# Chapter 13
# Teachers Learning from Teachers

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**Abstract** There is much debate within mathematics teacher education over ways in which professional and academic foci could be made to complement each other. On the one hand, teachers' craft knowledge is emphasized, mainly as this relates to the particular and local level of teaching; on the other hand, the importance of academic subject knowledge cannot be denied. In this chapter the focus will be on how to blend and balance the two through activities in which teachers learn from other teachers, particularly the co-learning of teachers and teacher educators. It will discuss professional relationships, reflective practice, community building, and research in practice. Examples of research-based programs involving *lesson study* (LS) and the *Learner's Perspective Study* (LPS) have moved the relevant research in this area to yet another level, in which theory and practice are combined. Projects such as these and others from diverse parts of the world will be presented and discussed.

## Introduction

Teaching is generally regarded as a complex and demanding profession that requires a mixture of subject knowledge together with theoretical and practical knowledge, skills and understandings. Teacher learning may originate from personal

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reflections on classroom experiences, professional readings, and other sources. However, the variations in teachers' learning sources have not been systematically documented and thus have had little input into the wider collective knowledge and theoretical underpinnings of teaching. Yet there exists a body of theoretical and teaching craft knowledge that is available to teachers (see, e.g., Wood, Jaworski, Krainer, Tirosh, & Sullivan, 2008). As well, focussing on a teacher's knowledge base reveals a multi-faceted, multi-sourced, highly interconnected mix that has defied the formation of widely accepted, common comprehensive frameworks. The confounding issues are whether and to what degree this knowledge is "private knowledge based on personal experience and only in the personal realm of thinking and acting," or is "knowledge coming from and staying in practice," or is "discursively generated, shared, and general knowledge" (Neubrand, Seago, Agudelo-Valderrama, DeBlois, & Leikin, 2009, p. 211).

There is a need to clarify the difference between teachers' theoretical knowledge and knowledge that arises from the teaching experience. It is common in education literatures for the term "craft knowledge" to be used to encapsulate the professional action-oriented knowledge used by teachers in their classroom teaching (Cooper & McIntyre, 1996).

> Craft knowledge describes the knowledge that arises from and, in turn, informs what teachers do. As such, this knowledge is to be distinguished from other forms of knowledge that are not linked to practice in this direct way … Neither is it knowledge drawn from theoretical sources. Professional craft knowledge can certainly be (and often is) informed by these sources, but it is of a far more practical nature than these knowledge forms. Professional craft knowledge is the knowledge that teachers develop through the processes of reflection and practical problem-solving that they engage in to carry out the demands of their jobs. (p. 76)

In contrast, theoretical knowledge generally lays down principles and frameworks derived from research studies that are often replicable and can be generalized to other contexts. This kind of knowledge is less focussed on the individual teacher or on small practical details required for teaching. Research has sought to identify and articulate the types of professional knowledge that a successful teacher would need. The seminal work of Shulman (1986) and colleagues proposed that a basis of professional knowledge would contain: (a) content knowledge both substantive and syntactic; (b) general pedagogical knowledge including generic principles of classroom management; (c) curriculum knowledge including materials and programs; (d) pedagogical content knowledge that for a given subject area included forms of representation, concepts, useful analogies, examples and demonstrations; (e) knowledge of learners; (f) knowledge of educational contexts, communities and cultures; and (g) knowledge of educational purposes.

A number of researchers have reflected upon Shulman's work in their studies regarding teachers' learning. For instance, Even and Tirosh (2008) claimed that in coining the term pedagogical content knowledge, Shulman contributed greatly to the discussion of what teachers needed to know about students' mathematical learning. On the other hand, pedagogical content knowledge has been the subject of much debate, particularly regarding its epistemological status (Ponte & Chapman, 2008). Although Shulman's work provided a suitable beginning for the growth of a

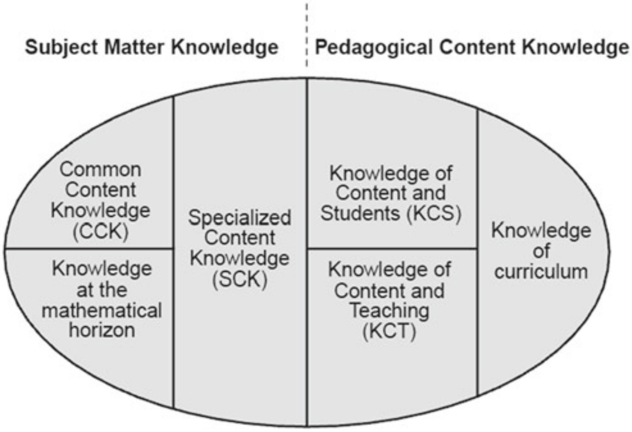



*Figure 13.1.* Domain map for mathematical knowledge for teaching (from Hill, Ball, & Schilling, 2008, p. 377).

framework, there has been considerable development by other researchers. For example, Hill, Ball, and Schilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the domain map for mathematical knowledge for teaching shown in Figure 13.1.

Tim Rowland and his colleagues (see, e.g., Rowland, 2008, 2009; Rowland, Huckstep, & Thwaites, 2005) suggested a framework that had four domains of knowledge: Foundation, Transformation, Connection and Contingency. This framework, which Rowland dubbed the "knowledge quartet," developed as the result of an analysis of data gathered from observations of prospective teachers, and it has now been applied to the work of practising teachers (Rowland, 2009). It drew attention to the importance of a teacher's knowledge at any given time, and also to the teacher's development of knowledge over time.

Shulman's (1986, 1987) categorization can be contrasted with the European focus on *the didactics of mathematics* (*didactique*), which is concerned with theoretical and practical issues surrounding mathematics curricula and teaching, and their relationships with learning. The European emphasis is on designing didactical situations which acknowledge and incorporate important transitions from mathematics itself to the ways in which that mathematics is brought to students in educational contexts.

Kilpatrick (2003) reported that *didactique* went beyond the art and science of teaching to include: learning and school systems; an intensive common epistemological analysis given to mathematical concepts and a shared methodology that is not to be found in U.S. research; the use of mathematics more extensively as a source of metaphors; a mode of analysis which proceeds from elaborate, a priori analyses to experimentation in the classroom. Sustained attention is to be given to classroom teaching and to the social context in which teaching and learning occur. It focusses intrinsically on the missing element for which Shulman introduced the term pedagogical content knowledge. There is considerable available research, conducted over several decades, focussing, in France, on didactical and adidactical situations (Brousseau, 1992), in Germany on the epistemological nature of teachers' thinking (Steinbring, 1998), and in the Netherlands, among researchers at the Freudenthal Institute (Gravemeijer, 1994a, 1994b, 2000; van den Heuvel-Panhuizen, 2001), on *Realistic Mathematics Education*.

The mixture of theoretical and practical learning that forms a teacher's knowledge base is open to many influences—from other teachers, friends, experiences in and out of school, subject associations, teacher-education programs, acknowledged professional experts, etc. The idea of teachers learning from teachers can conjure many different images. It is evident in the literature that there is no consensus regarding the use of terms such as teacher professional development and teacher professional learning. The terms are often used interchangeably and with little or no definition of their meanings (Even, 2008). Clements (2008) was critical of attempts to define professional development as the planned, formal activities and programs that teachers undertake to extend their professional learning, and professional learning as the individual growth of a teacher's expertise. Professional development can be the result of numerous activities that are neither planned nor ostensibly formal—such as classroom experiences, reading, and informal activities and experiences.

The term "professional development" of teachers has often, in the past, implied a deficit view of teachers, emphasizing elements of knowledge which teachers lack, or ways in which teachers need to be developed (Dawson, 1999; Hoyles, 1992; Ponte, 1994). The implication is that people who have access to theoretical knowledge (possibly teacher educators, or didacticians) are in a position to remedy the deficits in teaching by changing the practices of teachers. Such a view is simplistic, implying that those with the theoretical knowledge could translate that knowledge into classroom practice if given the opportunity to do so. It ignores the complexities of teaching practice—there are many factors which influence what teachers can do in the educational settings in which they work.

Simon (2008) wrote about two kinds of commonly available programs which influence learning and development of practising teachers—programs which focus on content *and* process, and those which are solely *process based*. According to Simon, programs which focus on content and process include "courses and workshops for teachers in which teacher educators aim to promote particular mathematical and pedagogical concepts, skills and dispositions" (p. 18). They can be considered as professional development programs in which teacher educators have an agenda for the learning of "participating teachers" (for example, for participants to become aware of research on students' strategies and errors in the teaching of algebra). With such programs it is typically assumed that participants' personal and professional learning will be stimulated. The process only category, according to Simon, includes programs such as *lesson study* (LS) and inquiry-based models, on which more will be said upon later in this chapter. In this category we would also include *developmental research* programs in which it is expected that participating teachers' engagement in research (possibly in partnership with teacher educators) will contribute to the improvement of teaching.

Teachers belong to communities situated within and around schools, and to educational systems created by the societies and cultures to which the schools belong (Wenger, 1998). The teachers construct professional identities within these socio-cultural and historical settings in line with the norms and expectations which prevail. The kinds of planned professional development programs that would be expected to occur in these settings and practices would include Simon's two categories of professional learning programs. Participation would give rise to situated learning arising from everyday interactions within particular environments. In all of these cases, a teacher's development would most likely be related not only to the programs but also to that teacher's prior knowledge and experience.

The study of mathematics teachers and mathematics teaching, and associated learning outcomes, will be resumed later in this chapter with the presentation of a range of programs relating to Simon's two categories. That further discussion will highlight the concomitant learning of teacher educators who work with teachers for the purpose of developing the quality of teaching and, therefore, learning. Before resuming, it will be appropriate to briefly discuss the forces and influences that exert pressure on the nature and delivery of these programs.

## Local, National, and Global Influences on Teachers, Teaching and Learning

Professional development programs and teacher professional learning are influenced in varying degrees by research across the field of education. These programs will be influenced by a mix of international, national and local research pressures and initiatives, the actual mix depending on contexts and personnel in any particular place at any particular time. An international influence could be the result of globalization; a national influence could be a perceived need to conform to a national standards document; and a local influence could be a school principal's desire to adopt an outcomes-based education approach within a school.

With the growth in communication technologies and stimulus to information flow, and the increased ease of overseas travel, it is common to hear that we live in a global world, and that the world has become a global village. Globalization has become a familiar, albeit imprecise, term associated with multiple and significant changes currently happening in all areas of social life (English, 2008; Stromquist & Monkman, 2000). Not surprisingly, education is also subject to forceful changes arising from globalization, particularly when the focus is on information flow and the possibilities for world-wide communication. Research in mathematics education is a global enterprise and as such is caught up in the wider movements that influence all educational research. Other chapters in this *Handbook* make it abundantly clear how Trends in International Mathematics and Science Study (TIMSS) and the Programme of International Student Assessment (PISA) are examples of programs whose influence has speedily transcended national boundaries.

English (2008), in her introduction to a handbook on research in mathematics education, stated: "In recent years, we have seen a major shift within the field of mathematics education from a mainly psychological and pedagogical perspective to one that encompasses the historical, cultural, social, and political contexts of both mathematics and mathematics education" (p. 4). It should be noted that globalization is not always equitable in that family and other local conditions can restrict access to information coming from, say, the Internet. This has been felt particularly in the experience of one of our authoring team who had difficulty accessing articles relating to this chapter because they appeared in books which were not available in her country.

One of the main difficulties in the dissemination of knowledge to teachers in some countries is the lack of an agreed means. We provide a concrete example from Iran, involving the quarterly journal *Roshd: Mathematics Education Journal*, which is one of 16 subject-bounded journals and 15 general magazines titled "Roshd" published by the Ministry of Education in Iran. One of the authors of this present chapter, Gooya, is the editor of the *Mathematics Education Journal*. Since 1996, a special section, titled "Teachers' Narrative," has been included in the *Journal* in order to disseminate the research findings of teacher researchers arising mainly from action-research projects conducted either locally or at district level. Teachers were also encouraged and assisted through personal communications to write scholarly papers, which were included in the *Journal*. Such publications sometimes generated workshops at annual national mathematics education conferences. The *Journal* had another section called "Viewpoints" in which teachers could share their ideas and receive feedback from their colleagues. The number of teachers communicating with this journal dropped sharply during the 2009–2010 academic year and this trend has continued. The editorial board investigated the reasons for the dramatic change and found that the formal educational system had announced that teachers could not get credit for their professional promotion by publishing in this or other similar journals. They could only get credit by publishing in university journals or journals of scientific societies approved by the Ministry of Science, Research and Technology (which is responsible for higher education and any forms of tertiary education). Thus, a single act by authorities could deny teachers the opportunities offered by the journals for disseminating their practical or craft knowledge.

It is interesting that this same kind of influence has been a reality in western academic circles for many years—where it is well known that getting a publication in a "top" journal (like, for example, *Journal for Research in Mathematics Education*, or *Educational Studies in Mathematics*) would be likely to "count" towards promotion, but a publication in a local "teaching" periodical would not. The message implicitly conveyed has been that publication in a peer-reviewed *research* journal is more important than publication in a periodical for which the readership is mainly school teachers.

Global forces should not be all powerful and should not completely mould local contexts into uniform shapes—that is because global forces do not take account for local realities. Education researchers have highlighted problems in adopting global programs because "pedagogical methods are culturally embedded, and transplanting them from one culture to another is not always feasible" (Hatano & Inagaki, 1998, p. 101). That said, there can be little doubt that, increasingly, local education contexts are being influenced by local, state and national authorities. For example, Japan, Malaysia and the UK have mandatory national curricula. Australia and the USA do not, but both may be moving towards getting one (see, e.g., Australian Curriculum Assessment and Reporting Authority (ACARA), 2010a, 2010b). Yet, independently of whether a national curriculum exists, local contexts cannot be fully understood without taking account of global influences. Stromquist and Monkman (2000) point to efforts of groups to recapture traditional values and identities as unintended effects of globalization and the reassertion of the importance of local contexts.

Is there a middle path to blend and balance the global and local forces through the activities of teachers learning from other teachers? It is within this interplay of the two forces that Robertson (1995) used the term "glocalization" to explain the process whereby the global and the local interpenetrate each other, creating a hybrid. This hybrid adapts and blends global trends with local conditions and options. In other words, global trends are contextualized into the specifics of local settings.

This interplay of global and local influences can be seen within the distinction made between formal research knowledge which is theoretical and able to be generalized across contexts and the practical knowledge of the teacher which is based at the particular and local context level (Fenstermacher, 1994). Teachers often concentrate on their own localized insights and improvements to practical—although published research can also be local in its focus. A survey of 282 research articles published between 1999 and 2003 in international journals, international handbooks of mathematics education, international mathematics education conference proceedings, and in national and regional sources revealed that more than 60% were small-scale qualitative studies of a single teacher or small group of less than 20 teachers, and that 72% were conducted by teacher educators studying teachers with whom they were working (Adler, Ball, Krainer, Lin, & Novotná, 2005). In a review of Australasian research between 2004 and 2007, Anderson, Bobis, and Way (2008) observed that "smaller-scale studies tended to rely on self-report data and that few incorporated significant amounts of observation data to help validate the self-reported findings … due to the labour-intensive and high cost involved when studies incorporate classroom observation" (p. 327).

The knowledge and results from many action research studies, conducted by teachers, have not been disseminated widely, and in such a circumstance any impact from a study is likely to have been confined within the local school or community. One result has been that teacher inquiry and practitioner research has been regarded "almost as second-level research paradigms in educational research, relevant mainly to improving professional practices rather than furthering the general field of education research and theory" (Lingard & Renshaw, 2010, p. 35). From this perspective of formal research, teachers could be seen as simply translators or interpreters of educational research completed elsewhere, or sometimes as merely the objects of formal research. One result of the fact that university- and system-based academics have often had greater access to power and resources than school-based teachers has been that many teachers have felt at liberty to ignore or reject academic research findings, which they perceive as coming from the "ivory tower." Moves towards research partnerships between teachers and teacher educators have changed this situation somewhat, however not completely, as will be seen in some of the examples which follow.

In the face of criticism, there has been increased support for the concept of "teacher-as-researcher," because of its focus on local issues and change. In the first *International Handbook of Mathematics Education,* Crawford and Adler (1996) argued that active teacher participation in research on their own professional practice was a pre-requisite to changing and improving student educational outcomes. They highlighted educational change and issues associated with the lack of dissemination of formal research findings, pointing out that often university research did not reach teachers and therefore did not have much chance of affecting teaching and learning in schools.

Since then, an International Group for the Psychology of Mathematics Education (PME) working group focussed on the "teacher as researcher in mathematics education," published a book of papers (Zack, Mousley, & Breen, 1997) germane to the teacher/teacher educator interface. And, since its first issue in 1998, the *Journal of Mathematics Teacher Education* (JMTE) has published many papers relating to teacher research, mostly written by teacher educators who work with teachers. Indeed, the first article in the first volume reported a study of the learning of teachers who explored questions relating to their own practice (Jaworski, 1998). We will briefly describe this project (the Mathematics Teacher Enquiry project) later in this chapter.

The practices of teacher research and some of the related issues for the learning and development of teachers were captured in the *Second International Handbook of Mathematics Education* in 2003, in which it was claimed that the roots of the teacher-as-researcher movement lay in a paradigm shift that focussed on teachers as knowers and thinkers. This shift grounded theory in practice and insisted that knowledge derived from research was necessarily personal. It was claimed that the value of knowledge arising from teachers' research into their own teaching "was accompanied by an explicit rejection of the authority of professional experts who produced accumulated knowledge in scientific settings for use by others in practical settings" (Breen, 2003, p. 528).

In 2005 an ICMI study conference on mathematics teacher education produced a publication focussed on teacher learning through research in practice (Even & Ball, 2009). One of the two main sections in this publication was devoted to *Teachers Learning in and from Practice*. As well, a first *Handbook of Mathematics Teacher Education* was published in four volumes, and each volume included chapters related to teacher research (Wood et al., 2008). The fourth volume was devoted to the learning of teacher educators who worked with teachers in various modes of practice-based activity.

The rise of the teacher-as-researcher movement was accompanied by a renewed focus on theory and theory development in mathematics education, evident in recent publications such as those mentioned above and in the *Second Handbook of*

*Research on Mathematics Education* (Lester, 2007), which devoted most of its first section to this theme. The 29th annual conference of PME held a special "Research Forum on Theories of Mathematics Education." Theories in mathematics education were emerging, not only global theories such as constructivism or socio-cultural theory, but also more localized theories in specific areas such as knowledge in teaching (cf., the knowledge quartet mentioned above), including the personal theories of teachers and teacher educators, which were mostly based on their experiences in practice. These theories often gained status through their use by members of the international community and through associated debates in scholarly publications and conferences (see, e.g., Niss, 2007). Gradually, as a result of such dissemination and debate, relationships between theory, knowledge, and practice have begun to emerge.

Teachers' theories which are tested in practice and are an influential part of that practice are often not articulated clearly. Nor are they always subjected to careful scrutiny outside a minority of theory-inclined mathematics education researchers. Teachers may develop teaching practices, and informal associations of ideas associated with their teaching, by being part of a community of teachers within a school or local area. Without the influence of more global theoretical teaching knowledge which teachers themselves embrace, both in their minds and in their professional behaviours, the teaching community may continue to perpetuate existing practices irrespective of how well, or otherwise, these practices are generating high quality student learning.

The "glocal" or balanced way was taken up by Lingard and Renshaw (2010), who entered the teacher-as-researcher debate by arguing that teaching should be both a research-informed and a research-informing profession. Not only should teachers have a "researchly disposition" but educational researchers should have a "pedagogical disposition" which entails a desire for multiple forms of dissemination. Lingard and Renshaw (2010) strongly supported the concept of co-learners and proposed the use of design research practices because, they maintained, these blend applied and theoretical positions and acknowledge teachers and academic researchers as equal partners in the production of knowledge. "Design research elevates the importance of teachers as research collaborators, not just at the local level in relation to context-specific professional practices, but in terms of developing more general insight and transferable knowledge about teaching and learning processes" (p. 36).

Jaworski (2004) made a distinction between design research and developmental research in terms of the degree of involvement of teachers. She argued that with design research, teachers often were included merely to test out designs developed by external researchers (see for example, Witmann, 1998) whereas, in developmental research, teachers were included in the decision-making process that generates a design. Cobb and colleagues, who have offered a range of activities in which the involvement of teachers can be seen to vary considerably (Cobb, Confrey, di Sessa, Lehrer, & Schauble, 2003), saw distinctions between design and developmental research as blurred.

In the *Second International Handbook of Mathematics Education,* Breen (2003) provided some examples of attempts to find connections between teacher education as a field of practice and as a field of research. One example was the spread of the Japanese process of *lesson study* (LS). Breen concluded his chapter with an appeal to mathematics education researchers to seek closer collaboration with teachers. Breen's appeal resonated with the general theoretical position emerging among mathematics educators (Even & Ball, 2009; Wood et al., 2008).

In the remainder of the chapter we explore professional relationships, reflective practices, and community building that have led to genuine learning on the part of both teachers and teacher educators. In the next section we consider relationships between research and development in mathematics teaching, focussing particularly on ways in which research can be seen to provide a basis for developing knowledge and practice in teaching.

## Research as a Basis for Learning in Teaching

Earlier in this chapter, in our brief discussion of pedagogical content knowledge and *didactique,* we reported some research studies that sought to identify and articulate better the types of professional knowledge that a successful teacher or teacher educator would need. We also referred to an existing division between research and craft knowledge and to various attempts to remove it. In this section, we examine studies that seek to maximize professional knowledge creation as the practices of researching and teaching become more coordinated and knowledge conversion from one practice to the other is encouraged by educational authorities (Ruthven & Goodchild, 2008). It will be seen that it is now well recognized, both inside and outside the mathematics education research community, that there is value in minimizing the gap between the theoretical expert and the classroom teacher by using research methodologies and practices that (a) place the teacher in the genuine role of a researcher, and (b) problematize the teaching process rather than simplify it (Pritchard & Bonne, 2007).

These desirable aims must be achieved in a wider context. Thus, for example, reflecting wider global trends in the period 2004–2007, the national governments in New Zealand and Australia promoted the development of accountability measures for funding and research, and this has been reflected in the Australasian mathematics education research output. According to Forgasz et al. (2008), there has been:

- A decrease in creative and idiosyncratic research and an increase in program research;
- A decrease in individual research and an increase in group or team research;
- A decrease in funding for basic research and an increase in funding for practice-oriented projects; and,
- A decreasing concern with the quantity of research and an increasing concern with the quality of research.

During the last decade there has been a steady increase in the number of publications reporting teacher-education research from around the world, and many of the publications are making clear the value of collaborative work among mathematics teachers or between teachers and researchers (Krainer & Wood, 2008). A variety of methodologies and organizational features can be identified in these studies and the research is contributing to teaching development and the associated professional learning of teachers.

Our discussion will be informed by the use of a framework developed by Jaworski (2003) based on research with teachers in which teachers took on a practitioner-researcher role (Jaworski, 1998, 2001). She suggested that the research itself can be an important mediating tool for teaching–learning development and proposed a framework for theorizing such mediation which consisted of four paired constructs:

- knowledge and learning,
- inquiry and reflection,
- insider and outsider,
- individual and community (Jaworski, 2003).

*Knowledge and learning* define an epistemological dimension in which participants bring their own thinking, beliefs and expertise to the research setting and learn through interactivity and dialogue within the community. *Inquiry and reflection* form a research dimension in which questions asked about practice and reflection on engagement in practice lead to new questions and new ways of doing and being. *Insider and outsider* recognizes the roles of teachers and teacher educators in processes of teaching development, both as insiders inquiring into their own practices and as outsiders researching the practices and development in teaching related to local and general knowledge (Bassey, 1995). *Individual and community* recognizes the importance of collaborative activity to the developmental enterprise and ways in which collaboration contributes to development for individual participants.

The term "developmental research" is sometimes used to refer to research which encourages development as well as documenting the developmental process. Stenhouse (1984) suggested that research is "systematic inquiry made public" (p. 120). Consistent with this point of view, we regard as research the activity of teachers who engage in systematic inquiry into their own practices and share their thinking and outcomes with other teachers and professionals. It is hard for teachers to take on researcher roles, since the practice of teaching is extremely demanding (McIntyre, 1997), and the nature of being a researcher can be perceived as not being within the accepted roles of a teacher. However, when collaborations are formed with university researchers, or teacher educators, the knowledge that both groups bring to the collaboration can enable a research or *inquiry* process to be established (Elliot, 1991; Jaworski, 1998, 2008).

In a developmental research project, development and research act as two sides of the same coin and participants are central players collaborating in action and outcome. Teachers are insider researchers, studying aspects of their own practice and of their students' learning. Teacher educators are often outsider researchers studying the development of teaching which arises through teacher research. They can also be insider researchers if they concomitantly study aspects of their own practices in promoting teaching. The inquiry processes that are involved can result in new knowledge in practice (insider research) and new knowledge about practice (outsider research). Outsider research can lead to more generalized knowledge available for inspection and critique in the academic community (Jaworski, 2003).

Developmental research can be seen as both a democratic and a critical approach to professional enhancement and improving practice (Goodchild, 2008). It is democratic when it includes participants in collaborative engagement and respect, valuing knowledge of different kinds from different sources (Herbert, 1989). It is critical when it encourages insight into and questioning of the processes and practices of its participants by the participants themselves (Carr & Kemmis, 1986). Collaboration is a basis for democratic engagement and inquiry provides the critical dimension.

Research into the professional practice of teaching, and teachers' learning about teaching, has suggested that engagement in inquiry processes can be a strong force for teaching development (Cochran Smith and Lytle, 1999; Jaworski, 1998; Wells, 1999). Cochran Smith and Lytle (1999) referred to inquiry as "stance." Teachers taking on an inquiry stance start to think differently about teaching and through their reflections on the teaching process are able to modify teaching in critical ways. Wells (1999) reported similarly, focussing particularly on the role of dialogue in encouraging new thinking and development. The collaborative nature of an inquiry process is central to teaching development. Teachers have the opportunity not only to inquire into their own practice and to modify practice (which is extremely hard to achieve alone) but conversations with their colleagues in an inquiry community enable both the encouragement of an inquiry approach and a sustaining of inquiry activity. If the inquiry community also includes university colleagues then the outside knowledge they bring of published research and theory can provide an important additional dimension (Jaworski, 2008).

Central to such an approach is the idea of creating or developing an "inquiry community" in which practitioners reflect on their own activities and, overtly, develop knowledge in practice. In order to theorize *inquiry community*, we might start from the concept of a *community of practice* (hereafter "CoP"), drawing on Wenger (1998). The term "community" designates a group of people identifiable by who they are in terms of how they relate to each other, their common activities and ways of thinking, beliefs and values. Activities are likely to be explicit, whereas ways of thinking, beliefs and values are more implicit. Wenger (1998) described community as "a way of talking about the social configurations in which our enterprises are defined as worth pursuing and our participation is recognizable as competence," and commented that "the social configurations in which our enterprises are defined" are the basis of practice (p. 5). In our field we might think of the practice of teaching mathematics. Teachers teaching mathematics within a school setting might be seen to form a community of mathematical teaching practice with its own norms and expectations and ways of being and doing. Mathematical knowledge provides a *foundation* for such practice (Rowland, 2008), being the basis of didactical knowledge and informing pedagogy.

Wenger has suggested that *belonging* to a CoP, that is having identity within a CoP, involves *engagement*, *imagination* and *alignment*. Thus, in practices of mathematics learning and teaching, participants engage in their practice alongside their peers, use imagination in drawing on their foundational knowledge and interpreting their own roles in the practice and align themselves with established norms and values. However, the expectation that teachers will align themselves with the practices in their school environment may not promote possibilities for development. Brown and McIntyre (1993), after gathering data through observations in classrooms and conversations with teachers, talked of classroom activity settling into “normal desirable states” (p. 54) in which teacher and students were comfortable with activity and expectations. Such normal “desirable states” may run counter to the need to develop students’ confidence in mathematics and strong conceptual understandings. A community of inquiry, therefore, seeks to challenge the status quo, not to change it overnight, but to start to question and to look critically at what alternatives might be possible; then to start to think and act differently. In such an inquiry approach, *alignment* becomes *critical*. This means that while aligning with the norms and expectations of the school environment, teachers might start to ask questions about ways in which teaching and learning are approached, and start to explore, and to inquire into alternative possibilities. The idea of critical alignment is central to that of an inquiry community (Jaworski, 2006).

## Learning of Teachers and Teacher Educators

> Mathematics teacher education is more difficult and complex than mathematics education, because it subsumes all of the latter. Likewise, research in mathematics teacher education is more difficult and complex than research in mathematics education. (Simon, 2008, p. 27)

This quotation from Simon recognizes that research in mathematics teacher education of necessity requires attention to several layers. Study of teacher learning (of mathematics teaching) requires within it a study of the concomitant learning of students in the mathematics classrooms where teachers teach (see Figure 13.1). Without the latter, a study of teacher learning is hollow. As Pring (2004) stated, “an action might be described as ‘teaching’ if, first, it aims to bring about learning, second, it takes account of where the learner is at, and, third, it has regard for the nature of what has been learnt” (p. 23). Thus, to study the learning of teachers, we have to attend to how they create opportunities for the specific students with whom they work, and how they consider the associated learning outcomes.

It is possible that the issue might indeed be even more complicated than this. Although it is possible to conduct research into teacher learning in the natural settings of teachers’ everyday classroom practices with their students, most often, in studying teacher learning, researchers focus on some teacher education program *designed to promote* development. Often, the people undertaking the research and reporting it in scholarly papers are themselves the teacher educators conducting the programs. As Chapman (2008) has pointed out, many such research reports focus on the nature of teaching and the learning of teachers, with no consideration given to the teacher educators’ own learning from their activity for promoting teachers’ learning. It is as if the practices of the teacher educators are not of critical concern.

Mathematics teacher educators ("MTEs") are themselves teachers, and in many ways their activities parallel the teaching activities of school teachers. They are professionals who work with practising teachers and/or prospective teachers to develop and improve the teaching of mathematics, just as school teachers work with students to develop students' mathematics knowledge and understanding. MTEs are often based in university settings with academic responsibilities. They are often both practitioners and researchers. They have to take account of what a teacher already knows, and does, and to have regard for the nature of what has to be learned (Pring, 2004). In their research roles, teacher educators have responsibility for conducting research into the education of teachers and such research can result in developing knowledge in practice for both groups of practitioners. Thus we might ask, *How do mathematics teachers and teacher educators learn and develop?*

- What forms of knowledge are important for teachers? For MTEs?
- In what ways does engagement in activity with teachers lead to learning and development for the MTE and vice versa?
- What programs in mathematics teacher education have been significant for the learning and development of teachers and MTEs?

Figure 13.2 suggests related aspects of teacher and MTE knowledge.

Both groups have knowledge of mathematics, pedagogy, etc., as shown in B (in Figure 13.2). This knowledge may take different forms for each group, but it nevertheless provides a basis for communication through common areas, experience and interests. In addition, each group brings its own specialist knowledge as shown in A and C. Educators do not generally have the knowledge indicated in C and teachers generally do not have that indicated in A. A surrounding rectangle (not shown) might represent the deep complexity of educational environments in which teaching development is situated.

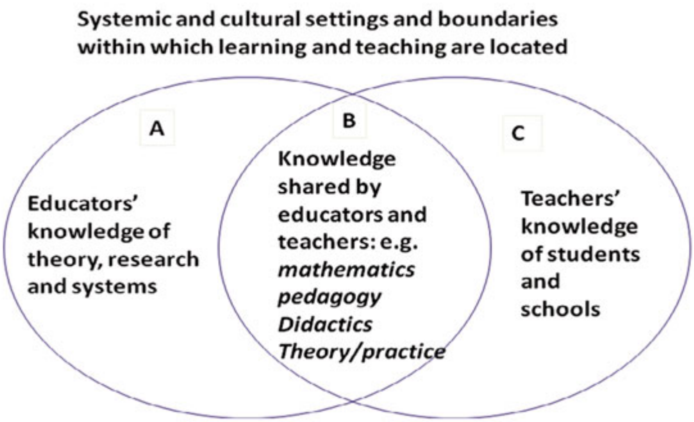



*Figure 13.2.* Related aspects of mathematics teacher and mathematics teacher educators' knowledge (from Jaworski, 2008, p. 336).

In the rest of the chapter we present a range of examples of projects and programs which illuminate the concepts discussed above. Our focus is on research in mathematics teacher education which has revealed and/or contributed to development in teaching mathematics and in which teachers and teacher educators have learned from each other. We draw particularly on projects with which we are familiar through our own engagement as researchers and practitioners with mathematics teachers in diverse parts of the world, seeking to develop mathematics teaching practice. We consider the development of teaching knowledge for both teachers and teacher educators.

In what follows, we use the structure of three main headings and a number of subheadings. The three main headings correspond to teachers learning from teachers as a result of participating in: (a) large-scale projects; (b) small-scale professional learning; and (c) preservice programs. These will be applied loosely as some studies could appear under more than one heading. Under the structure, in each of the modes of teachers' learning it is likely that teacher educators will be involved, sometimes as leaders in the education of teachers and sometimes as researchers. Such roles are not unproblematic and so we take up issues of teacher educators' roles and indeed teacher educators' learning alongside those relating to the learning of teachers.

## Teachers Learning from Teachers in Large-Scale Projects

In this section, recent studies arising from large-scale professional development or research projects are presented. The adjective "large-scale" was considered to include those studies that drew upon systemic, state-wide, or multi-country projects as well as studies or projects in local areas that involved schools and non-school environments such as universities. The focus for these projects is their impact on the professional learning of teachers, on curriculum reform and on improved student outcomes.

All of the programs described in this section except the *Learner's Perspective Study* (LPS) possessed, to varying degrees, the following common features concerning the working process and the results (hereafter "CFPR" for common features of process and results). To avoid duplication, these common features will be assumed and only unusual or unique aspects will be highlighted. What were the common features? Firstly all programs incorporated workshops involving mathematics teachers and MTEs. These workshops were conducted at universities, schools or other institutions. All involved teachers conducting research into aspects of their own practice within their own schools, and communicating their activities and findings in the workshops. All involved MTEs who contributed, to the workshops, relevant material from research and other literature related to the teachers' own explorations, or expectations arising from mandated curriculum reform. Common features of the reported results were learning improvement of teachers in developing knowledge of theories and research, and insights into new approaches in the classroom. MTEs developed greater awareness of teachers' ways of thinking and of the challenges and limitations within schools and classrooms. Thus mathematics teachers and MTEs modified their ways of being and thinking to accommodate those of other teachers and MTEs, and this accommodation resulted in challenges to existing practices and new ways of perceiving each other. This growth of awareness led to a greater depth of understanding between mathematics teachers and MTEs which enabled them to deal with the issues that arose and to work towards productive development.

The theoretical ideas involving community of inquiry and critical alignment were the basis for two government funded, 4-year projects in Norway: "Learning Communities in Mathematics" (LCM) and "Teaching Better Mathematics" (TBM). The first involved 14 didacticians of mathematics (mathematics educators) from one university and about 30 teachers from 8 local schools in exploring the development of mathematics teaching in their schools; the second, building on the first, involved a consortium of 5 universities in different cities in Norway and schools local to each, extending the developmental process across the country. These programs followed CFPR with the workshops using collaborative inquiry-based activities between the mathematics teachers and didacticians. An uncommon feature involved the design of teaching and video-recording of innovative activities in classrooms. These video records formed part of a large bank of data from all aspects of the project which was a source of analysis for didacticians as outsider researchers in relation to a range of research questions.

The results of the program were very positive in the areas described in CFPR. Publications from the LCM project documented the learning processes in which both teachers and didacticians were engaged (e.g., Jaworski et al., 2007). The project demonstrated that learning in both groups was necessary in order to form a community of inquiry, and when there appeared to be a conflict a sincere desire to make the project work led to activity to resolve the conflict (Jaworski & Goodchild, 2006). The stakes were important for both groups and both groups felt ownership of and responsibility for the activity involved, albeit in differing ways. The implications for other programs lie in the relationships that evolved and the ways in which the program managed to foster equity. This is an important challenge for all those currently engaged, or about to be engaged, in teacher education programs.

Resonating with research in many western countries, Australasian research literature has focussed on the structures and findings of a number of large early numeracy programs—such as the Australian *Count Me In Too* (CMIT) project in the state of New South Wales, the *Early Numeracy Research Project* (ENRP) in the state of Victoria, and New Zealand's *Numeracy Development Projects* (NDP). These projects were funded by governments seeking to establish research priorities and methodological approaches. The projects aimed to deliver professional development teaching programs using a variety of strategies that included MTEs and extensive use of ICT while improving student achievement with early mathematical concepts. When these three programs were compared, researchers were able to extract common structures as well as identify the unique aspects of each project. Each featured: (a) a research-based framework for children's mathematical learning; (b) the use of individual student thinking assessment interviews; and (c) intensive whole-school professional development programs (Bobis et al., 2005). The process described in CFPR was evident in the workshops, and the results listed gains in knowledge, and improvement of relationships between teachers and MTEs.

Each of these large early numeracy programs had unique features, and we will consider just one, CMIT, as an example. The program regarded the identification, sharing and activation of knowledge of how children learn mathematics as a long-term, whole-school, classroom-based learning process. The interplay of researcher knowledge and teacher knowledge was an expectation of the CMIT program which used a design research model (Cobb, 2003) that collapsed the four groups of insiders and outsiders into one group of co-learners (the academic facilitators; the Departmental consultants (who were mostly former teachers); the teachers; and the students). The unique feature of using an on-going evaluation process conducted by external researchers (outsiders) meant that insights developed in collaboration with teachers and MTEs were used to "feed forward" into the theory development and instructional design loops that were implemented by the teachers and MTEs. Thus, theoretical knowledge was shared with teachers as active learners in their schools to be trialled and developed with their colleagues with the participation of their students. It was regarded as a factor in keeping the program dynamic and sustainable.

An extension to CMIT was the large system-wide *Counting On* (CO) program, also based in NSW. CO was designed to support the professional learning of teachers in identifying and addressing the learning needs of those students in the middle years who were having difficulties with early mathematical concepts and skills. The process and results of CFPR were recorded in a number of external evaluation studies (White, 2008, 2009, 2010). These evaluations used a framework of five critical levels (Guskey, 2000): participants' reaction; participants' learning; organizational support and change; participants' use of new knowledge and skills; and student learning outcomes. All evaluations reported positive teacher reactions and gains in organizational support, teacher learning, teacher use of new knowledge, and student achievement outcomes.

CO also had other unique features concerning the workshops, model of dissemination; and the greater autonomy given to the teachers. Each participating school sent a volunteer teacher (facilitator) to a 2-day training course. The facilitator then returned to organize and run the program in the school, supported with resources (publications, website, DVDs, and money), with mentoring being available through a Departmental consultant. Although this might first appear to be an application of a "train-the-trainer" model, the correct term is a "facilitated model," as the quality of the program was dependent on the school facilitators and their skills in leading their teams as they conducted their research and developed teaching strategies according to their needs and context. Whereas cascade models of train-the-trainer suffer from "dilution" as the process moves from level to level, by contrast the facilitated model has the potential to be better (but also worse) than what was provided with original facilitator workshops.

With CO, the school team was expected to operate using the *lesson study* (LS) model developed in Japan to enable and encourage collaborative professional learning and sharing between teachers (Stigler & Hiebert, 1999). A more detailed description of the LS model will be given later in this chapter (see Figure 13.4).

The "feed forward" in the CO process provided an excellent example of the interplay between teachers and MTEs in developing teaching that led to the improvement of student learning outcomes. For example, in response to teacher concerns involving student interactions with, and attitudes towards, mathematics word problems, Newman's diagnostic error analysis procedure (Newman, 1977; 1983; Clements, 1980) was introduced to the program. Teachers worked with MTEs to develop strategies to remedy the student difficulties revealed by this form of analysis. Initially in many classrooms, teachers displayed the diagnostic questions as a hand-made poster and the prompts were used to assist a problem-solving process. After teacher requests, a professionally designed poster was produced for dissemination throughout all schools by the NSW Department of Education Curriculum Support Directorate (see Appendix 1). Another difficulty reported by teachers was how to assist students who could not transform (or "mathematize") written mathematics problems into a suitable procedure. Teacher material involving the use of what are known as "tape diagrams" was developed by teachers and MTEs as a pedagogical strategy which assisted the teachers. Tape diagrams are visual representations (see Appendix 2) that are used extensively in Japanese schools (Murata, 2008). The success of the collaboration and co-learning between the mathematics teachers and MTEs in sharing a common goal were evident in the completed evaluation reports.

The next two cases, situated in Brunei Darussalam and Iran, shared similar CFPRs with other programs, but exhibited uniqueness in the roles of mathematics teachers, researchers and MTEs, which became blurred and interchangeable. In Brunei Darussalam, the *Active Mathematics in Classrooms* (AMIC, see Figure 13.3) was a national project designed to provide upper-primary teachers with ongoing professional learning and support (Mardiah & Shimawati, 2004; White, 2004b).

An unusual feature involved 14 practising primary teachers (called "the writers")—who were enrolled in an upgrading B.Ed program at Universiti Brunei Darussalam. These teachers adopted the roles of mathematics teachers and MTEs at different times. The writers developed, trialled, and revised AMIC workshop notes and materials for the nine topics, under the supervision of their MTE. After developing the 9 AMIC workshop units, the 14 writers then led trial workshops in which 10 future "AMIC workshop teacher leaders" (each representing a school) participated. Following these workshops the materials were revised and were then published by the Ministry of Education (Hafizah & Rosmawati, 2003; Haslina, 2003; Kamsiah, 2003; Lim & Zarinah, 2003; Maria & Ramnah, 2003; Mohammad Ariffin, 2003; Norjah, Rozaimah, & Tini, 2003; Rozina, 2003; Yunaidah, 2003). The school leaders then conducted the workshops in their schools with the help of the writers. This cycle continued, with teachers from other schools being involved, and a widening number of AMIC "graduates" becoming workshop teacher leaders.

Due to the unique geographical spread of schools, initial AMIC workshops were conducted in five schools and involved 60 upper-primary teachers. Thus a community of practice was formed in each school involving teachers, teacher leaders, writers and the MTE. Results resonated with the CFPR.

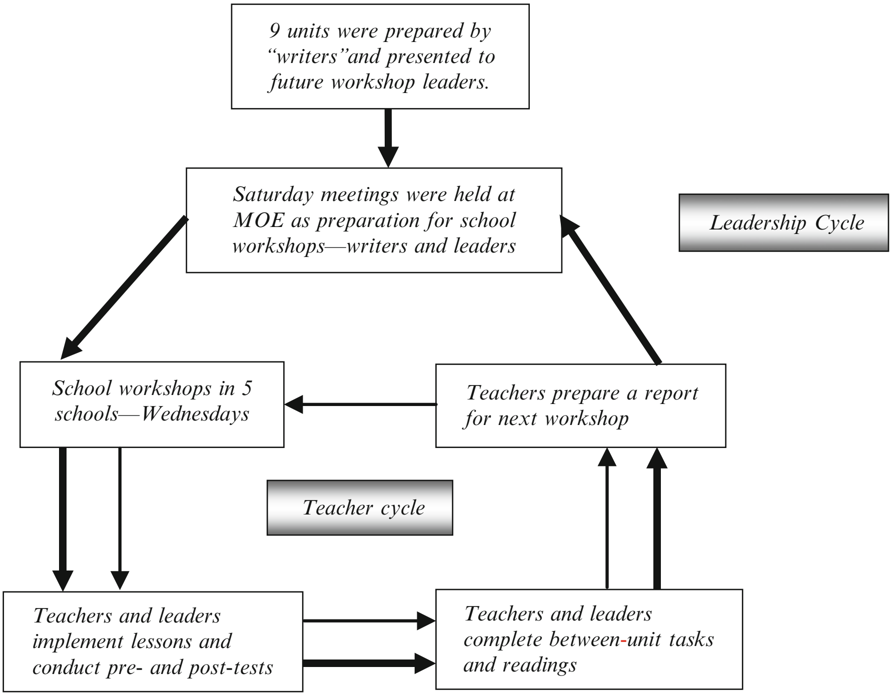



*Figure 13.3.* AMIC cycle for teacher writers, workshop leaders and classroom teachers (from White & Clements, 2005, p. 152).

The next example in this section relates to a national mathematics curriculum reform process in Iran that developed as a response to significant changes in secondary school education during the early 1990s. This reform process and the new curriculum challenged mathematics teachers and in particular, those who had taught geometry, and only geometry, for years. In response to teacher concerns, 11 national teacher-education sessions were planned and delivered by MTEs between 1994 and 1999. One session relating to geometry was the most controversial (Gooya, 2007). The direction and purpose of high school geometry had changed and there was an increase in the number of mathematics teachers involved in teaching geometry. Many of the new teachers were female. Previously, in Iran, geometry had been a male-dominated subject, and there was a concern that it might lose status if it became accessible to both male and female mathematics teachers and students (Gooya & Zangeneh, 2005). The most notable implication of this event for teachers' learning and their professional practices was that young mathematics teachers' views and insights about their own mathematics learning evolved and their self-confidence towards teaching geometry, in particular, was greatly improved.

Sharing the CFPR, from the outset the intention was for teachers and MTEs to work together and to integrate theoretical and practical knowledge with the teachers' craft knowledge. The results were those listed in CFPR. As well, small research projects were conducted to reveal teachers' concerns. One involved an action-research approach in which a number of graduate students, and one of the present authors (Gooya) worked with mathematics teachers from different cities. These projects involved mathematics teachers and MTEs collaborating to a degree that in some cases blurred the line between "insiders" and "outsiders," particularly when the teacher was also an MTE (Gooya, 2006). This happened quite naturally as the collaborations became more genuine and more meaningful for both groups.

The final study in this section is a large-scale study that differs from the earlier ones in the focus upon the relationships between teachers and researchers. In its original form, the *Learner's Perspective Study* (LPS) sought to document the classroom practices of competent mathematics teachers and to identify the meanings that participants held for those practices and the meanings that arose out of those practices (Clarke, 2001a, 2001b; Clarke, Keitel, & Shimizu, 2006; Clarke, Shimizu et al., 2006; Shimizu, 2002). LPS was originally a nine-country study (Australia, Germany, Hong Kong, Israel, Japan, the Philippines, South Africa, Sweden and the USA) of learner practices within the practices and meanings associated with "well-taught" Grade 8 mathematics lessons. LPS sought to uncover and to make explicit the cultural values and beliefs that framed the educational endeavours of teachers, researchers and policy makers in each country in order to contribute to the optimization of their effectiveness as sites for learning while acknowledging that optimization is shaped by the cultures of those classrooms.

LPS collected data using video and various texts such as classroom dialogue ("public" and "private"), teacher and student written material, and teacher and student post-lesson reconstructive interviews. The collaboration and sharing between MTEs and teachers through the post-lesson video-stimulated interviews contributed to accounts of the practices of classrooms and reflected teachers' intentions, actions and classroom consequences of these actions. The study challenged international comparative research practices (see, e.g., Stigler & Hiebert, 1999) by developing ways to accommodate the cultural differences through attending more closely to context and voice. The roles of teachers and learners in the examination of practice were explained using attempts to include the realities of political, societal needs and cultural plurality that were present in any particular classroom. "Teachers in Australia, Japan, The Philippines and South Africa face very different challenges with regard to cultural diversity of the communities they serve—class size, instructional resources, and societal and political priorities" (Clarke, Shimizu et al., 2006, p. 378). Many participating teachers described their participation as a powerful professional development experience. There is anticipation that value will accrue from research reports with different cultural authorship.

Although there were some differences in the last study (LPS) considered in this section, all the studies were explored in relation to the growth in learning of teachers and MTEs while they were involved in large-scale projects. The next section looks at smaller-scale studies.

## Teachers Learning from Teachers in Small-Scale Professional Learning Projects

In this section, smaller studies involving learning by mathematics teachers and MTEs are considered. These studies included such things as teaching experiments, self studies, and small-group learning communities. They relied generally on self-reported data.

The structure of this section has two subsections. The first involves studies conducted within schools in which teachers worked individually or collaboratively to improve teaching practice with the aim of improving student learning outcomes. The second involves studies which focussed on the impact of teacher-development programs and activities which occurred both within and away from the school, and in which teachers from different schools worked together.

**Knowledge growth within the school context.** In order to explore the complexities inherent within school contexts, studies have used a range of samples of large to small numbers of mathematics teachers, sometimes involving MTEs. This then permitted the collection and analysis of rich, detailed data from multiple sources. Small sample studies can contribute to the building of a larger data set from which a synthesis across cases can form a more convincing body of evidence.

Hunter (2008, 2010) reported on a one-year-long study which involved four primary school teachers and herself, as the teacher–educator–researcher. The five participants worked together as a collaborative partnership to investigate how communication and participation patterns in the classroom might be best constituted to support student engagement in efficient and correct mathematical reasoning discourses. The study was conducted in a New Zealand primary school where the majority of students were of Pasifika or New Zealand Maori ethnic groupings. Data were collected both from study-group sessions, which took place regularly throughout the year, and from classrooms, through videotapes, done by the teachers, and researcher observations. Interviews with the teachers and reflective diaries also provided important forms of data.

Hunter (2008, 2010) described, powerfully, the gradual and, sometimes, circuitous and challenging journey through which one of the teachers, Moana, in a culturally responsive manner, shifted her positioning in the classroom culture from teacher in control of the discourse, to participant in, and facilitator of, the discourse. Inquiry of both teachers and the MTE was facilitated by the use of a specifically designed "Communication and Participation Frame." A community of inquiry focussed on how to structure and support the development of communication and participation patterns in their mathematics classrooms. The MTE inquired both into the teacher's learning and into her own practice as a colleague and supporter of the teachers in their journey of change of their pedagogical practices. Evident in this project was "a notion of teaching as learning in practice" through the overt use of "inquiry" in mathematics learning, mathematics teaching and "the development of practices of teaching in communities involving teachers and [teacher] educators" (Jaworski, 2006, p. 187).

In recent years, there has been a growing interest, particularly in western countries, in how Japanese teachers learn from each other when they are involved in the process of *lesson study* (LS). LS became highly visible beyond Japanese shores and strongly associated with mathematics education initially due to the influence of a number of American researchers and writers collaborating with Japanese counterparts (see, e.g., Fernandez, 2002; Fernandez & Yoshida, 2004; Lewis & Tsuchida, 1998; Shimizu, 1996, 1999a, 1999b; Stigler & Hiebert, 1998, 1999). This interest was stimulated by the publication of results from the Third International Mathematics and Science Study (TIMSS). The TIMSS Video Study made clear that differences did, in fact, exist not only in the mathematical achievement of American and Japanese students, but also the manner in which students were taught. One important result was a better understanding of the Japanese problem-solving teaching methods which improved student achievement on complex and novel mathematical problems. These teaching methods are now globally recognized as models for teaching that resonate with constructivist philosophical principles (Isoda, 2007).

Simon (2008) considered Japanese LS as having only process goals as there was an expectation that teachers would learn through engaging with the process and so the content was not specifically defined. LS provided a process whereby teachers could develop their professional learning and skills in order to improve classroom teaching and the learning outcomes of their students. The LS process enables and encourages collaborative professional learning and sharing between teachers and MTEs. The focus is upon the lesson instead of starting from learning theories and then trying to apply them to the classroom (Stigler & Hiebert, 1998).

LS spread throughout the world and particularly the Asia Pacific region—it has had a global influence upon the teaching of mathematics. The spread of LS has received support through the growth of information communication technologies and the ease of international travel. For example, the World Association of Lesson Studies (WALS: http://www.worldals.org/) was formed and this promoted LS at many levels from systems to individual schools across a range of countries. Another project, one which was supported by the Asian Pacific Economic Cooperation (APEC: http://hrd.apecwiki.org/index.php/Lesson_Study#Lesson_Study_in_Mathematics), was designed to encourage the spread of LS across the region. More recently LS has been promoted by the Southeast Asian Ministers of Education Organisation (SEAMEO) through their regional centres of excellence such as the Regional Centre for Education in Science and Mathematics (RECSAM) in Penang, Malaysia, and the Regional Centre for Quality Improvement of Teachers and Educational Personnel (QITEP) in Yogyakarta Indonesia (Hartono, 2010; Muchtar & Sutarto, 2010). However, different countries have adapted aspects of LS to their context (Isoda, Stephens, Ohara, & Miyakawa, 2007), so that: "The term Lesson Study has become an umbrella term for a variety of adaptations or glocal responses" (White & Lim, 2008, p. 916).

The distinctive steps and iterative nature of the LS process are illustrated in Figure 13.4. It should be noted that some "glocal" model studies do not include an iteration process. Re-teaching is a common LS feature in the USA, but is only an

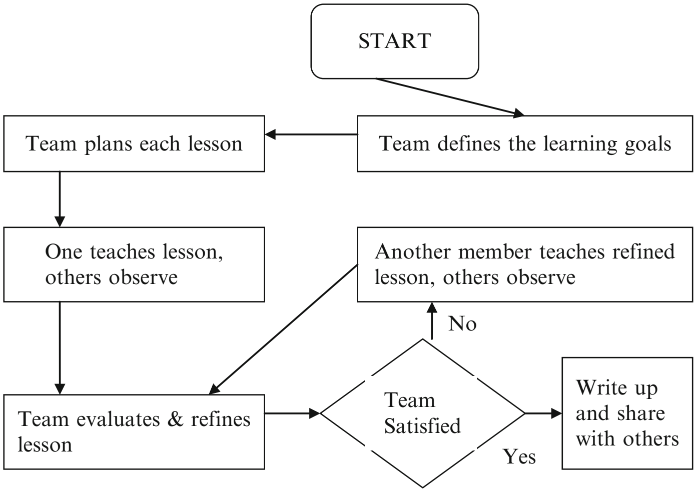



*Figure 13.4.* LS cycle (from White & Lim, 2007, p. 568).

occasional feature of Japanese LS (Lewis et al., 2009). Other LS studies differ over whether teachers are volunteers or are conscripted, and for other studies there are considerable differences in the composition of the teams (Hart et al., 2011). There are some "glocal" models that have a content focus and serve as counter-examples to Simon's assessment of LS as having only process goals. The following discussion will attempt to describe just three from a large number of available studies in order to highlight the growth in learning of teachers and MTEs, as well as the vast differences in the forms by which Japanese LS has been applied. We discuss studies from the USA (Lewis, Perry, & Hurd, 2009), from Australia (White, 2004a, 2006; White & Southwell, 2003; Southwell & White, 2004) and from Malaysia (Chiew, 2009; Lim, White, & Chiew, 2005; White & Lim, 2007).

The context of the US study was that it was part of a 2-week summer workshop in a North American school district. In Australia, the LS was situated within the context of a state-wide change of a mathematics syllabus, and the Malaysian context was as part of a doctoral study of a two-school professional development program.

The US study involved one team of five teachers, a teacher-coach, an MTE and two researchers. The use of an experienced and knowledgeable mathematics teacher as a coach in LS groups was regarded as important to the effectiveness and sustainability of the program (Lewis et al., 2009). The Australian study involved a large number of teams situated in schools scattered around the state, each team comprising five to six teachers, of whom one served as the facilitator. Access was limited to only one MTE and only one external researcher for all the teams. The Malaysian study consisted of one team of six and another team of eight teachers, with both teams having access to an MTE and a researcher. Membership of the US and Australian teams was voluntary, but the Malaysian participants had been directed to join by their school leaders.

In the US study the teachers chose to focus on the goal of helping primary school students indentify and express, mathematically, patterns, by working collaboratively through a sequence of activities. There were two iterations involving fourth-grade classes. The Australian teachers chose their own focus area within the new syllabus, and conducted varying numbers of iterations, depending upon the team. The Malaysian teachers were influenced by their head of mathematics in their choice of topic and iterations.

Data in the US study were collected through videotaping, field notes and artefacts such as student work samples. The Australian study collected data through questionnaires, interviews and document analysis, and the Malaysian study used observations, interviews and questionnaires.

The US study reported three types of changes produced by LS: changes in teachers' knowledge and beliefs, changes in professional community, and changes in teaching-learning resources. The Australian study reported improvements to teachers' learning and use of new knowledge, the establishment of stronger and on-going professional relationships among team members, and increased recognition and organizational support from the school leadership. The Malaysian results were mixed depending upon differences in the degree of administrative or executive support which directly affected teacher commitment in both schools.

This short discussion has highlighted the diverse range of LS "glocal" models. The studies reported improvements in the learning of teachers. Other studies, such as those reported by Arbaugh (2003) and Slavit and Nelson (2010), have generated similar benefits among teachers participating in LS groups. However, none of the three discussed here reported improvements in the learning of the MEs or researchers.

The existence of various variations to the Japanese LS model around the world has led to many other designs of professional development programs which although resonating with some of the aspects of the LS process cannot be strictly classed as an LS model. For example, in modern Iran there are examples of teachers collaborating and learning from each other within approaches and structures that resemble aspects of Japanese LS. In 1960, the teachers' council of an elementary school in a small Iran–Iraq border town called Paveh, located in the west end of Iran, began to discuss teaching methods and curriculum organization with respect to mathematics in Grades 4, 5 and 6. The council agreed to make changes and all members of the council signed an agreement. An analysis of the minutes of one of these meetings has indicated that the process of planning was similar to LS (Gooya, 2010). However, the activities that were planned and implemented would be best seen as reflecting local insights into how practice might be improved, and the aim was not to develop generally applicable findings.

**Knowledge growth in and beyond school contexts.** Research studies have explored the impact of professional development offered beyond the school context as teachers attended meetings, workshops or courses with teachers from other schools. In many studies there has been a synergy between out-of-school activity and related activity taking place in school.

In the Mathematics Teacher Enquiry project that ran for two years, Jaworski (1998) described teachers' learning through engaging in research into aspects of their own teaching, and the concomitant learning of the MTEs conducting the project, and studying the developmental processes involved. The project brought teachers and MTEs together in both school and university environments in which they developed mutual respect and common understandings. The program successfully used workshops in schools and university, focussing on CFPR. Thus MTEs and mathematics teachers worked together as colleagues and co-researchers in a joint professional environment which was theorized subsequently, by the MTEs as a "community of inquiry" (Jaworski, 1998, 2006).

In the nation of Colombia, a program was developed and implemented with the acronym PROMESA (Creating Science and Mathematics Connected Learning Experiences that Open Opportunities for the Promotion of Algebraic Reasoning—in Spanish, the corresponding acronym is PROMICE) (Agudelo-Valderrama & Vergel, 2009a, 2009b). In PROMESA, school mathematics and science teachers, and teacher educators, worked together as a developing community of inquiry with the shared aim of promoting students' meaningful and connected learning of mathematics and science. Eleven well-qualified mathematics and five science teachers, at three schools which served students from disadvantaged socioeconomic communities in Bogotá, and two teacher educators, worked together over a 14-month period on issues that they had identified after discussions among themselves and with their school principals. Following the teachers' participation in a series of workshops, which provided ample opportunity for analysis and discussion of ways and means of connecting science and mathematics in their schools, the teachers from each school organized themselves into sub-groups (each sub-group had one science and two mathematics teachers). These sub-groups then worked collaboratively with the teacher-education researchers on the processes of designing, implementing and documenting classroom innovations. The purpose of these innovations was always to engage the students in connected science and mathematics learning experiences which would generate opportunities for the promotion of algebraic reasoning. During the process, the teachers and the teacher educators met regularly, at each school and at the university, for whole-group-discussion and sharing sessions.

Throughout the project, data on teachers' knowledge, conceptions, beliefs and attitudes with respect to school mathematics and science, and of teaching specific concepts, were gathered using a variety of data collection methods. The students of the participating teachers were also involved in the study, and data were collected, by the teachers, in a longitudinal study of two Grade 8 groups, from different schools.

In this project, the teachers and teacher educators were both insiders and outsiders: as insiders, teachers inquired into their own thinking and their understandings of specific mathematics and science concepts. They participated in planned interviews and kept diaries in which they reflected on their own teaching practices, and expressed their feelings. As outsiders, they inquired into their students' thinking and learning in relation to the classroom work. The teacher educators were also simultaneously insiders and outsiders: as insiders, they inquired into their own practices in order to be in a better position to make informed decisions; they acted as supporters and orchestrators in the complex task of researching the various contexts and issues which emerged during the project. At the same time, they acted as MTEs in a collaborative and supportive manner. As outsiders, they inquired into the development of the teachers' thinking and teaching practices, for "in order to fulfil the task of collaborators and members of a community of inquiry, insights into the teachers' thinking processes were key" (Agudelo-Valderrama & Vergel, 2009a, p. 33). The teachers and MTEs both inquired into the students' learning, and sought to enhance and maximize student learning, which was their ultimate shared goal.

In this Colombian study there was considerable evidence of improved student learning and gain in the students' sense of purpose related to learning. The evaluations which project members carried out indicated that participants grew in their appreciation of the connections between science and mathematics knowledge and their enacted co-teaching practices during the project. Agudelo-Valderrama and Vergel (2009a) emphasized the important professional lessons MTEs learned in relation to various areas of their roles and duties as teacher educators.

Implications of the study were identified for those intending to participate in programs of initial mathematics and science teacher education, for continued professional learning, and for Local Education Authorities. As in the study by Lewis et al. (2009), there emerged issues that related not only to the sustainability of teacher professional learning, but also to possibilities and barriers affecting teacher participation in professional learning projects. For example, Agudelo-Valderrama and Vergel (2009a) drew attention to the need to establish a coherent policy in relation to the administration of school staffing and participation in inservice professional learning programs. The head teachers in the project found it difficult to allow the participating teachers to attend a weekly one-hour group meeting, citing lack of staffing and the requirements of staff management policies. As a result, the regular work sessions of teachers and teacher educators at school sites had to take place after school hours and prevented teachers from engaging in further collaborative work at the end of the 14-month period, despite this having been included as a requirement in the design of the project. Nevertheless, many of the participating teachers expressed their willingness to continue working with the researchers in order to write papers, to report on their classroom project findings and on their own learning, and to prepare these reports for publication.

## Learning of Preservice Teachers and Teacher Educators

This section summarizes a collection of studies which explored teachers involved in preservice programs (including early-career teachers), focussing on the learning of all participants. It gives a brief discussion of some research models that relate the learning of teacher educators to the learning of preservice and early-career teachers with whom they worked.

In the *Second International Handbook of Mathematics Education,* the issue of teachers as mentors to early career teachers and their roles in relation to university MTEs with whom they worked was explored. Three levels of knowledge for teachers, mentors and MTEs were suggested as follows.

*Level 1.* Mathematics and the provision of classroom mathematical activities for students' effective learning of mathematics. This included socio-cultural mathematics education, such as the wider influences on pupils' learning, and reasons why pupils need to learn mathematics.

*Level 2.* Mathematics teaching and ways in which teachers think about developing their approaches to teaching.

*Level 3.* The roles and activities of teacher-educators in contributing to developments in (1) and (2) and including constraints on teacher education and how they can be tackled (Jaworski, 2001; Jaworski & Gellert, 2003).

Each of the first two levels incorporates those below it. Teachers operate largely (but not exclusively) at Level 1, mentors at Level 2, incorporating Level 1, and teacher educators at Level 3, incorporating Levels 1 and 2. What this framework misses is the areas of knowledge indicated earlier in this chapter in Figure 13.2—that is teachers' knowledge of students and schools and teacher educators' knowledge of theory, research and educational systems. What we recognize in considering such a different framework is the complexity of the developmental scene and the areas of knowledge on which it draws. These areas of knowledge are far from distinct and it seems important to recognize that preservice and early-career teachers and the MTEs share the knowledge in complex ways.

Perks and Prestage (2008) recognized links between their own knowledge as MTEs and the knowledge of the preservice teachers whom they taught. They offered a model for teacher learning in a teacher-education program aimed at preservice mathematics teachers, and a version of the same model aimed at teacher educators (see Figures 13.5 and 13.6). In the first case, teacher learning draws on teachers' knowledge of classroom events, professional traditions, learner knowledge and practical wisdom. The parallels for teacher educator learning draw on mathematics education sessions in the teacher-education program, professional traditions, own learner knowledge as a classroom teacher, and practical wisdom. Perks and Prestage commented particularly on the teacher educators who had experience of being teachers themselves in earlier professional practice—but of course this is not the case with all teacher educators.

An alternative way of seeing relationships between teacher educator learning and teacher learning was offered by Zaslavsky (2008), and is represented in Figure 13.7. The main idea is that the educator (or facilitator) designs activities to promote teachers' learning and then s/he learns from reflecting on the teachers' activities.

These models from Perks and Prestage and from Zaslavsky suggest that MTEs learn through engagement in and reflection on their own practice, in working with teachers, and there are parallels with teachers' learning through practice (see Even & Ball, 2009).

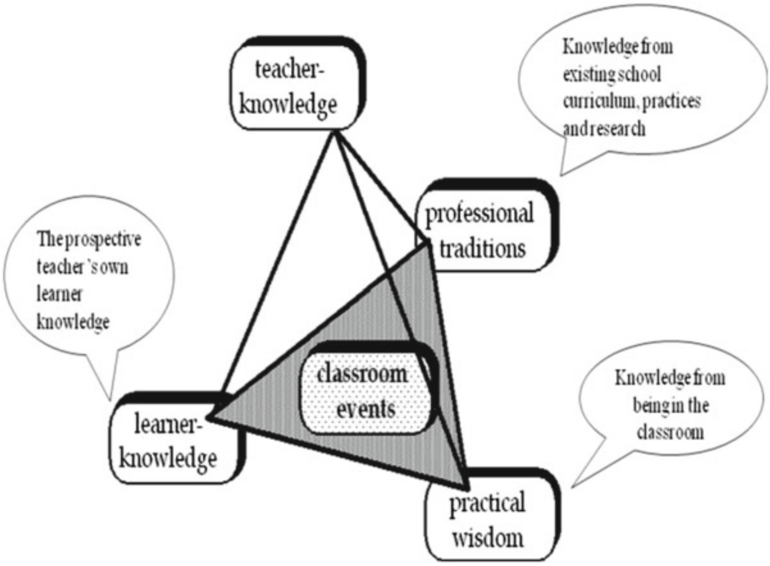



*Figure 13.5.* Teacher learning in a teacher-educator program with preservice mathematics teachers (from Perks & Prestage, 2008, p. 270).

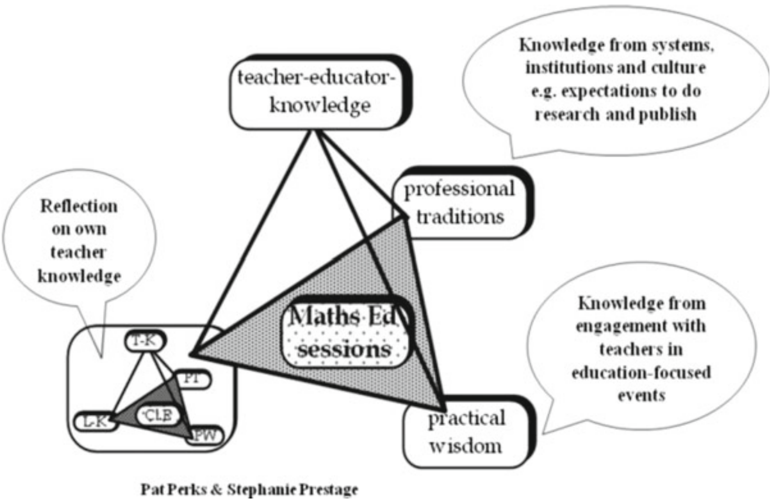
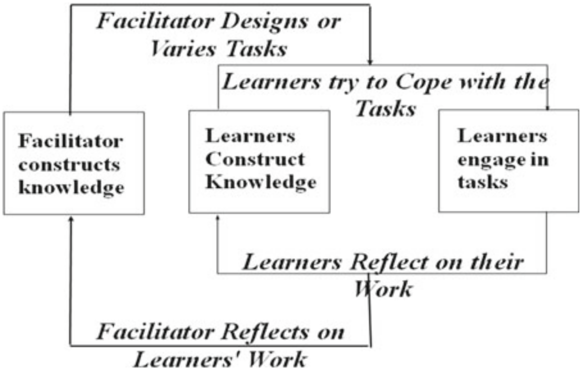



*Figure 13.6.* Teacher-educator learning in a teacher-educator program with preservice mathematics teachers (Perks & Prestage, 2008, p. 271).

In some parts of the world, programs have been especially designed for the learning of MTEs—an example of this was the *Manor* program in Israel (Even, 2008). This program included an introduction to research, theoretical ideas and issues related to practice, and provided professional opportunities for prospective MTEs to

*Figure 13.7.* Relationships between teacher-educator learning and teachers' learning (from Zaslavsky, 2008, p. 95).




engage with teachers in professional development programs. In doing so, the program modelled ways in which those becoming teacher educators might themselves work with teachers.

These three examples related to taught programs that developed the learning of MTEs. The programs with Perks and Prestage (2008) and with Zaslavsky (2008) were for the education of teachers in which MTEs learned overtly through scrutiny of their own practice. In the third case, Even (2008) described a program for MTE learning with a model that could be adapted for teacher learning. These fit into what Simon (2008, p. 18) called teacher-education programs with *content and process* goals where there is something to be taught and teachers are expected to learn.

## Concluding Comments

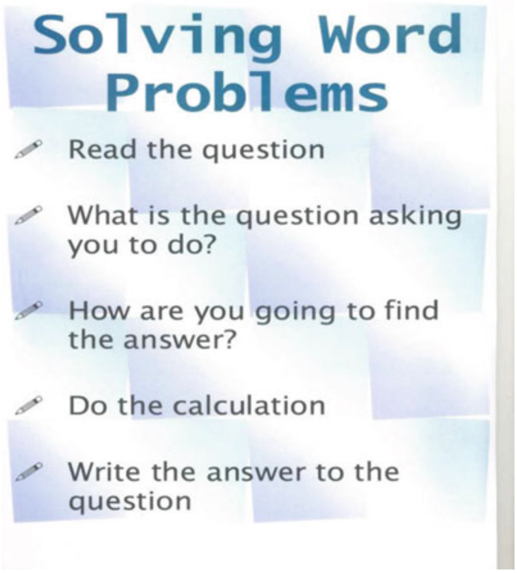
This chapter has attempted to convey the complexity and diversity of research focussed upon teachers learning from teachers. Our range of examples reveals the complexity of settings in which teachers learn, and the related knowledge that grows through the various developmental programs. This complexity is influenced by both global and local forces, such as the recent pressure on teachers to meet different demands imposed on them either directly by politicians and national laws such as value-added and No Child Left Behind in the United States of America, or indirectly by politicians and policy makers such as in Iran (Gooya, 2011).

Central to all the settings described were the relationships between mathematics teachers and MTEs which varied according to the nature of the program. Within three sections used to group programs of similar features we have further used the framework of Jaworski and also Simon's distinction between programs to illuminate certain important issues. In some, MTEs had a greater teaching role in guiding teachers in relation to pre-defined content, be it mathematical, didactical or pedagogical. In others, MTEs and teachers worked together in developmental roles, often in inquiry-based practices and sometimes using LS models; teachers often worked together to design their own developmental activities. Both the learning of teachers and the learning of MTEs were addressed. We can see a range of similarities and differences between the knowledge that these two groups bring to the learning interface. Importantly, neither group had all the knowledge that was needed for the development of teaching, but working together they could become a unified, powerful developmental force. Undoubtedly, both learned from each other as a result of their interactions in a research process.

Mutual respect and collaboration allow the input of critical elements of knowledge, often by MTEs, that are seen to be valuable to developmental practice. Although this input might take place in out-of-school contexts, it is within the in-school situations that knowledge can be tested and developed in practice. Here teachers' knowledge is pre-eminent and MTEs have much to learn about the systemic factors and issues that influence what can happen in schools and what is needed to put research-based knowledge into practice.

## Appendix A. Appendix 1

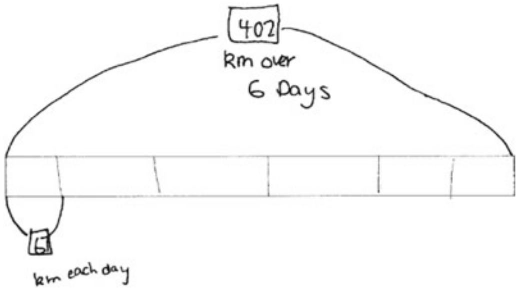
The classroom poster (following Newman's error analysis procedure): New South Wales Department of Education and Training Curriculum Support Directorate




## Appendix B. Appendix 2

A tape diagram for the problem:
Sue paddled 402 km along a river in her canoe over 6 days.

She paddled the same distance each day.
How far did Sue paddle each day?

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