics and/or teaching practices” (p. 308).

A program such as Pairform@nce would be in line with the evolutions in the field of teacher education, by considering teachers as *actors* of their own development.

Window 10: Pairform@nce, Promoting Teachers Collaborative Work on Resources

Pairform@nce is a French national inservice teacher education program featuring paths available on an online platform [http://national.pairformance.education.fr/] (Gueudet & Trouche, 2012b). Each path is structured in seven stages, combining face-to-face sessions and distance work: (1) Introduction to the training session; (2) Selection of teaching contents and organization of teams; (3) Collaboration and self-development; (4) Collaborative design of a lesson; (5) Trial of the lesson in each teacher's class; (6) Shared reflection about feedbacks of class experience; (7) Evaluation of the session. This organization seems to be close to what Fugelstadt et al. (2010, p. 297) describe as an "*inquiry cycle* … seen as consisting of the main steps: plan, act, observe, reflect and feedback."

Each stage comes with specific resources, suggestions for teacher activities, and collaboration tools. On the program's platform (see Figure 24.13), the seven stages are accessible on the left side; and some collaborative tools, like chat or forum, are accessible on the right side. Depending on the designer's choices, the tools may be specific to each stage of the path. The middle of the page displays path contents, and guidelines for the work of the participants.

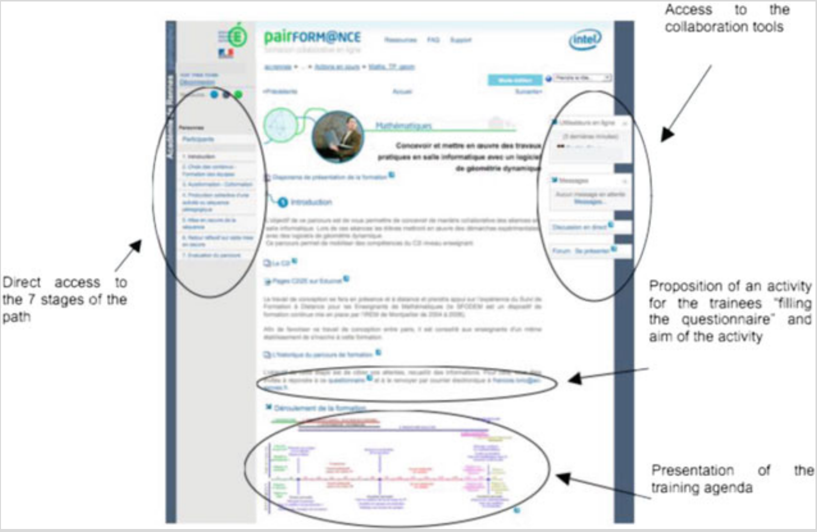

Figure 24.13. Presentation of the first stage of a training path on the Pairform@nce platform.

Analyzing the development and effects of a teacher education program constituted a "burning question," according to Mousley et al. (2003), who stated that:

> Most reporting of uses of technology in mathematics teacher education—as in teacher education more generally and school and adult education—is descriptive; such reporting, however, generally concentrates on how specific tools were used, rather than on how learning took place and the broader question of how teachers learn. (p. 425)

Ten years after, it seems that new projects are looking more carefully at the effects of what is actually done in the programs (see, e.g., Sacristán, Parada, Sandoval, & Gil, 2009; Soury-Lavergne, Trouche, Loisy, & Gueudet, 2011). Analyses of recent programs indicate that the following approaches can assist in providing valid feedback mechanisms.

- The importance of the *collective* teamwork of teacher education students for fostering their involvement in the process of designing and implementing resources in their own classrooms.
- The importance of the work on resources for supporting evolution of practices, confirming the importance of what Koehler et al. (2007) called *design talk*—that is to say, "the kinds of conversations that occur in design teams as they struggle with authentic problems of technology integration in pedagogy" (p. 741).
- The complexity of designing a development pathway that needs to be strong enough to support teachers' work, yet open enough to allow for teacher creativity.
- The necessity of conceiving a teacher education program as a "lived" entity that needs to be permanently renewed by the actors involved (both the teacher educators and the preservice and inservice teacher education students).
- The necessity of accompanying such lived entities by hybrid teams which associate researchers, designers and teacher educators and teachers with the program at stake.
- The importance of tracking the work of teacher educators and teacher education students for long enough to be able to catch real changes (a) during a program, (b) immediately after the program, and (c) one or more years later.
- Another way of monitoring the effects of a program is for outsiders to keep in touch with the continuing work of participants by means of questionnaires, interviews, "visits" of resources, and classroom observations, and for insiders to become reflective practitioners through the use of *logbooks* or diaries (prepared by teacher educators and the teachers themselves).

Networking and Professional Geneses

We agree with Grugeon et al. (2010), that "research about teacher development courses in technology and mathematics is still in infancy" (p. 343). For us, it is more than a matter of merely developing "appropriate" courses—it is a matter of *supporting* the course of teacher development. The move seems to be clearly from "teacher education for technology integration," to "teacher (co)-education in/to designing–appropriating resources (integrating technologies under various forms) for teaching mathematics." From this point of view, there has been certainly a profound evolution since the *Second Handbook*. The institutional recognition of the complexity of teaching in complex environments (continuous evolution, abundance of resources) has led to emergent forms of teacher education programs where task design, development of reflexivity (e.g., via case studies) and collaborating, play a crucial role.

New technological means have been part of these metamorphoses: for example, the role of videos for sharing and analyzing practices; or the role of distant platforms for collaborating and continuing work. The possibilities of networking appear as a major support for such evolutions, with this networking involving teachers and trainers, and also researchers in many experimental contexts.

It seems to be a time of blending: face-to-face with distance; communities involving teacher education students–teacher educators–researchers, etc. These metamorphoses renew the regard for teacher education, considered more as a *professional genesis*, resulting in teachers (individually and collectively) acting with/on resources.

Conclusion

We have come here to the end of our journey through the "mathematics education with technology" universe. We made four stops, successively visiting policies (including curricula and assessment); available resources; learning and teaching spaces; and finally teacher education strategies. It is time to close our journey's logbook, keeping in mind the main impressions.

The first impression is that the landscape we discovered through the opened windows is a *complex* one. Technology represents a deep change in mathematics learning and teaching conditions; educational policies can draw on it, but also must face associated evolutions. The two dimensions that we have distinguished in this chapter, namely the *top-down/bottom-up* dimension and the *access/support* dimension, helped us to analyze these policies. But we found that it was not always possible to characterize policies according to these dimensions. In the same country, the official institution can support the design and/or availability of resources by communities of teachers, and at the same time develop and/or provide "official resources." Also, the involvement in the design and in the provision of resources can lead some teachers to make career switches, (for example, taking on responsibilities in a district). Thus, policies do not seem to move along the neat straight lines sketched in our model.

The second impression is that technology could enlarge the digital divide between developed and developing countries. It is certainly naïve to imagine that the worldwide profusion of resources solves the essential problem of access. Access includes access to machines and access to Internet; and that is not the case in many regions. Moreover, access is dependent on official recommendations: if

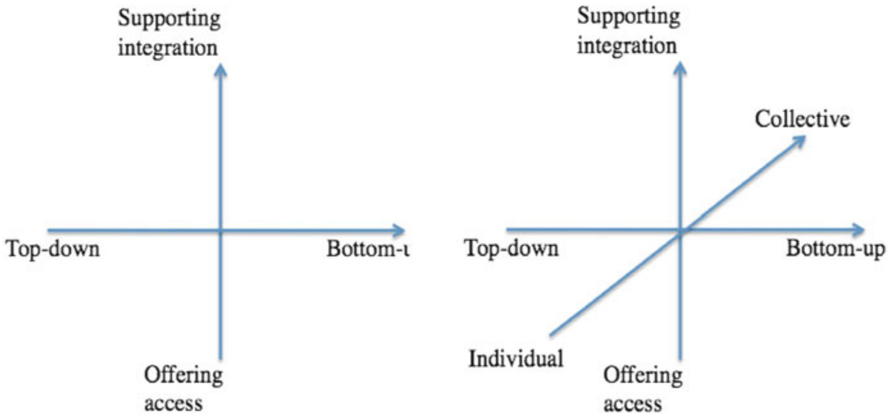

Figure 24.14. From two policy dimensions to three policy dimensions.

policies offer access to *poor* resources, this access naturally leads to dead-ends for mathematics education.

The third impression came from considering the mathematics and technology education universe as a 3D space—adding to the previous two dimensions a third axis positing an *individual/collective* dimension (Figure 24.14). For instance, the *EMAT* activities, in Mexico, were designed with the aim of developing individual learning, while most *Enciclomedia* resources were meant to be presented to the collective classroom. Our journey reveals an evolving landscape where work on resources (designing as well as offering, using or adapting) seems to be increasingly collective. We could thus complement the initial two axes with a third one, representing an individual/collective evolution, and can represent the paradigm's shift concerning the production of resources for teaching as a move in this 3D space.

Our fourth impression is that resources are never *finished*, but always involve, in the design processes, an appropriation process—individual, as well as collective—leading to a renewal of resources. Monitoring this permanent move towards increased *quality* is an essential policy issue.

Finally our journey evidenced a need for a deep reflection on what *initial* resources are required to learn and teach mathematics in technology-rich environments. How can we best give access to and support the appropriation of such critical resources? Which are the missing resources, and how can we initiate and support their design? Such reflections, which may guide future policies, do not seem to exist yet.

Each of mathematics, education, and technology is a rich world. The combination of these three worlds constitutes a very complex universe. We have tried to explore this universe. A single journey always gives a limited access to the visited universe. We are conscious of this limitation. Other chapters in this *Handbook* have enlarged this visit, and supported our reflection about what was, what is, what could be, and what should be.

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