Chapter 2
Theories for Studying Social, Political and Cultural Dimensions of Mathematics Education

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Abstract In mathematics education research and practice today we notice a change in the multiplicity of approaches that allow us to widen our perspectives on diverse social, political and cultural dimensions of mathematics education. This chapter provides an overview of trends and a critical discussion of the use of theories to approach, discuss and critique research and practices in mathematics education, particularly with attention to social, political and cultural dimensions.

Introduction

All research is built around a set of assumptions about the world and how it should be understood and studied. Researchers who study the social, political and cultural dimensions of mathematics education ground their work in a range of assumptions about the nature of knowledge and truth (epistemology) and being (ontology). These understandings are typically implicit, yet they inform the overarching stance of the researcher. Researchers, whether or not they acknowledge or discuss their stance, choose theories that are appropriate to their own view of the world and these, in turn, influence the kinds of projects the researchers undertake. Each perspective allows us to enrich our understandings of the diverse social, political and cultural dimensions of mathematics education. How those dimensions are conceptualized in contemporary research is the focus of this chapter.

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The word "theory" carries with it various meanings, all of which take theory as something one sees or recognizes. The Greek roots of the word connect it to seeing. A description of a researcher's theoretical perspective, then, recognizes that the researcher looks at the researched situation from a particular vantage point. Clearly, certain vantage points that may be available to others will not be available to us. As researchers, we can choose from various vantage points and thus, ultimately, work to initiate change in what we see in the researched situation. Frameworks and models refer to conceptualizations of classes of situations, which we may compare to a situation we see in a researched situation. Thus the frameworks and models we bring with us as researchers affect the locus of our attention and affect what we see in a research context. Jablonka and Bergsten (2010) illustrate different strategies of theorizing in mathematics education in terms of their intertextuality, that is, engagement with and reference to previous work, and "relational density," that is, the extent to which relations between key concepts are established. They distinguish "ad-hoc constructions," "theory conglomerates" and "local models" from proper theories. For this chapter, we will subsume such frameworks and models that refer to previous research and make explicit their intellectual roots under the word "theory," though we are aware that there are differences among the various ways of thinking about theory.

What our exploration in this chapter seeks to do is offer an assemblage of theoretical vantage points that have been used by researchers in mathematics education in contemporary times. Arguably, among the differing perspectives, "incommensurability" (Cobb, 2007) will be a feature, which will prevent us from "providing warrants for our field's identity and intellectual autonomy within apparently broader fields such as education, psychology, or mathematics" (Silver & Herbst, 2007, p. 60). We begin from the position that the wide range of theories, characteristic of the research field today, does not symbolize a field marked by disarray, tensions and contradictions. Rather, what we wish to portray is a vibrant and diverse field, comprising influential perspectives, all of which have important things to tell us about the shape and character of mathematics education. Each perspective allows mathematics education to develop a vision of what to work toward.

Our concern, initially, is to investigate the potential of theories that have their intellectual roots outside the field of mathematics education to advance our perspectives on diverse social, political and cultural dimensions of mathematics education. We are also interested in the ways our researchers use them. In the first part of this chapter we locate trends in theorizing in mathematics education in relation to a widening of perspectives that call our attention to social, political and cultural dimensions. In locating such trends, we focus our attention on well-established theories that have been developed outside the field of mathematics education and their adoption, assimilation and potentials that are hoped for. In the second part of the chapter we review some of the work in mathematics education that has advanced our knowledge of social, political and cultural dimensions. Again, we look at work that has made use of theories developed in other fields, in particular in social linguistics and sociology as well as in postmodern analyses. We support the view of Sriraman and English (2010) to the effect that advancement in the field has often been initiated by adoption and assimilation of new theoretical vantage points that

have their intellectual roots outside the field of mathematics education. However, as discussed in the third part of the chapter, there are important theories developed within mathematics education.

It becomes clear from our overview that mathematics education is no longer only concerned with the technologies of learning and teaching in institutionalized pedagogic settings. It includes researching mathematics education in sites beyond the classroom (e.g., local communities and families, workplaces, policy making, the media, textbook production) and research activities that describe and theorize these practices, including research that is directed towards studying the social, economic and political conditions and consequences of those practices.

Trends and Advances in Theorizing

Trends

Mathematics education is at the intersection of many disciplines including socio-cultural disciplines, language, mathematics, and politics. There is a smorgasbord of theories that researchers might draw upon productively from these disciplines, because each discipline also carries a variety of theories. With this diversity at our disposal, it is instructive to note which disciplines and which theories are being taken up. Tsatsaroni, Lerman, and Xu (2003), when reporting their investigation of theories in mathematics education, noticed a social turn. They noted that where once inspiration for researchers was drawn primarily from psychology, a turn to the social enabled the exploration of a broader range of research questions and issues. New perspectives, topics and methodologies arose, and influential journals (e.g., *Educational Studies in Mathematics;* the *Journal of Mathematics Teacher Education;* and the *Journal for Research in Mathematics Education*) were noticeably now more inclusive of non-traditional frames. These non-traditional frames had enabled researchers to attend to previously unseen aspects of practice.

In addition to traditional psychological and mathematics theories, a growing variety of psycho-social, sociological, socio-cultural, (social) linguistic and semiotic theories have been referred to in conference proceedings and journal articles. Also, reference to recent broader theoretical currents, such as feminism and post-structuralism has been made in the more recent publications (since the time of Tsatsaroni, Lerman and Xu's analysis). During the same period the total numbers of traditional psychological and mathematical papers did not decrease, and Jablonka and Bergsten (2010) have referred to the addition of a "social branch" rather than a social turn. Sub-fields of mathematics education grow in parallel and eventually constitute their own discourses, without one dominating or being privileged.

Inspired by Tsatsaroni et al.'s (2003) investigation, we identify expansions of theorizing in the proceedings of four recent annual conferences of the International Group for the Psychology of Mathematics Education (PME), 2007, 2008, 2009 and 2010. While three of these conferences had no special theme, PME 2009 was subtitled "In Search for Theories in Mathematics Education." We have chosen

PME principally because this is the most established organization that organizes regular conferences and thus reflects changes in what is to be considered as mainstream. We did not anticipate that all innovations in theorizing would emerge within this context, as we are well aware that such innovations take seed in edited volumes, anthologies as well as at conferences that are specifically devoted to exchanging and developing alternative views. In relation to investigating the social and political dimensions of mathematics education, the Mathematics Education and Society conferences provide such forums (see, for example, Gellert, Jablonka, & Morgan, 2010; Matos, Valero, & Yasukawa, 2008).

We compiled a list of names of theories, frameworks and authors associated with socio-linguistic, socio-cultural, sociological and postmodern theories and searched proceedings by using a global document search function. Raw numbers from this search are shown in Tables 2.1, 2.2, and 2.3. If a search term only occurred in the reference list, the paper was not included in the count. For some searches we used word roots in order to capture variations. For example, Vygotsk' captures "Vygotsky" and "Vygotskian." Similarly, sociol' captures "sociological," "sociology," and other variations. We are aware of other classifications of theories from those used to construct the tables.

Table 2.1
Number of PME Papers Mentioning Vygotskian and neo-Vygotskian Theories

Search Terms for Vygotskian and Neo-Vygotskian Theories	PME 2007	PME 2008	PME 2009	PME 2010
Vygotsk [y]	14	18	34	15
[Jean] Lave	7	8	15	9
[Etienne] Wenger	14	12	12	10
[Barbara] Rogoff	2	2	3	
Psycholinguist [ics]				
Activity theory	2	3	12	13
[Yrjö] Engeström				7

Table 2.2
Number of PME Papers Mentioning Sociological Theories

Search Terms for Sociology	PME 2007	PME 2008	PME 2009	PME 2010
Sociol [ogy/ogical]	3	33	16	8
Intellectual Roots of Contemporary Sociological Theories, by Authors:				
[Émile] Durkheim			1	
[Karl] Marx	1		3	3
[Max] Weber	1		1	

(continued)

Table 2.2
(continued)

Search Terms for Sociology	PME 2007	PME 2008	PME 2009	PME 2010
[Edmund] Husserl	1		1	2
[Alfred] Schütz			2	
[Talcott] Parson, [Louis] Althusser, [Antonio] Gramsci, [Eric Olin] Wright, [Georg] Simmel, [George Herbert] Mead, [Herbert] Blumer, [Erving] Goffmann, [Harold] Garfinkel				
Neofunctionalism:				
[Niklas] Luhmann			1	
Neofunctionalis [m/t]				
Critical Theory and Conflict Theory:				
Critical Theor [y]			2	
Conflict Theor [y]			1	1
Frankfurt School			1	
[Max] Horkheimer			1	
[Theodor] Adorno			1	
[Herbert] Marcuse			1	
[Erich] Fromm			1	
[Charles Wright] Mills				
[Pierre] Bourdieu	2	1	7	5
Analytic Sociology of Conflict:				
[Analytic] Sociology of Conflict, [Ralf] Dahrendorf, [Randall] Collins				
Theories of Evolution, Modernity and Globalization:				
[Anthony] Giddens			1	
Structuration Theory				
[Jürgen] Habermas		1	5	1
[Theory of] Communicative [Action]		12		
[Ulrich Beck]				
[Reflexive] Modernization			1	
Risk Society				1
Symbolic Interactionism and Phenomenology:				
Symbolic Interactionism		3	2	1
[Patricia Hill] Collins, [Dorothy E.] Smith				
Phenomenology	2	8	5	3
[Peter] Berger		1	1	
[Thomas] Luckmann			1	
[Max Van] Manen		1		
Rational Choice [Theories]				
Sociology of Education, of Mathematics Education:				
[Michael] Young				
[Michael] Apple			1	
[Basil] Bernstein	1	2	6	1
[Paul] Dowling	1		1	1

Table 2.3
Number of PME Papers Mentioning Literary Theory, Discourse Analysis, Social Linguistics, Positioning Theory and Postmodern Approaches

Search Terms	PME 2007	PME 2008	PME 2009	PME 2010
Literary Theory, Discourse Analysis, Social Linguistics:				
Critical Discourse Analysis			3	
Discourse Analysis	4	4	1	5
[Mikhail] Bakhtin	1	4	3	5
[Norman] Fairclough			3	
[Michael] Halliday	2		1	
[Ruqaiya] Hasan			2	
[J.R.R.] Martin		3		
[Gunther] Kress	1		4	
Positioning Theory:				
Positioning theory		2	4	
Social psychology				
[Rom] Harré	1		2	
Foucault and Postmodern Approaches:				
[Michel] Foucault, Foucauldian	1	2	4	4
Feminis [m/t]				
Psychoanaly [tic theory]	1	2	2	1
[Slavoj] Žižek			1	
[Jacques] Lacan		1	1	3
[Deborah] Britzman, [Elizabeth] Ellsworth				

There are a number of limitations we need to make explicit with respect to our use of PME proceedings. Although annual PME conferences are recognized as important international conferences for mathematics education researchers, they do not fully capture the research being undertaken by mathematics educators worldwide. Papers provided by researchers from non-English-speaking countries are published less frequently in PME proceedings relative to those of English-speaking researchers. In addition, the kinds of classrooms depicted in research reported in PME proceedings tend to reflect a prototypical mathematics classroom which is not representative of classrooms throughout the world. Skovsmose (2006) has suggested that 90% of mathematics classroom research represents only ten per cent of the classrooms in the world.

It is important for us to clarify that in our analysis of the PME volumes we were not seeking to identify papers that failed to make explicit the theory that underpinned the work. Rather, we wondered if it is possible to characterize, without explicit reference to any intellectual tradition, some research in mathematics education as adopting a sociological, political or postmodern perspective by asking research questions that bear testimony to the "spirit" of a theory. Nevertheless, we agreed that it is important to identify and make explicit one's theoretical perspective

because attention to this detail makes for richer, more thoughtful interpretation. In addition, if readers outside an esoteric circle are to be addressed (which is necessary for dissemination), then the conceptual underpinning should be articulated.

We need to make clear, too, that page restrictions for papers in PME proceedings act as constraints for researchers. Theoretical frameworks were usually presented in a succinct format or not at all. Although many papers provided hints at the standpoint taken, the implicit nature of this evidence made it difficult to provide absolute characterizations of the field. There was evidence, however, from those reports which declared their positions, that the PME conference proceedings under investigation were open to a range of theoretical and methodological standpoints. That is to say, a diverse and complex array of theoretical frameworks informed inquiry. Specifically, although there are many references to cultural studies and a range of social practice theories such as symbolic interactionism, activity theory, situated learning and social constructivism, a relatively small number of studies were informed by postmodern and sociological theories.

Given that sociology challenges many assumptions of psychology, reference to sociological theories could indeed have been expected to be uncommon in PME proceedings. Tables 2.1, 2.2, and 2.3 also reveal the "white spots," that is when we did not find a reference to a theory we searched for. These white spots could indicate that the respective theories were not being integrated into the mainstream. However, in some cases, lack of such reference could also mean that although a well-established researcher from mathematics education, who has built from and elaborated a theory that has its roots outside the field, is cited, any reference to the original sources is not seen as essential anymore. However, in our investigation we were less interested in the proportions of different branches of theorizing, and more interested in how theories from sociology, linguistics, activity theory, positioning theory, situated cognition and postmodern theories were used, and to what effect.

Adoption and Assimilation of Established Theories

One difference between the work of mathematics educators and the theorists from whom we draw is that most of these theories are oriented to describing and analyzing practice, while in mathematics education there is a sense that we have to prescribe or at least identify good practice. We think that this tension is central to many of the challenges mathematics educators have when applying theories which emerge from other disciplines. As criteria for usefulness, in a technical sense, of theorizing can only be framed in relation to a given practice, there would not be any innovative or critical potential if identifying good practice were the only *raison d'être* of research. A sometimes-observed hostility towards theory in mathematics education research is based on a misreading of theory as mere contemplation and speculation. Theorizing includes systematization of and critical reflection upon practice that opens up new views. Seen in this way, theorizing is indispensable for the advancement of a field.

In considering the range of theories available to mathematics education researchers, there are a number of decisions researchers have to make with regard to theory. First, one chooses theory that enables one to address the research question, but often the theory is instrumental in formulating the question as well. Second, when choosing a well-developed theory, one chooses aspects of that theory for focus. Third, it is important to consider the "translation" of the theory that was birthed in a specific context to the context of mathematics education. Fourth, there are decisions about how much attention to give to the theory when writing about the research, including the possibility of not recognizing that the research has a perspective that is sociocculturally and politically relevant. For the last three of these choices, there are continua—for example, a researcher might take one concept from a theory, more of the theory, or much of the theory. There is yet another possibility—taking two or more theories in some kind of hybridization. Moreover, in our view, importing a theory from a different tradition of research is already a form of hybridity. The recontextualization of theories from outside our field necessarily involves a change in the criteria for what counts as advancement, a shift in focus and in meaning.

In our conversations about the PME papers we considered possible ways of misusing theories. One could, for example, use a single concept from a theory and thus miss some central ideas of the theory. This could be done with intention or with naïveté (and we acknowledge that there are only degrees of naïveté, for no one can be said to know everything about a theory). We agreed that for a misread of a theory to be deemed heresy, it would have to be an intentional twisting of the theory, but then we wondered how to distinguish between heresy and "moving theory forward," both of which turn and/or move theory. Picking up on single concepts from a theory can be productive, but it may not be. Productive, deliberate re-interpretation and expansion of theory based on some principles might be called heresy or development, depending on one's point of view. Hybrids from different theoretical sources can be promising in bringing together ideas that seemed apart, but also limiting by distorting the spirit of the individual theories. The strategy might amount to a pastiche or a conglomerate, and perhaps even to an anti-theoretical bricolage. This brings forward the important question of how we might judge the qualities theories bring to our field.

When reading a selection of PME contributions, we were interested in which aspects of the theories were used, whether or not the papers included reflection on the challenges of applying these theories in particular research contexts, how the researchers described the motivation for their choice of theory, the extent to which the data interpretation drew on the theory, the extent of the description of the theory, and who the paper cites in the description of the theory—the major theorists from outside mathematics education, or mathematics educators applying the theory in our field.

From our reading, we see that "networking" theories remains a challenge for research in mathematics education. We found examples of this challenge in our reading of promising contributions that could form a starting point for moving the discipline forward. We found some innovative study designs that attempted to achieve some theoretical combinations that looked entirely novel. But in the examples of theoretical combinations, one theory often dominated. The assumptions

shared by the individual theories were not elaborated. In many reported empirical studies, the motivation for theory choice is not made explicit. While the "novelty" of an approach might be mentioned, the promises of a new theory in relation to other approaches that did not carry the same promises tended not to be discussed. We also noted a geographical distribution of branches of theorizing and of innovation. This is of course due to the physical closeness of experts in a location, for example supervisors, but also to the cultural situatedness of traditions. In the PME papers we also identified some contributions that fully exploited the potential of theories and sought to advance our understanding of the field. Furthermore, some authors alerted readers to the potential of a whole branch of theories.

Examples of Providing New Terrain

Our selection of papers from the PME proceedings for further discussion was guided by the number of theories the papers connected with, by promising titles, and surprising combinations of references. We were also careful to review plenary papers and research forums. In our selection, we also have taken the number of references made in the same paper as an indication of an extended discussion of a theory, though these numbers are not represented in the tables above.

A theoretical paper by Brown (2008), for example, provides a critical analysis of Luis Radford's cultural theory of objectification. Using ideas drawn from a range of postmodern sources, Brown offered a critique of the way in which Radford conceptualized the notions of culture and subjectivity in his theoretical development. Brown drew specifically on discursive approaches to knowledge and subjectivity to develop his critique. Working from the premise that Radford's cultural theory of objectification "perhaps provides the most sustained and substantial excursion" (p. 209) into the area of cultural and historical dimensions of mathematical objects, Brown attempted to unsettle some of the foundations on which that theory is built.

Breen's plenary paper at the PME 2007 conference at the end of his term as President of PME (Breen, 2007) is suffused with ideas from enactivism and psychoanalytic theory and these provided a springboard for Breen's reflections of and hopes for mathematics education. What the paper revealed particularly is that theoretical border-crossing into enactivism and psychoanalytic ideas requires a shift in thinking and in attitude and, in that sense, the sensibilities of the theory may have been lost on some readers. Such a shift offers readers new understandings about mathematics education and its situatedness with institutions, history, and cultural fields. It also draws our attention to our ultimately compromised stance in everything we do and say within mathematics education. As a plenary paper the content could be deemed highly influential. It opened up theoretical discussion for the discipline. Of course, readers could choose to dismiss his theoretical tools or they could choose to pick up snippets of ideas that suited them. Alternatively, readers could assess his theoretical apparatus as a key resource for interrogating and understanding the dynamics and politics of mathematics education.

At the PME 2009 conference, a research forum on sociological theories in mathematics education was held. The overall agenda of the forum was researching possibilities of how more equitable outcomes may be achieved in mathematics education, as no research group in mathematics education, least of all the leading international group, can ignore the social disadvantages reproduced in mathematics classrooms in most countries of the world (Lerman, 2009). The contributors explored how research in mathematics education has made, and could make, use of sociological theories in shaping research questions and methodologies that contribute to the agenda. The forum also discussed the ideologies at work in research designs. At the PME 2010 conference the discussion group on mathematics education and democracy (Mattos, Batarce, & Lerman, 2010) also provided new terrain that is not genuinely linked with psychology. The members of the discussion group included as their theoretical underpinnings Karl Marx's concept of commodity, Jean Baudrillard's concept of sign value as well as the work of Jacques Derrida. One key issue for discussion was the constitution of mathematics knowledge as universal need in today's society and the role of mathematics education in the constitution of such an ideology.

In a plenary address at PME 2009, Morgan's (2009) account of her evolving research program provided a window into how tools and ideas from linguistics might connect with a researcher's agenda. She described how Pimm's (1987) book title connected to the questions that dominated her thinking about mathematics education, and how this connection drew her to systemic functional linguistics (SFL). From this, she became interested in Fairclough's work because it helped her move beyond description to the judgment of mathematical texts. Fairclough's work connects Halliday's to critical social theorists including Foucault and Bourdieu. Not surprisingly, Morgan was next drawn to Bernstein's theory to help her understand the social context of mathematics discourse. In her work with Evans (Evans & Morgan, 2009), she noted that discursive approaches address some of the classic dilemmas in sociology and social theory: structure versus action, order versus conflict, and official versus deviant perspectives. Morgan's path is illustrative of the connections among the three strands we are using to divide up theories that attend to social, political and cultural dimensions of mathematics education.

Opening Up New Perspectives

We now turn to include mathematics education literature beyond the recent PME proceedings to outline the way theory from outside the field has been used to move the field forward through accounting for social, cultural and political dimensions of mathematics education. We divide this work into the three broad areas, discourse analysis, sociology and postmodern approaches, though we know that these three areas are interconnected. Indeed, our work on these overviews reminded us of these intersections and the related difficulty of categorizing work. However, we are also aware of other approaches to knowledge development that provide alternatives to cognitivism and are compatible with socio-cultural learning theories.

Enactivist and complexity theories, for example, add a new twist to the influences of the social in highlighting the dynamic and interactive adaptations of the learner, and address questions of "being" rather than "knowing." An insightful application of enactivism for education can be observed in the work of Davis and Simmt (2003). A similar biological metaphor is used by Radford, Edwards, and Arzarello (2009) in their embodied theory. Ideas are not held by individuals but are embodied by human beings with normal human cognitive capacities living in a culture, situated in and productive of larger, social, cultural and historical thinking. Mathematical thinking, learning and communication involve different semiotic systems and multiple modalities of expression including gesture, speech, written inscriptions, and physical and electronic artefacts, all of which are integral to the cognitive process (Radford, 2009). The conceptualizations of one person are not assessed as a measure of "fit" or "match"; rather they are said to be viable (or otherwise) in relation to another's conceptualizations.

Discourse Analysis

Theory from linguistics has been instrumental in illuminating interpersonal interaction within the contexts of mathematics teaching and learning and, in particular, the positioning of students and teachers in relation to others and the discipline. However, discourse analysis is not limited to linguistics. As articulated by Ryve (2011), mathematics education draws on various theorizations of discourse to illuminate multiple perspectives on mathematics teaching and learning. Ryve's analysis of numerous articles in mathematics education journals shows that the concept of discourse is too-often undertheorized in research reporting. However, there are strong examples of productive use of various forms of discourse analysis in the recent years of mathematics education scholarship, which we overview below.

Halliday (1978) called the discipline-specific use of language employed in mathematics communication "the mathematics register." With the increasing use of this term the fuzzy boundaries of the register are becoming exposed, drawing attention to the goals of mathematics educators. Mathematicians speak and write differently from mathematics teachers and learners. Pimm (2007) and Barwell (2007) have commented on this distinction in response to research that seems to blur this line. Herbel-Eisenmann, Wagner, and Cortes (2010) clarified their analysis of mathematics classroom discourse as investigations of "the mathematics classroom register." Even so, any classroom or any interaction has its own peculiar forms, so it is not possible to delineate "the" register accurately.

Following Halliday's (1985) social semiotics, a powerful body of tools for understanding how people use language for various purposes and effects in discourse—called systemic functional linguistics (SFL)—has been developed. Though various scholars used SFL tools before, Morgan (2006) contributed an introduction of social semiotics to mathematics education, with the purpose of demonstrating its tools and of identifying research questions that these tools can help to answer. They are useful

for analyzing transcripts to identify who or what is doing things in learning contexts, the objects of mathematics in these contexts and the relationships at work.

For example, Nachlieli and Herbst (2007) used such tools to identify the particular utterances that related to assumptions in the proof discourse they analyzed. Both Mesa and Chang (2008) and Wagner and Herbel-Eisenmann (2007) used a narrower tool set within the rubric of SFL, as they used Martin and White's appraisal linguistics to understand the engagement of mathematics learners in classrooms.

Mathematics educators are using approaches to discourse analysis in addition to social semiotics. For example, Both Mesa and Chang (2008) and Wagner and Herbel-Eisenmann (2009) complemented their SFL work with positioning theory, which is another form of analyzing discourse from a social psychology perspective. Hegedus and Penuel (2008) used Goodwin's participation frameworks to analyze discourse specific to a mathematics learning with wireless technology, aiming to document how students' identity shifts during the course of a class. Carlsen (2010) used Linell's dialogical approach to study interaction and its effect on meaning. Black et al. (2010) used Gee's approach to discourse analysis, which takes a broader view of discourse, to identify the interconnecting stories at work in students' accounts of their mathematical narratives.

In addition to using tools to identify features of language for understanding what is happening in mathematics learning contexts, linguistics and other domains provide theory for understanding in general the connections between language and thinking. This kind of theory is often called "discursive psychology." Some examples of using a linguist's observations to support one's line of attention in mathematics education include the following. Leung and Or (2007) used Michael Halliday to support their claim that language choices shape human experience. Similarly, Sakonidis and Klothou (2007) used Gunther Kress to substantiate their observation that students' writing is not necessarily read in the intended way by their assessors. De Freitas (2009) used Fairclough's "critical discourse analysis" to locate the structuring of power relationships in mathematics classrooms in the language choices.

Sfard (2008), in developing her own model to explain how communication and cognition are co-implicated in mathematics learning, invented the word "commognition" to denote this inherent connection. Barwell (2009) drew on discursive psychology to critique Sfard's use of examples to develop her model, and pointed to the general challenge of drawing inferences from excerpts of mathematics learning situations. Indeed, most analysis of mathematics education discourse works with texts identified by researchers, perhaps because these chosen texts exemplify a particular distinction or phenomenon. Alternatively, if one works from a large body of diverse classroom texts, which linguists call a "corpus" (e.g., Herbel-Eisenmann et al., 2010; Wagner & Herbel-Eisenmann, 2007), it is possible to identify features of the discourse in general. This corpus linguistics work does not undermine the importance of in-depth analysis of isolated excerpts. Nevertheless, it is important to be careful about warrants for claims made from examples of mathematics teaching and learning discourse.

Discourse analysis appears in yet other forms of mathematics education work. In researched professional development contexts, mathematics educators have been directing the attention of teachers to forms of discourse analysis. Herbel-Eisenmann supported a group of mathematics teachers in their action research projects that focussed on aspects of discourse (e.g., Herbel-Eisenmann & Cirillo, 2009). Zolkower and de Freitas (2010) guided teachers in deconstructing transcripts of their mathematics teaching to increase their awareness of semiotic choices available to them.

De Freitas (2010) used critical discourse analysis to study the classroom discourse and interaction patterns of two secondary school mathematics teachers of senior classes in Canada. She employed Fairclough's understanding that language not only produces meaning but also positions speakers in specific relations of power. The purpose was to understand the way in which teachers' subjectivity is constituted and enacted, in brief and often spontaneous and contradictory speech acts. The task demands thinking about text and context in classroom interaction as intersecting rather than separated. In the analysis de Freitas showed how one teacher, Mark, repeatedly used metaphors that signified an antagonistic relationship between students and texts, and embedded many references to sports throughout his lessons. She demonstrated how the other teacher, Roy, continuously made reference to the difficulty of learning calculus, choosing to exclude discourse from other texts that spoke calculus into existence in other ways.

Both analyses highlighted what teachers choose to say and the way in which they say it, and the power relations that descend from those linguistic decisions. In particular, the analyses provided counter-narratives about classroom discourse, pointing to the regulatory power of teacher discourse in providing access to mathematics, by shedding light on those students who were included within and those who were positioned outside of the text. Importantly, through the fine-grained reading that unpacked hidden relationships and regulatory practices operating within the classroom, de Freitas demonstrated the way in which the discursive practices of the two teachers contributed to the kind of thinking that is possible within the classroom.

The number of edited collections that have focussed on discourse in mathematics education in recent years points to the importance of discourse analysis within the field. Chronaki and Christiansen (2005) presented a collection of varied perspectives used to theorize communication and this collection also addressed associated political issues. Moschkovich (2010) likewise assembled multiple perspectives on language and mathematics education and identified new directions for research. This volume featured different authors from the Chronaki and Christiansen volume, demonstrating the depth of the field within mathematics education. Herbel-Eisenmann, Choppin, Wagner, and Pimm (2011) brought into conversation mathematics education research that focussed on equity and on discourse to show how these two are inherently connected. Barwell, Barton, and Setati (2007) edited a special issue of *Educational Studies in Mathematics* focussed on a narrower discourse-related issue—mathematics learning in multilingual contexts. There is an active group of scholars working together to focus on multilingual contexts, many of whom gathered for an ICMI study conference in 2011.

Sociology

Our study of trends in mainstream research as reflected in the PME conferences has shown that employing Vygotskian and neo-Vygotskian theories, as well as a general reference to the label "socio-cultural," has become common. Vygotsky is often cited only in the text, as it is usual with references to classical works and names that stand for an intellectual tradition (e.g., as the "Vygotskian paradigm" or "Vygotskian approach"). Similarly, activity theory is often referred to without specific references. Clearly, socio-cultural perspectives based on those theories on learning have been integrated into mainstream. These perspectives have considerably advanced the field of mathematics education by drawing attention to socially and culturally specific experiences among learners of mathematics.

However, relations to social structures remain under-theorized in those approaches. Learning within a community of practice does not occur in isolation from the power relations that operate within that practice. Conceptualizing learning through legitimate peripheral participation does not necessarily help to understand stratification of achievement in mathematics classrooms, especially in relation to social and economic class, race and gender (Ensor & Galant, 2005; Huzzard, 2004). How does social structuration come about in communities of practice? Daniels (2001) suggested that the theories of situated knowledge and learning should be related to a political analysis of power and control. A potential of productive interaction between socio-cultural perspectives on learning and sociological theories was pointed out by de Abreu (2008), who suggested that while cultural psychology allows for, if not draws attention to, the diversity among learners in their socially and culturally specific experiences, apprenticeship models are limited in conceptualizing consequences of macro-social structures on learning.

In many studies of mathematics classroom interaction, reference is made to symbolic interactionism and phenomenology, the reference sometimes being mediated through the works of mathematics educators who have followed these perspectives in their works (see, e.g., Yackel & Cobb, 1996). However, not all sociologists accept phenomenology (and its offspring ethnomethodology) and symbolic interactionism as genuine sociological theorizing. Although both share an anti-positivistic paradigm and common assumptions about the task of focussing on understanding of how meanings are developed and shared, they have been criticized for focussing merely on micro-level small group social interactions as well as for non-attention to the unintentional "hidden" consequences of actions or to the constraints of socio-political structures on people's actions.

But there are also important differences between the two traditions. Symbolic interactionism is interested in how the participants define the situation and come to make sense through the process of interaction, while ethnomethodology is interested in uncovering the taken-for-granted values, norms and rules that already operate in the interaction. Classroom studies based on symbolic interactionism illuminate how (flexible) role expectations and meanings are established through a "negotiation of meaning," with a focus on how students act in situations that demand new interpretations.

This leads to more insightful interpretations of what happens in "inquiry-based" learning situations than in classrooms with more apparent role-asymmetries (Voigt, 1996). Ethnomethodology acknowledges that the taken-for-granted rules are functional and that the participants' interpretations might be limited. Voigt (1984), for example, showed that teachers and students enact subconscious practices or "routines" when structuring the process of developing new knowledge interactively. He also pointed out that in the interactive construction of new meaning through an elicitation pattern, which starts with a teacher's open question (a new "task"), there is no shared frame of reference from the outset. For the students the new question is ambivalent, and this ambivalence is only retrospectively (reflexively) reduced when the official solution is institutionalized. This ambivalence causes a problem, especially with contextualized mathematics as a starting point for developing new concepts and methods. The issue has been taken up by researchers who are interested in the effects of "invisible" pedagogic practices on the stratification of achievement (see e.g., Jablonka & Gellert, 2011).

In order to explore the stratification of achievement in mathematics classrooms, one has to acknowledge that patterns of classroom interaction are functional in terms of the goals of the institution and are not accomplished at the initiative of the participants in a single classroom. Theorizing the reproduction of inequalities through mathematics education is the most obvious agenda of genuine sociological approaches. Advances have been made through employing the works of Bourdieu and Bernstein. In PME conferences, references to Bourdieu and Bernstein are not very common and remain often on a general level, with the exception of the research forum on sociological theories mentioned above. Bernstein's notion of visible pedagogy invites didactization, as can be seen by Sullivan's (2008) reference to Bernstein, pointing to the necessity of making explicit the criteria for evaluation when implementing non-routine tasks in classrooms. Aaron (2008) innovatively employed Bourdieu's notion of "symbolic economy" when analyzing students' views of classroom work in geometry lessons in order to conceptualize differences in the students' identities.

In an illuminative investigation of unequal achievement in mathematics secondary education in Victoria, Australia, Teese (2000) drew on Bourdieu. His analysis shows that much of the students' success at different levels of the mathematics curriculum depended on their personal characteristics, such as organizational skills, study habits, concentration and academic self-esteem. Teese argued that the discriminating potential of mathematics education is implicit in a curriculum hierarchy that raises the demands over successive levels of mathematics that call more and more on embedded scholastic attitudes and behaviours. It can be taken as a measure of the implicit cultural homogeneity of the mathematics curriculum as a whole—based on sequenced and overlapping content and shared conceptual emphasis—that the average social level of students rises at each level of performance. Teese's investigation showed the potential of data analysis from a consistent theoretical vantage point.

Bernstein's work offers a broad range of interrelated notions that are incorporated into a complex theoretical body. Most prominently in references feature the concepts of recontextualization, horizontal and vertical discourse, classification and framing, and visible and invisible pedagogy. Increasing numbers of researchers in

mathematics education have extended, developed and critically engaged with that body of theory. Dowling's (1998) study of school mathematics texts and some of the methodological tools developed in that study provided a major contribution to sociological theorizing in mathematics education.

Uncovering the ideologies behind different mathematics curricula and scrutinizing the ways in which what types of knowledges are constructed for which groups of learners remains a major task for sociological approaches. Noss et al. (1990) provided a collection of sociological analyses of curriculum, Ernest (2009) attempted a critique of ideology in mathematics, science and technology education research and its globalization, Ensor and Galant (2005) reviewed studies from the South African context. Analysis of the development and effects of policy discourses in mathematics education is another domain of study to which sociological theory provides powerful tools.

Valero and Zevenbergen (2004) located approaches, sometimes also subsumed under the label "socio-cultural," which acknowledged that both mathematics education and research in the field are not only social but also political practices. Depending on the political, economic and social conditions, these practices exercise power in different forms. Institutions that contribute to the reproduction of power, as for example schooling, can be analyzed as political institutions. Perspectives explicitly sharing the acknowledgement of this fact can be described as "socio-political." Such approaches have moved beyond the tools made available within classical sociology and cultural psychology to explore the power dynamics within social interactions. They ground their investigations on the premise that the practices and processes of mathematics education are inherently political. Skovsmose (2009), for example, wrote, in his critique of mathematical rationality, of the symbolic power of mathematics.

Martin (2010) observed that race still remains under-theorized in mathematics education, as disparities in achievement are often taken as reflecting race effects rather than as consequences of the racialized nature of the students' mathematical experiences. A similar point has been made (e.g., Skovsmose, 2007) about many, mostly quantitative, studies of unequal attainment in relation to social and economic background that treat the students' background merely as an input variable. Similarly, Gutiérrez and Dixon-Román (2011) demonstrated how "gap-gazing" constructs those who do not achieve as deficitarian, while not addressing the ideological underpinnings of the goals of mathematics education. Chronaki (2011) argued that curriculum politics act as "ideological state apparatus" (Althusser, 1971), regulating the micro-level of mathematics education by creating micro-spaces, for example in the form of didactic innovations. She argued that hegemonic discourses of equity construct subjects with static identities as marginalized and voiceless. Chronaki observed that such discourses are underpinned by constructivist and socio-cultural approaches that overemphasize the "autonomous subject" who makes rational decisions. This points to the potential of employing psychoanalytic and poststructuralist theories, and clearly challenges psychological theorizing.

Researching unequal access to mathematical practices and discourses that provide cultural and symbolic capital might leave the conception of curriculum untouched. However, exposing the forms of mathematics privileged in a curriculum is an outcome

of an analysis of the functionality of curriculum as well as of mathematics. Pais and Valero (2011) argued that mathematics education in many places must be understood within a capitalist economic and neo-liberal political setting that calls for quality and equity yet serves particular interests in these settings.

Critique and a wish to contribute to an agenda for social change is an important agenda in sociologically-oriented research in mathematics education as it was outlined by Noss et al. (1990). The fact that this is a political agenda does not mean that the research is more value-loaded than any other, but rather that the values are made more explicit.

Postmodern Approaches

Seminal edited volumes written during the last decade (e.g., Bishop, Clements, Keitel, Kilpatrick, & Leung, 2003; Boaler, 2000; Lester, 2007; Sriraman & English, 2010), although making important contributions to the discipline, did not reflect the impact and take-up of postmodern theory within mathematics education. Given that an increasing number of researchers are interested in what these social theories might mean for mathematics education, we look at the origins of and the assumptions underpinning this theoretical movement and the way in which the movement promotes local voice and critical thinking, even as it holds critical thinking itself up for scrutiny.

The specific traditions of psychology and sociology provide a bedrock of concepts and theories for the study of mathematics education from a postmodern perspective. Psychology has informed a psychoanalytical turn, designed to unsettle fundamental modernist assumptions concerning identity formations. For example, Brown and McNamara (2010) drew on the work of Lacan to investigate how preservice teachers use language to describe the world around them and how they see themselves fitting in. Sociology has helped seed poststructuralist work that aimed at drawing attention to the ways in which power works within mathematics education, at any level, and within any relationship, to constitute identities and to shape proficiencies. Walshaw (2004a), for example, built on the work of Foucault to explore the ways in which teaching practice is inherently political.

Like analyses of a modernist persuasion, at the heart of postmodern analyses lies an interest in understanding contemporary social and cultural phenomena (e.g., Brown, 2008; de Freitas & Nolan, 2008; Walls, 2009 Walshaw, 2004b, 2010). Postmodern analyses chart teaching and learning, and the way in which identities and proficiencies evolve, tracking reflections, investigating everyday classroom activities and tools, analyzing discussions with principals, mathematics teachers, students, and educators, and mapping out the effects of policy, and so forth. The point of departure from modernist narratives is derived from assumptions about the nature of the reality being studied, assumptions about what constitutes knowledge of that reality, and assumptions about what are appropriate ways of building knowledge of that reality. As a result of these specific understandings, the lived contradictions of mathematics processes and structures are able to be explored.

Poststructuralists and psychoanalysts share some fundamental assumptions about language, meaning and subjectivity. They see language as fragile and problematic and as *constituting* social reality rather than *reflecting* an already given reality. What is warranted at one moment of time, may be unwarranted at another time (see Walshaw, 2007). The claim is that because the construction process is ongoing, we do not have access to an independent reality. Hanley (2010) demonstrated that point in her exploration of the way in which teachers make sense of and enact curriculum reform. She showed that although teachers attempted to put into practice what they learned through a professional development project, what was learned and practised in professional development initiatives was never fully cashed in as educational capital within the classroom.

Objectivity is not the only concept that postmodern theorists take issue with. They debate conventional understandings of reason, insisting that rationality is always relative to time and place. They prefer to think in terms of "local" determinants, fallibility and contingency. Underwriting their projects is a "decentred self"—a self that is an effect of discourse which is open to redefinition and which is constantly in process. This point was given expression by Walls (2010) in her investigation of the "good" teacher, in a setting of compulsory standardized testing. Walls drew on the idea of teacher identity as a process embedded in discourse, to explore teachers' struggle for self and to investigate how systemic forces, in a culture of teacher accountability, are lived by teachers as individual dilemmas.

What is apparent in mathematics education research that draws on postmodern theories is a move towards exploring tentativeness and developing scepticism of the particular principles and methods that put a shine on essentialist and absolutist tendencies. What such theories also do is require researchers to consider the implicit assumptions that guide their work. The point was emphasized by Adler and Lerman (2003), who argued that there are moral obligations, and hence, ethical issues at stake in any research practice. More critical debate and evaluation of the competitive work of researchers is needed particularly at this moment of time when political influences on research are becoming deeply entrenched. In Adler and Lerman's view, such influences were "insufficiently problematized in the mathematics education community" (p. 457).

De Andrade (2008), reporting on a research project undertaken in Brazil, drew on the work of Foucault and used a form of discourse analysis to look at the relationship between research and classrooms in mathematics education. Derrida's ideas of deconstruction also informed the methodology, by providing a vehicle for keeping "the system in play," "in process," and "to set up procedures to continuously demystify the realities we create, to fight the tendency for our categories to congeal" (p. 60). Employing these ideas, De Andrade (2008) set up contradictions between model classrooms as depicted in mathematics education research, and the kinds of classrooms in which teachers in Brazil sometimes find themselves teaching. Two major themes could be discerned in De Andrade's paper: the subjectivity of learners in actual classrooms and the inherently political nature of research. These themes are in keeping with other work based in the field that draws on Foucault's framework. Such a framework provides the means to explore the relationship between power and knowledge. It is also able to signal that the views of teachers and researchers are

always enmeshed in sites of knowledge production that are unavoidably political. In drawing on the work of Foucault, De Andrade dealt with meaning construction in a way that acknowledged the researcher's own complicity in the analysis.

Stentoft and Valero (2010) investigated the fragility of mathematical learning. Their discussion expressed a poststructuralist imagination that took seriously the notion that language constitutes social reality rather than reflects an already given reality. In developing an understanding of the "noise" symptomatic of everyday classrooms, Stentoft and Valero (2010) challenged interpretations of the practices within what are typically characterized as "pure mathematics classrooms." Their theoretical approach used precepts that are, in tenor, at odds with the presuppositions that ground the rational autonomous learner.

In a discussion on undermining traditional approaches to learning, Stentoft and Valero (2010) drew attention to the interrelatedness as well as the fragility of classroom discourse, identity and learning. They argued that these three elements together constitute the landscape within which a student's sense-of-self as learner is formed. In their discursive analysis, they case studied mathematics classroom interactions at a Danish teacher training college. Underlying the analysis was an intent to avoid mere descriptions of classroom life, but rather, to unpack how students and teachers were involved with constructing multiple identities over the course of a mathematics lesson. The intent was also to make clear how learning mathematics and constructing mathematical knowledge in the classroom is inextricably caught up in the discursive practices of the classroom.

Bibby (2010) used concepts from psychoanalytic theory to explore the pedagogical relation. She drew on the concepts of the oedipal family and the Oedipus complex to unpack relationships to mathematics, particularly as they are constituted in primary schools. Post-Freudian psychoanalytic theories of authority provided her with conceptual tools to investigate the way in which mathematics, with an emphasis of rules, speed and correct answers, is characterized as masculine in traditional school mathematics pedagogy. Taking care not to essentialize gender, Bibby unpacked the ideational fiction of binary characterization, and proffered, instead, masculinity and femininity, boy and girl, as "elements within gender." She drew on research data to unpack some of the potential consequences of differentiating mathematics as an unemotional, authoritative, rational, systematic and logical set of values and practices, away from so-called feminine qualities such as warmth, emotional attunement, and creativity. Specifically, she explored the tensions that result from fictions that allow for the deployment of masculinity in the discursive construction of mathematics and investigated the consequences for teachers and students living with the effects of these splits in policy and practice.

Theories from Within Mathematics Education

In addition to the theories mathematics education researchers import from other domains, there are theories that were born within mathematics education itself in order to overcome the limitations of a purely psychological paradigm. For example,

as mentioned previously, Sfard (2008) developed her own model for describing the interaction between communication and cognition. This model has subsequently been used by other researchers as a theoretical perspective on cognition. Because the connection of communication and cognition could have been theorized outside of the context of mathematics education, Sfard's theory is an example of the way researchers in our field develop theory using contexts from mathematics education and theory from other domains. This is a strong example, because others have taken up her theory.

Renert and Davis (2010) proposed an integral perspective for exploring knowledge production. They developed a model that integrated self, culture, and nature, through which a plurality of perspectives could be entertained. Their proposal was towards an evolutionary perspective, one that is inclusive of the contributions of traditional, modernist and postmodern perspectives. They showed how each perspective leads to different views about the kinds of tools that mathematics uses and each makes it possible for certain understandings to be entertained and legitimated. Their integral perspective valued the enacted, creative and dynamic dimensions of mathematics and was focussed on the health and harmony of the entire system. Renert and Davis applied these ideas to their work with experienced middle-school teachers. Their work demonstrated how teachers are crucial participants in the creation of mathematical possibilities. They suggested that teachers might engage students more meaningfully with mathematics by elaborating the specific, by using active language, and by allowing them to engage with multiplicity and plurality in discourse, meaning-making and interpretation.

There are other theories that have arisen in mathematics education for which the unique context of mathematics education is a necessary aspect of the theory. These include work that is critical of mathematics education and its position and role in society. One of the more established, but also very diverse, of these approaches is ethnomathematics. In PME proceedings, the term was referred to in 10 papers in the volumes from 2010, but only in three papers from 2009 and in one paper from 2007. Ethnomathematics is concerned with practices and activities of marginalized groups, that can be identified as mathematical, but which are not institutionalized as mathematics.

The term ethnomathematics is a label used for the theoretical underpinnings as well as for the product of an analysis of the mathematical nature of such activities. It emerged from a critique of both a Eurocentric gaze in popular history of mathematics as well as an elitist pedagogic model together with a deficitarian perspective on the knowledge of students with a cultural frame not in line with the official school culture. Thus it allows seeing the political dimension of mathematics education derived from and designed for the hegemonic sectors of society. The spirit of the approach to both research and education is a commitment to inclusion. Ethnomathematics brought into attention the cultural embeddedness of mathematical knowledge and of mathematics education.

There are, of course, other forms of criticism of mathematical practices and mathematics education practices. For example, much equity work in mathematics education has criticized the way mathematics is taught and has based this

criticism on sociological and political perspectives. There has also been criticism of mathematical rationality, and its effects on society. Skovsmose's work, partly inspired by critical theory and critical pedagogy, drew many to contribute to the project of critical mathematics education (see Alrø, Ravn, & Valero, 2010). His analysis of the formatting power of mathematics itself ought to be viewed as a contribution to the sociology of mathematics, a branch of sociological theorizing that has a great potential to be further developed. It is necessary for scrutinizing traditional dogmas about mathematical knowledge production and applications. Understanding the relationships between different practices that include mathematics and exhibit different knowledge structures and discourses, in school and outside school, remains a major concern (Jablonka, 2003; Jablonka & Gellert, 2007).

Though theories have emerged within mathematics education, as described above, these theories still connect with theories outside mathematics education. For example, ethnomathematics research is informed by ethnographic traditions. And critiques of mathematics use approaches that have been developed outside mathematics education. In short, there are no theories that are absolutely independent from other theories. There are no distinct theories; there are only relations among theories.

Conclusion

In this chapter we have presented a critical investigation of contemporary theoretical trends in international research in mathematics education. Our attempt at mapping the field by broad strokes has allowed us to grasp the current state of play in theory selection, to understand how particular theories gain ascendancy, and to see how differing theories are acted upon in varying research projects. What has been revealed is a vibrant international research community that validates a wide range of theoretical perspectives, each of which informs the production of new knowledge relevant to mathematics education.

Though there is vibrancy and growth in our field attributable to socio-cultural and political perspectives, it is important to recognize that the field itself is dominated by one language and also by certain cultural practices, some of which are related to that language. Our review of the field follows developments in the literature published in English because this volume is in English and because English is the primary linguistic medium for developments in our field. However, we recognize that there is good work in other languages that addresses socio-cultural and political dimensions of mathematics education. The dominance of English in our field is, of course, a characteristic that relates to social, cultural and political forces. There is some scholarship that addresses this characteristic of our field (e.g., Barton, 2008; Skovsmose, 2006), but more often this characteristic is addressed in researchers' descriptions of the limitations of their work.

As all of the perspectives we have discussed not only challenge the assumptions of psychology but are also based on partly conflicting assumptions, the question of

the complementarity or juxtaposition of analyses informed by these perspectives becomes important. Research in mathematics education is diversified and the domain might be characterized as a collection of different approaches within rival discourses with little or no dialogue. If dialogue is avoided, it is then not the explanatory power of the diverse theories that prompt reception and dissemination, but the power relations among the researchers within competing discourses. However, many researchers acknowledge the potential of asking for alternative interpretations of the same empirical field from different theoretical vantage points, for these can bring tensions to the foreground. This will only happen, of course, if the theoretical underpinnings are well understood and their implications for the research design well articulated. That is particularly the case if interpretations based on distinct theories are controversial.

What can we learn from this profile? We can find out about the specific theorists who are currently influential in the field. We can learn about the way in which ideas about theory in mathematics education change. But we can do more—we can draw on the insights that our exploration offers to inform the debate about those things that are most important in mathematics education. From our interrogation we see signs of a shift away from cognitive psychology and evidence of critical questioning, of the creation of new ideas, and new ways of doing things, as well as a tolerance for multiplicity. All of these observations will contribute to the development of a body of professional knowledge in our discipline, informed by theory rather than driven by policy. We believe the international research community holds the reins of exciting potential for further development of leading edge knowledge in mathematics education.

Serious engagement with the work produced from different vantage points and openness towards different views can counteract the establishment of closed circles of academic inquiry, often labelled under a common term and declaring other projects as irrelevant to their own. We share the belief that an analysis of the situation of mathematics education needs to include critical reflection of its practices. It is only in its difference to practice as unmediated and often unconscious action, that theory transcends practice.

References

Aaron, W. R. (2008). Academic identities of geometry students. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), *Proceedings of the Joint Conference PME32–PMENAXXX for the Psychology of Mathematics Education* (Vol. 2, pp. 1–8). Morelia, Mexico: PME.

Adler, J., & Lerman, S. (2003). Getting the description right and making it count: Ethical practice in mathematics education research. In A. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Second international handbook of mathematics education* (pp. 441–470). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Alrø, H., Ravn, O., & Valero, P. (Eds.). (2010). *Critical mathematics education: Past, present and future*. Rotterdam, The Netherlands: Sense Publishers.

Althusser, L. (1971). Ideology and ideological state apparatuses. In L. Althusser (Ed.), *Lenin and philosophy and other essays* (Ben Brewster, Trans.) (pp. 121–176). London, UK: New Left Books.

Barton, B. (2008). *The language of mathematics: Telling mathematical tales*. New York, NY: Springer.

Barwell, R. (2007). Semiotic resources for doing and learning mathematics. *For the Learning of Mathematics, 27*(1), 31–32.

Barwell, R. (2009). Researchers' descriptions and the construction of mathematical thinking. *Educational Studies in Mathematics, 72*(2), 255–269.

Barwell, R., Barton, B., & Setati, M. (2007). Multilingual issues in mathematics education: Introduction. *Educational Studies in Mathematics, 64*(2), 113–119.

Bibby, T. (2010). What does it mean to characterize mathematics as "masculine"? Bringing a psychoanalytic lens to bear on the teaching and learning of mathematics. In M. Walshaw (Ed.), *Unpacking pedagogy: New perspectives for mathematics classrooms* (pp. 21–41). Charlotte, NC: Information Age.

Bishop, A., Clements, M. A., Keitel, C., Kilpatrick, J., & Leung, F. (Eds.). (2003). *Second international handbook of mathematics education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Black, L., Williams, J., Hernandez-Martinez, P., Davis, P., Pampaka, M., & Wake, G. (2010). Developing a "leading identity": The relationship between students' mathematical identities and their career and higher education aspirations. *Educational Studies in Mathematics, 73*(1), 55–72.

Boaler, J. (Ed.). (2000). *Multiple perspectives on mathematics teaching and learning*. Westport, CT: Ablex Publishing.

Breen, C. (2007). On humanistic mathematics education: A personal coming of age? In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 3–16). Seoul, Korea: PME.

Brown, T. (Ed.). (2008). *The psychology of mathematics education: A psychoanalytic displacement*. Rotterdam, The Netherlands: Sense Publishers.

Brown, T., & McNamara, O. (2010). *New teacher identity and regulative government: The discursive formation of primary mathematics teacher education*. New York, NY: Springer.

Carlsen, M. (2010). Appropriating geometric series as a cultural tool: A study of student collaborative learning. *Educational Studies in Mathematics, 74*(2), 95–116.

Chronaki, A. (2011). Disrupting "development" as the quality/equity discourse: Cyborfs and the subalterns in school technoscience. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 3–19). New York, NY: Springer.

Chronaki, A., & Christiansen, I. (Eds.). (2005). *Challenging perspectives on mathematics classroom communication*. Charlotte, NC: Information Age Publishing.

Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspectives. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 3–38). Charlotte, NC: Information Age/NCTM.

Daniels, H. (2001). *Vygotsky and pedagogy*. London, UK: Routledge Falmer.

Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education, 34*(2), 137–167.

de Abreu, G. (2008). From mathematics learning out-of-school to multicultural classrooms: A cultural psychology perspective. In L. English, M. Bartolini Bussi, G. Jones, R. Lesh, & B. Sriraman (Eds.), *Handbook of international research in mathematics education* (2nd ed., pp. 354–386). New York, NY: Taylor and Francis.

De Andrade, S. (2008). The relationship between research and classroom in mathematics education: A very complex and multiple look phenonemon. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), *Proceedings of the Joint Conference PME32 – PMENAXXX for the Psychology of Mathematics Education* (Vol. 2, pp. 57–64). Morelia, Mexico: PME.

De Freitas, E. (2009). Decoding the discourse of difficulty in high school mathematics. In M. Tzekaki, M., Kaldrimidou, & H. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, p. 365). Thessaloniki, Greece: PME.

De Freitas, E. (2010). Regulating mathematics classroom discourse: Text, context and intertextuality. In M. Walshaw (Ed.), *Unpacking pedagogy: New perspectives for mathematics classrooms* (pp. 129–151). Charlotte, NC: Information Age.

De Freitas, E., & Nolan, K. (Eds.). (2008). *Opening the research text: Critical insights and in(ter)ventions into mathematics education*. Dordrecht, The Netherlands: Springer.

Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths/pedagogic texts*. London, UK: Falmer.

Ensor, P., & Galant, J. (2005). Knowledge and pedagogy: Sociological research in mathematics education in South Africa. In R. Vithal, J. Adler, & C. Keitel (Eds.), *Researching mathematics education in South Africa. Perspectives, practices and possibilities* (pp. 281–306). Cape Town, South Africa: HSRC Press.

Ernest, P. (2009). Globalization, ideology and research in mathematics education. In M. Setati, R. Vithal, C. Malcolm, & R. Dhunpath (Eds.), *Researching possibilities in mathematics, science and technology education* (pp. 3–36). New York, NY: Nova Science Publishers.

Evans, J., & Morgan, C. (2009). Sociological frameworks in mathematics education research: Discursive approaches. In M. Tzekaki, M. Kaldrimidou, & C. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 223–226). Thessaloniki, Greece: PME.

Gellert, U., Jablonka, E., & Morgan, C. (Eds.). (2010). *Proceedings of the Sixth International Mathematics Education and Society Conference: Berlin, March 20–25, 2010*. Berlin, Germany: Freie Universität Berlin.

Gutiérrez, R., & Dixon-Román, E. (2011). Beyond gap gazing: How can thinking about education comprehensively help us (re)envision mathematics education? In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 21–34). New York, NY: Springer.

Halliday, M. (1978). Sociolinguistic aspects of mathematics education. In M. Halliday (Ed.), *Language as social semiotic: The social interpretation of language and meaning* (pp. 194–204). London, UK: Edward Arnold.

Halliday, M. (1985). *An introduction to functional grammar*. London, UK: Edward Arnold.

Hanley, U. (2010). Teachers and curriculum change: Working to get it right. In M. Walshaw (Ed.), *Unpacking pedagogy: New perspectives for mathematics classrooms* (pp. 3–19). Charlotte, NC: Information Age.

Hegedus, S., & Penuel, W. (2008). Studying new forms of participation and identity in mathematics classrooms with integrated communication and representational infrastructures. *Educational Studies in Mathematics, 68*(2), 171–183.

Herbel-Eisenmann, B., Choppin, J., Wagner, D., & Pimm, D. (Eds.). (2011). *Equity in discourse for mathematics education: Theories, practices, and policies*. New York, NY: Springer.

Herbel-Eisenmann, B., & Cirillo, M. (Eds.). (2009). *Promoting purposeful discourse: Teacher research in mathematics classrooms*. Reston, VA: National Council of Teachers of Mathematics.

Herbel-Eisenmann, B., Wagner, D., & Cortes, V. (2010). Lexical bundle analysis in mathematics classroom discourse: The significance of stance. *Educational Studies in Mathematics, 75*(1), 23–42.

Huzzard, T. (2004). Communities of domination? Reconceptualising organisational learning and power. *The Journal of Workplace Learning, 16*(6), 350–361.

Jablonka, E. (2003). Mathematical literacy. In A. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second international handbook of mathematics education* (pp. 77–104). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Jablonka, E., & Bergsten, C. (2010). Theorising in mathematics education research: Differences in modes and quality. *Nordisk matematikkdidaktikk: Nordic Studies in Mathematics Education, 15*(1), 25–52.

Jablonka, E., & Gellert, U. (2007). Mathematisation–demathematisation. In U. Gellert & E. Jablonka (Eds.), *Mathematization and de-mathematization: Social, philosophical and educational ramifications* (pp. 1–18). Rotterdam, The Netherlands: Sense Publishers.

Jablonka, E., & Gellert, U. (2011). Equity concerns about mathematical modelling. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 221–234). New York, NY: Springer.

Lerman, S. (2009). Sociological frameworks in mathematics education research. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, p. 217). Thessaloniki, Greece: PME.

Lester, F. K. (Ed.). (2007). *Second handbook of research on mathematics teaching and learning*. Charlotte, NC: Information Age/NCTM.

Leung, A., & Or, C. (2007). From construction to proof: Explanations in dynamic geometry environment. In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 177–184). Seoul, Korea: PME.

Martin, D. (2010). Not-so-strange bedfellows: Racial projects and the mathematics education enterprise. In U. Gellert, E. Jablonka, & C. Morgan (Eds.), *Proceedings of the Sixth International Mathematics Education and Society Conference: Berlin, March 20–25, 2010* (pp. 57–79). Berlin, Germany: Freie Universität Berlin.

Matos, J. P., Valero, P., & Yasukawa, K. (Eds.). (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference Albufeira, Portugal, February 16–21, 2008*. Lisbon: Centre de Investigacao em Educacao, Universidade de Lisboa and Department of Education, Learning and Philosophy, Aalborg University.

Mattos, A. C., Batarce, M. S., & Lerman, S. (2010). Discussion group: Mathematics education and democracy. In M. M. Pinto & T. F. Kawasaki (Eds.), *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, p. 338). Belo Horizonte, Brazil: PME.

Mesa, V., & Chang, P. (2008). Instructors' language in two undergraduate mathematics classrooms. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), *Proceedings of the 32nd Conference of the International Group for the Psychology of Mathematics Education held jointly with the 30th Conference of PME-NA* (Vol. 3, pp. 367–374). Morelia, Mexico: PME

Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics, 61*(12), 219–245.

Morgan, C. (2009). Understanding practices in mathematics education: Structure and text. In M. Tzekaki, M. Kaldrimidou, & C. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 49–64). Thessaloniki, Greece: PME.

Moschkovich, J. (Ed.). (2010). *Language and mathematics education: Multiple perspectives and directions for research*. Charlotte, NC: Information Age Publishing.

Nachlieli, T., & Herbst, P. (2007). Students engaged in proving—participants in an inquiry process or executers of a predetermined script? In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 9–16). Seoul, Korea: PME.

Noss, R., Brown, A., Drake, P., Dowling, P., Harris, M., Hoyles, C., & Mellin-Olsen, S. (Eds.). (1990). *The political dimensions of mathematics education: Action & critique*. London, UK: Institute of Education.

Pais, A., & Valero, P. (2011). Beyond disavowing the politics of equity and quality in mathematics education. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 35–48). New York, NY: Springer.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. London, UK: Routledge.

Pimm, D. (2007). Registering surprise. *For the Learning of Mathematics, 27*(1), 31.

Radford, L. (2009). Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings. *Educational Studies in Mathematics, 70*(2), 111–126.

Radford, L., Edwards, L., & Arzarello, F. (2009). Introduction: Beyond words. *Educational Studies in Mathematics, 7*(2), 91–95.

Renert, M., & Davis, B. (2010). Life in mathematics: Evolutionary perspectives on subject matter. In M. Walshaw (Ed.), *Unpacking pedagogy: New perspectives for mathematics classrooms* (pp. 177–199). New York, NY: Information Age Publishing.

Ryve, A. (2011). Discourse research in mathematics education: A critical evaluation of 108 journal articles. *Journal for Research in Mathematics Education, 42*(2), 167–198.

Sakonidis, H., & Klothou, A. (2007). On primary teachers' assessment of pupils' written work in mathematics. In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 153–160). Seoul, Korea: PME.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge, UK: Cambridge University Press.

Silver, E. A., & Herbst, P. (2007). Theory in mathematics education scholarship. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (2nd ed., pp. 39–68). Charlotte, NC: NCTM/IAP.

Skovsmose, O. (2006). Research, practice, uncertainty and responsibility. *Journal of Mathematical Behaviour, 25*(4), 267–284.

Skovsmose, O. (2007). Students' foregrounds and the politics of learning obstacles. In U. Gellert & E. Jablonka (Eds.), *Mathematization and de-mathematization: Social, philosophical and educational ramifications* (pp. 81–94). Rotterdam, The Netherlands: Sense Publishers.

Skovsmose, O. (2009). *In doubt: About language, mathematics, knowledge and life-worlds*. Rotterdam, The Netherlands: Sense Publishers.

Sriraman, B., & English, L. (2010). Surveying theories and philosophies of mathematics education. In B. Sriraman & L. English (Eds.), *Theories of mathematics education: Seeking new frontiers* (pp. 7–32). Berlin, Germany: Springer.

Stentoft, D., & Valero, P. (2010). Fragile learning in the classroom: Exploring mathematics lessons within a pre-service course. In M. Walshaw (Ed.), *Unpacking pedagogy: New perspectives for mathematics classrooms* (pp. 87–107). Charlotte, NC: Information Age.

Sullivan, P. (2008). Designing task-based mathematics lessons. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), *Proceedings of the Joint Conference PME32 – PMENAXXX for the Psychology of Mathematics Education* (Vol. 1, pp. 133–138). Morelia, Mexico: PME.

Teese, R. (2000). *Academic success and social power. Examinations and inequality*. Melbourne, Australia: Melbourne University Press.

Tsatsaroni, A., Lerman, S., & Xu, G.-R. (2003, April). *A sociological description of changes in the intellectual field of mathematics education research: Implications for the identities of academics*. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago.

Valero, P., & Zevenbergen, R. (2004). Introduction: Setting the scene of this book. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 1–4). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Voigt, J. (1984). *Interaktionsmuster und Routinen im Mathematikunterricht. Theoretische Grundlagen und mikroethnographische Falluntersuchungen*. Weinheim, Germany: Beltz.

Voigt, J. (1996). Negotiation of mathematical meaning in classroom processes: Social interaction and learning mathematics. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 21–50). Mahwah, NJ: Lawrence Erlbaum.

Wagner, D., & Herbel-Eisenmann, B. (2007). *Discursive tools for suppressing and inviting dialogue in the mathematics classroom*. Proceedings of the 29th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Lake Tahoe, USA.

Wagner, D., & Herbel-Eisenmann, B. (2009). Re-mythologizing mathematics through attention to classroom positioning. *Educational Studies in Mathematics, 72*(1), 1–15.

Walls, F. (2009). *Mathematical subjects: Children talk about their mathematics lives*. Dordrecht, The Netherlands: Springer.

Walls, F. (2010). The good mathematics teacher: Standardized mathematics tests, teacher identituy, and pedagogy. In M. Walshaw (Ed.), *Unpacking pedagogy: New perspectives for mathematics classrooms* (pp. 65–83). Charlotte, NC: Information Age.

Walshaw, M. (2004a). The pedagogical relation in postmodern times: Learning with Lacan. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 121–139). Greenwich, CT: Information Age.

Walshaw, M. (Ed.). (2004b). *Mathematics education within the postmodern*. Greenwich, CT: Information Age.

Walshaw, M. (2007). *Working with Foucault in education*. Rotterdam, The Netherlands: Sense Publishers.

Walshaw, M. (Ed.). (2010). *Unpacking pedagogy: New perspectives for mathematics classrooms*. Charlotte, NC: Information Age.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458–477.

Zolkower, B., & de Freitas, E. (2010). What's in a text? Engaging mathematics teachers in the study of whole-class conversations. In U. Gellert, E. Jablonka, & C. Morgan (Eds.), *Proceedings of the Sixth International Mathematics Education and Society Conference* (pp. 508–518). Berlin, Germany: Freie Universität Berlin.