Chapter 12
Linking Research to Practice: Teachers as Key Stakeholders in Mathematics Education Research

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Abstract Teachers are regarded as having a major role in the development of mathematics teaching and students' learning. Nevertheless, in much mathematics education research, teachers are viewed as recipients, and sometimes even as means to generate or disseminate knowledge, thus conserving a distinctive gap between research and practice. The theme of this chapter is to regard teachers as key stakeholders in research (i.e., as (co-)producers of professional and/or scientific knowledge) in order to make the link between research and practice more fruitful for both sides. After exploring the concept of stakeholder, the authors present five international examples, all of them involving teachers researching their own or their colleagues' practice. An analysis of the commonalities and differences among these examples reveals the presence of three important dimensions of research where teachers are key stakeholders: reflective, inquiry-based activity with respect to teaching action; a significant action-research component accompanied by the creation of research artefacts by the teachers (sometimes assisted by university researchers); and the dynamic duality of research and professional development. This chapter illustrates how traditional barriers between research and practice are being replaced by synergistic interactions between the two, enabling the intersection of the two worlds.

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Introduction: Teachers' and Researchers' Diverse Worlds

In most cases, the worlds of teachers and researchers differ greatly, even if there are also cases where they work together so closely that the traditional roles begin to blur. Nevertheless, mathematics education research and mathematics teachers' practices can overlap considerably, as is reflected in the case of *students' mathematical thinking*. Let us start with a concrete example. When middle-school students deal with identities like $(a+b)^2=a^2+2ab+b^2$, a variety of errors or misconceptions appear. For instance, many students come up with a^2+b^2 as the result of expanding $(a+b)^2$. Several researchers have published studies on this phenomenon (see, e.g., Davis, Jockusch, & McKnight, 1978; Kieran, 2007; Kirshner & Awtry, 2004; Matz, 1982). As well, most teachers are aware of this phenomenon and have developed—consciously or subconsciously—strategies for dealing with it. They have worked out ways to support students' thinking and re-designed their introduction to the topic in order to decrease the likelihood that this error will occur. Some teachers might have been influenced by mathematics educators' research, and a few of them might have even collaborated very closely with them. In addition, some teachers are highly respected researchers in their own right. However, the picture is even more complex than this since there is considerable variety within the worlds of teachers and researchers.

The variation within the *world of mathematics teachers* can be illustrated by their ways of dealing with the identity $(a+b)^2=a^2+2ab+b^2$ in their classroom. For example, teacher Anna might show on the blackboard that $(a+b)(a+b)$ just leads to the identity; then the identity is written with colors and some similar examples are given. Anna covers the topic in one hour because she believes that dealing with algebra software systems at a later grade will be much more effective. Björn, who spends four to five lessons on the topic, regards this identity as essential and aims at offering his students rich learning opportunities in order that they will remember very well the identity and its generation. He builds on links to geometry—interpreting $(a+b)(c+d)$ as expanding the size of the rectangle (a, c) to $(a+b, c+d)$—and then encourages the students to find the identity themselves. Cecile has a flexible strategy and only decides on her concrete teaching design after some repetition work where she develops a sense of her students' pre-knowledge and interests on this issue. Davido always starts a larger unit with a diagnostic test in order to know all his students' mathematical abilities. According to his findings, he forms three to five ability groups in class with different tasks and task levels, supporting in particular those who might have problems in meeting the minimum standards. And there are many other approaches, including fostering students' own ways to reach their goals (and perhaps documenting their progress in a portfolio), as well as training students by "teaching to the test."

The diversity of students' knowledge and interests in a classroom, the (subjectively) giant obstacles to overcome, and the always missing time for dealing with these challenges are some major factors that contribute to the complexity and unpredictability of teaching. "How great it would be," said Maria, an experienced mathematics

teacher in a professional development course, "if I had videotapes of all interesting mathematical situations in my classroom, and the time to analyze them with colleagues; however, I have to react immediately to each error, and this causes errors on my part too."

This marks a good counterpart to the *world of mathematics education researchers* dealing with students' knowledge, be it an identity like $(a+b)^2=a^2+2ab+b^2$, the characteristics of a proof in geometry, or advanced stochastical thinking. Although the identity above is a very tiny piece within mathematics, research on it is abundant, with new ways of framing the research emerging over time. For example, the researcher, Albert, might investigate the challenges to the student who comes up with $(a+b)^2=a^2+b^2$ and then construct a mind-map about her algebraic thinking, explaining her difficulties on a theoretical basis. Bruno investigates how students' mathematical ideas are dealt with by other classmates and by the teacher, and how the negotiation of meaning takes place or the didactical contract is generated. Corinne is interested in the interplay between a particular student's mathematical abilities and interests. And there are many other perspectives that researchers might take, including large-scale investigations of students' answers on national standard tests, "design research" activities with small samples of teachers, evaluations of studies dealing with students' and teachers' mathematical growth, and systematic reflections by teacher educators on their learning processes while leading intervention projects.

We can summarize the situation as follows: Even if focussed on a very specific topic (e.g., students' thinking), *mathematics education research as well as mathematics teaching is highly diverse*. Much empirical and theoretically-based knowledge is produced by the scientific community and much, mostly unpublished, knowledge is produced by the rich experiences of thousands of teachers. From this it is also clear that the communication and possible collaboration between teachers and researchers is diverse.

The major question is: How can mathematics education research have an impact on mathematics classrooms, on students' learning, abilities, beliefs, and interests? And how can researchers benefit from the rich body of knowledge and subjective theories teachers have? *And who is responsible for dealing with this question?*

Regarding Teachers as Key Stakeholders in Research

Researchers and teacher educators neither have the role nor the capacity to influence directly mathematics teaching on a large scale. Their major impact on teaching seems to be related to the *production of relevant knowledge* and generating opportunities for teachers (and to some extent also for other relevant groups like principals) to confront this knowledge with their existing knowled*ge*. In general, teachers are regarded as key persons of educational change (e.g., Fullan, 1993). This view is largely supported by research evidence. For example, an analysis of student learning over many large-scale projects (Hattie, 2003) shows that teachers' impact

on students' learning is high: identified factors that contribute to major sources of variation in student performance include students (50%) and teachers (30%) as the most important factors, whereas home, schools, principals, peer effects (altogether 20%) play a less important role (see, e.g., Pegg & Krainer, 2008). Research on "successful" schools shows that such schools are more likely to have teachers who have continual substantive interactions (Little, 1982) or that inter-staff relations are seen as an important dimension of school quality (Pegg, Lynch, & Panizzon, 2007; Reynolds et al., 2002).

The implication of this research is that approaches with the most potential to bring about genuine improvement in learning mathematics are those that resonate with teachers—with their interests, beliefs, emotions, knowledge, and practice—as well as those that encourage further collaboration among them. Krainer and Llinares (2010) have emphasized that "it is desirable to use the synergy of teachers' expertise and therefore to engage them in research activities and to support action research, among others, with the goal that some of them might develop deeper interest in research and thereby to enlarge the scientific community" (pp. 704–705). The idea of viewing teachers as experts and competent partners in research is not new at all. For example, in the literature, they are regarded as *researchers* (e.g., Altrichter, Feldman, Posch, & Somekh, 2008; Crawford & Adler, 1996; Stenhouse, 1975), *reflective practitioners* (e.g., Schön, 1983), and *experts* (e.g., Bromme, 1992). *Intervention research* with teachers as partners and *action research* by teachers or teacher educators is becoming more prominent in mathematics teacher education (see, e.g., volumes 6.2 and 9.3 of *JMTE* in 2003 and 2006). Lesson study, as a teacher-led professional development approach, has a long tradition in Japan and has begun over the last decade to spread to other countries (see, e.g., Hart, Alston, & Murata, 2011). Recently, the fourth volume of the *First Handbook of Mathematics Teacher Education* (Jaworski & Wood, 2008) drew attention to the crucial importance of activity involving *learning and self-reflection* for both teachers and teacher educators.

It is the ethical responsibility of a scientific community and at the same time a wise strategy to raise questions (see Krainer, 2011) such as: How does our knowledge get known, used, and reflected upon by relevant people and institutions? How can their experiences, which form a new kind of knowledge, be fed back to the researchers? What can be done by researchers apart from writing papers and giving talks—predominantly within the scientific community—and from teaching classes of student teachers and offering professional development courses? It cannot be taken for granted that the majority of those to whom research might possibly be addressed do in fact read the tremendously increasing number of research papers and that traditional teacher education is a viable means to link research results with the challenges of practice.

There have been efforts by individual researchers and groups to raise this issue, for example, in a conference on "*Systematic Cooperation between Theory and Practice in Mathematics Education*" (Bazzini, 1994), in papers like the "Dialogue between theory and practice in mathematics education" (Steinbring, 1994), in a special issue of *Educational Studies in Mathematics* on connecting research, practice, and theory (Even & Ball, 2003), in the chapter "Mathematics Teacher Education" in

the *International Encyclopedia of Education* (Krainer & Llinares, 2010), and most recently in this chapter in the *Third International Handbook of Mathematics Education.* Not long ago, an initiative to create stronger links between researchers and practitioners was undertaken by the National Council of Teachers of Mathematics (2010) as researchers and practitioners met together to create a research agenda consisting of questions deemed most critical to conduct collaborative research. Despite these efforts and continuous claims on the importance of the role of teacher-researcher collaboration, teachers are still often seen as more or less passive recipients of researchers' knowledge production and sometimes as a means (e.g., as data supplier) to help produce knowledge. What is missing, in particular, is a systematic effort by the scientific community (such as societies, commissions, universities, research groups) to analyze and promote the potential role of teachers in research and its benefit for teachers and researchers.

In the 1980s, an interesting change of paradigm started in *management strategy* (in particular in the USA). The traditional view was the *shareholder approach*, which regarded it the duty of management to protect the interests of the shareholder, basically in order to avoid having poor social performance hurt the company financially. Management aimed at satisfying clients, consumers, society, etc., by specific strategies (e.g., public relations). In contrast, Freeman (1984) and others developed a stakeholder approach, defining "*stakeholder*" as "*any group or individual that can affect or is affected by the achievement of a corporation's purpose*" (Freeman, 2004, p. 229). The approach dealt with the practical concerns of managers—"How could they be more effective in identifying, analyzing and negotiating with key stakeholder groups?" (p. 230). The stakeholder idea is connected to ethics and values, which are regarded as equally important as the business itself (see also Krainer, 2011). The main message is that "*looking at the whole system*" *(of interests) is a benefit for all parts of the system aiming at sustainable development.*

The mathematics education research "enterprise," whose "business" includes the improvement of the teaching and learning of mathematics, is distinctly unlike a corporation in many respects. Nevertheless, the similarities between the two can be useful, including the presence of a multitude of stakeholders. It is not just researchers who have a stake in the research enterprise, even if they are generally considered to have the most expertise in research (e.g., with respect to theory, methodology, etc.) and tend to set the trajectories for research. In addition, they are assumed to form their decisions not only for the sake of the scientific community but more broadly for society too. Nevertheless, the research enterprise in mathematics education has other stakeholders: for example, students, teachers, parents, principals, superintendents, mathematicians, teacher educators, educational publishers, test-developers, firms, education policy-makers, and even the whole society can be regarded as "stakeholders" of the joint societal enterprise of promoting students' mathematical knowledge. They all have an effect on students' knowledge and at the same time they are affected by their knowledge. But of all these stakeholders, it is the teacher who *can affect to the greatest extent the achievement of one of the main purposes of the research enterprise, that is, the improvement of students' learning of mathematics.*

The scientific community needs to regard teachers not just as stakeholders, but also as "*key stakeholders*" of research. At least five aspects should be discussed when analyzing the role of teachers with regard to the production and dissemination of scientific knowledge—an activity central to the research enterprise and one by which its participants aim at contributing to the improvement of students' learning of mathematics. The first three aspects suffice if researchers are mainly attempting to optimize their own interests as researchers and seeing the production of knowledge being predominantly done within the scientific community (excluding practitioners like teachers and non-researching teacher educators). With regard to these three aspects, teachers are seen as "stakeholders" in that they have a stake in the results of research, which can inform them about elements of student learning; but they are not seen as *key stakeholders*—a term that we reserve for the fourth and fifth aspects. The fourth aspect deals with embracing teachers as experts who are principally able to contribute heavily to the quality of research, and the fifth aspect regards them as co-producers of scientific knowledge. The following presents a brief sketch of these five aspects.

Teachers as Means

For most of the research where the beliefs, knowledge, and practice of students and/or teachers are the focus, a collaboration with teachers is needed. They supply data, which are analyzed by the researchers. It should be a viable standard to provide involved teachers with a rationale for the research and its possible implications for teachers' work before the collection of data, and a summary of the research and its relevant findings after it. For example, it would be of interest to teachers to read which different ways of introducing algebraic identities (like $(a+b)^2=a^2+2ab+b^2$) different teachers use, and what the rationale behind their approaches is, probably accompanied by comments, evidence, and suggestions from the authors of the study.

Teachers as Recipients

The primary responsibility of teachers is to *teach* their students, not to read research papers, and there is some evidence that most teachers don't read such papers very often (Zeuli, 1994). Strategies by members of the scientific community in order to increase teachers' interest in reading research papers are manifold (see, e.g., Debien, 2010; Shearer, Lundeberg, & Coballes-Vega, 1997). Some scholarly journals have sections that are specifically intended to share research with teachers and some teacher journals have sections devoted to "research in practice." Many researchers publish additional papers with a clear practice-oriented focus in journals widely read by teachers, write practice-focussed summaries and put them on Web sites and

in teacher journals, write papers and give talks about the results of research studies that would be of interest to teachers, use pieces of research in teacher education and, for example, engage student or practising teachers in short parts of this research (e.g., having teachers construct a multiple-choice item on "$(a+b)^2=\ldots$" with the correct answer and three other tempting answers, and having them estimate the distribution of answers of students in their class).

Teachers as Alumni

Teachers are regarded as life-long learners, having spent a considerable amount of time at teacher-education institutions. Hopefully, while there, they were confronted with a selection of interesting activities in the context of research and came to realize that research is fascinating, and that it provides insights and thereby a strong basis for understanding their own thinking and their students' thinking. Thus, they might have developed a kind of "inquiry stance" that could increase their interest in trying out small pieces of research in their classrooms, in looking for contact with teacher educators and researchers, or being open to offers from the wider scientific community. Teachers' calls to university partners like, "Do you have news about research on students' algebraic thinking?" or "Are you running another interesting project?" would be indicators of teachers' inquiry stance and former teacher educators' success at evoking such interests.

Teachers as (Co-)Producers of Professional Knowledge

Teachers deal on a daily basis with students' thinking, their beliefs and conceptions, errors and ideas, interests and fears, emotions and cognitions, views of mathematics and mathematics teaching, etc. They can be regarded as experts on students' subject-related learning. On each curricular topic they teach for a long period, they develop specific expertise; however, it varies from teacher to teacher, dependent on pedagogical, didactical, and mathematical abilities and interest. Teachers who share their experiences with peers (e.g., within the context of joint lesson study or other kinds of professional development) are more likely to intensify their abilities and interest. For example, teachers having extensively discussed their approach to the introduction of identities like $(a+b)^2=a^2+2ab+b^2$ and its effect on students' learning surely develop forms of professional knowledge and subjective theories about students' algebraic thinking of interest to researchers. In particular, reflecting on the growth of students' and/or teachers' knowledge might be a beneficial endeavour for both parties. Through being involved in such projects, bridges between teachers and researchers might be built—bridges to link professional and scientific knowledge, which are not easy to separate in many cases, anyway.

Teachers as (Co-)Producers of Scientific Knowledge

There is evidence of research where teachers are equal partners or even central figures. For example, there are research projects where teachers not only help to gather data but also give advice concerning the design of research and the refinement of methods. Studies have been carried out where teachers design the research themselves, collect data, and are engaged in the analysis and interpretation of data, as well as in the process of formulating and disseminating the results. There are projects where the people involved decide intentionally to avoid the distinction between teachers and researchers since both are regarded as researchers, with differentiated roles in the research process. The presence or absence of teachers in a research project is not an indicator of research quality per se. In contrast, bringing in additional perspectives, data, and forms of communicative validation can be regarded as a feature enriching scientific research. Having teachers participating in such kinds of research, the dissemination into practice is facilitated.

In particular in the field of student- and practising-teacher education, there is a considerable amount of research where those who are educating the teachers are also those who are carrying out the research. A special kind of such research includes those projects where teachers or teacher educators investigate their own practice in order to improve it (action research). An example of research with regard to teaching algebra might be an action-research project within the framework of a professional development program where teachers try out and investigate new ways of algebra teaching (e.g., a different approach for dealing with $(a+b)^2=a^2+2ab+b^2$), finally producing small case studies of their experiences. The teacher educators support teachers' innovations and investigations and probably investigate their own growth and support processes in order to improve them—a kind of second-order research (see, e.g., Altrichter et al., 2008). In addition, or alternatively, they might write a cross-case study on teachers' approaches and growth or/and investigate students' thinking together with the teachers, or/and write together a handbook for teachers with learning units based on examples and reflections from the project.

The question of how intensively researchers regard teachers and others as *key stakeholders* is an expression of the intended and/or lived relationship between teachers and researchers. This means that our view of "teachers as stakeholders" is about "us," about our beliefs and roles, about our understanding of "research."

In the following section, we aim at providing examples of research projects, where teachers are regarded as *key stakeholders* in research, in the sense that teachers and researchers (or teachers with other teachers) act as co-producers (or as producers) of professional and scientific knowledge.

Five International Examples

In reviewing the international mathematics education research literature, we sought approaches to linking research and practice that were innovative and where collaborative research partnerships had a clear focus on teachers researching their

practice. Several examples presented themselves, including successful recent endeavours in both developed and developing countries (see, e.g., OECD, 2011). Space constraints, however, restricted our selection to five examples, each of which offers specific insights into the diverse ways in which teachers engage as key stakeholders in research. Two are large nation-wide programs (Japan, China), while three (USA, Norway, Canada) are initiatives that are much smaller in scale and do not claim to be widespread within their given country. We note that the terminology used for *teacher* and *researcher* is not uniform across the five examples. We have tried to respect the nomenclature adopted by the authors of the reports of each specific example by using the same terms that they have employed for teacher and researcher. We have also strived, by means of various forms of contact with individuals involved in the projects and programs described herein, to do justice to all examples and to represent their multifaceted dimensions as fairly and as accurately as possible.

The USA Example

In a 4-year project (2004–2008) led by Beth Herbel-Eisenmann, teacher researchers collaborated with university researchers in reflecting on their own teaching and in conducting cycles of action research that focussed on improving the mathematical discourse of their classrooms (Herbel-Eisenmann, 2010). Eight mathematics teachers from grades 6 to 10, whom Herbel-Eisenmann had met through her work in her university position, were interested in learning more about classroom discourse, and they agreed to be the teacher-researcher participants in the project. These teacher-researcher volunteers came from a variety of types of school settings—rural, urban, and suburban. They had teaching experiences that varied from 4 to 23 years and taught from a variety of different curricular materials. Herbel-Eisenmann, together with several graduate students, served as the university researchers over the life of the project.

At the beginning of the project, the group agreed that the primary goal of their activity, for both the university researchers and the teacher researchers, would be to learn about, reflect upon, and change mathematical discourse in classrooms. The book *Promoting Purposeful Discourse* (edited by Herbel-Eisenmann and Cirillo, 2009) provides a reflective narrative of the details of the project, including timeline, details of the study-group activities of the eight teacher researchers, data generation and analysis phases of the action-research projects, write-ups by the teachers on their own research projects, and reflections on the experience from both the university researchers and teacher researchers.

The first year of the project was spent gathering baseline data on the teachers' practices, beliefs, and patterns of discourse in their classrooms. Each teacher researcher had one of his/her classrooms videotaped for an entire week, for four different weeks over a six-month time period in the school year. When classes were not being taped, the teachers and university team met and analyzed mathematical tasks and shared artefacts from their teaching in the study group. The university

researchers provided quantitative and qualitative discourse analyses of the taped classroom episodes for the teachers. Discussions were then held in which the teacher researchers reacted to the videotapes, discussed them, and had the opportunity to provide interpretations which differed from those given by the university researchers. The collaboration of university researchers and teacher researchers extended to all aspects of this project, planning, readings, data analyses, reflective writing, and developing the action-research projects themselves.

The interactions between university researchers and teacher researchers were designed to develop a community of trust and support. The main goal of the project was to give teachers the opportunity to find *their own* research voice, to tap the researchers within themselves, in order to gather evidence to help change their practice. As such there were multiple levels of "research" occurring within this project that linked research and practice, research by the teacher researchers themselves as well as research by the university researchers.

Early on in the project, the teachers were asked to create belief maps, professed beliefs about what was closest to their heart in their teaching, and then to write journal entries about these professed beliefs. Compact versions of their belief maps were created for continued reference throughout the project, so that both the university and teacher researchers could continually look for congruence between professed beliefs about teaching, and actual behavior in the classrooms by the teacher researchers. Throughout the project the teacher researchers were continually provided with prompts for creating reflective journal entries. Questions were posed after study group discussions that led to journal entries. Teacher researchers were encouraged to write journal entries on what they were learning from the discussions on their classroom videotapes. Discussions were also punctuated with commentary related to the readings on classroom discourse in which they were engaged. The habit of becoming a reflective practitioner, keeping a journal, and reflecting on their practice, was being instilled in these teacher researchers throughout this project.

As the readings, discussions, and shared classroom video segments progressed, the teacher researchers began to identify "performance gaps" that they noticed between what they claimed were their professed beliefs, and what they actually did while teaching in their classrooms. This process provided the seeds and incubation time for the teacher researchers to identify their own research questions to investigate during their cycles of action research throughout the last two years of the project. The teachers noted the importance of wait time—not only after questions are posed, but also after a student responds. They realized that wait time was critical to provide opportunities for richer, deeper student discourse about mathematical content in the classroom.

The teacher researchers found that the process of revoicing students' comments, suggestions, and questions proved to be a powerful tool for improving content discourse. Another primary focus of their work was on improving classroom discourse for social purposes. They found there was a critical need to provide a safe classroom environment for students to share their thinking, solutions, and ideas and to feel comfortable to ask questions of the teacher and of one another.

Promoting Purposeful Discourse included reflective research chapters written by each of the eight teacher researchers in which they documented and shared their

passage through cycles of action research—what they did in their own research projects and what they learned throughout the four years of the project. The teacher researchers made their own choices on how they wished to approach writing up their action-research experience. In addition to their chapters, the teacher researchers had opportunities to present their work and experiences at several meetings and conferences attended by other teachers and mathematics education researchers. The topics investigated by the teacher researchers in their action-research projects included: increasing student participation in conceptual discourse; attending to particular performance gaps in their classroom practices uncovered in their belief maps; working towards giving students more ownership in the mathematical discourse in classes; revoicing student questions; addressing vagueness in classroom mathematical discourse; and improving listening to students' mathematical discourse.

Several things are of particular note in this long-term effort by Herbel-Eisenmann to link research and practice. There were multiple levels of research linked to practice that were created and continued over the entire project. Throughout the project the university researchers were investigating what moves, actions, and support structures might be helpful to create an environment where teachers could become researchers. The university researchers also kept reflective journals throughout the project, and met to discuss and plan study group meetings based on their own research observations of the group. The teacher researchers were conducting research on their classroom discourse behaviours, and on patterns of student discourse interactions in their classrooms as they developed their own action-research projects. A "linking" of research and practice occurred continually throughout this project within the discussions and reflective activity of the community meetings of the study group. Ultimately, the reflective story of this project as captured in *Promoting Purposeful Discourse* provided yet another level of research itself. It presented both a meta-reflection by the university researchers that identified the major themes, trends, and activities of the project, along with the stories told by the teacher researchers as they described their own action-research projects.

The Norwegian Example

The three-year Learning Communities in Mathematics (LCM) Project (2004–2007) was a research and development project that brought together teachers and didacticians to work together as both practitioners and researchers (Jaworski et al., 2007). It involved a team of 14 didacticians (the term that the team preferred to use for the teacher educators), which included 5 doctoral students, working with 8 schools (including primary, lower, and upper secondary) with a minimum of three teachers from each school (Jaworski, 2006). Schools volunteered to be part of the project as a result of an invitation from Agder University College in Norway where Barbara Jaworski, who led the project, held a faculty position.

The motivating principle on which the didacticians and teachers agreed to work together was the desire to develop better learning environments for mathematics

students at the levels of schooling with which each teacher was associated. In fact, co-learning was central to this project. Jaworski (2011) cited Wagner (1997) to make the point:

> In a co-learning agreement, researchers and practitioners are both participants in processes of education and systems of schooling. Both are engaged in action and reflection. By working together, each might learn something about the world of the other. Of equal importance, however, each may learn something more about his or her own world and its connections to institutions and schooling. (p. 16)

Workshops at the college were an important tool in the co-learning process. During the first two years, six workshops were held per year, and four during the third year. Workshops were three and a half hours in length and consisted of both plenary and small group activity. Plenary input from both didacticians and teachers included introducing mathematical tasks (usually by the didacticians), reporting about classroom activity (mainly by the teachers), and reporting from small group activity (by all). Small group activity included working on mathematical tasks, usually followed by didactical discussions in which both teachers and didacticians participated.

The teachers in the school teams worked together on designing tasks for the classroom. Didacticians were available to discuss the ideas that the teachers had generated, as well as to observe the classroom unfolding of the activities. Three didacticians were associated with each school to discuss the planned activities, to provide support, and to collect data.

All classroom lessons related to the project, as well as the workshop sessions, were videotaped. Jaworski (2006) stated that "the data and its analysis was largely owned by didacticians, with video data also providing a source for teachers to review classroom activity and reflect on teaching" (p. 11). All data were available to all of the didacticians of the project; in addition, the teachers had access to the data for their school should they so wish. The video data also proved to be a valuable resource within both the workshops and school settings as a tool for reflecting on developing student thinking within classroom activity. The video data were not related to particular research questions; rather research questions evolved through activity and data were used according to need. Jaworski (2008) pointed out that, as the didacticians followed up initial research questions in analysis of data and writing of papers, more refined questions emerged which then fed into future activity and further research.

At the heart of this collaborative project was the resolve to frame it around an inquiry-based approach within communities of practice. Inquiry, which involved questioning, exploring, investigating, and researching within everyday practice, was conceptualized at three levels:

1. Inquiry in mathematics: (a) teachers and didacticians exploring mathematics together in problems and tasks in workshops; (b) pupils in schools learning mathematics through exploration in tasks and problems in classrooms;
2. Inquiry in teaching mathematics: teachers using inquiry in the design and implementation of tasks, problems, and mathematical activity in classrooms in association with didacticians;

3. Inquiry in developing the teaching of mathematics: teachers and didacticians researching the processes of using inquiry in mathematics and in the teaching and learning of mathematics.

This emphasis on inquiry was, in the words of those who were asked to evaluate the LCM project at its close (Skovsmose & Säljö, 2007), a "challenge to the traditional notion of school mathematics in Norway … the inquiry approach explicitly and radically breaks with this [traditional] conception of learning mathematics; the power of the [LCM] project has to do with how the inquiry approach informs and comes to be a part of reformed classroom practices" (p. 11).

Within the LCM project, an inquiry community for the project at large had been created, but it could not be separated from the established communities of which project members were a part. According to Jaworski (2008), "teachers participated in the day-to-day life of their schools and, integrally, explored the use of inquiry-based tasks in their classrooms and observed their students' mathematical activity and learning; didacticians collected and analyzed data and wrote research papers, as expected of university academics and, integrally, explored the design of tasks for workshops and their work with teachers in school environments to support teachers in their project activity" (p. 320). But even more importantly, Jaworski emphasized that the alignment of both didacticians and teachers with their respective communities was a "critical alignment." By this she meant that they did so with a critical attitude whereby they questioned, explored, and sought alternatives while engaging, so as to "have possibilities to develop and change the normal states" (p. 314). Teachers and didacticians had engaged in a research activity that yielded evidence of both teachers' learning and didacticians' associated learning. Jaworski (2008, p. 326) argued that "seeing the enterprise in terms of an activity system made it possible to pick out elements in their complexity and trace developmental patterns for participants in the project (see Goodchild & Jaworski, 2005; Jaworski & Goodchild, 2006)."

The Canadian Example

In 1989, the CIRADE research centre attached to the Université du Québec à Montréal established research links with some schools. Over the years, the research engaged in at these schools began on to take on a distinctive shape where the emphasis was clearly on collaboration between teachers and researchers—research was being conducted "with" rather than "on" teachers. The example presented herein involved a group of teachers at one of these research schools and some of the CIRADE university researchers, led by Nadine Bednarz, who collaborated with that school (Bednarz, 2004). The collaborative project that emerged was one that combined professional development with supported action research in the classroom.

A group of first-grade teachers approached the researchers because they were having difficulty conceptualizing a way in which they might implement a ministerial-mandated, problem-solving approach to the teaching of mathematics in their

classes. The questions that the teachers put to themselves were the following: Is it possible to adopt a problem-solving process with young children? What does such an approach mean, and how can it be developed? These questions provided the basis for a collaborative research project that initially lasted for a year, but was extended for three more years. The team consisted, at first, of four first-grade teachers, a remedial teacher, and two researchers, but then brought in teachers from second and third grade during the following years. During the course of the project's being extended to the second and third grades, the mathematical content was also extended.

The design of problem situations, and ways in which to intervene with the children, was the central focus of the meetings that took place between the teachers and researchers. The dimension of professional development, referred to as *reflection on action* by Bednarz, was constituted by the discussions regarding the problem situations, the strategies used by the children, their approaches and ways of reasoning, and the teachers' management of the activity in the classroom context. In the process of reflecting, other questions of a more general nature arose among the teachers regarding problem solving and its integration into their practice. The research dimension was also fuelled by the joint construction of these problem situations, in particular by a reflection on the ways in which the problem situations were enhancing the mathematical learning of the children.

Over the course of the four years during which the joint process of constructing teaching situations occurred, approximately 1 day per month was given to reflection. In addition, one day of assessment was also included at the end of each year in order to review the outcomes of the project. As described by Bednarz (2004):

> The reflective activity was conducted in such a way as to encourage a planned, regular alternation between classroom experience and review of this experience. Work was performed in groups using accounts of the in-class activities, the difficulties arising in context, the records of statements by the children, and the difficulties they encountered. This review of the experience took different forms and served as a starting point for developing a new intervention sequence. This reflective activity thus developed around the meanings that the teacher developed in context and indeed imparted meaning to the situations or actions put forward. (p. 7)

Researchers and teachers interacted and jointly explored teachers' practice and engaged in the reflective review of that practice. The regular meetings of researchers and practitioners permitted, according to Bednarz (2004), the creation of an "interpretive zone" around the practice that was the subject of the exploration. This reflective activity was deemed to serve a dual function: "It is an opportunity for professional development through reflective review of the practice, with the objectives of clarifying, making explicit, and improving understanding of this practice—hence, of ultimately contributing to its restructuring; it is a research opportunity, as this meeting zone (interpretive zone) constitutes material for analysis to be used for investigating a certain object of interest to practice-related knowledge" (p. 11). In addition, Bednarz argued that, in the process of joint reflection on their action in collaboration with the researchers, the teachers were co-constructing new knowledge about their practice.

By the end of the project, several professional artefacts had been produced by the teachers: a collection of activities, observation grids, and classroom materials for the

school. Jointly, the teachers and researchers produced a book containing mathematical games for first graders, as well as videos of classroom teaching and student engagement in problem solving. Scientific publications were also produced by the researchers based on analyses of video recordings of in-class situations, the records of students' statements, and audio recordings of reflection-oriented meetings between researchers and teachers. Some of these analyses dealt with teaching situations and their potential for stimulating children's learning (Bednarz, 1996; Bednarz, Dufour-Janvier, Poirier, & Bacon, 1993; Poirier & Bacon, 1996); the process of co-construction that took place and the respective contributions (Bednarz, Poirier, Desgagné, & Couture, 2001); and the structuring of a teaching situation over time and the principles that guided this restructuring (Poirier, Bourdage, & Bednarz, 1999).

To close, we note that Bednarz (2004) argued that collaborative research such as that engaged in within this project not only contributed to the growth of knowledge for the research community but also, and equally importantly, to the professional development of the teachers involved. Moreover, she emphasized that the need of the researcher to integrate the practitioner in the construction of practice-related knowledge was based on "the idea of better understanding the reasoning that supports his or her [the teacher's] practice; … the teacher is considered as a partner in the inquiry 'with' whom one looks into the practice, who contributes in joint reflection (with the researcher) to the development of the practice" (p. 6).

The Japanese Example

This fourth example—on which, there are more details in Krainer (2011)—is unique in that it is not an approach initiated by a teacher educator or researcher, but rather is a longstanding, nation-wide approach conducted by teachers for teachers: Japanese lesson study. In their brief history of Japanese *lesson study*, Fernandez and Yoshida (2004) indicated that the origins can be traced back to the early 1900s. In the 1960s, teachers started combining lesson study (*jugyokenkyu*) and school-based inservice professional development (*konaikenshu*). Recognizing the value of *konaikenshu*, in the 1970s the Japanese government started supporting these grassroots activities. This support—small financial and other incentives—still exists today. Lesson study is by far the most common *konaikenshu* activity.

There are manifold versions and sizes of Japanese lesson study. They range from small-scale in-school initiatives with from four to six teachers to large-scale nationwide ones with hundreds of participants, many travelling long distances. However, a typical *lesson study process* (Fernandez & Yoshida, 2004; see also Hart et al., 2011) contains four to six steps, with a study lesson (*kenkyujugyo*) as the centerpiece of a lesson study (*jugyokenkyu*):

Step 1: Collaboratively planning the study lesson
Step 2: Seeing the study lesson in action
Step 3: Discussing the study lesson
Step 4: Revising the lesson (optional)

Step 5: Teaching the new version of the lesson (optional)
Step 6: Sharing reflections about the new versions of the lesson

Many schools solicit the support of an external advisor (most often instructional superintendents, sometimes experienced teachers on leave, or university staff). Schools often organize their *konaikenshu* work around a lesson study open house (*kokaijugyo*). Here well-developed ideas are shared with visitors (mostly teachers and other educators from neighboring schools). When distinguished guests take part (e.g., an external advisor), their reactions are paid considerable attention, often indicating very clear and pragmatic missions [e.g., Mr. Saeki's statement in Fernandez & Yoshida (2004): "A lesson cannot just start with giving students a problem on a sheet of paper"; teachers need to pay "attention to connecting lessons to students' prior knowledge" (p. 202)]. In many cases, lesson study open houses are followed by a joint celebration in the evening (with a mixture of relaxed socializing and exchanging opinions not articulated at the formal meeting). Some schools even produce written reports about their work (*kenkyukiyo no matome*). In the early 1990s, for example, the National Institute for Educational Research compiled every year over 4,000 reports written by teachers (see Fernandez & Yoshida, 2004, p. 213, referring to Sato, 1992).

The vast majority of elementary schools and many middle schools in Japan conduct *konaikenshu* (in all subjects). In contrast, very few high schools are engaged. In principle, *konaikenshu* activities are voluntary; in reality however, they are regarded as quasi-required. However, and most importantly, many teachers find *konaikenshu*, in particular lesson study, highly beneficial. Three mathematics teachers' opinions might give a flavor of their high regard for lesson studies:

> Developing a great lesson is an ideal thing but I think the best thing about the lesson study experience is that it gives you a chance to reflect about and rethink your own teaching. … I think even if it is a short period of time, having a place where everybody gets together and discusses instruction very seriously is an extremely valuable experience. … Anyway, lesson study can help teachers develop strong relationships, something I think is really important for all teachers. (Fernandez & Yoshida, 2004, p. 17)

It is common for individual teachers to belong to more than one lesson study group. In addition to within-school lesson study groups, autonomous cross-school study groups (regional study groups and teacher clubs) are also organized by teachers or unions (sometimes funded; in most cases membership fees are collected). A system of regular teacher rotations allows lesson study groups to learn from each other.

There are several features that are regarded as *key elements*—and at the same time as *success factors*—of lesson study. Murata (2011) highlighted five key characteristics. Lesson study: is centered on teachers' interests, is student focussed, has a research lesson, is a reflective process, and is collaborative. Further named key elements (see also Fernandez & Yoshida, 2004) are that lesson study: has its roots in strong movements (e.g., child-centred and problem-solving-based learning), regards teaching as a complex and profound enterprise (being not a one-way—and only a didactic—path, but a two-way integration of student ideas and content exploration),

is part of a culture of school-based professional development, is a way of enculturing novice teachers by serious academic activity, and is a way of improving yourself by looking at others (*Hito no furi mite waga furi naose*), with no end to improving teaching (indicating a culture of life-long effort and continuous further development). In addition, it should be stressed that lesson study is an autonomous and sustained effort by the teaching profession for the teaching profession. It has a process and also a product dimension (lesson plans and books, indicating a rich body of knowledge), and has created a language of its own (indicating the status of a well-developed profession). It is supported by townships, boards of education, the ministry, etc., indicating a culture of trust in teachers.

While the lesson study movement has become very popular internationally, the way in which it is practised in Japan is quite different from its many applications in Western countries. For example, the recent book *Lesson Study Research and Practice in Mathematics Education. Learning Together*, edited by Hart et al. (2011), addressed research and practice in 16 different locations (mostly in the USA). Due to the lack of experienced lesson study teachers and teacher educators, and lacking prior participation in the whole culture of *konaikenshu* activities, teacher educators act as initiators of lesson studies and support practitioners or student teachers in the practice of lesson study. This is in contrast to the Japanese lesson study approach where teachers themselves are the initiators and school externals (e.g., teacher educators at universities) are invited. Because other countries lack the grassroots teacher movement on which the Japanese lesson-study system builds, the initiating role taken on by school externals in adaptations of lesson study should not be considered too surprising.

The Chinese Example

In China, at the turn of the millennium, the National Mathematics Curriculum Standards (NMCS) were issued, and this ushered in a new set of curriculum guidelines emphasizing creative thinking, problem solving, and mathematical exploration (Huang & Bao, 2006). That document presented a challenge to teachers, who experienced difficulty in implementing these changes, as well as to mathematics educators who wanted to be able to assist in this endeavour. To address the problem, Chinese scholars developed an innovative model of inservice teacher education, called the *Keli* approach.

According to Huang and Bao (2006), development of the new model was to include the following key features:

> First, it is necessary to have expert input in order to upgrade teacher ideas, in a context of peer support; second, it is necessary to include the whole process of action, follow-up, and reflection; and third, it is necessary to form a community, which consists of experts, researchers and teachers. Thus, the program of in-service teacher education, called Xingdong Jiaoyu (Action Education) has been created. In this program, a community consisting of teachers and experts and researchers is formed, and the teachers improve their teaching action and upgrade their professional theory through unfolding the Keli process in cooperation with the members of the community. (p. 284)

Li, Huang, Bao, and Fan (2011) emphasized that innovative approaches to teachers' professional development in China establish direct connections with teachers' practices and what they try to do in their own classrooms. The *Keli* approach is no exception.

The implementation of the *Keli* approach in a school or school district usually unfolds in three phases: (a) familiarization and focussing; (b) a cycle of teaching, reflection, and revision; and (c) disseminating the *Keli* process and the exemplary lesson.

During the first phase, "familiarization and focussing," teachers' approaches are updated and they are introduced to the procedures of developing an exemplary lesson, usually by some experts. Within the Chinese educational system, an expert or master teacher is one who holds a senior rank:

> The conditions for being a senior secondary teacher include 5 years or more serving as a secondary school teacher at the intermediate level or being the holder of a PhD and demonstrating the ability to take the responsibility of senior secondary teacher. Moreover, the candidates should (a) have either systematic and sound fundamental theory and subject content knowledge, plentiful teaching experience and good teaching effectiveness, or specialize in political and moral education and classroom management, and achieve high performance and acquire rich experience; (b) engage in education research on secondary education and teaching and write an experience summary, scientific report, or research paper on the integration of theory and practice at a certain academic level or make remarkable contributions to the improvement of other teachers' academic levels and teaching abilities. (Huang, Li, & He, 2010, p. 295)

At a certain moment a collaborative group, which consists of researchers and interested teachers, is formed. Huang and Bao (2006) provided a couple of examples: one study group consisted of two researchers (one from the District Education Institute and the other from a Teachers College/Normal University) and the teachers from one school; another group consisted of two professors, a PhD holder from the Shanghai Academy of Education Sciences, three PhD candidates from East China Normal University, and researchers from a local educational institute, together with the mathematics teachers from one secondary school. The study group members then decide on a particular research question related to one of the challenging areas of the curriculum, which thereby becomes the focus for the construction of the exemplary lesson.

During the second phase, "teaching, reflection, and revision," an exemplary lesson is developed through a cycle of three teaching stages and two reflection stages. At the outset one teacher—often someone with considerable teaching experience—is selected for all three teaching stages. The first stage involves the teacher designing the lesson by him/herself and then presenting it to a class of students, with all the members of the *Keli* group observing. This is followed by a first feedback meeting immediately after the lesson, which focusses on the teacher's rationale for the design of the lesson, commentary from the group, and suggestions for revision of the lesson. Group members may work together at developing a new and improved version of the lesson.

Following this first reflection stage and the subsequent revision of the lesson, the teacher then presents the lesson to other classes of students at the same grade level

within the same school—once again being observed by the *Keli* group. After that second round of teaching, further reflection by the group takes place, which focusses on the promising features of the exemplary lesson and on the differences that remain between its design and what is considered to be effective practice according to the new curricular guidelines. An additional revision is made and a third teaching stage follows.

The third phase, "disseminating the *Keli* process and the exemplary lesson" involves writing a lesson description that can be shared with the public. According to Huang and Bao (2006), this description focusses on the following aspects: "(a) how the learning styles and teaching strategies have been changed in the classroom; (b) how the teacher's conception of teaching and ways of developing a lesson have been updated to meet the new ideas of the new NCMS curriculum; and (c) challenges faced during the process of *Keli* or the reflections occurring during *Keli*" (p. 286). Huang and Bao emphasized, as well, that teachers collaborate with the researchers and university members of the *Keli* group in the writing of the report. Once the report has been completed for publication, a video case study is produced for eventual use in teacher-education programs; it includes the main sections of the lesson, the reflections by members of the group, and an analysis of the lesson in both quantitative and qualitative terms.

In the example of the *Keli* group provided by Huang and Bao (2006), teachers were asked to keep a diary. Some of the commentary that they entered emphasized in particular the value they found in the process of reflecting on the lesson immediately afterwards, revising it, and then redelivering it. One teacher, who was interviewed on this point, stated that, "Traditionally, without follow-up action, the same content will probably be taught 4 years later, so there is only a little impression about how the content was handled before. Then the lesson will be re-designed repeatedly. Nowadays, the lesson plan was revised three times, and the lesson will be observed and reflected two times; it is definitely helpful" (p. 293).

One of the researchers from a Normal University, who had about 10 years teaching experience and held a PhD specializing in mathematics education, reflected on the role he played in this group and in others like it: "It is an important phase to summarize the particular implementation of *Keli*. … At this stage, I usually play a key role in helping them in theorizing and abstracting such as how to organize events to support the main findings, how to effectively organize a paper suitable for publication" (p. 294).

A final issue concerns the commonalities and differences between the *Keli* approach and Japanese Lesson Study. According to Huang and Bao (2006):

> The common features of both Japanese Lesson Study and the Keli process are their common concern with practical issues and the attention both pay to developing a particular lesson through collaborative lesson planning, classroom observation and post-lesson discussion to tackle the particular issues in question. However, the Chinese Keli process emphasizes the expertise stemming from experts, the revision of lesson design and the consequent new action. (p. 295)

Experts thus would seem to have a much more involved role in the *Keli* approach than is the case in Japan where it is the teachers who choose the goals they wish to

pursue and the ways of achieving them within their lessons and, in fact, control the entire lesson study process—even if external experts are sometimes invited to join a given Japanese study-lesson sequence.

Discussion: Three Dimensions Central to these Examples

In the first section of this chapter, we situated our perspective on teachers as key stakeholders within two broader contexts, one related to the general notion of stakeholder and the other related to a distinction between teacher as *stakeholder* and teacher as *key stakeholder* in mathematics education research. The term "key stakeholder" was adopted in reference to research where the teacher is considered a co-producer of professional and/or scientific knowledge. In the subsequent presentation of examples drawn from the international corpus of research in mathematics education, we synthesized five cases of research where the teacher participants had a "key stakeholder" role to play. Reflecting upon these examples and focussing on their commonalities and differences allows us now to draw out some of the important dimensions of this research. These dimensions include the following: reflective, inquiry-based activity with respect to teaching action; a significant action-research component accompanied by the creation of research artefacts by the teachers (sometimes assisted by the university researchers); and the dynamic duality of research and professional development.

Reflective, Collaborative, Inquiry-Based Activity with Respect to Teaching Action

All five of the examples presented in the previous section involved sustained reflection on teaching action. Although the specific focus and form of the reflection varied from one example to the other, the importance of this dimension cannot be overemphasized. Let us look more closely at the ways in which reflection was engaged in across the example-set.

The underlying assumption of the USA example was that teachers can improve their practice by studying what they do, learning how to do it better, and sharing their experiences with others in the field. Reflection was considered an essential part of this overall process:

> From the outset of the project, the teacher researchers engaged in many kinds of reflection. Some activities that the teacher researchers cited as provoking especially meaningful reflection included creating belief mappings, juxtaposing their belief mappings with classroom videotapes [of their own teaching], and incorporating ideas from the study-group readings into their own daily practice. (Herbel-Eisenmann, Cirillo, & Otten, 2009, p. 211)

Before beginning the project work, the teachers had not yet made explicit to themselves the beliefs that they thought drove their instructional practice. They were asked to create belief maps, which were a kind of semantic net that described "what was closest to their hearts" when they practised their teaching of mathematics.

According to the university researchers, "the increased awareness gained from developing a belief mapping enabled the teacher researchers to identify what they *wanted* to happen (and why) [in their classrooms] and to continually examine whether what they *wanted* to happen was *actually* happening" (p. 212). The continuous examining and reflecting on their practice in relation to what they had described in their belief mappings, which occurred over the duration of the project, took place largely as the teacher researchers watched and reviewed videotaped lessons. Everyone in the group watched, discussed, and reflected upon the videotapes of all the teacher researchers' classroom teaching, with a particular focus on the discourse of both teacher and students. Teachers talked about how the various forms of reflection they were engaging in were enabling them to transform their thinking about their practice and described their increasing awareness as they constantly revisited their belief mappings throughout the project. In particular, the reflections that were encouraged during the project meetings helped the teachers to develop their own ideas for their action-research projects, of which more will be said shortly.

The joint reflective activity in the Canadian example, which alternated between classroom experience and review of that experience, focussed in particular on the difficulties that arose for the teachers, and for the children, as they attempted to put into practice the novel situations that they had co-constructed during the previous meeting sessions involving teachers and university researchers. This reflective activity often centred on the didactical and pedagogical principles that were underpinning the teachers' practices. For example, the teachers focussed on issues such as having the maximum number of children active, getting the children to be organized, and having the children see different ways of solving a problem and listening to different points of view (Bednarz, 2004). This kind of collective review of their practice then served as a starting point for developing new teaching sequences in the next cycle of reflective activity.

Although the Norwegian example was similar to the Canadian one in that it alternated between school activity where innovation could take place and workshops where both the design of tasks and reflective discussion occurred, the focus of the reflections was somewhat different. At the base of the Norwegian project was the principle of co-learning inquiry: people learning together through inquiry, where both didacticians and teachers were engaged in action and reflection, so as to learn not only something about the world of the other but also more about his or her own world. According to Jaworski (2008), one of the reasons for introducing inquiry as a tool was to challenge the normal state of school mathematics teaching and to question what that teaching was achieving. She emphasized that in an inquiry community, participants are not satisfied with the normal state, but approach their practice with a questioning attitude, "to start to explore what else is possible; to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to provide answers to them" (p. 314). Thus, teachers' reflections during the workshops centred on questioning, exploring, and seeking alternatives to their usual approaches to teaching mathematics.

One of the distinguishing features of the Chinese approach (sometimes also a part of Japanese lesson study) is the form that the reflection takes—one involving

successive iterations of a lesson. Reflections that are based on the observation of a lesson and which focus on how the lesson could be improved, which in turn feed into the revising of the lesson and the teaching of the new version, are then followed by further shared reflections about the new version. According to Huang and Bao (2006), the reflections centre in particular on the promising features of the lesson and on the differences which remain between its design and what is considered to be effective practice according to the new curricular guidelines that emphasize creative thinking, problem solving, and mathematical exploration.

The shared reflections that take place during Japanese lesson study tend to focus on the well-developed foundational principles of Japanese mathematics teaching, such as paying attention to connecting lessons to students' prior knowledge, engaging students intellectually with important mathematics, having clear and explicit goals that address student understanding and performance, and ensuring that a given lesson fits into an overall unit within a specific grade level (Fernandez & Yoshida, 2004; for more discussion of these principles, see Corey, Peterson, Lewis, & Bukarau, 2010).

In all five examples, we noted the role of the discussions and joint activities which served to link teachers' practice to the reflective review of that practice. In some of the examples, these conversations involved teachers and university researchers; in others, teachers with teachers. But in all cases, the reflective activity was used as a vehicle for teachers' clarifying and making explicit certain aspects of teaching practice. It thereby constituted a form of professional development, which is further discussed below.

The Action-Research Dimension: Teachers as Researchers

Action research is generally defined as "systematic inquiry into one's own practice for the purpose of learning about and changing one's practice in order to better support students' learning" (Herbel-Eisenmann, 2009, p. 7; see also Altrichter et al., 2008; Benke, Hospesová, & Tichá, 2008; Krainer, 2006). Action research challenges the assumption that knowledge is separate from and superior to practice. Atweh (2004) has argued that action research serves as a conduit between theory and practice because it bridges the gap between the two. In action research, the production of local knowledge is seen as equally important as general knowledge. All of the examples that are offered in our chapter of this volume present various approaches to action research, the most significant variation being between Japanese lesson study where teachers carry out the activity autonomously with, in some cases, externals (e.g., university researchers) being invited, and the other examples where the university researchers initiate the activity and support teachers engaged in action research.

The most extensive and nationally widespread version of action research by teachers is practised in Japan within the framework of "lesson study" with its systematic reflection of practitioners on action. The teachers in a lesson study context are collaborative researchers who collect data, interpret it, and write down their experiences

in papers and books. In many cases, in order to increase the effectiveness of the outcomes or the dissemination of knowledge, experienced others ("critical friends") are invited. Their role varies tremendously. They might participate in order to observe (primarily as learners), to give occasional feedback, to present an invited reaction, to give input, to (co-)investigate students' growth, or to (co-)investigate lesson-study participants' growth. However, in general, lesson study in Japan is initiated, done, reflected, and transferred to written artefacts by teachers for teachers, in an investigative attitude towards their own practice.

Jaworski (2011) in discussing teachers as researchers, distinguished between, on the one hand, research programs in which teachers research their own practice within collaborative teacher practitioner-university didactician groups and, on the other hand, research initiatives by teachers where they are the designers of the research. The example of Japanese lesson study is clearly of the latter type, with teachers designing the research, carrying it out, and producing artefacts to be shared with other teachers. However, the other examples presented within this chapter do not fall neatly into Jaworski's former category. Some traverse the two. For instance, the USA example involved a collaboration of teachers and university researchers. But it was the teachers who selected aspects of their classroom discourse that they wanted to change and then designed and carried out cycles of action research occurring over more than a year, during which time they studied the impact of the changes on students' social and mathematical experiences.

Each teacher in the project then wrote up an account of his/her action-research project in separate chapters of a book which documented the overall project (see Herbel-Eisenmann & Cirillo, 2009). In their action research, the teacher researchers collected their own videotapes and other artefacts of practice and used these to engage in systematic inquiry related to their goals. Their earlier belief-mapping schemas were used as the standards by which the teacher researchers evaluated their own teaching.

Although, within the Canadian example, it was the teachers at the research school who approached the university researchers and asked for their assistance in a project that they themselves initiated, it was not the teachers who designed the research. This was a joint collaborative venture involving both university researchers and teachers. The products of the collaborative action research described in the Canadian example consisted of a collection of activities, of observation grids, and of classroom materials for the school. In addition, several videos related to the situations tested out in class were produced by the university researchers, in collaboration with the teachers; these videos were to serve as material for preservice and inservice teacher education, as well as for a number of research publications written primarily by the university researchers.

Similarly, the research on their own practice that was carried out by the teachers of the Norwegian project was also designed in collaboration with the university researchers with whom they worked, yielding products much like the Canadian example. The Chinese example of teachers researching exemplary lessons, in collaboration with university researchers, also yielded research reports, written jointly by the teacher researchers and the university researchers, and video case studies for eventual use in teacher education programs.

The five examples thus present a picture of action research that includes the co-production of professional and scientific artefacts. The ways in which the action research was carried out, and the artefacts produced, can be characterized as a continuum ranging between two poles: one pole where the work is collaborative and shaped by input from both university researchers and teachers and where researchers and teachers together design, implement, and report findings of their research, but where the university researchers also write additional articles of a scholarly nature; the other pole where teachers collaborate with other teachers doing this work. The USA example was one that clearly straddled both poles with its teacher-initiated action-research studies of an individual nature and teacher-written publications on that research, but within a supportive collaborative framework involving other teachers and university researchers.

The Dynamic Duality of Research and Professional Development

The vision that teachers conducting research constitutes a form of professional development presents a powerful image. In the words of Cochran-Smith and Lytle (1993): "Because teacher research challenges the dominant views of staff development and preservice training as transmission and implementation of knowledge from outside to inside schools, it has the potential to reconstruct teacher development across the professional life span so that inquiry and reform are intrinsic to teaching" (cited in Herbel-Eisenmann, Cirillo, & Males, 2009, p. 219). In an interview just before the USA project came to an end, when teachers were asked how they felt about not being told what to do for their action-research studies, most responded that it was quite different from any of their other professional development experiences. "To have your ideas taken seriously and to be supported in what you think is best over a long time" was, in the words of one of the teacher participants, a foreign but rewarding experience. Although teacher action research is still quite rare in the field of mathematics education, and it is even rarer for it to be viewed as a form of professional development, especially in the USA, the examples presented in this chapter are not unique. In Australia, for example, a model of professional development, titled Improving Teaching Approaches to Mathematics (Pegg & Panizzon, 2011), has been elaborated to underpin the process whereby teachers work collaboratively, with support from university practitioners, in developing and researching strategies to address issues that they have identified and which are relevant to their own teaching contexts. In Austria, several programs have been launched where teachers are supported in carrying out action-research projects, writing reflective papers, and in forming learning communities at their schools or in their districts (see, e.g., Krainer, 2011).

Although much has already been said in this discussion section with respect to the importance of reflective activity within the five examples, its role in relation to professional development has not yet been articulated. Bednarz (2004) drew our attention to the ways in which reflection on action constitutes professional development.

In so doing, she emphasized the relevance of the knowledge that the practitioner constructs and develops throughout the course of his or her teaching experience, which then feeds into the knowledge constructed during the action-research experience. Furthermore, the shared reflection that occurs within the context of the research experience, with or without the university researcher's contribution (as is quite often the case with Japanese lesson study), renders explicit the knowledge that might otherwise remain implicit. In the group construction process, a variety of resources are brought into play, all of them nourishing the professional development that is inherent to the situation—professional development that, according to Bednarz, is as significant for the university researcher as for the teacher. More specifically and based on her experience with the Canadian project, Bednarz (2004) noted the following components of the process of collaborative research that she viewed as contributing to the teachers' professional development:

- A deeper reflection on mathematical content (learning situated in practice), where teachers have the opportunity, during the discussions around the teaching situations and the productions of children, to improve their understanding of the mathematical concepts at play;
- A new awareness of the nature of mathematical activity, where the collaborative research process is also the occasion to debate what mathematical activity means;
- New ways to look at children's statements, where teachers have the opportunity, during the discussions on the teaching situations and productions of students, to develop new ways to look at children's productions, to take some distance, to consider different ways to solve a problem;
- Reflection on the didactical variables involved in a given task and their influence, where the analysis of tasks moves away from superficial aspects and towards student reasoning, thereby encouraging the seeing of complexity;
- Teaching strategies, where the arguments underlying decision making are rendered explicit, thereby opening up other points of view;
- An evolving relationship to the teaching of mathematics, where a changing relationship with teaching "know-how" is encouraged.

Jaworski (2008) has described, in relation to the Norwegian project, the professional development that occurred both for the teacher researchers and the university researchers (didacticians): "For example, teachers suddenly came to see, through their study of students' thinking and activity in algebra, how they could explore in their school environment ways to develop teaching and learning; didacticians saw the nature of a task that could lead to teachers' effective recognition of the nature of school goals for students' development and learning in mathematics" (p. 326).

The fact that the professional development that takes place in these types of projects occurs not just for the teachers but also for the university researchers of the project is a very important point. The initiators of these projects (usually university researchers) also experience professional development and growth in these collaborative research efforts—that part is seldom carefully documented or written about. More recently, Makar and O'Brien (2012) discussed the transformative nature of

collaborative research, the changes in identity, and the growth in the participation and perspectives of both teachers and researcher that developed over a 6-year, design research project on inquiry-based teaching. The teachers in Makar and O'Brien's project experienced an "identity renegotiation" as they became aware of and then acknowledged their research contribution to the project. Meanwhile, the researcher documents her own professional growth as a collaborative researcher, and what she is learning from the teachers in the project. Makar and O'Brien refer to this as *reflexivity,* the joint contributions and joint benefits of teachers and researchers engaged in collaborative research.

In the Japanese example, the research that is associated with the lesson study process goes hand in hand with professional development and is in fact part of the culture of school-based professional development. The professional development aspect of lesson-study activity is also captured by one of its key elements in that it is viewed as "a way of improving yourself by looking at others (*Hito no furi mite waga furi naose*), with no end to improving teaching (indicating a culture of life-long effort and continuous further development)" (Murata, 2011, p. 10).

In the Chinese example, the direct link between professional development and research involving teachers' practices and what they try to do in their own classrooms was an explicit focus, according to Li, Huang, Bao, and Fan (2011). More specifically, the entire research process of action, follow-up, and reflection, as well as the necessity of forming a community consisting of experts, researchers and teachers, is considered integral to the professional development approach adopted in China.

In their reflective discussions and their written research chapters, the teacher researchers in the USA project identified three major factors that transformed their own practice with regard to discourse in their classrooms and which constituted a form of professional development for them: (a) the influence of the readings and research literature, (b) the importance of reflection by the teachers—both in study-group discussions and written reflections in journal entries, and (c) the power available within a collaborative community of teachers to support one another in this kind of effort by teacher researchers. The creation of belief maps and subsequent opportunities to reflect on the videotapes they made of their practice proved to be transformative for the teacher researchers. Just seeing the data alone was not sufficient to change practice—the teacher researchers said that opportunities to reflect and to discuss with the study group whether those beliefs were actually being implemented in their classrooms was critical to making changes in their practice.

Extrapolating from the research by Herbel-Eisenmann and her colleagues suggests that, for professional development to have the potential to help teachers transform their practice, consideration of whether the following conditions are in place would be useful. Having a supportive, safe, community for the teacher researchers to share and discuss, maintained over a very long period of time, was clearly a decisive piece in this research effort. In addition to the safe harbour of the community of practice, the opportunity was provided for the teacher researchers to select from a collection of thoughtfully chosen readings that linked to the project goals and to their own practice. Open discussions and analyses of the video data were conducted

jointly during group meetings of the university and teacher researchers. And finally, these teacher researchers had the opportunity to write their own stories in their own ways, supported by the university researchers in the process. The teachers' voice was crucial to the success of the work in this project.

Closing Remarks

This chapter has attempted to close the distinctive gap between research and practice that exists in much of the mathematics education research literature by viewing teachers as key stakeholders in research—stakeholders who co-produce professional and scientific knowledge—rather than as "recipients of research," and sometimes even "means" to generate or disseminate knowledge. We presented five examples, drawn from individual and nation-wide projects around the world, examples that offered the potential to link research and practice in clear and explicit ways. Our analysis of these projects revealed three salient dimensions to research where the teacher is considered a key stakeholder: (a) teacher reflection, (b) teachers in the role of researchers themselves, and (c) the multi-leveled professional development experience within the research process for both teacher researchers and university researchers. The (co-)production of professional and scientific knowledge, which cut across all three of these dimensions in the examples presented, is considered a critical aspect of the notion of the "teacher as key stakeholder" in research, an aspect to which we now briefly return.

The (co-)production of professional and scientific knowledge is clearly linked with writing papers and thus making one's findings open for public discussion and critique (Krainer, 2006). In general, this is rather more difficult for teachers than for teacher educators and researchers who live in a "culture of publishing." Despite the diversity between teachers' and researchers' worlds, discussed earlier in this chapter, all five of the approaches that were presented were able to bridge these worlds and, as well, succeeded in promoting teachers' writing down of the findings of their inquiries and investigations. This promotion was done for several reasons: systematic reflection by teachers on their own work creates new knowledge which in turn positively influences their (future) teaching and enhances the quality of teaching. Writing down is an additional opportunity to learn; written artefacts increase the opportunities for communicating and cooperating with interested people (teachers, theoreticians, administrators); written artefacts help to make teachers' professional knowledge more visible and accessible, and thus contribute to the further development of the teaching profession as a whole; these artefacts also give teacher educators and researchers an additional opportunity to learn from teachers. Teachers' own investigations increase their interest in research, in reading research papers, and in collaborating in research projects, thus building further bridges between research and practice.

The challenge now for all of us in the international mathematics education community is to consider how further to promote and systematize collaborative research

work among teachers, with or without university researchers, in ways that will reflect and build upon what has been documented in the five examples presented in this chapter. Given the potential for professional growth from the expanded roles for both classroom teachers and researchers alike, and the growing documentation of the long-term benefits for researchers, teachers, and their students from such collaborative research, a case can be made that all countries should consider implementing a *systematic* integration of linked research and practice. Collaborative research with teachers has heretofore arisen on a case-by-case basis, and somewhat haphazardly, especially in the western countries where it has occurred. We feel that every country could benefit by implementing its own national commitment to linked inquiry. As has been illustrated in examples discussed in this chapter, promoting a national effort and national discourse around creating stronger links between research and practice is not only possible, but can also be rewarding for all concerned. These examples can thus serve both as inspiration and model for truly bridging the gap between mathematics education research and practice. The crucial element is to regard *researchers as key stakeholders in practice* and *teachers as key stakeholders in research*.

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References

Altrichter, H., Feldman, A., Posch, P., & Somekh, B. (2008). *Teachers investigate their work; An introduction to action research across the professions* (2nd ed.). London, UK: Routledge [German original: Altrichter, H. & Posch, P. (1990). *Lehrer erforschen ihren Unterricht*. Bad Heilbrunn, Germany: Klinkhardt. Chinese translation 1997, Taipei, Taiwan: Yuan-Liou.].

Atweh, B. (2004). Understanding for changing and changing for understanding. Praxis between practice and theory through action research in mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 187–206). New York, NY: Kluwer Academic Publishers.

Bazzini, L. (Ed.). (1994). *Proceedings of the 5th International Conference on Systematic Cooperation between Theory and Practice in Mathematics Education (SCTP 5) in Grado: Theory and Practice in Mathematics Education*. Pavia, Italy: ISDAF.

Bednarz, N. (1996). Language activities, conceptualization and problem solving: The role played by verbalization in the development of mathematical thought by young children. In H. M. Mansfield, N. A. Pateman, & N. Bednarz (Eds.), *Mathematics for tomorrow's young children: International perspectives on curriculum* (pp. 228–239). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Bednarz, N. (2004). Collaborative research and professional development of teachers in mathematics. In M. Niss & E. Emborg (Eds.), *Proceedings of the 10th International Congress on*

Mathematical Education (CD version) (pp. 1–15). Copenhagen, Denmark: IMFUFA, Roskilde University.

Bednarz, N., Dufour-Janvier, B., Poirier, L., & Bacon, L. (1993). Socioconstructivist viewpoint on the use of symbolism in mathematics education. *The Alberta Journal of Educational Research, 29*(1), 41–58.

Bednarz, N., Poirier, L., Desgagné, S., & Couture, C. (2001). Conception de séquences d'enseignement en mathématiques: Une nécessaire prise en compte des praticiens. In A. Mercier, G. Lemoyne, & A. Rouchier (Eds.), *Le génie didactique: Usages et mésusages des theories de l'enseignement* (pp. 43–70). Brussels, Belgium: De Boeck Université.

Benke, G., Hospesová, A., & Tichá, M. (2008). The use of action research in teacher education. In K. Krainer & T. Wood (Eds.), *Participants in mathematics teacher education: Individuals, teams, communities and networks (International handbook of mathematics teacher education)* (Vol. 3, pp. 283–307). Rotterdam, The Netherlands: Sense Publishers.

Bromme, R. (1992). *Der Lehrer als Experte. Zur Psychologie des professionellen Wissens*. Bern, Switzerland: Huber.

Cochran-Smith, M., & Lytle, S. L. (1993). *Inside/outside: Teacher research and knowledge*. New York, NY: Teachers College Press.

Corey, D. L., Peterson, B. E., Lewis, B. M., & Bukarau, J. (2010). Are there any places that students use their heads? Principles of high-quality Japanese mathematics instruction. *Journal for Research in Mathematics Education, 41*, 438–478.

Crawford, K., & Adler, J. (1996). Teachers as researchers in mathematics education. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 1187–1205). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Davis, R. B., Jockusch, E., & McKnight, C. (1978). Cognitive processes in learning algebra. *Journal of Children's Mathematical Behavior, 2*(1), 10–320.

Debien, J. (2010). *Répertorier les modalités favorisant une démarche de développement professionnel chez les enseignants de mathématique de niveau secondaire* (Master's thesis). Université du Québec à Montréal, Département de mathématiques.

Even, R., & Ball, D. L. (Eds.). (2003). Connecting research, practice and theory in the development and study of mathematics education. *Educational Studies in Mathematics* (special issue)*, 54*(2–3).

Fernandez, C., & Yoshida, M. (2004). *Lesson study: A Japanese approach to improving mathematics teaching and learning.* Mahwah, NJ: Lawrence Erlbaum (Reprint by Routledge, 2009).

Freeman, R. E. (1984). *Strategic management. A stakeholder approach*. Boston, MA: Pitman.

Freeman, R. E. (2004). The stakeholder approach revisited. *Zeitschrift für Wirtschafts-und Unternehmensethik (zfwu), 3*(5/2004), 228–241.

Fullan, M. (1993). *Change forces. Probing the depths of educational reform*. London, UK: Falmer Press.

Goodchild, S., & Jaworski, B. (2005). Identifying contradictions in a teaching and learning development project. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 41–47). Melbourne, Australia: International Group for the Psychology of Mathematics Education.

Hart, L. C., Alston, A., & Murata, A. (Eds.). (2011). *Lesson study research and practice in mathematics education: Learning together*. Dordrecht, The Netherlands: Springer.

Hattie, J. A. (2003, December). Teachers make a difference: What is the research evidence? *Building teacher quality: What does the research tell us?* Paper presented at a Conference held at the Australian Council for Educational Research. Retrieved from http://research.acer.edu.au/research_conference_2003/4.

Herbel-Eisenmann, B. (2009). Introduction to the project, the people, and the reflective activities. In B. Herbel-Eisenmann & M. Cirillo (Eds.), *Promoting purposeful discourse. Teacher research in mathematics classrooms* (pp. 3–28). Reston, VA: National Council of Teachers of Mathematics.

Herbel-Eisenmann, B. (2010). Discourse analysis: A catalyst for reflective inquiry in mathematics classrooms. In *Linking research and practice: The NCTM Research Agenda Conference report* (pp. 36–37). Reston, VA: National Council of Teachers of Mathematics.

Herbel-Eisenmann, B., & Cirillo, M. (Eds.). (2009). *Promoting purposeful discourse: Teacher research in mathematics classrooms*. Reston, VA: National Council of Teachers of Mathematics.

Herbel-Eisenmann, B., Cirillo, M., & Males, L. (2009). An argument for taking up similar work. In B. Herbel-Eisenmann & M. Cirillo (Eds.), *Promoting purposeful discourse. Teacher research in mathematics classrooms* (pp. 219–232). Reston, VA: National Council of Teachers of Mathematics.

Herbel-Eisenmann, B., Cirillo, M., & Otten, S. (2009). Synthesizing the bases of purposeful discourse: Reading, reflecting, and community. In B. Herbel-Eisenmann & M. Cirillo (Eds.), *Promoting purposeful discourse: Teacher research in mathematics classrooms* (pp. 205–217). Reston, VA: National Council of Teachers of Mathematics.

Huang, R., & Bao, J. (2006). Towards a model for teacher professional development in China: Introducing *Keli*. *Journal of Mathematics Teacher Education, 9*, 279–298.

Huang, R., Li, Y., & He, X. (2010). What constitutes effective mathematics instruction? A comparison of Chinese expert and novice teachers' views. *Canadian Journal of Science, Mathematics and Technology Education, 10*, 293–306.

Jaworski, B. (2006). Developmental research in mathematics teaching and learning: Developing learning communities based on inquiry and design. In P. Liljedahl (Ed.), *Proceedings of the 2006 Annual Meeting of the Canadian Mathematics Education Study Group* (pp. 3–16). Calgary, Canada: CMESG.

Jaworski, B. (2008). Development of mathematics teacher educators and its relation to teaching development. In B. Jaworski & T. Wood (Eds.), *The mathematics teacher educator as a developing professional (International handbook of mathematics teacher education)* (Vol. 4, pp. 335–361). Rotterdam, The Netherlands: Sense Publishers.

Jaworski, B. (2011). Situating mathematics teacher education in a global context. In N. Bednarz, D. Fiorentini, & R. Huang (Eds.), *International approaches to professional development of mathematics teachers* (pp. 2–51). Ottawa, Canada: Presses de l'Université d'Ottawa.

Jaworski, B., Fuglestad, A.-B., Bjuland, R., Breiteig, T., Goodchild, S., & Grevholm, B. (2007). *Learning communities in mathematics*. Bergen, Norway: Caspar Forlag.

Jaworski, B., & Goodchild, S. (2006). Inquiry community in an activity theory frame. In J. Novotná, H. Moraova, M. Kratka, & N. Stelikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 353–360). Prague, Czech Republic: International Group for the Psychology of Mathematics Education.

Jaworski, B., & Wood, T. (Eds.). (2008). *The mathematics teacher educator as a developing professional (International handbook of mathematics teacher education)* (Vol. 4). Rotterdam, The Netherlands: Sense Publishers.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707–762). Greenwich, CT: Information Age Publishing.

Kirshner, D., & Awtry, T. (2004). Visual salience of algebraic transformations. *Journal for Research in Mathematics Education, 35*, 224–257.

Krainer, K. (2006). Action research and mathematics teacher education. *Journal of Mathematics Teacher Education, 9*, 213–219.

Krainer, K. (2011). Teachers as stakeholders in mathematics education research. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 47–62). Ankara, Turkey: International Group for the Psychology of Mathematics Education.

Krainer, K., & Llinares, S. (2010). Mathematics teacher education. In P. Peterson, E. Baker, & B. McGaw (Eds.), *International encyclopedia of education* (Vol. 7, pp. 702–705). Oxford, UK: Elsevier.

Li, Y., Huang, R., Bao, J., & Fan, Y. (2011). Facilitating mathematics teachers' professional development through ranking and promotion practices in the Chinese mainland. In N. Bednarz, D. Fiorentini, & R. Huang (Eds.), *International approaches to professional development of mathematics teachers* (pp. 72–87). Ottawa, Canada: Presses de l'Université d'Ottawa.

Little, J. W. (1982). Norms of collegiality and experimentation: Workplace conditions of school success. *American Education Research Journal, 19*, 325–340.

Makar, K., & O'Brien, M. (2012). Blurring the boundaries: The transformative nature of research participation. In W. Midgley, P. A. Danaher, & M. Baguley (Eds.), *The role of participants in education research: Ethics, epistemologies, and methods*. London, UK: Routledge.

Matz, M. (1982). Towards a process model for high school algebra errors. In D. Sleeman & J. S. Brown (Eds.), *Intelligent tutoring systems* (pp. 25–50). London, UK: Academic Press.

Murata, A. (2011). Introduction: Conceptual overview of lesson study. In L. C. Hart, A. Alston, & A. Murata (Eds.), *Lesson study research and practice in mathematics education. Learning together* (pp. 1–12). Dordrecht, The Netherlands: Springer.

National Council of Teachers of Mathematics. (2010). *Linking research and practice: The NCTM Research Agenda Conference Report*. Reston, VA: Author.

OECD. (2011). *Strong performers and successful reformers in education*. Paris, France: Author.

Pegg, J., & Krainer, K. (2008). Studies on regional and national reform initiatives as a means to improve mathematics teaching and learning at scale. In K. Krainer & T. Wood (Eds.), *Participants in mathematics teacher education: Individuals, teams, communities and networks (International handbook of mathematics teacher education)* (Vol. 3, pp. 255–280). Rotterdam, The Netherlands: Sense Publishers.

Pegg, J., Lynch, T., & Panizzon, D. (2007). *An exceptional schooling outcomes project: Mathematics*. Brisbane, Australia: Post Press.

Pegg, J., & Panizzon, D. (2011). Collaborative innovations with rural and regional secondary teachers: Enhancing student learning in mathematics. *Mathematics Education Research Journal* (special issue on "Mathematics Education in Rural Schools: Evidence-Based Approaches"), *23*(2). doi:10.1007/s13394-011-0009-0.

Poirier, L., & Bacon, L. (1996). Interaction between children in mathematics class: An example concerning the concept of number. In H. M. Mansfield, N. A. Pateman, & N. Bednarz (Eds.), *Mathematics for tomorrow's young children: International perspectives on curriculum* (pp. 166–174). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Poirier, L., Bourdage, N., & Bednarz, N. (1999). Un lien possible entre la recherche en didactique des mathématiques et la pratique de classe: la recherche collaborative. In F. Jacquet (Ed.), *Les liens entre la pratique de la classe et la recherche en didactique des mathématiques. Actes de la CIEAEM 50* (pp. 193–197). Neufchâtel, Switzerland: Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques.

Reynolds, D., Creemers, B., Stringfield, S., Teddlie, C., & Schaffer, G. (Eds.). (2002). *World class schools. International perspectives on school effectiveness*. London, UK: Routledge.

Sato, M. (1992). Japan. In H. B. Leavitt (Ed.), *Issues and problems in teacher education: An international handbook*. New York, NY: Greenwood Press.

Schön, D. (1983). *The reflective practitioner: How professionals think in action*. London, UK: Temple-Smith.

Shearer, B. A., Lundeberg, M. A., & Coballes-Vega, C. (1997). Making the connection between research and reality: Strategies teachers use to read and evaluate journal articles. *Journal of Educational Psychology, 89*, 592–598.

Skovsmose, O., & Säljö, R. (2007). *Report on the KUL-projects: Learning Communities in Mathematics and ICT in mathematics learning.* Retrieved from http://www.navimat.dk/uploads/39600/Report-KULfinal_OSK_RS_okt07.pdf.

Steinbring, H. (1994). Dialogue between theory and practice in mathematics education. In R. Biehler, R. W. Scholz, R. Sträßer, & B. Winkelmann (Eds.), *Didactics of mathematics as a scientific discipline* (pp. 89–102). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Stenhouse, L. (1975). *An introduction to curriculum research and development*. London, UK: Heinemann.

Wagner, J. (1997). The unavoidable intervention of educational research: A framework for reconsidering research-practitioner cooperation. *Educational Researcher, 26*(7), 13–22.

Zeuli, J. S. (1994). How do teachers understand research when they read it? *Teaching and Teacher Education, 10*, 39–55.