Chapter 12
Boiler Circuits and Steam Generation

As described in the previous chapter, furnaces and boilers are manufactured in a wide range of types and sizes. Regardless of steaming capacity, the steam generation process in any boiler is the same. Heat released in the furnace by the combustion of a fuel is transferred to water in tubes in which steam is formed. Boilers are distinguished one from another by their physical size, the balance between radiative and convective heat transfer from the furnace to the water tubes, the rate of heat release and steam production, the method of separation of the steam and water phases and the operating pressure and temperature of the steam produced.

The simulation objective will vary, depending on the purpose to which the simulation will be put. Smaller boilers in process plants are generally part of the site's energy recovery and generation, and the operational interest in the boilers is frequently their integration with the site's overall steam and fuel supply systems. The detailed behaviour of the boiler plant itself is of secondary interest, and a lower level of simulation detail will suffice. The operational interest for a power utility plant will be operator training and the development and pre-commissioning tuning of a plant's automation and control system. Increasingly this interest is being shown by the process industries and therefore even for industrial boilers, the detailed behaviour of the boiler plant, during both normal and abnormal operations, will be of interest and may require a more precise treatment of the boiler and furnace dynamics.

12.1 Power Generation Utility Boilers

Large steam power-generating plants fired with fossil fuels are built in any of two basic configurations. In regions with relatively low fuel costs, boilers below about 600 MW have usually been of the drum type, with one or two horizontally mounted steam/water separating drums mounted at the top elevation of the waterwall. Drum boilers are limited in rating to less than 700 MW or so because of the high costs of the drums and of the structure needed to support them. Some reduction in structural

P. O'Kelly, *Computer Simulation of Thermal Plant Operations*,
DOI 10.1007/978-1-4614-4256-1_12,

costs in very large boilers can be gained by using twin but smaller drums. These boilers operate with continuous circulation of water from the drum to the bottom of the evaporator (the riser) and thence back to the drum. They are limited to operating pressures less than 19 MPa, and the fluid emerging from the top of the riser contains up to 33 % steam by mass.

If fuel costs are relatively high or if advantage is to be taken of the improved efficiencies achievable at higher steam temperatures, once-through boilers may be preferred for units with rated outputs above 400 MW. These plants do not maintain a circulation flow during normal loaded operation but instead introduce feedwater directly into the riser where it is raised first to saturation temperature and then converted progressively to 100 % steam with some superheating while still in the riser. There being no water phase left in the riser exit flow, there is no need for water/steam separation, and a drum is not required. However, when starting up, these boilers operate in recirculating mode and during this phase water/steam separation is required.

Once-through plants are built as *subcritical*, with rated steam conditions below the critical pressure (22.4 MPa), or *supercritical*. Operating conditions of supercritical plant vary throughout the world, being typically around 25 MPa (3,700 psi) and 537°C (1,000°F) in the USA and 28.5 MPa and 620°C (1,100°F) in Europe. Future designs, termed ultra-supercritical (USC), are planned with operating pressures up to 34 MPa and temperatures up to 700°C [93]. The two most common types are Benson (licence held by Siemens AG, Germany) and Sulzer (Sulzer Bros., Switzerland). The Benson design is intended for once-through operation at supercritical pressures. The Sulzer design was originally developed as a means of circumventing the high-water-purity requirement of the Benson design and was intended to operate at subcritical pressures with a small continuous recirculation flow. Continuous blowdown from the water separator assists water quality control. This arrangement is still in use and maintains a riser exit mass fraction of around 97%. This type of plant is frequently used with lignite (brown coal) firing in which furnace exit temperatures must be held below the ash fusion temperature to avoid serious slag formation problems in the upper furnace and superheating regions. The following table summarises some salient features and merits of each type.

The choice of a particular configuration will always be influenced by construction costs but may include operational considerations, for example, the ash slagging issue.

12.1.1 Boiler Design Parameters

The vertical profile of heat flux to the waterwalls is not uniform and exhibits a peak in the region of the burners. In most furnaces these are located in the lower half with the elevation of the uppermost burner row about 1/3 to 1/2 the height of the furnace. The waterwall tubes are welded together to form a membrane wall, with a tube centre line spacing typically around 1.5–2 times the tube outer diameter.

	Drum boiler	Once-through
Capital cost	Ultimately limited by cost of drum and support structure	High cost of special steel thick-walled tubing less important at higher ratings
		Sensitivity to water purity penalises cost by requiring expensive demineralising plant
Construction features	Riser formed from large number of smooth-bore tubes	Small number of spiral-wound tubes in lower evaporator section with a larger number of smooth vertical tubes in the upper section
		May use rifled tubes to enhance heat transfer in the spiral-wound section
Efficiency	Maximum efficiency limited by attainable pressure and temperature ($< p_{crit}$)	High efficiencies achievable at supercritical pressures and temperatures
Operations	Start-up and load-change rates limited by drum stress limits (<50°C/h)	Fast start-up and shutdown due to less restrictive stress limits
	Simple operation and control	Complex start-up procedure and on-load control
	Normally operated with fixed pressure and temperature	Well suited to sliding pressure operation

The design of the heat flux distribution attempts to minimise large temperature differentials which could cause uneven expansion and thermal stresses within the walls. Because of their higher operating temperatures, once-through boilers use spiral-wound tubing in the lower furnace area—the region of highest heat flux—to assist this objective. Spiral-wound tubes are longer than vertically arranged tubes for the same heat transfer area. The tubes being longer, the residence time of the riser fluid in the tubes is greater. The gravitational head, expressed in terms of the resolved vertical component of the flow, is $g \sin\beta$ where β is the angle of inclination of the tubes to the horizontal. Typical inclination angles are 10–20° giving tube lengths 3–5 times the vertical height. In other boiler designs, the riser tubes are grouped into separate banks in order to optimise heat flux and temperature distribution.

Basu et al. [15] quote recommended values of flow velocities and specific mass flows (kg/s/m^2) for various boiler configurations. The following table presents selected extracts from that reference. All figures are quoted on a per tube basis.

Natural- and assisted-circulation boilers—water velocities

Drum pressure MPa	4–6	10–12	14–16	17–19
Water velocity at riser inlet m/s	0.5–1.0	1.0–1.5	1.0–1.5	1.5–2.0
Downcomer inlet velocity m/s	≤3	≤3.5	≤3.5	≤4

Once-through boilers—specific mass flow rates [kg/s/m^2] in the lower radiative region

	Vertical tube panel		
	Single pass	Multi pass	Spiral-wound
Subcritical	1600–2700	1200–2000	1500–2500
Supercritical	2100–2700	1600–2000	2000–3000

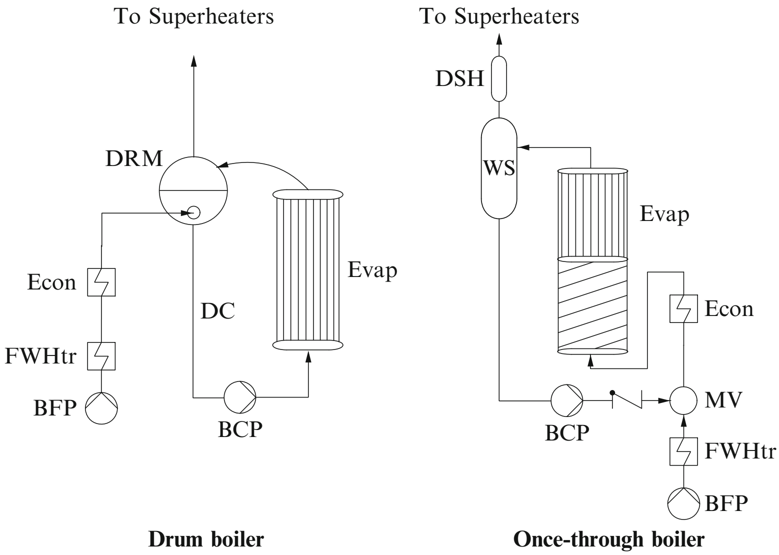
These guidelines can be used to determine the principal design parameters of an evaporator, given its type and rating. This can be useful if detailed plant data is not available.

Example 1: Supercritical Benson boiler

Rated MCR	435 MW
Rated steam flow	350 kg/s
Rated operating pressure	24.5 MPa (242 bar)
Rated steam temperature at boiler exit	589°C
Water spec. vol. at riser inlet v'	0.001556 m^3/kg
Riser inlet temperature (from economiser)	290°C
Target mass flow density ϕ	2,000 kg/s/m^2
Riser inlet flow velocity $= \phi\, v'$	3.1 m/s
Assumed tube ID	0.036 m
Inlet flow rate per tube $= \phi\, A$	2.04 kg/s
Number of riser tubes	172

Example 2: Natural-circulation drum boiler

Rated MCR	500 MW
Rated steam flow	400 kg/s
Rated operating pressure	18.24 MPa (180 bar)
Rated steam temperature at boiler exit	540°C
Water spec. vol. at riser inlet v'	0.00104 m^3/kg
Riser inlet temperature (from economiser)	320°C
Target circulation ratio	7:1
Target circulation flow rate	2,940 kg/s
Target mass flow density ϕ	1,500 kg/s/m^2
Riser inlet flow velocity $= \phi\, v'$	1.56 m/s
Assumed tube ID	(i) 0.054 (ii) 0.064 m
Inlet flow rate per tube	(i) 3.45 (ii) 4.83 kg/s
Number of riser tubes	(i) 852 (ii) 609



DRM Steam drum
WS Water separator
BCP Boiler circulationpump
BFP Boiler feedwaterpump
Evap Evaporator
DC Downcomers
Econ Economiser
FWH Feedweaterheater
MV Mixing vessel
DSH Desuperheater

Fig. 12.1 Water circuit arrangements of a power plant boiler

12.1.2 Evaporator Water Circuits

The general arrangement of the evaporator circuit is shown by Fig. 12.1 for a drum-type and a once-through boiler.

Drum Boilers

Wet steam emerging from the riser is separated into its steam and water components in the drum(s). The steam phase leaves the drum as saturated steam and flows to the superheaters while the water phase is mixed with incoming feedwater and recirculated. Water travels from the drum to the bottom of the boiler via the downcomer tubes, a set of typically 3–9 large pipes set into the bottom of the drum. The entry orifice to each downcomer tube is fitted with a coarse grid of crossed plates to prevent the formation of vortices (vortex suppressors). The minimum water

Fig. 12.2 Tubes attached to a lower boiler header (courtesy of Foster-Wheeler corp.)

volume permitted in the drum during operation is chosen to ensure the absence of vortices which could draw steam into the downcomers. The presence of a vapour phase in the downcomers can adversely affect the circulation flow and initiate hydrodynamic flow instability.

The downcomer pipes receive no heat from the furnace but lose heat to ambient. They terminate at the bottom of the boiler in the bottom header(s) (Fig. 12.2) from which the flow divides into a large number of parallel streams (individual riser tubes) which ascend back to the boiler drum. These are usually divided into sets of vertical banks of tubes, distributed around and forming the walls of the furnace. A number of boiler circulation pumps (BCPs) (usually less than four) may be installed between the bottom header(s) and the riser banks. A boiler with circulation pumps is termed an *assisted-circulation* boiler, and the pumps run continuously. One without pumps is termed a *natural-circulation* boiler, the circulation flow being induced by the buoyancy of the steam phase in the risers. Circulation is maintained around the loop during all phases of operation, except of course when the plant is out of service. While this may seem a trite observation, it highlights the need, from a modelling perspective, to ensure validity of the model when flow is zero. In a drum boiler with assisted circulation, water will leave the riser with a steam mass fraction of up to around 33 % at rated output. The riser exit mass fraction in a natural-circulation boiler is less, at around 12–18 % at full load. In these boilers, to reduce the pressure losses around the circuit, the riser and downcomer tubes are of larger diameter than those in assisted-circulation boilers.

The ratio of the full-load steady-state steam flow leaving the evaporator—equal to the steam production rate—to the total circulation flow is the *circulation ratio*. It is the inverse of the mean steam mass fraction at the riser exit and is typically around 3:1 for an assisted-circulation boiler, 7:1 for natural circulation and 10:1 for an industrial boiler.

The vertical height differential of the riser tubes (the difference between the top and bottom elevations) will typically be of the order of 20–50 m for a drum boiler and up to 90 m for large supercritical boilers, which tend to be built in tower configuration. Tube numbers vary in the range of 150–1,200 with an outer diameter of between 35 and 85 mm. The wall thickness depends on the tube internal diameter and operating pressure and usually lies in the range 3–6 mm. The heat transfer area is large, and the tubes are exposed on the furnace side to radiant and convective heat transfer from the furnace chamber. In consequence of the vertical head, a static pressure differential exists between top and bottom of the riser (of the order of 300–800 kPa prior to boiling when full of water, reducing with the formation of steam), creating local variations of saturation conditions which will influence the local formation of steam, particularly under the low-pressure conditions during start-up.

Feedwater is introduced into the drum where it mixes with the drum water contents. The sub-cooling effect of the incoming feedwater keeps the water temperature in the drum below the saturation temperature. This and the higher static pressure at the riser inlet ensure that the riser inlet flow is sub-cooled. It should be noted that prior to first boil-off, there will be no feedwater flow to the drum, unless needed to make up water lost through the drum blowdown which, in some plants, is a normal part of the start-up procedure in order to maintain the necessary boiler water quality.

Once-Through Evaporators

The flow up the riser is progressively heated by heat absorbed from the tube walls. In a drum boiler under load, the riser flow reaches the top of the riser with up to around 2/3 water content. In a once-through boiler operating in once-through mode, the riser converts completely to steam somewhere before the riser exit. The point along the riser flow path at which the water phase disappears (steam mass fraction becomes 1) is termed the dry-out point. During start-up, shutting down and at low load, dry out will not be achieved, and the riser exit flow will need separation of the phases. A vertical water/steam separator vessel is provided between riser exit and inlet, with a forced recirculation path back to the riser inlet. During operation with recirculation, feedwater is introduced into and mixed with water leaving the separator vessel in a mixing chamber below the bottom of the water separator. In some designs, the mixing chamber may be located upstream of the economiser. The mixture is passed to the bottom of the riser and ascends in the usual way but now forced by the feedwater pumps, an arrangement called *forced circulation* operation. The total flow to the riser is the sum of the feedwater flow from the feed pump(s) and the recirculation flow from the boiler recirculation pump. At least a minimum flow to the evaporator tubes is guaranteed at all times to ensure adequate cooling of the economiser and boiler tubes. Circulation around the separator loop is maintained up to around 40–50 % load (start-up and low-load operation) or at some predetermined steam pressure (typically around 12–14 MPa) at which time the riser exit flow to the water separator is close to or has passed the dry-out point. At this stage, the recirculation flow is stopped, the recirculation line isolated and once-through operation commenced. Without the recirculation mixing, the riser inlet temperature is the final feedwater temperature (economiser outlet) which, to avoid steam formation in the economiser, will be less than the local saturation temperature and again, the incoming water is sub-cooled.

In boilers subject to frequent starts and shutdowns, an alternative recirculation arrangement may be used whereby the liquid phase from the water separator is passed through a recuperative heat exchanger before being passed to the deaerator. This arrangement can improve overall efficiencies by recovering blowdown losses. The return of the drain to the deaerator raises the deaerator water temperature and reduces bled steam demand, both by the deaerator and HP feedwater heaters [94].

Steam Formation

As the water flows upwards along the riser tubes, it absorbs heat from the tubes and its enthalpy increases. Since it is moving upwards, static pressure will decrease with height and with it the saturation temperature. If sufficient heat is absorbed from the tubes, the water enthalpy will be increased to the local saturation enthalpy. At this point, the flow is no longer sub-cooled and the addition of any further heat will cause boiling to commence. The point of transition between saturated water and saturated two-phase fluid is denoted the "boiling boundary". This is not a sharp demarcation point and is preceded by a sub-cooled zone in which small steam bubbles form.[1] If insufficient heat is absorbed by the water flow, because either the inlet enthalpy or the riser tube temperature is too low, the riser flow will not reach saturation enthalpy and will remain sub-cooled over its complete length. This is the case during start-up prior to the first boil-off and can also be established some time after a unit trip.

Experimental data indicates that local evaporation commences in the riser before the bulk flow has reached saturation enthalpy. Due to the high heat flux at the tube/fluid interface, the fluid temperature at the interface will be higher than the bulk fluid temperature, and steam bubbles will form at this interface before bulk boiling commences. The bubbles detach from the tube wall and are entrained by the upward riser flow. The fluid velocity is highest and hence the static pressure is lowest along the centre line of the riser flow. The steam bubbles from the tube interface will move towards this region of local low pressure. Thus, although steam is produced predominantly at the tube wall interface, the riser flow in cross section

[1] A more detailed discussion of this and other boiling processes is given in Chap. 13.

shows a higher concentration of steam bubbles towards the centre line than at the interface. Bubbles coalesce into clumps and large voids, passing through the various flow regimes described in Chap. 13.

Above the boiling boundary the riser flow develops into two phases as an increasing fraction of the water flow converts to steam. This gives rise to increases in,

(a) The two-phase pressure loss along the riser attributable to the effective reduction of the water-phase flow area by the steam phase.
(b) The volume occupied by the steam phase (increases in the ratio of $v'' : v'$) with a corresponding displacement of water towards the upper reaches of the riser.
(c) The upward buoyancy force due to the reduction in fluid density.
(d) The two-phase heat transfer due to the effects of local turbulence caused by the production and movement of the steam bubbles.

In the once-through evaporator, as the dry-out point is approached, the wall heat transfer coefficient decreases markedly, causing an attendant sharp rise in tube wall temperature. Beyond the dry-out point, the steam is single phase and the addition of further heat increases its temperature.

Various valves attached to the evaporator circuit include

- Safety valves
- Blowdown valves
- Waterwall upper and lower header drains
- Water separator drains
- Boiler circulation drains

12.2 Modelling Treatment

The primary interest of the evaporator model is the calculation of the profiles of riser fluid and metal temperatures, circulation flow rate, steam production rate and the relative fractions of water and steam phases in the riser tubes.

Many of the operational characteristics of the evaporator loop result from its essentially distributed nature, that is, the influence of processes arising from variation of state and property variables in and across the direction of flow. By treating only single-dimensional variations in the axial flow direction, we will tacitly assume uniform fluid properties normal to the direction of flow.

The differential equations determining the dynamic behaviour of the evaporator system are derived by developing the non-stationary mass, momentum and energy balances for the drum or water separator, the downcomer and riser and the overall circulation flow. The following table summarises the salient differences of drum and once-through boilers from the modelling perspective.

		Once-through mode	
	Drum	Recirculation	Once-through
Pressure defined by	Drum	Water separator	FW pumps
Circulation	Natural or assisted	Forced	Forced
Feedwater to	Drum	Riser	Riser
Riser flow =	Downcomer flow	Recirc + feedwater	Feedwater

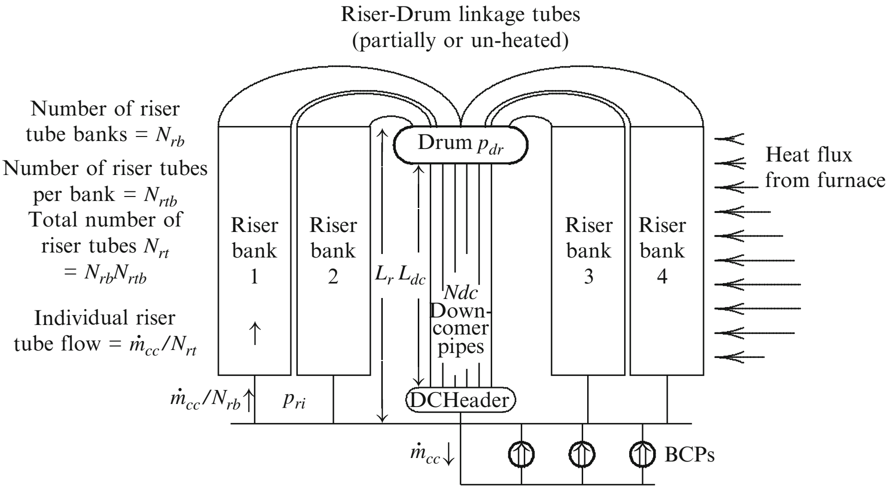


Fig. 12.3 Schematic representation of drum boiler circulation paths

12.2.1 Drum Boilers

The total circulation mass flow $\dot{m}_{cc}$ is obtained from the balance of individual pressure rises and drops around the evaporator circuit.

$$\text{drum static head} - \text{riser static head} + \text{circulation pumps developed head}$$
$$= \text{downcomer head loss} + \text{riser head loss (one/two phase)}.$$

With reference to Fig. 12.3, the first term is simply $g\,L_{dc}/v_{dc}$. The riser static head is identical, with the downcomer water specific volume v_{dc} replaced by the mean riser specific volume v_{rm}. L_r denotes the vertical length of the riser and L_{dc} that of the downcomer. The net buoyancy head is then $g\,(L_{dc}/v_{dc} - L_r/v_{rm})$. The effective geodetic heights of both riser and downcomer may be taken as equal by referring each to the same upper and lower datums. The tubing linking the riser exit to the drum may be taken as contributing zero net height.

The circulation flow is assumed evenly distributed across the downcomer tubes and the individual downcomer tube flow $\dot{m}_{dc} = \dot{m}_{cc}/N_{dc}$. The downcomer head loss is the friction loss per tube $k_{dc}\dot{m}_{dc}^2$. The friction loss coefficient is given from Eq. 6.6 as

$$k_{dc} = \frac{\xi L_{dc}\, v_{dc}}{2\, d_{dc} A_{xdc}^2}$$

A_{xdc} is the flow cross-sectional area of a downcomer pipe and ξ is the Fanning friction factor. Alternatively, Zlin [47] Chap. 8, quotes

$$k_{dc} = \left(\xi \frac{L_{dc}}{d_{dc}} + \sum \xi_M \right) \frac{\nu^2}{2} \rho_l,$$

where ν is the flow velocity = $\dot{m} v / A_{xdc}$. The friction factor ξ is calculated from

$$\xi = 0.25\, [\log(3.7\, d_i / k)]^{-2}$$

and k is the hydraulic roughness [mm].

The downcomer bank pressure loss coefficient is

$$K_{dc} = \frac{1}{N_{dc}^2} k_{dc}.$$

Figure 12.3 depicts the riser as divided into four parallel and, for the present purpose, identical banks of N_{rtb} tubes per bank. In general, it may be divided into any number N_{rb} of unequal banks. This is consistent with the usual construction of boilers in which the riser may be divided into front, rear, left and right sidewalls and upper, lower, screen and sundry other tube bank designations. If N_{rt} is the total number of riser tubes (= $N_{rb} N_{rtb}$), the mass flow per tube is $\dot{m}_{rt} = \dot{m}_{cc} / N_{rt}$. The single-phase pressure gradient along the riser is $k_{1\phi} \dot{m}_{rt}^2$ where the pressure loss coefficient is

$$k_{1\phi} = \frac{\xi\, v_r}{2\, d_{i,r} A_{x,r}^2}.$$

The riser head loss is composed of the single-phase friction drop in the sub-cooled zone (water) plus the two-phase pressure drop in the saturated or boiling zone. Two-phase effects are accommodated by multiplying the single-phase pressure loss coefficient $k_{1\phi}$ by an appropriate two-phase multiplier, calculated as a function of local flow conditions.[2] For example, using the Doležal multiplier (Eq. 13.3),

$$\phi_{lo}^2 = 1 + x \left(\frac{v''}{v'} - 1 \right) + 8 \frac{\dot{q} v''}{k_{r,1ph} \mathbf{r} \nu_0}.$$

Assuming identical thermal and flow conditions in each tube, the per tube pressure loss coefficient $k_{1\phi}$ in the riser converts to a bank-based coefficient $K_{1\phi}$ as

$$K_{1\phi} = k_{1\phi} / N_{rt}^2.$$

[2] Refer to Sect. 13.1.5.

As discussed in Sect. 7.2.1, the BCP head-flow characteristic can be approximated by a parabolic curve passing through selected points on the test characteristic.

$$\Delta p_{bcp}(n_{bcp}) = dp_0(\omega^2) - k_{bcp}\dot{m}_{cc}^2$$

dp_0 is the zero-flow head of the pump and is proportional to the square of the pump rotational speed ω. n_{bcp} denotes the number of BCPs in service.

Combining these terms gives

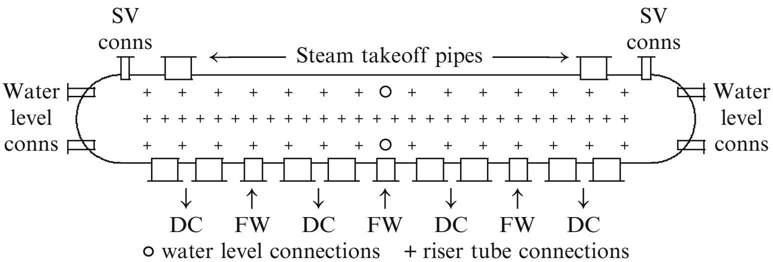
$$g\left[\frac{L_{dc}}{v_{dc}} - \frac{L_r}{v_{rm}}\right] + dp_0(\omega_{bcp}^2) - k_{bcp}(n_{bcp})\dot{m}_{cc}^2 = K_{dc}\dot{m}_{cc}^2 + K_{1\phi}\phi_{lo}^2\dot{m}_{cc}^2. \quad (12.1)$$

This quadratic equation can be solved for the total circulation flow $\dot{m}_{cc}$, given the combined head/flow characteristic of the in-service BCPs.

12.2.2 The Boiler Steam Drum

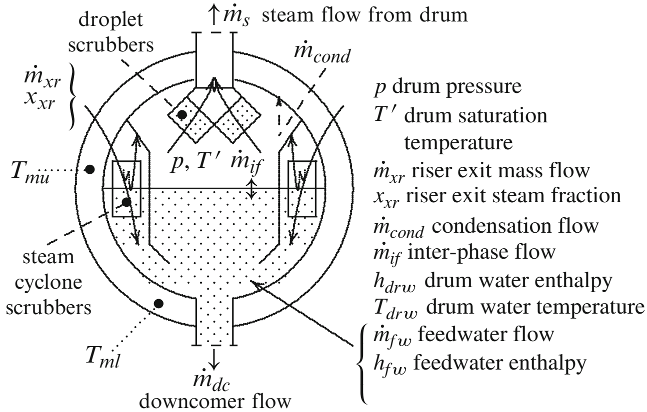
A boiler may be equipped with one or more steam drums, depending on rating and construction type. The drum serves several purposes:

(a) Separates the water and vapour phases of the fluid emerging from the riser.
(b) Mixes the incoming feedwater with water in the drum to provide the water source for the evaporator.
(c) Establishes a reference point for control of the water mass within the evaporator system.
(d) Provides a droplet-free supply of steam to the main steam system.



From the simulation point of view, the drum provides the thermodynamic boundary conditions for both the evaporator and the downstream steam system and, most importantly, steam pressure and its rate of change. In the following discussion, the drum is treated as an isolated component, connected to its interfacing systems by conditions defined at its boundaries. Although the drum is relatively long in comparison with its diameter, the distributed effects of the complex flow patterns within the drum will be ignored. For example, it will be assumed that the incoming feedwater mixes immediately and perfectly with the existing water phase to produce a single representative drum water temperature. In the real drum, this is not the case, and temperature variations exist throughout the water, both lengthwise along the drum and vertically.

Uneven patterns of flow to the downcomers and emerging from the risers produce local variations of water level along the length of the drum. To detect these it is usual practice to provide sets of tapping points for water-level measurements at a minimum of three locations along the drum—one at each end and one in the middle. It has been observed during operation that the level profile varies with operating condition such as load, but it is difficult to identify generalised repeatable and predictable patterns as they depend strongly on plant construction details. An important influence parameter is the method of connection of feedwater piping to the drum. The preceding diagram shows feedwater connection stubs interleaved between the downcomers. An alternative method is to connect the feedwater inflow to one end of the drum, in larger plants via two stubs, and distribute it along the drum through perforations in the pipe(s), under the normal water level.



The water and steam phases occupy well-delineated spaces which meet at the surface of the water space. Given the large mass and energy flows within the drum and their turbulent conditions, this interface surface is unlikely to be either smooth or steady. However, for this level of modelling, it will be expedient to treat it as smooth and sufficiently well defined to serve as a measurable line of separation.

Differing approaches can be taken to the calculation of the thermodynamic state of the drum contents. The simplest is to treat them as a saturated homogeneous two-phase mixture of liquid and vapour phases coexisting in thermodynamic equilibrium. The temperature of the mixture is the saturation temperature corresponding to the drum pressure. Mass and energy balances for the contents can then be written in terms of the net mass $\Sigma(\dot{m})$ and enthalpy $\Sigma(\dot{m}\,h)$ flows crossing the boundary.

This approach of necessity ignores any inter-phase mass or energy transfer processes which might be taking place within the drum.

The Homogeneous Contents Approach

The mass balance taken over the drum contents, assumed homogeneous, yields

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \Sigma(\dot{m}), \tag{12.2}$$

where M is the total mass of the drum contents. The net mass balance of all flows crossing the drum's boundary is

$$\Sigma(\dot{m}) = \dot{m}_{fw} + \dot{m}_{rx} - \dot{m}_{dc} - \dot{m}_s - \dot{m}_{bl}$$

$\dot{m}_{bl}$ is the blowdown flow from the drum. This is taken from the drum as part of the chemical water treatment system and is used to reduce dissolved solids in the boiler water. It may be an intermittent or continuous flow, depending on the quality of the boiler make-up water.

The energy balance in turn yields

$$\frac{\mathrm{d}(M\,h)}{\mathrm{d}t} = \Sigma(\dot{m}\,h) - V\frac{\mathrm{d}p}{\mathrm{d}t} + \dot{q}_{hx},$$

where h is the mixture mean specific enthalpy, V is the drum volume and p is the drum pressure. $\dot{q}_{hx}$ is the total heat transferred across the boundary. $V\,\mathrm{d}p/\mathrm{d}t$ is the rate of change of compression energy to which only the compressible part of the drum contents contributes. If x_s is the mass fraction of the vapour phase and v'' is the specific volume of saturated vapour, V is replaced by the drum steam-phase volume $V_s = x_s M\,v''$.

This equation can be expanded to

$$h\frac{\mathrm{d}M}{\mathrm{d}t} + M\frac{\mathrm{d}h}{\mathrm{d}t} = \Sigma(\dot{m}\,h) - V_s\frac{\mathrm{d}p}{\mathrm{d}t} + \dot{q}_{hx}$$

from which

$$M\frac{\mathrm{d}h}{\mathrm{d}t} + x_s M\,v''\frac{\mathrm{d}p}{\mathrm{d}t} = \Sigma(\dot{m}\,h) - h\Sigma(\dot{m}) + \dot{q}_{hx}. \tag{12.3}$$

Equations 12.2 and 12.3 contain four unknowns, M, h, p and x_s, leaving two further equations to find. The first of these is provided by the assumption that the drum contents exist in a state of thermodynamic equilibrium. This allows the replacement of h by its representation in terms of x_s and the saturation specific enthalpies of the liquid h' and vapour h'' phases.

$$h = x_s h'' + (1 - x_s)h'.$$

We could then write

$$\begin{aligned}\frac{\mathrm{d}h}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t}(x_s h'' + (1-x_s)h') \\ &= x_s \frac{\mathrm{d}h''}{\mathrm{d}t} + h'' \frac{\mathrm{d}x_s}{\mathrm{d}t} + (1-x_s)\frac{\mathrm{d}h'}{\mathrm{d}t} - h' \frac{\mathrm{d}x_s}{\mathrm{d}t} \\ &= x_s \frac{\mathrm{d}h''}{\mathrm{d}t} + (h''-h')\frac{\mathrm{d}x_s}{\mathrm{d}t} + (1-x_s)\frac{\mathrm{d}h'}{\mathrm{d}t} \\ &= x_s \frac{\mathrm{d}h''}{\mathrm{d}t} + \mathbf{r}\frac{\mathrm{d}x_s}{\mathrm{d}t} + (1-x_s)\frac{\mathrm{d}h'}{\mathrm{d}t},\end{aligned}$$

where $\mathbf{r} = h'' - h'$. Since h' and h'' are functions of the single variable p, the derivatives of h' and h'' may be written in terms of $\mathrm{d}p/\mathrm{d}t$ using their partial derivatives.

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \left(x_s \frac{\partial h''}{\partial p} + (1-x_s)\frac{\partial h'}{\partial p} \right) \frac{\mathrm{d}p}{\mathrm{d}t} + \mathbf{r}\frac{\mathrm{d}x_s}{\mathrm{d}t}.$$

This equation would allow the elimination of $\mathrm{d}h/\mathrm{d}t$ from Eq. 12.3 but introduces a new time derivative $\mathrm{d}x_s/\mathrm{d}t$ for which an explicit expression must be found. While this can be found from the definition of x_s the resulting equations become quite cumbersome, and an alternative approach should be sought.

The Separated Phases Approach

We first observe that the assumed homogeneity of the drum contents is a poor representation of reality. There are clear divisions between the liquid and vapour phases which occupy different spaces at different temperatures. The flows from the drum to the downcomers and to the steam system are of different phases and specific enthalpies, and the individual terms making up $\Sigma(\dot{m}\, h)$ cannot be expressed simply in terms of the mixture enthalpy h. Further, the assumption of saturation conditions in the drum obviates the need for a separate energy balance, and therefore, the energy equation 12.3 is unnecessary. Were we to relax the saturation assumption, we would need a separate energy balance for each phase to define the conditions for return to saturation and would need to describe a mechanism for mass and energy exchange between the phases.

These issues can be avoided if we proceed as follows. The vapour-phase mass fraction x_s is defined to be

$$x_s = 1 - \frac{M_w}{M}. \tag{12.4}$$

The total mass of the drum contents M is given from Eq. 12.2. The mass of the liquid (water) phase M_w is obtained from a separate water-phase mass balance.

$$\frac{\mathrm{d}M_w}{\mathrm{d}t} = \Sigma(\dot{m})_w, \tag{12.5}$$

where

$$\Sigma(\dot{m})_w = \dot{m}_{fw} - \dot{m}_{dc} + (1 - x_{rx})\dot{m}_{rx} - \dot{m}_{bl} + \dot{m}_{if}$$

and $(1 - x_{rx})\dot{m}_{rx}$ is the water fraction of the flow emerging from the riser(s). The total mass of the drum contents is the sum of the two individual phase masses $M_s + M_w$. The mass of steam M_s is therefore

$$M_s = M - M_w.$$

Differentiating this equation with respect to time gives

$$\frac{\mathrm{d}M_s}{\mathrm{d}t} = \frac{\mathrm{d}M}{\mathrm{d}t} - \frac{\mathrm{d}M_w}{\mathrm{d}t}$$

from which we obtain

$$\frac{\mathrm{d}M_s}{\mathrm{d}t} = x_{rx}\dot{m}_{rx} - \dot{m}_s - \dot{m}_{if} - \dot{m}_{cond} = \Sigma(\dot{m})_s$$

$\dot{m}_{cond}$ is the rate of condensation on the inner surface of the upper drum in contact with steam. Heat transfer from the steam to the wall is $\dot{q}_{hxs}$ where

$$\dot{q}_{hxs} = \alpha_{hxs} A_{hxs}(T' - T_{dru}) = \dot{m}_{cond}\,\mathbf{r},$$

where α_{hxs} and A_{hxs} are the heat transfer coefficient and contact surface area between the steam and drum wall, respectively. T' is the saturation temperature, and T_{dru} is the temperature of the upper wall in contact with the steam. Therefore,

$$\dot{m}_{cond} = \dot{q}_{hxs}/\mathbf{r}$$

$\dot{m}_{if}$ is the interface flow (condensation or evaporation) passing between the two phases across the steam-water interface. This is a complex mechanism of relatively little operational consequence. Here it is heuristically represented as being proportional to the difference between saturation and drum water temperatures.

$$\dot{m}_{if} = k_{if}(T' - T_w).$$

Since the drum water mean temperature will usually be less than the saturation temperature due to sub-cooling by the feedwater, this representation produces a predominantly condensation interface flow. However, it will almost always be small and of no consequence to normal operations. Its largest contribution will appear during large transient variations of drum pressure and after shutdown of the plant when, together with condensation on the drum wall surface, it will contribute to the slow decrease of drum pressure.

An alternative expression for $\mathrm{d}M_s/\mathrm{d}t$ is, with $M_s = V_s/v''$,

$$\begin{aligned}\frac{\mathrm{d}M_s}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{V_s}{v''}\right) \\ &= \frac{1}{v''}\frac{\mathrm{d}V_s}{\mathrm{d}t} - \frac{V_s}{v''^2}\frac{\mathrm{d}v''}{\mathrm{d}t} \\ &= \frac{1}{v''}\frac{\mathrm{d}V_s}{\mathrm{d}t} - \frac{V_s}{v''^2}\frac{\partial v''}{\partial p}\frac{\mathrm{d}p}{\mathrm{d}t}\end{aligned}$$

from which we have

$$-\frac{V_s}{v''^2}\frac{\partial v''}{\partial p}\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{1}{v''}\frac{\mathrm{d}V_s}{\mathrm{d}t} + \Sigma(\dot{m})_s.$$

From Eq. 4.34,

$$\frac{\partial v''}{\partial p} = -\frac{v''}{\gamma\, p},$$

which leads to

$$\frac{V_s}{\gamma\, p}\frac{\mathrm{d}p}{\mathrm{d}t} = v''\Sigma(\dot{m})_s - \frac{\mathrm{d}V_s}{\mathrm{d}t}. \tag{12.6}$$

The volume of the steam phase equals the total drum volume V_{drm} less the volume of the water phase. Then with $V_w = v' M_w$,

$$V_s = V_{drm} - V_w = V_{drm} - v' M_w.$$

Differentiating this equation with respect to time gives

$$\frac{\mathrm{d}V_s}{\mathrm{d}t} = -v'\frac{\mathrm{d}M_w}{\mathrm{d}t}$$

assuming v' is constant in the short term. After substitution for $\mathrm{d}V_s/\mathrm{d}t$, Eq. 12.6 may be written.

$$\left(\frac{V_s}{\gamma\, p}\right)\frac{\mathrm{d}p}{\mathrm{d}t} = v''\Sigma(\dot{m})_s + v'\Sigma(\dot{m})_w.$$

Noting that

$$\frac{\mathrm{d}(\ln\, p)}{\mathrm{d}t} = \frac{1}{p}\frac{\mathrm{d}p}{\mathrm{d}t}$$

this last equation may be written

$$\frac{\mathrm{d}(\ln\, p)}{\mathrm{d}t} = \frac{\gamma}{V_s}\left[v''\Sigma(\dot{m})_s + v'\Sigma(\dot{m})_w\right] \tag{12.7}$$

and $p = \exp(\ln\, p)$.

The right-hand side of Eq. 12.7 depends implicitly on p as the flows to and from the drum depend on drum pressure. As this analysis treats the drum as an isolated system, it assumes that these interfacing variables are defined by external systems which are aware of the drum pressure.

Drum Water Energy Balance The temperature of the drum water phase is determined from the mixing of all water flows into and out of the drum. Then,

$$\frac{\mathrm{d}(M_w h_w)}{\mathrm{d}t} = \Sigma(\dot{m}h)_w - \dot{q}_{hxw}, \tag{12.8}$$

where net energy flow into and from the drum water is

$$\Sigma(\dot{m}h)_w = \dot{m}_{fw}h_{fw} + (1 - x_{rx})\dot{m}_{xr}h' - \dot{m}_{dc}h_w - \dot{m}_{bl}h_w + \dot{m}_{if}(h'' - h')$$

for which we assume that the water phase of the riser flow is saturated. During start-up prior to first boil-off, this will not be the case, and the actual riser exit enthalpy must be used in place of h'.

During start-up and initial pressure raising, feedwater flow to the drum is minimal, comprising the mass flow of steam filling and pressurising the steam system piping, including superheaters and headers, and blowdown to control water chemistry. The normal sub-cooling effects of the feedwater are therefore minimal during this phase. The static head in the riser will allow the water phase of the flow exiting the riser to reach a higher temperature than the drum saturation temperature, and part of the riser flow will evaporate in the drum ($\dot{m}_{if}$). The remainder will mix with the drum water and increase its temperature.

The heat transfer to the lower wall of the drum in contact with the water phase is

$$\dot{q}_{hxw} = \alpha_{drw}A_{drw}(T_{drw} - T_w),$$

where α_{drw} and A_{drw} are the heat transfer coefficient and contact surface area between the drum water and wall, respectively.

Equation 12.8 expands to

$$\frac{\mathrm{d}(M_w h_w)}{\mathrm{d}t} = h_w\frac{\mathrm{d}M_w}{\mathrm{d}t} + M_w\frac{\mathrm{d}h_w}{\mathrm{d}t}$$

from which we have

$$\begin{aligned} M_w\frac{\mathrm{d}h_w}{\mathrm{d}t} &= h_w\frac{\mathrm{d}M_w}{\mathrm{d}t} + \Sigma(\dot{m}h)_w - \dot{q}_{hxw} \\ &= -h_w\Sigma(\dot{m})_w + \Sigma(\dot{m}h)_w - \dot{q}_{hxw} \end{aligned}$$

or

$$M_w\frac{\mathrm{d}h_w}{\mathrm{d}t} + h_w\Sigma(\dot{m})_w = \Sigma(\dot{m}h)_w - \dot{q}_{hxw}. \tag{12.9}$$

After expansion of the two Σ terms, Eq. 12.9 can be written

$$M_w \frac{\mathrm{d}h_w}{\mathrm{d}t} + h_w(\dot{m}_{fw} + (1 - x_{rx})\dot{m}_{rx} + \dot{m}_{if}) \\ = \dot{m}_{fw}h_{fw} + (1 - x_{rx})\dot{m}_{rx}h' + \dot{m}_{if}\mathbf{r} - \dot{q}_{hxw} \qquad (12.10)$$

from which the drum water temperature $T_w = h_w/c_w$. c_w is the specific heat capacity of water.

Drum Wall Metal Temperatures The wall of the drum in a high-pressure boiler is extremely thick, around 200 mm for a 500 MW plant. Like all thick-walled components subject to high temperatures, the maximum temperature differentials through the wall must be limited in order to avoid excessive stresses. This imposes maximum rates of warming during start-up, and the rate of change of drum saturation temperature is usually limited to a maximum of 50–70 °C per hour. This can be the most significant factor in determining the minimum cold start duration. Operating staff and supervisory systems are provided with measurements of drum metal temperatures and their upper/lower differentials for monitoring purposes.

The drum is also subject to bending stress induced by the differential between the temperatures of the upper and lower hemispheres of the drum. The upper part of the drum is in contact with steam with poor heat transfer, and the lower is in contact with lower temperature water but with good heat transfer. Temperature differentials can be calculated independently for the upper and lower sections of the drum. For simulation purposes, a simplified representation of these temperatures is usually sufficient. The derivation of a set of simple equations to calculate two representative temperatures T_{dru} and T_{drl} is outlined, using the heat transfer terms $\dot{q}_{hxw}$ and $\dot{q}_{hxs}$ mentioned in the preceding derivation. The calculation includes a simplified provision for conduction from the upper to the lower sections as well as ambient heat loss through the drum insulation.

$$c_m M_{mu} \frac{\mathrm{d}T_{dru}}{\mathrm{d}t} = -\lambda A_{xm}(T_{dru} - T_{drl}) + \dot{q}_{hxs} - \dot{q}_{amb},$$

$$c_m M_{ml} \frac{\mathrm{d}T_{drl}}{\mathrm{d}t} = \lambda A_{xm}(T_{dru} - T_{drl}) + \dot{q}_{hxw} - \dot{q}_{amb},$$

where M_{mu} and M_{ml} are the masses of metal in the upper and lower hemi-cylindrical halves and c_m is the specific heat capacity of the metal (0.465 kJ/(kg C) for steel).

After substitution of the various heat transfer terms, we obtain

$$c_m M_{mu} \frac{\mathrm{d}T_{dru}}{\mathrm{d}t} = -\left(\lambda A_{xm} + \alpha_{hxs}A_{hxs} + \lambda_{insul}\frac{A_{insul}}{d_{insul}}\right)T_{dru} \\ + \lambda A_{xm}T_{drl} + \alpha_{hxs}A_{hxs}T' - \lambda_{insul}\frac{A_{insul}}{d_{insul}}T_{amb} \qquad (12.11)$$

for the drum upper metal temperature and

$$c_m M_{ml} \frac{\mathrm{d}T_{drl}}{\mathrm{d}t} = -\left(\lambda A_{xm} + \alpha_{hxw} A_{hxw} + \lambda_{insul} \frac{A_{insul}}{d_{insul}}\right) T_{drl} + \lambda A_{xm} T_{dru} + \alpha_{hxw} A_{hxw} T_w - \lambda_{insul} \frac{A_{insul}}{d_{insul}} T_{amb} \qquad (12.12)$$

for the drum lower metal temperature. These equations are linear in the two temperatures and can be solved by any of the usual methods.

The indicative stresses in both the upper and lower sections can be estimated from the instantaneous difference between the representative wall temperature and the appropriate reference source temperature, steam saturation or drum water temperature.

Water Temperature at the Downcomer Outlet

Water enters the downcomer at a temperature close to the drum or water separator water temperature. Assuming that no steam has been entrained from the drum or separator vessel, the fluid entering the downcomer is always water.

In Sect. 5.2.3, the general one-dimensional energy equation was resolved into

$$c_p \frac{\partial T}{\partial t} + c_p \nu \frac{\partial T}{\partial z} = v \left(\frac{\partial p}{\partial t} + \frac{\partial p}{\partial z}\right) - \frac{v}{A} \alpha_{hx} A_{hx} (T - T_{ref}) + v\nu \left.\frac{\partial p}{\partial z}\right|_{loss},$$

which, with water as the working fluid, may be written for the downcomer as

$$c_w \frac{A_{dc}}{v_{dc}} \frac{\partial T_{dc}}{\partial t} + c_w \dot{m} \frac{\partial T_{dc}}{\partial z} + \alpha \pi d_{dc} \, T_{dc} = \alpha \pi d_{dc} \, T_{amb}. \qquad (12.13)$$

The boundary conditions are defined by the initial temperature profile $T_{dc}(z,0)$ for all z at t $=$ 0 and the downcomer inlet temperature $T_{dc}(0,0) = T_{drw}$.

In Sect. 5.2.3, the analytic solution $T_{dc}(z,t)$ of Eq. 12.13 for all (z,t) with the given boundary conditions was shown to be

$$T_{dc}(z,t) = T_{dc}(z,0)\mathrm{e}^{-\tau_t t} + T_{amb}(1 - \mathrm{e}^{-\tau_t t}) + (T_{dc}(0,0) - T_{amb})(\mathrm{e}^{-\tau_z z} - \mathrm{e}^{-\tau_t t}) H(t - z/\nu), \qquad (12.14)$$

where

$$\tau_t = \frac{\alpha \pi d_{dc}}{c_w \frac{A_{dc}}{v_{dc}}}, \qquad \tau_z = \nu \, \tau_t$$

and the flow velocity is

$$\nu = \frac{v_{dc} \dot{m}_{dc}}{A_{dc}}.$$

The downcomer outlet temperature $T_{dcx}(t) = T_{dc}(L_{dc}, t)$ is obtained from Eq. 12.14 by setting $z = L_{dc}$.

While this gives a true reproduction of the transport or residence time of the water in the downcomer, this time (of the order of a few seconds) is insignificant compared to the other thermal delays within the flow loop. For example, for a downcomer 30 m long and a typical flow velocity of 10 m/s, the transit time is 3 s. Not much heat will be lost to the pipe wall, but any changes of drum water temperature will be delayed by 3 s before entering the riser. The downcomer outlet temperature therefore can be approximated effectively and more simply as follows. Given that the volume V_{dc} in the downcomer is constant, the mass of water in the downcomer is

$$M_{dc} = \frac{V_{dc}}{v_{dc}}.$$

Ignoring water compressibility, mass flow in = mass flow out. From the energy balance, we then have for the water in the downcomer

$$M_{dc}\,\frac{\mathrm{d}}{\mathrm{d}t}h_{dc} = \dot{m}_{dc}(h_{drw} - h_{dcx}) - \dot{Q}_{amb}, \tag{12.15}$$

where the ambient heat loss from the downcomer is

$$\dot{Q}_{amb} = \alpha_{dc,amb} A_{dc}(T_{dc} - T_{amb}).$$

In Eq. 12.15, h_{dc} is the mean or representative water enthalpy in each downcomer tube, while h_{dcx} is the enthalpy at the downcomer exit. A second relationship between these two variables is required in order to complete the calculation. This may be established in various ways but most conveniently by setting the outlet enthalpy equal to the mean enthalpy.

$$h_{dc} = h_{dcx}.$$

With the substitution $T_{dcx} = c_w h_{dcx}$ and $T_{drw} = c_w h_{drw}$, assuming the water specific heat c_w is the same for both, we can write

$$M_{dc}c_w\frac{\mathrm{d}T_{dcx}}{\mathrm{d}t} + (\dot{m}_{dc}c_w + \alpha_{dc,amb}A_{dc})T_{dcx} = \dot{m}_{dc}c_w T_{drw} + \alpha_{dc,amb}A_{dc}T_{amb}.$$

The solution of this equation can be expressed as that returned by the library function `fnlag`, introduced in Sect. 3.3.2. With $T_{dcx,targ}$ the target (steady-state) value of the solution and τ_{dcx} the associated time constant, the downcomer exit temperature T_{dcx} is returned as

$$T_{dcx} = \mathtt{fnlag}(T_{dcx}, T_{dcx,targ}, \tau_{dcx}), \tag{12.16}$$

where

$$T_{dcx,targ} = \frac{\dot{m}_{dc} c_w T_{drw} + \alpha_{dc,amb} A_{dc} T_{amb}}{\dot{m}_{dc} c_w + \alpha_{dc,amb} A_{dc}},$$

$$\tau_{dcx} = \frac{M_{dc} c_w}{\dot{m}_{dc} c_w + \alpha_{dc,amb} A_{dc}}.$$

The inclusion of ambient losses ensures this formulation remains valid for zero downcomer flow.

12.3 Steam Generation in the Riser

Starting from the basic conservation equations, we will now derive of a set of equations which describes the dynamic behaviour of the key state and subsidiary variables of the riser. This applies equally to the risers in both drum and once-through configurations. It will be assumed that the direction of flow is upwards, inclined to the horizontal at an angle β.

The development is presented for a single tube. The multiple tubes of a real riser are handled by either assuming that all tubes are identical and simply summing the single tube result over all tubes or by grouping similar tubes into banks, within each of which all tubes can be regarded as identical. Individual banks can differ, one from another.

Two different methods are employed. The first uses spatial discretisation and explicit integration of each of the conservation equations. The second applies the serial method, described in Sect. 5.4.1, which leads to almost the same result but with an important and significant difference.

In each derivation all variables are functions of the axial ordinate z and time t. As this is the evaporator we can assume that the flow can be single or two phase (liquid and vapour) and, if two phase, the fluid is in the saturated state.

12.3.1 The Profile of Specific Mass Flow from the Mass Balance

The mass balance equation is

$$\frac{\partial \phi}{\partial z} = \frac{1}{v^2}\frac{\partial v}{\partial t} = \frac{1}{v^2}\left(\frac{\partial v}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial v}{\partial x}\frac{\partial x}{\partial t}\right).$$

Since

$$v = x\,v'' + (1-x)v'$$

we have

$$\frac{\partial v}{\partial x} = v'' - v'$$

and, with $\partial v/\partial p$ approximated by $x\, \partial v''/\partial p$,

$$\frac{\partial \phi}{\partial z} = \frac{1}{v^2}\left(x \frac{\partial v''}{\partial p}\frac{\partial p}{\partial t} + (v'' - v')\frac{\partial x}{\partial t} \right). \tag{12.17}$$

This equation quantifies the influence of both pressure and steam fraction changes on the local flow rate. It shows, for example, that since $\partial v''/\partial p$ is negative, an increase in local pressure induces a decrease in local mass flow to a degree proportional to the local steam content and inversely proportional to the local pressure. It also shows that, for pressures for which $v'' \gg v'$ (less than, say, 20 MPa), a local increase in steam content, caused, for example, by increased local steam generation, will cause an increase in local mass flow inversely proportional to the local fluid specific volume. This effect is particularly pronounced at the lower pressures and steam mass fractions encountered during start-up. At the onset of boiling, the small local change in steam mass fraction will cause a large increase in mass flow which can destabilise the calculation by causing an excessively large change in ϕ within one time step.

The term $\partial x/\partial t$ can be eliminated as follows.

The riser fluid mass fraction is defined in terms of local fluid enthalpy as

$$h = xh'' + (1 - x)h' = h' + x\mathbf{r},$$

where the latent heat of evaporation $\mathbf{r} = h'' - h'$ is a function of local pressure. The partial differentials of h with respect to time t and space z are

$$\frac{\partial h}{\partial t} = \frac{\partial h'}{\partial t} + x\frac{\partial \mathbf{r}}{\partial t} + \mathbf{r}\frac{\partial x}{\partial t} \tag{12.18}$$

and

$$\frac{\partial h}{\partial z} = \frac{\partial h'}{\partial z} + x\frac{\partial \mathbf{r}}{\partial z} + \mathbf{r}\frac{\partial x}{\partial z}. \tag{12.19}$$

The time derivative can written

$$\begin{aligned}\frac{\partial h}{\partial t} &= \frac{\partial h'}{\partial t} + x\frac{\partial h''}{\partial t} - x\frac{\partial h'}{\partial t} + \mathbf{r}\frac{\partial x}{\partial t} \\ &= (1 - x)\frac{\partial h'}{\partial t} + x\frac{\partial h''}{\partial t} + \mathbf{r}\frac{\partial x}{\partial t} \\ &= \left((1 - x)\frac{\partial h'}{\partial p} + x\frac{\partial h''}{\partial p} \right)\frac{\partial p}{\partial t} + \mathbf{r}\frac{\partial x}{\partial t}\end{aligned}$$

giving

$$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial p}\frac{\partial p}{\partial t} + \mathbf{r}\frac{\partial x}{\partial t}$$

or

$$\frac{\partial x}{\partial t} = \frac{1}{\mathbf{r}}\frac{\partial h}{\partial t} - \frac{1}{\mathbf{r}}\frac{\partial h}{\partial p}\frac{\partial p}{\partial t}.$$

Therefore,

$$\begin{aligned}(v'' - v')\frac{\partial x}{\partial t} &= \left(\frac{v'' - v'}{h'' - h'}\right)\frac{\partial h}{\partial t} - \left(\frac{v'' - v'}{h'' - h'}\right)\frac{\partial h}{\partial p}\frac{\partial p}{\partial t} \\ &= \frac{\partial v}{\partial h}\frac{\partial h}{\partial t} - \frac{\partial v}{\partial h}\frac{\partial h}{\partial p}\frac{\partial p}{\partial t}.\end{aligned}$$

Equation 12.17 can now be written

$$v^2\frac{\partial \phi}{\partial z} = \left(x\frac{\partial v''}{\partial p} - \frac{\partial v}{\partial p}\right)\frac{\partial p}{\partial t} + \frac{\partial v}{\partial h}\frac{\partial h}{\partial t}.$$

But

$$x\frac{\partial v''}{\partial p} = \frac{\partial v}{\partial p}$$

leaving

$$v^2\frac{\partial \phi}{\partial z} = \frac{\partial v}{\partial h}\frac{\partial h}{\partial t}. \tag{12.20}$$

12.3.2 The Profile of Pressure from the Momentum Balance

The discussion presented in Sect. 5.2.2 revealed that wave effects are introduced through the momentum equation via local compression of the steam phase. While wave effects create the conditions for choked flow and shock effects and are important *per se*, they have little relevance to the dynamics of steam generation in the evaporator under any but extreme operational conditions. From a computational perspective, we should remove local compression effects and reduce the momentum equation to

$$\frac{\partial p}{\partial z} = -\left(\frac{\partial \phi}{\partial t} - \phi^2\frac{\partial v}{\partial h}\frac{\partial h}{\partial z} - \left.\frac{\partial p}{\partial z}\right|_L - \rho\, g\, \sin\beta\right) \tag{12.21}$$

that is, the profile of pressure along the riser is statically defined by friction losses, gravity and local thermal effects. The term $\partial\phi/\partial t$ accounts for flow inertia.

In steady state $\partial\phi/\partial t = 0$, and the distribution of pressure is defined by geodetic head differences and flow losses. The volume expansion term $\phi^2\frac{\partial v}{\partial h}\frac{\partial h}{\partial z}$, although a non-zero contribution if $\partial h/\partial z \neq 0$, is generally small and can be neglected for most purposes.

The following is to be noted regarding units. If a consistent set of units has been used, each term in Eq. 12.21 is expressed in units of Pascals. Terms which include constants may be calculated implicitly in kiloPascals (kPa) if the constants are expressed in non-consistent units. For example, use of $g = 9.80665\,\mathrm{m/s^2}$ in this equation is consistent and renders the geodetic height contribution in Pascals. For water ($\rho \approx 1{,}000\,\mathrm{kg/m^3}$), this yields $\Delta p = \rho\, g = 9{,}806.65\,\mathrm{Pa/m} = 9.80665\,\mathrm{kPa/m}$. By contrast, the volume expansion term is typically of the order of only a few hundred Pa per metre of riser length.

12.3.3 The Profile of Fluid Enthalpy from the Energy Balance

Ignoring kinetic and gravitational potential energy, the energy conservation equation 5.7

$$\frac{v}{A}\frac{\mathrm{d}\dot{q}}{\mathrm{d}z} = \left(\frac{\partial h}{\partial t} + v\frac{\partial h}{\partial z}\right) - v\frac{\partial p}{\partial t} + v\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} + g\,\sin\beta\right)$$

may be reduced to

$$v\frac{\partial h}{\partial z} = \frac{v}{A}\frac{\mathrm{d}\dot{q}}{\mathrm{d}z} - \frac{\partial h}{\partial t} + v\frac{\partial p}{\partial t}. \tag{12.22}$$

The term $\partial h/\partial t$ accounts for thermal inertia. The energy balance in the evaporator is driven by the heat transfer term $\mathrm{d}\dot{q}/\mathrm{d}z$. In an open-flow system such as this, the compression energy term $v\,\partial p/\partial t$ is relatively small and can be neglected.

In steady state, $\partial p/\partial t = 0$ and $\partial h/\partial t = 0$ and mass flow ϕ will be uniform along the flow path ($\partial\phi/\partial z = 0$). Equation 12.22 then indicates that the spatial gradient of enthalpy $\mathrm{d}h/\mathrm{d}z$ is defined by the heat transferred to the flow $\mathrm{d}\dot{q}/\mathrm{d}z$, as expected.

$$\dot{m}\frac{\mathrm{d}h}{\mathrm{d}z} = \frac{\mathrm{d}\dot{q}}{\mathrm{d}z}.$$

12.3.4 Solution of the Conservation Equations

The conservation equations applied to the riser are summarised as follows.

Mass:

$$v\frac{\partial\phi}{\partial z} - \frac{1}{v}\frac{\partial v}{\partial h}\frac{\partial h}{\partial t} = 0. \tag{12.23}$$

Momentum:

$$\frac{\partial p}{\partial z} + \frac{\partial \phi}{\partial t} - \phi^2 \frac{\partial v}{\partial h} = \rho g \sin \beta + \left.\frac{\partial p}{\partial z}\right|_L . \tag{12.24}$$

Energy:

$$-v\frac{\partial p}{\partial t} + \frac{\partial h}{\partial t} + \phi v \frac{\partial h}{\partial z} = \frac{v}{A}\frac{\mathrm{d}\dot{q}}{\mathrm{d}z}. \tag{12.25}$$

Heat transfer is defined by the supplementary equation

$$\frac{\mathrm{d}\dot{q}}{\mathrm{d}z} = -\alpha_{hx} a_{hx} (T_{fl} - T_w), \tag{12.26}$$

where a_{hx} is the heat transfer area per unit length. The flow specific volume v is given from the saturation properties relationship

$$v = v(p, h) \tag{12.27}$$

and the fluid temperature from

$$T_{fl} = T(p, h). \tag{12.28}$$

These equations can be solved numerically via an appropriate time and space discretisation scheme. Since the local compression term has been removed from the momentum equation, the solution will not include high-speed acoustic effects and will allow the use of longer time steps.

The solution of these equations is no small task for a large number of riser cells. We will now look at the serial solution method as an alternative approach.

12.3.5 Application of the Serial Method of Solution

The serial method of solution of the three conservation equations was described in Sect. 5.4.1 where the following three equations were derived.

$$p_j^{n+1} = p_{j-1}^{n+1} + \left[f_{1,j} + \phi \frac{\partial v}{\partial h} f_{2,j} \right] \Delta z, \tag{12.29}$$

$$h_j^{n+1} = h_{j-1}^{n+1} + \left[v f_{1,j} + f_{2,j} \right] \Delta z, \tag{12.30}$$

$$\phi_j^{n+1} = \phi_{j-1}^{n+1} + \frac{1}{\theta \Delta t} \left[S_\rho^n - \rho_j^{n+1} \right] \Delta z. \tag{12.31}$$

The functions f_1 and f_2 were given as

$$f_1(z) = \frac{1}{\theta \Delta t}\left[S_\phi^n - \phi^{n+1}\right] + (\Delta p_{loss} - g\,\rho \sin\beta)^n\,, \tag{12.32}$$

$$f_2(z) = \frac{1}{v\theta \Delta t}\left[S_h^n - h^{n+1} + v^{n+1}p^{n+1}\right] + \frac{1}{\phi}\frac{1}{A}\frac{\mathrm{d}\dot{q}}{\mathrm{d}z} + v\Delta p_{loss} \tag{12.33}$$

and the recursive functions are

$$S_\rho^{n+1} = \frac{1}{\theta}\rho^{n+1} - \frac{1-\theta}{\theta}S_\rho^n \tag{12.34}$$

with the boundary condition

$$S_\rho^0 = \rho^0(z),$$

$$S_\phi^{n+1} = \frac{1}{\theta}\phi^{n+1} - \frac{1-\theta}{\theta}S_\phi^n \tag{12.35}$$

with the boundary condition

$$S_\phi^0(z) = \phi^0(z),$$

and

$$S_h^{n+1} = \frac{1}{\theta}\left[h^{n+1} - v^{n+1}p^{n+1}\right] - \frac{1-\theta}{\theta}S_h^n \tag{12.36}$$

with the boundary condition

$$S_h^0(z) = h^0(z) - v^0 p^0(z).$$

Together with the properties relationship

$$v^{n+1} = v(p^{n+1}, h^{n+1})$$

these equations may be applied to the steam generation process directly in this form. In keeping with the discrete-space formulation, it will be necessary to divide the flow path into spatial cells for which the tube wall temperature and fluid properties (including steam mass fraction) will be calculated. Boundary conditions at each end of the flow path will be defined by the configuration of the steam generation system, configured either as a circuit (drum boiler or once-through unit operating in circulation mode with a water separator) or for once-through flow.

The term $(S_u - u)/\Delta t$, with $u = \phi$, p or h, introduces an important difference to the earlier formulation represented by Eqs. 12.23–12.25. It can be interpreted as a numerical approximation to the derivative of u. Differentiation can act as a destabilising influence as it amplifies small numerical differences. If Eqs. 12.31–12.33 are used as they stand, the implied differentiation may cause numerical instability. Where signal differentiation is used in digital process control, it is always combined with a low-pass filter. A similar technique can be applied here by averaging the derivatives over consecutive time steps. If k steps are used, then $1/(\theta \Delta t)$ can be replaced by $1/(k\,\theta \Delta t)$ at the cost of some loss of dynamic accuracy.

Computation experience suggests that a riser cell length of around 1 m is a good compromise between complexity and accuracy and ensures stable and reliable results. A riser 20 m long would then be represented by 20 cells, one 40 m long by 30–40 cells, and so on. The cell length defines the minimum resolution of the location of the boiling boundary and the volume of the fluid converting *instantaneously* from the liquid to the vapour state. The larger this volume, the greater the effect of phase change on fluid flow and volume displacement from the cell. Both of these can have an adverse effect on the stability and accuracy of the solution.

Treatment of Phase Change Within the Flow

Equation 12.31 is suitable for single-phase flow without phase change. The following approach is more appropriate to the treatment of two-phase flows.

With $a = x\partial v''/\partial p$ and $\bar{v} = v'' - v'$ Eq. 12.17 can be written

$$a\frac{\partial p}{\partial t} + \bar{v}\frac{\partial x}{\partial t} = v^2\frac{\partial \phi}{\partial z}. \tag{12.37}$$

With the time derivatives replaced by their first-order discrete equivalents this becomes

$$a\frac{p^{n+1} - p^n}{\Delta t} + \bar{v}\frac{x^{n+1} - x^n}{\Delta t} = v^2\frac{\mathrm{d}\phi}{\mathrm{d}z}$$

from which

$$\begin{aligned} a\,p^{n+1} + \bar{v}\,x^{n+1} &= a\,p^n + \bar{v}\,x^n + \Delta t v^2\frac{\mathrm{d}\phi}{\mathrm{d}z} \\ &= a\,p^n + \bar{v}\,x^n + \Delta t v^2\frac{\mathrm{d}}{\mathrm{d}z}[\theta\,\phi^{n+1} + (1-\theta)\phi^n] \\ &= a\,p^n + \bar{v}\,x^n + (1-\theta)\Delta t v^2\frac{\mathrm{d}\phi^n}{\mathrm{d}z} + \theta\Delta t\,v^2\frac{\mathrm{d}\phi^n}{\mathrm{d}z}. \end{aligned}$$

Grouping terms denominated at the $(n+1)$th and n-th steps gives

$$\begin{aligned} \theta\,\Delta t\,v^2\frac{\mathrm{d}\phi^{n+1}}{\mathrm{d}z} - a\,p^{n+1} - \bar{v}\,x^{n+1} &= -(1-\theta)\Delta t\,v^2\frac{\mathrm{d}\phi^n}{\mathrm{d}z} - a\,p^n - \bar{v}\,x^n \\ &= -S_\rho^n \end{aligned}$$

from which

$$\theta\,\Delta t\,v^2 \frac{\mathrm{d}\phi^{n+1}}{\mathrm{d}z} = a\,p^{n+1} + \bar{v}\,x^{n+1} - S_\rho^n \tag{12.38}$$

with

$$S_\rho^n = (1-\theta)\Delta t\,v^2 \frac{\mathrm{d}\phi^n}{\mathrm{d}z} + a\,p^n + \bar{v}\,x^n.$$

We can then write

$$\begin{aligned} S_\rho^{n+1} &= (1-\theta)\Delta t\,v^2 \frac{\mathrm{d}\phi^{n+1}}{\mathrm{d}z} + a\,p^{n+1} + \bar{v}\,x^{n+1} \\ &= -\theta\,\Delta t\,v^2 \frac{\mathrm{d}\phi^{n+1}}{\mathrm{d}z} + a\,p^{n+1} + \bar{v}\,x^{n+1} + \Delta t\,v^2 \frac{\mathrm{d}\phi^{n+1}}{\mathrm{d}z} \\ &= S_\rho^n + \Delta t\,v^2 \frac{\mathrm{d}\phi^{n+1}}{\mathrm{d}z}. \end{aligned}$$

Equation 12.38 can now be written

$$\Delta t\,v^2 \frac{\mathrm{d}\phi^{n+1}}{\mathrm{d}z} = -\frac{1}{\theta} S_\rho^n + \frac{1}{\theta}\left(a\,p^{n+1} + \bar{v}\,x^{n+1}\right) \tag{12.39}$$

with

$$\begin{aligned} S_\rho^{n+1} &= \frac{1}{\theta} S_\rho^n + \frac{1}{\theta}\left(a\,p^{n+1} + \bar{v}\,x^{n+1} - S_p^n\right) \\ &= -\frac{(1-\theta)}{\theta} S_\rho^n + \frac{1}{\theta}\left(a\,p^{n+1} + \bar{v}\,x^{n+1}\right) \\ &= -\frac{(1-\theta)}{\theta} S_\rho^n - \frac{1}{\theta}\left(v''(x^{n+1} - x^n) - v'\,x^{n+1}\right). \end{aligned}$$

Again replace $\mathrm{d}\phi/\mathrm{d}z$ in Eq. 12.39 by its discrete-time equivalent to obtain

$$\frac{\theta\,\Delta t\,v^2}{\Delta z}(\phi_j^{n+1} - \phi_{j-1}^{n+1}) = a\,p^{n+1} + \bar{v}\,x^{n+1} - S_\rho^n$$

from which

$$\phi_j^{n+1} = \phi_{j-1}^{n+1} + \frac{\Delta z}{\theta\,\Delta t\,v^2}\left(a\,p^{n+1} + \bar{v}\,x^{n+1} - S_\rho^n\right).$$

Finally, replace a by $x\,\partial v''/\partial p = -x\,v''/p$ to give

$$\phi_j^{n+1} = \phi_{j-1}^{n+1} + \frac{1}{v^2}\frac{\Delta z}{\theta\,\Delta t}\left[v''(x^{n+1} - x^n) - S_\rho^n\right]. \tag{12.40}$$

Boundary Conditions at the Riser Inlet

The inlet boundary conditions p_0, h_0 and ϕ_0 are defined by interfacing systems. With these known and using the profile of riser metal temperature known from the previous time step, solution of this system of equations will yield the complete picture of riser fluid conditions at the (n+1)th time step, up to and including the last riser cell which defines the riser discharge conditions.

The following table summarises the identification of riser inlet conditions for each of the two possible plant configurations, circulation loop and once-through.

Circulation loop

Inlet enthalpy $h_0 = h_{ri}$	The downcomer outlet enthalpy or circulation pump discharge enthalpy
Inlet flow ϕ_0	The per riser tube specific flow
Inlet pressure $p_0 = p_{ri}$	The drum or water separator pressure plus the downcomer gravity term minus the downcomer friction loss

Once-through

Inlet enthalpy $h_0 = h_{ri}$	The enthalpy of the feedwater to the boiler
Inlet flow ϕ_0	The per tube specific flow, calculated as total feedwater ϕ divided by the number of riser tubes
Inlet pressure $p_0 = p_{ri}$	The feedwater pressure at the boiler inlet

12.3.6 Calculation of the Riser Tube Wall Temperatures

We will assume that each riser tube wall cell can be adequately represented by a single metal temperature T_m. A simple heat balance on the j-th cell gives

$$\begin{aligned} c_m M_m \frac{\mathrm{d}T_{m,j}}{\mathrm{d}t} &= \dot{q}_{frn,j} - \dot{q}_{tf,j} + \lambda A_x (T_{m,j} - T_{m,(j+1)})/dz \\ &\quad + \lambda A_x (T_{m,(j-1)} - T_{m,j})/dz - \alpha_{hxf} A_{tf} (Tm_j - T_{f,j}) \\ &= \dot{q}_{frnR,j} + \lambda_{frn} A_{frn} (T_{frn} - T_{m,j}) \\ &\quad + \lambda A_x (T_{m,j} + T_{m,(j+1)} - 2\, T_{m,j})/dz \\ &\quad - \alpha_{hxf} A_{tf} (Tm_j - T_{f,j}) \end{aligned}$$

where
- $\dot{q}_{frnR,j}$ radiant heat to the j-th riser tube cell
- $\dot{q}_{tf,j}$ convective heat from furnace to the j-th riser tube cell
- A_x longitudinal heat flow cross-sectional area
- A_{frn} outer tube area receiving convective heat from the furnace
- A_{tf} inner tube area between tube and riser fluid
- T_{frn} furnace temperature used for convective heat transfer
- $T_{f,j}$ temperature of the riser fluid in the j-th cell
- α_{frn} convective heat transfer coefficient furnace and riser tube
- α_{hxf} convective heat transfer coefficient riser tube/fluid

λ is the coefficient of thermal conductivity of the riser tube and c_m the specific heat capacity of the tube metal. M_m is the mass of the tube metal per unit length. δz is centre line spacing of adjacent cells, assumed the same for all cells.

As usual we replace the time derivative by its discrete-time equivalent. Using a fully implicit formulation and with

$$D = \frac{\Delta t}{c_m M_m} \qquad \text{and} \qquad X = \alpha_{frn} A_{frn} + \alpha_{hxf} A_{hxf}$$

we obtain

$$-D\frac{\lambda\, A_x}{\delta z}\, T^{n+1}_{m,(j-1)} + T^{n+1}_{m,j}\left[1 + D\left(\frac{\lambda\, A_x}{\delta z} + X\right)\right] + D\frac{\lambda\, A_x}{\delta z}\, T^{n+1}_{m,(j+1)}$$
$$= T^{n}_{m,j} + D(\dot{q}_{frnR,j} + \alpha_{hxf} A_{hxf} T_{f,j} + \alpha_{frn} A_{frn} T_{frn,j}). \tag{12.41}$$

For a set of N_{rsz} riser cells, this may be expanded into a tridiagonal matrix equation with boundary conditions defined by the temperatures of the components to which the riser tubes connect at each end. At the inlet end this will be the metal temperature of the boiler lower header. At the outlet it will be the drum metal (upper or lower) or water separator.

The structure of the coefficient matrix is defined by its main and off-diagonal elements.

Main diagonal $\quad 1 + D\left(\frac{\lambda\, A_x}{\delta z} + X\right)$

Lower off-diagonal $\quad -D\frac{\lambda\, A_x}{\delta z}$

Upper off-diagonal $\quad D\,\frac{\lambda\, A_x}{\delta z}$

If longitudinal conduction is ignored, the calculation of the cell metal temperatures reduces to a single equation for each, decoupled from its neighbours. However, inclusion of conduction is certainly a better representation of reality, and significant spatial variations of riser metal temperatures should be expected. This is particularly the case in the once-through configuration, with notably higher temperatures in the vicinity of the dry-out zone and beyond. An additional benefit brought by the longitudinal conduction terms is improved numerical stability of the calculation, particularly in the vicinity of dry out.

12.3.7 Shrink and Swell of Drum Level

In a drum boiler the water level in the drum is known to exhibit predictable but large variations in response to changes in furnace firing rates or steam flows. When they occur during normal operations, these variations tend to occur in a direction opposite to the drum water-level control system expectations and can create level control difficulties. These drum-level variations are collectively referred to as "shrink and swell" and are caused principally by changes in the volume of the steam phase in the riser, with some less significant contribution from any steam phase which might be present within the drum water. Riser-based effects occur either as the result of changes in the rate of steam production or changes in pressure local to the point of steam production.

Two distinct versions of this swell behaviour can be distinguished. The first is the rapid and in some plants extreme rise in drum level which can follow the commencement of boiling. Water-level increases in excess of 50 % of drum diameter can occur which, if not correctly anticipated, can lead to a furnace trip on high drum level. Similar but less severe events can occur at higher pressures following changes in furnace firing configuration.

Immediately prior to initial boil-off, the riser is full of sub-cooled water, close to the saturated state. If a small increase in heat absorbed by an incremental volume of water increases its enthalpy beyond saturation, a bubble of steam is formed. At or near atmospheric pressure the volume occupied by a unit mass of steam is some 1,700 times that of the same mass of water. In the absence of any flow inertia this volume of water would be displaced immediately from the riser into the drum by the steam bubble, causing a rise in drum level. This potentially very large flow is restricted by the inertia and friction losses of the riser fluid and a much smaller displacement actually occurs. The constraint on the local displacement flow causes the local pressure to increase, with an attendant rise in local saturation enthalpy and temperature. At the inception of boiling, the good heat transfer between the riser tube wall and fluid will keep their temperature differential small and the initial boiling will cease, temporarily. As the riser tube temperature continues to increase this pattern will be repeated, with further displacements of water to the drum. Steam produced in the riser is partially reabsorbed by cooler riser fluid and partially transported to the drum where its accumulation raises the drum pressure. Increasing drum pressure raises the saturation enthalpy and temperature everywhere along the riser and inhibits further steam production until more heat is transferred from the furnace through the riser tubes by the increasing tube temperature. Eventually, a stable rate of steam production is achieved, with a raised but stable drum level.

If the riser consisted of a single tube, this process would cause the drum level to increase by a succession of small discrete increments. However, the riser consists of a large number of tubes, each of which exhibits this behaviour but to a slightly different extent and at a slightly different time. The net effect is to smear the individual effects together resulting in a smooth change of drum level.

As the pressure increases, the relative displacement of water from the riser decreases with the increasing saturated steam specific volume. The ratio of the specific volumes of steam and water decreases from around 1,700 at atmospheric pressure to 1,236 at 140 kPa, 841 at 200 kPa, 176 at 10,000 kPa and 88.9 at 18,000 kPa.

The second manifestation of drum swell is associated with momentary changes of drum pressure which can occur at any pressure and during any phase of plant operations. Any pressure change causes an immediate contraction (pressure increase) or expansion (pressure decrease) of the steam bubbles in the riser (and in the drum, if any) with a consequent change in flow rate to the drum. In both natural- and assisted-circulation systems the total circulation flow is also influenced by changes in the 2-phase pressure drop in the circuit with changes in local mixture quality. Thus a decrease in pressure causes an increase in riser steam-phase volume and an acceleration of flow to the drum, exacerbated by a decrease in flow from the drum to the downcomers due to the (now) higher 2-phase pressure loss in the circuit. The net mass gain in the drum increases the level. A pressure increase has the opposite effect on both of these influences, and the drum level decreases.

Equation 12.17 identifies and quantifies each of these influences as being dependent on the rate of change of pressure and steam mass fraction.

$$\frac{\partial \phi}{\partial z} = \frac{1}{v^2}\left(x \frac{\partial v''}{\partial p}\frac{\partial p}{\partial t} + (v'' - v')\frac{\partial x}{\partial t} \right).$$

12.4 Numeric Example

Figure 12.4 presents the results of computation of an idealised boiler start-up from first light-off to achievement of normal operating pressure. The computation uses the drum model described in Sect. 12.2.2 integrated with the serial solution of the riser as described in Sect. 12.3.5 with $\phi(z)$ calculated using Eq. 12.40. The magnitude of the riser swell contribution was adjusted to produce a drum-level increase of some 50 % of the drum diameter.

Figure 12.5 shows an enlarged section of the preceding figure around the moment of first boil-off. The sudden expulsion of fluid from the riser into the drum and the resulting rise in drum level are clearly evident, as are three distinct expulsion phases, of diminishing magnitude, as the formation of the vapour phase spreads through the riser.

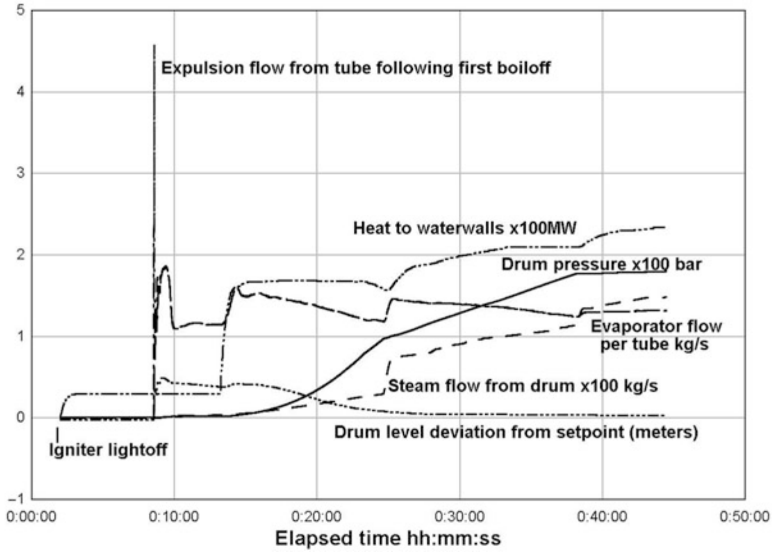


Fig. 12.4 Drum boiler start-up from first light-off

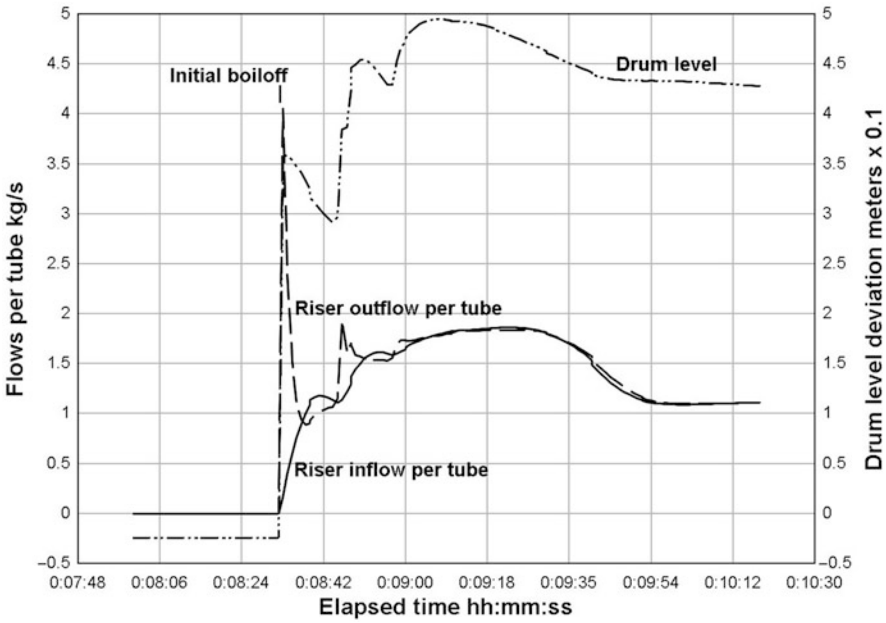


Fig. 12.5 Drum boiler start-up—Expanded view around first boil-off

12.5 Other Boiler Models

The analysis method outlined above is obviously not the only way in which the dynamic behaviour of these components can be modelled and alternatives can be found in the relevant literature, for example [59, 60]. Many of these models are of low order and are intended for use in control system investigations and development. They usually represent the entire boiler as an integration of furnace (heat release related directly to fuel flow), evaporator, often divided into two or three zones covering sub-cooled water, a two-phase saturated fluid zone and, for once-through evaporators, a superheated single-phase steam zone. The drum or water separator is treated as the location of water-steam separation and defines the system pressure and water mass. Depending on the purpose to which the model is to be put, the model may show more detail in one or other of these regions.

Of particular note is the model of a drum boiler developed by Åström and Bell. In a series of three papers [55–57], the authors have described the evolution of a compact low (4th)-order integrated model of the evaporator and drum of a typical drum boiler. Despite its small size, this model is capable of good reproduction of the salient features of a drum boiler over a reasonable range of its on-load operation. It is a practical and useful model for cases in which the detailed behaviour of the boiler itself is not the primary interest but rather its integration with its control systems and interaction with interconnected systems.