Chapter 16
A Portfolio Approach to Multi-product Newsvendor Problem with Budget Constraint*

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Abstract This chapter investigates a portfolio approach to multi-product newsvendor problem with budget constraint, in which the procurement strategy for each newsvendor product is designed as portfolio contract. A portfolio contract consists of a fixed-price contract and an option contract. We model the problem as an expected profit-maximization model, and propose an efficient solution procedure after investigating the structural properties of the model. We conduct numerical studies to show the efficiency of the proposed solution procedure, and to compare three models with different procurement contracts, i.e., fixed-price contract, option contract, and portfolio contract. Numerical results are shown to demonstrate the advantage of the portfolio model, and sensitivity analysis is provided for obtaining some managerial insights.

Keywords Newsvendor • Option contract • Portfolio • Budget constraint • Multiple products

16.1 Introduction

Multi-product constrained newsvendor problem is a classical inventory management problem, which was firstly introduced by Hadley and Whitin (1963). After Hadley and Whitin's seminal work, many researchers have investigated different models and

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solution methods for multi-product newsvendor problems. Khouja (1999) presented a good literature review on the research. Due to the difficulty of solving large-scale problems, most recent works have focused on developing efficient solution methods, e.g. Lau and Lau (1996), Erlebacher (2000), Vairaktarakis (2000), Moon and Silver (2000), and Abdel-Malek et al. (2004). To address nonnegativity constraints of the order quantities, Abdel-Malek and Montanari (2005) proposed a modified Lagrangian-based method by analyzing the solution space. Zhang et al. (2009) provided an exact solution method for the problem with any continuous demand distribution. Zhang and Hua (2008) proposed a unified algorithm for solving a class of convex separable nonlinear knapsack problems, which include the singly constrained multi-product newsvendor problem with box constraints. Zhang and Du (2010) studied the multi-product newsvendor problem with limited capacity and outsourcing. Zhang (2012) analyzed structural properties of the multi-product newsvendor problem with multiple constraints, and proposed a multi-tier binary solution method for solving the exact solution. Zhang (2011) investigated a multi-product newsvendor problem with limited capacity in the presence of mixed deterministic and stochastic demands.

In the classical newsvendor problem, the product is procured from the supplier with fixed-price contract. Under this procurement strategy, the retailer will undertake the salvage loss resulting from lower realized demand. To avoid this risk, the retailer always does not order enough inventories to maximize the supply chain's total profit under the fixed-price contract (Cachon 2003). In order to maximize the supply chain's total profit, and share the risk raised from demand uncertainty with supply chain partners, some different contract types are used for encouraging the retailer to increase the order in supply chain management practice, such as buy back contracts, revenue sharing contracts, quantity flexibility contracts, sales rebate contracts, and quantity discount contracts (Cachon 2003). These contracts are labeled as "flexibility contract", in which a fixed amount of supply is determined when the contract is signed, but the amount to be delivered and paid for can differ from the quantity determined upon signing the contract. In comparison with fixed-price contracts, these flexibility contracts not only coordinate the supply chain, but also have sufficient flexibility (by adjusting parameters) to allow for any division of the supply chain's profit between suppliers and retailers. For more details of these flexibility contracts, please refer to Cachon (2003).

Option contract is one type of flexibility contracts (Martínez de Albéniz and Simchi-Levi 2005), which is defined as an agreement between the retailer and the supplier, in which the retailer pre-pays a reservation cost up-front for a commitment from the supplier to reserve certain order quantity. If the retailer does not execute the option, the initial payment is lost. With option contract, the retailer can purchase any amount of supply up to the option reservation level by paying an execution cost for each unit purchased. In other words, option contract provides the retailer with flexibility to adjust order quantity depending on the realized demand, and, hence, the inventory risk can be lowered for the retailer by utilizing the flexibility of option contract.

There are mainly two branches for the research on option contracts in supply chain management literature: One perspective is supply chain coordination with option contracts, e.g., Barnes-Schuster et al. (2002), Wu et al. (2002), Kleindorfer and Wu (2003), Wu and Kleindorfer (2005), Wang and Liu (2007), Gomez_Padilla and Mishina (2009), and Zhao et al. (2010). The other is on a single firm's optimal procurement decisions given particular contractual terms, e.g., Cohen and Agrawal (1999), Marquez and Blanchar (2004), Wang and Tsao (2006), and Boeckem and Schiller (2008), etc. In the research, some combinations of different contracts, such as fixed-price contract, and option contract, have been investigated.

In addition, some research on option contracts also took into account spot market since it is another source of supply for commodity products, e.g., Martínez de Albéniz and Simchi-Levi (2005), Aggarwal and Ganeshan (2007), and Fu et al. (2010). Spot market is a supply market in which products are sold for cash and delivered immediately. Contracts bought and sold on spot market are immediately effective. For some products, spot market can be used by the firm to purchase at any time; however, the product price on spot market is random. Over the last years, the emergence of the business-to-business trading exchange has transformed the procurement strategies, which provides spot market where buyers and sellers can trade products any time at online markets (Aggarwal and Ganeshan 2007). As Carbone (2001) reported, 50% of Hewlett-Packard's procurement cost was spent on fixed-price contract, 35% in option contracts, and the remaining was left to the spot market.

Up to now, all the existing works on the combination strategies of different contracts focused on single product setting. We have not found any research on multi-product demand management with the combination of fixed-price contract and option contract. In this chapter, we introduce a portfolio approach for managing multi-product newsvendor problem with budget constraint, in which each product can be procured with a portfolio contract consisting of a fixed-price contract and an option contract. The dual contracts for each product in the problem make the optimal ordering decisions more challenging in multi-product setting. On one hand, the use of option contract for lowering the overage cost should be properly balanced against the additional cost of using the option contract since unit reservation plus execution cost of option contract is typically higher than unit cost of a fixed-price contract. On the other hand, the total budget should be well allocated to different products for signing the fixed-price contracts and option contracts.

The overall objective of the newsvendor is to decide the optimal quantities of portfolio contracts for maximizing the total expected profit. We establish the structural properties for the optimal decisions of the proposed profit-maximization model, and develop an efficient solution procedure for the studied problem. Numerical results are shown to demonstrate the advantage of the portfolio model, and sensitivity analysis is provided for obtaining some managerial insights.

The rest of the chapter is organized as follows: Section 16.2 describes the problem formulation. In Sect. 16.3, the properties of the optimal solution are established, and an exact solution procedure is developed. Section 16.4 is dedicated to numerical studies for demonstrating the advantage of the portfolio contract model, and obtaining some managerial insights from sensitivity analysis. Section 16.5 briefly concludes the chapter and provides some future research directions.

16.2 Problem Formulation

We consider the following multi-product newsvendor problem. A retailer sells n different products with stochastic demands over a single period, and each product can be acquired from the suppliers by signing a portfolio contract, which includes a fixed-price contract and an option contract. In the fixed-price contract, the retailer pay unit fixed cost for procuring each product; in the option contract, the retailer pays unit reservation cost up-front for a commitment from the supplier, then the retailer can pays unit execution cost for procuring each product under the commitment level. If retailer does not exercise the option, the initial payment is lost. The retailer has limited budget for signing the portfolio contracts. In the following, we use i to be the index of product $1,\ldots,n$.

The cost parameters used in this chapter are summarized in the following:

$p_i =$ Unit selling price for product i;
$s_i =$ Unit salvage value for product i;
$c_i =$ Unit procurement cost of fixed-price contract for product i;
$v_i =$ Unit reservation cost of option contract for product i;
$w_i =$ Unit execution cost of option contract for product i;
$B =$ Total budget available for signing the portfolio contracts.

To avoid the trivial case, we assume that $p_i > c_i > s_i$ for $i = 1,\ldots,n$. Typically, the total cost of the option contract (reservation plus execution cost) is assumed to be larger than the cost of fixed-price contract, i.e., $v_i + w_i > c_i$; otherwise, the fixed-price contract is dominated by the option contract, and, hence, the fixed-price contract will never be engaged in the problem. We also assume that the reservation cost of option contract is smaller than the pure procurement cost of the fixed-price contract, i.e., $v_i < c_i - s_i$; otherwise, the option contract will be dominated by the fixed-price contract because the fixed-price contract always has the lower costs whether the product can be sold or not. From these two assumptions, i.e., $v_i + w_i > c_i$ and $v_i < c_i - s_i$, we have $s_i < w_i$, which implies that the retailer will not have an opportunity to make profit by executing an option contract in order to obtain the product salvage value.

The retailer makes quantity decisions of the portfolio contracts to fulfill n independent and stochastic demands. Let D_i denote the random demand for product $i = 1,\ldots,n$, which has continuous probability density function $f_i(\cdot)$, cumulative distribution function $F_i(\cdot)$, and reverse distribution function $F_i^{-1}(\cdot)$. It is not uncommon to assume that all demands are nonnegative, thus, we can assume that $F_i(x) = 0$ for all $x < 0$, and $F_i(0) \geq 0$. This assumption does not rule out normal distribution as well as many other distributions with negative support values, since the distributions with negative support values should be approximated as nonnegative demand distributions in practice (Zhang and Du 2010).

The retailer's decisions are made in two stages. At the first stage, the retailer receives demand forecasts for all products, and determines a fixed-price contract quantity x_i, and an option contract quantity y_i to be signed. At the second stage, all demands are realized and the retailer exercises the quantity $\min((D_i - x_i)^+, y_i)$ of product i from the option contract to satisfy the demands for maximizing the revenue, where $(\cdot)^+ = \max\{\cdot, 0\}$.

We are ready to present profit-maximization model (denoted as problem P):

$$\mathrm{Max}\pi(x,y) = \sum_{i=1}^{n} \pi_i = \sum_{i=1}^{n} E_i \begin{bmatrix} p_i \min(D_i, x_i + y_i) + s_i(x_i - D_i)^+ \\ -c_i x_i - v_i y_i - w_i \min((D_i - x_i)^+, y_i) \end{bmatrix}, \tag{16.1}$$

Subject to

$$\sum_{i=1}^{n} (c_i x_i + v_i y_i) \le B, \tag{16.2}$$

$$x_i \ge 0,\ y_i \ge 0,\ i = 1, \ldots, n. \tag{16.3}$$

For each product $i = 1, \ldots, n$, the first term $p_i \min(D_i, x_i + y_i)$ in (16.1) is the selling revenue, the second term $s_i(x_i - D_i)^+$ is the salvage value, the third term $c_i x_i$ is the acquisition cost with the fixed-price contract, the fourth term $v_i y_i$ is the option reservation cost, and the last term $w_i \min((D_i - x_i)^+, y_i)$ is the option execution cost. Equation (16.2) specifies the budget constraint on the quantities of the portfolio contracts. Note that the execution costs are excluded from the budget constraint because they are not needed to pay when signing the contracts at the first stage. Equation (16.3) gives the nonnegative constraints on the order quantities.

By using the formula $\min(D_i, x_i + y_i) = x_i + y_i - (x_i + y_i - D_i)^+$ and integration by parts formula $\int_0^x (x - z)\mathrm{d}F(z) = \int_0^x F(z)\mathrm{d}z$, the expected profit of problem P can be rewritten as:

$$\begin{aligned} \pi(x,y) &= \sum_{i=1}^{n} E_i \begin{bmatrix} p_i(x_i + y_i - (x_i + y_i - D_i)^+) + s_i(x_i - D_i)^+ \\ -c_i x_i - v_i y_i - w_i(y_i - (x_i + y_i - D_i)^+ + (x_i - D_i)^+) \end{bmatrix} \\ &= \sum_{i=1}^{n} \begin{bmatrix} (p_i - c_i)x_i + (p_i - w_i - v_i)y_i \\ -(p_i - w_i)\int_0^{x_i + y_i} (x_i + y_i - z_i)\mathrm{d}F_i(z_i) - (w_i - s_i)\int_0^{x_i} (x_i - z_i)\mathrm{d}F_i(z_i) \end{bmatrix} \\ &= \sum_{i=1}^{n} \begin{bmatrix} (p_i - c_i)x_i + (p_i - w_i - v_i)y_i \\ -(p_i - w_i)\int_0^{x_i + y_i} F_i(z_i)\mathrm{d}z_i - (w_i - s_i)\int_0^{x_i} F_i(z_i)\mathrm{d}z_i \end{bmatrix}. \end{aligned} \tag{16.4}$$

16.3 Properties and Solution Procedure

In this section, we first establish some structural properties for the optimal decisions, and then we develop an efficient solution method for the studied problem.

16.3.1 Properties of the Optimal Solution

Beginning with the objective function, we have the following proposition:

Proposition 1. *The expected profit function π is jointly concave in x_i and $y_i, i = 1,\ldots,n$.*

Proof. Since

$$\begin{cases} \partial\pi/\partial x_i = (p_i - c_i) - (p_i - w_i)F_i(x_i + y_i) - (w_i - s_i)F_i(x_i) \\ \partial\pi/\partial y_i = (p_i - w_i - v_i) - (p_i - w_i)F_i(x_i + y_i) \end{cases}, \quad i = 1,\ldots,n,$$

we have

$$\begin{cases} \partial^2\pi/\partial x_i^2 = -(p_i - w_i)f_i(x_i + y_i) - (w_i - s_i)f_i(x_i) \le 0 \\ \partial^2\pi/\partial y_i^2 = \partial^2\pi/\partial x_i\partial y_i = -(p_i - w_i)f_i(x_i + y_i) \le 0 \end{cases}, \quad i = 1,\ldots,n,$$

and $\partial^2\pi/\partial x_i\partial x_j = \partial^2\pi/\partial y_i\partial y_j = \partial^2\pi/\partial x_i\partial y_j = 0$ for $i \neq j$, $i, j = 1,\ldots,n$. Thus, the Hessian matrix of the objective function is negative semi-definite. □

Since π is concave and the feasible domain of the problem is convex, the Karush–Kuhn–Tucker (KKT) conditions are necessary and sufficient for optimality. Let $\lambda \ge 0$, $\alpha_i \ge 0$, and $\beta_i \ge 0$, $i = 1,\ldots,n$, be the dual variables corresponding to the constraints in (16.2)–(16.3), respectively. Then (x_i, y_i), $i = 1,\ldots,n$, is optimal if and only if there exists nonnegative dual variables λ, α_i, and β_i, $i = 1,\ldots,n$, such that

$$(p_i - c_i) - (p_i - w_i)F_i(x_i + y_i) - (w_i - s_i)F_i(x_i) - \lambda c_i + \alpha_i = 0, \quad i = 1,\ldots,n, \tag{16.5}$$

$$(p_i - w_i - v_i) - (p_i - w_i)F_i(x_i + y_i) - \lambda v_i + \beta_i = 0,\ i = 1,\ldots,n, \tag{16.6}$$

$$\sum_{i=1}^{n} (\alpha_i x_i + \beta_i y_i) = 0, \tag{16.7}$$

$$\lambda \left(B - \sum_{i=1}^{n} (c_i x_i + v_i y_i) \right) = 0. \tag{16.8}$$

To solve these KKT conditions, we first investigate how to solve (16.5)–(16.7) with any given $\lambda \ge 0$, and then we illustrate how to decide the optimal value for λ. Denote by $(\tilde{x}_i^\lambda, \tilde{y}_i^\lambda, \tilde{\alpha}_i^\lambda, \tilde{\beta}_i^\lambda, \lambda)$, $i = 1,\ldots,n$, an solution of (16.5)–(16.7), then we have the following propositions:

Proposition 2. *For any given $\lambda \ge 0, (\tilde{x}_i^\lambda, \tilde{y}_i^\lambda), i = 1,\ldots,n$, satisfies*

$$\begin{cases} \tilde{x}_i^\lambda = F_i^{-1}\left(\min\left\{ \left(\frac{w_i - (1+\lambda)(c_i - v_i)}{w_i - s_i} \right)^+, \left(\frac{p_i - (1+\lambda)c_i}{p_i - s_i} \right)^+ \right\} \right) \\ \tilde{y}_i^\lambda = \left(F_i^{-1}\left(\left(\frac{p_i - w_i - (1+\lambda)v_i}{p_i - w_i} \right)^+ \right) - F_i^{-1}\left(\left(\frac{w_i - (1+\lambda)(c_i - v_i)}{w_i - s_i} \right)^+ \right) \right)^+ \end{cases}.$$

Proof. The proof of this proposition is presented in Appendix.

This proposition characterizes the optimal solution of (16.5)–(16.7) with any given $\lambda \geq 0$, and also indicates the optimal solution to the problem without budget constraint (denoted as problem P1). Denote by $(\tilde{x}_i, \tilde{y}_i)$, $i = 1, \ldots, n$, an optimal solution to problem P1, by simply setting $\lambda = 0$ in Proposition 2, then the optimal values of $(\tilde{x}_i, \tilde{y}_i)$, $i = 1, \ldots, n$ are given as follows:

$$\begin{cases} \tilde{x}_i = F_i^{-1}\left(\min\left\{\frac{w_i+v_i-c_i}{w_i-s_i}, \frac{p_i-c_i}{p_i-s_i}\right\}\right) \\ \tilde{y}_i = \left(F_i^{-1}\left(\frac{p_i-w_i-v_i}{p_i-w_i}\right) - F_i^{-1}\left(\frac{w_i+v_i-c_i}{w_i-s_i}\right)\right)^+. \end{cases} \tag{16.9}$$

From the result of Proposition 2, we also know that the difference between the optimal solution of the constraint problem and that of the unconstraint problem increases too, when λ increases.

Before discussing the result in (16.9), we first investigate the optimal unconstrained orders under pure fixed-price contract (FC) and pure option contract (OC). If there is no budget constraint and only fixed-price contract is used, then we can solve the optimal unconstrained order $\tilde{x}_{i,\mathrm{FC}}$ from

$$\partial\pi/\partial x_i = (p_i - c_i) - (p_i - w_i)F_i(x_i + y_i) - (w_i - s_i)F_i(x_i) = 0,$$

by setting $y_i = 0$, and, hence, $\tilde{x}_{i,\mathrm{FC}} = F_i^{-1}\left(\frac{p_i-c_i}{p_i-s_i}\right)$, $i = 1, \ldots, n$. If there is no budget constraint and only pure option contract is used, then we can solve the optimal unconstrained order $\tilde{y}_{i,\mathrm{OC}}$ from

$$\partial\pi/\partial y_i = (p_i - w_i - v_i) - (p_i - w_i)F_i(x_i + y_i) = 0,$$

by setting $x_i = 0$, and, hence, $\tilde{y}_{i,\mathrm{OC}} = F_i^{-1}\left(\frac{p_i-w_i-v_i}{p_i-w_i}\right)$, $i = 1, \ldots, n$. Note that the option contract (v_i, w_i) can also be viewed as a fixed-price contract with unit purchase cost $c_i' = v_i + w_i$, and unit salvage value $s_i' = w_i$. Thus, $\tilde{y}_{i,\mathrm{OC}}$ and $\tilde{x}_{i,\mathrm{FC}}$ have the same form, i.e., $\tilde{y}_{i,\mathrm{OC}} = F_i^{-1}\left(\frac{p_i-w_i-v_i}{p_i-w_i}\right) = F_i^{-1}\left(\frac{p_i-c_i'}{p_i-s_i'}\right)$.

Let us discuss the relationship among the unconstrained solution $\tilde{x}_{i,\mathrm{FC}}$, $\tilde{y}_{i,\mathrm{OC}}$, and the optimal unconstrained order of portfolio contract, $(\tilde{x}_i, \tilde{y}_i)$ presented in (16.9). If $\frac{p_i-w_i-v_i}{p_i-w_i} > \frac{p_i-c_i}{p_i-s_i}$, then the mathematical transform gives $\frac{p_i-w_i-v_i}{p_i-w_i} > \frac{p_i-c_i}{p_i-s_i} > \frac{v_i+w_i-c_i}{w_i-s_i}$, and we have $\tilde{x}_i = F_i^{-1}\left(\frac{w_i+v_i-c_i}{w_i-s_i}\right)$ and $\tilde{y}_i = F_i^{-1}\left(\frac{p_i-w_i-v_i}{p_i-w_i}\right) - F_i^{-1}\left(\frac{w_i+v_i-c_i}{w_i-s_i}\right)$; thus, the total order quantity of portfolio contract is $\tilde{x}_i + \tilde{y}_i = F_i^{-1}\left(\frac{p_i-w_i-v_i}{p_i-w_i}\right)$; otherwise, we have $\frac{p_i-w_i-v_i}{p_i-w_i} \leq \frac{p_i-c_i}{p_i-s_i} \leq \frac{v_i+w_i-c_i}{w_i-s_i}$, and, hence, $\tilde{x}_i = F_i^{-1}\left(\frac{p_i-c_i}{p_i-s_i}\right)$ and $\tilde{y}_i = 0$; thus the total order quantity of portfolio contract is $\tilde{x}_i + \tilde{y}_i = F_i^{-1}\left(\frac{p_i-c_i}{p_i-s_i}\right)$. Therefore, the total optimal unconstrained order of portfolio contract can be expressed as

$$\tilde{x}_i + \tilde{y}_i = \max\left\{F_i^{-1}\left(\frac{p_i - c_i}{p_i - s_i}\right), F_i^{-1}\left(\frac{p_i - w_i - v_i}{p_i - w_i}\right)\right\} = \max\left\{\tilde{x}_{i,\mathrm{FC}}, \tilde{y}_{i,\mathrm{OC}}\right\}.$$

From the proof of Proposition 2, we have

$$\tilde{x}_i^{\lambda} + \tilde{y}_i^{\lambda} = \max\left\{ F_i^{-1}\left(\frac{p_i-(1+\lambda)c_i}{p_i-s_i}\right)^+, F_i^{-1}\left(\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}\right)^+ \right\}.$$

Since $\lambda \geq 0$, we know $\tilde{x}_i^{\lambda} + \tilde{y}_i^{\lambda} \leq \tilde{x}_i + \tilde{y}_i$, which means that the total optimal unconstrained order of portfolio contract is an upper bound for the total optimal order of portfolio contract in problem P. Thus, the maximum of the optimal unconstrained order under pure fixed-price contract and the optimal unconstrained order under pure option contract is an upper bound for the optimal total order of portfolio contract in problem P.

Denote by (x_i^*, y_i^*), $i = 1, \ldots, n$, an optimal solution to problem P, and λ^* the corresponding optimal value of λ, we have the following proposition:

Proposition 3. *(a) If* $\sum_{i=1}^{n}(c_i\tilde{x}_i + v_i\tilde{y}_i) \leq B$, *then* $x_i^* = \tilde{x}_i$ *and* $y_i^* = \tilde{y}_i$, $i = 1, \ldots, n$; *(b) If* $\sum_{i=1}^{n}(c_i\tilde{x}_i + v_i\tilde{y}_i) > B$, *then* $\sum_{i=1}^{n}(c_i x_i^* + v_i y_i^*) = B$.

Proof. (a) This property is obvious since the budget constraint is not active. It is also easily verified that $(x_i^*, y_i^*) = (\tilde{x}_i, \tilde{y}_i)$ with $\lambda^* = 0$ satisfy the condition in (16.8).

(b) If $\sum_{i=1}^{n}(c_i x_i^* + v_i y_i^*) < B$, according to $\sum_{i=1}^{n}(c_i\tilde{x}_i + v_i\tilde{y}_i) > B$, there must exist at least one $k \in \{1, \ldots, n\}$ such that $c_k x_k^* + v_k y_k^* < c_k\tilde{x}_k + v_k\tilde{y}_k$. Since $\sum_{i=1}^{n}(c_i x_i^* + v_i y_i^*) < B$, the slackness condition $\lambda(B - \sum_{i=1}^{n}(c_i x_i + v_i y_i)) = 0$ in (16.8) implies $\lambda^* = 0$, and this further means $(x_i^*, y_i^*) = (\tilde{x}_i, \tilde{y}_i)$, $i = 1, \ldots, n$, which violates $c_k x_k^* + v_k y_k^* < c_k\tilde{x}_k + v_k\tilde{y}_k$. Thus, we have $\sum_{i=1}^{n}(c_i x_i^* + v_i y_i^*) = B$. □

Property (a) indicates that the optimal solution to problem P is the same as that of problem P1 if budget constraint is inactive. Property (b) illustrates that the budget must be fully utilized at the optimal solution if the budget constraint is binding, i.e., $\sum_{i=1}^{n}(c_i\tilde{x}_i + v_i\tilde{y}_i) > B$.

16.3.2 Solution Procedure

Before developing the solution procedure, we first prove the following result:

Proposition 4. $c_i\tilde{x}_i^{\lambda} + v_i\tilde{y}_i^{\lambda}$ *is nonincreasing in* λ.

Proof. The proof of this proposition is presented in Appendix.

Fig. 16.1 Main steps of Algorithm 1

Step 0: Calculate $(\tilde{x}_i, \tilde{y}_i)$, $i = 1, \ldots, n$, from Eq (16.9).

Step 1: If $\sum_{i=1}^{n}(c_i\tilde{x}_i + v_i\tilde{y}_i) \le B$, then

let $(x_i^*, y_i^*) = (\tilde{x}_i, \tilde{y}_i)$, $i = 1, \ldots, n$, goto *Step* 6.

Step 2: Let $\lambda_L = 0$, $\lambda_U = \max_{i=1,\ldots,n}\left(\frac{p_i - c_i}{c_i}, \frac{p_i - w_i - v_i}{v_i}\right)$.

Step 3: Let $\lambda = (\lambda_L + \lambda_U)/2$;

Calculate $\tilde{x}_i^\lambda$ and $\tilde{y}_i^\lambda$, $i = 1, \ldots, n$, from Proposition 2.

Step 4: If $\sum_{i=1}^{n}(c_i\tilde{x}_i^\lambda + v_i\tilde{y}_i^\lambda) < B$, then let $\lambda_U = \lambda$, goto *Step* 3;

If $\sum_{i=1}^{n}(c_i\tilde{x}_i^\lambda + v_i\tilde{y}_i^\lambda) > B$, then let $\lambda_L = \lambda$, goto *Step* 3.

Step 5: Let $x_i^* = \tilde{x}_i^\lambda$ and $y_i^* = \tilde{y}_i^\lambda$, $i = 1, \ldots, n$.

Step 6: Output x_i^* and y_i^*, $i = 1, \ldots, n$, stop.

Proposition 4 provides a good property with which the optimal value of λ can be found without using any linear search method. We can develop an efficient way to decide the optimal value for λ when the budget constraint is binding.

If the budget constraint is binding, according to Proposition 3(b), we know that $\sum_{i=1}^{n}(c_i x_i^* + v_i y_i^*) = B$, which implies $\lambda^* \le \bar{\lambda} \equiv \max_{i=1,\ldots,n}\left(\frac{p_i - c_i}{c_i}, \frac{p_i - w_i - v_i}{v_i}\right)$. Otherwise, when $\lambda^* > \max\left\{\frac{p_i}{c_i} - 1, \frac{p_i - w_i}{v_i} - 1\right\}$, $i = 1, \ldots, n$, from the proof of Proposition 2, we know $\tilde{x}_i^{\lambda^*} = \tilde{y}_i^{\lambda^*} = 0$, $i = 1, \ldots, n$, which violates the necessary condition $\sum_{i=1}^{n}(c_i x_i^* + v_i y_i^*) = B$. Thus, according to the results in Propositions 2–4, we can determine λ^* by applying a binary search method over the interval $\lambda \in [0, \bar{\lambda}]$, and simultaneously solve the optimal solution to problem P.

Main steps of the solution procedure for solving the optimal solution to problem P are summarized in Algorithm 1 (as shown in Fig. 16.1).

In Algorithm 1, we first solve problem P1 (*Step* 0) to obtain $(\tilde{x}_i, \tilde{y}_i)$, $i = 1, \ldots, n$. Then we judge whether $(\tilde{x}_i, \tilde{y}_i)$, $i = 1, \ldots, n$, leads to a binding budget constraint or not (*Step* 1). If the budget constraint is inactive at $(\tilde{x}_i, \tilde{y}_i)$, $i = 1, \ldots, n$, then we let $(x_i^*, y_i^*) = (\tilde{x}_i, \tilde{y}_i)$, $i = 1, \ldots, n$. Otherwise, we apply the binary search procedure over interval $[\lambda_L, \lambda_U]$ to determine (x_i^*, y_i^*) $(i = 1, \ldots, n)$ (*Steps* 2–5). *Step* 6 outputs the optimal solution to problem P. Since we do not assume any specific property on demand distribution, our approach is applicable to any continuous demand distribution.

The computational complexity of Algorithm 1 is analyzed as follows. The complexity of Steps 0–2 is $O(n)$. The search of λ within the interval $[0, \bar{\lambda}]$ in Steps 3–5 needs $\log_2(\bar{\lambda}/\varepsilon)$ iterations, where ε is the error target for the binary search procedure. Take $\bar{\lambda} = 10^{10}$ and $\varepsilon = 10^{-6}$ as an example, the number of iterations for determining λ is $\log_2(\bar{\lambda}/\varepsilon) = 36.8414 \approx 37$. The computation procedure in each step of Steps 3–5 has complexity $O(n)$. So the computational complexity of Steps 3–5 is $O(\log_2(\bar{\lambda}/\varepsilon)n)$. The complexity of Step 6 is $O(n)$. Thus, Algorithm 1 has computational complexity $O(\log_2(\bar{\lambda}/\varepsilon)n)$, which is polynomial in the number of products.

16.4 Numerical Studies

In this section, numerical results are provided to show the efficiency of the proposed solution procedure, and to compare three models with different procurement contracts, i.e., fixed-price contract (FC), option contract (OC), and portfolio contract (PC). Sensitivity analysis is also provided for obtaining some managerial insights. The two pure contract models (i.e., fixed-price contract, pure option contract) are easily obtained from the portfolio contract model by setting $y_i = 0$ or $x_i = 0$, $i = 1,\ldots,n$, respectively. The portfolio contract model should dominate the two pure contract models since the optimal solutions to the two pure contract models are both feasible solutions to the portfolio contract model.

Before presenting numerical results, we first briefly illustrate how to solve pure fixed-price contract model and pure option contract model. The two pure contract models can be reformulated as minimizing $-\pi_{\text{FC}}(x)$ and $-\pi_{\text{OC}}(y)$, respectively:

$$\text{(FC) Min} - \pi_{\text{FC}}(x) = -\left[\sum_{i=1}^{n}(p_i - c_i)x_i - (p_i - s_i)\int_0^{x_i} F_i(z_i)\mathrm{d}z_i\right],$$

$$s.t. \sum_{i=1}^{n} c_i x_i \le B,\ x_i \ge 0,\ i = 1,\ldots,n.$$

$$\text{(OC) Min} - \pi_{\text{OC}}(y) = -\left[\sum_{i=1}^{n}(p_i - w_i - v_i)y_i - (p_i - w_i)\int_0^{y_i} F_i(z_i)\mathrm{d}z_i\right],$$

$$s.t. \sum_{i=1}^{n} v_i y_i \le B,\ y_i \ge 0,\ i = 1,\ldots,n.$$

Since the two pure contract models are the special cases of problem P where $y_i = 0$ or $x_i = 0$, $i = 1,\ldots,n$, according to Proposition 1, we know $\pi_{\text{FC}}(x)$ is concave in x, and $\pi_{\text{OC}}(y)$ is concave in y. Thus the objective functions of PC and OC models, i.e., $-\pi_{\text{FC}}(x)$ and $-\pi_{\text{OC}}(y)$, are both convex. It is obvious that the objective functions of PC and OC models are both separable, therefore the two pure contract models can be viewed as the class of convex separable nonlinear knapsack problems studied by Zhang and Hua (2008).

These knapsack problems have two important characteristics: positive marginal cost (PMC) and increasing marginal loss-cost ratio (IMLCR). PMC means that the budget occupancy increases in order quantity, which is guaranteed by the positive

Table 16.1 Parameters and solutions for the illustrative example

i	p_i	s_i	c_i	v_i	w_i	μ_i	σ_i	$x^*_{i,\mathrm{FC}}$	$y^*_{i,\mathrm{OC}}$	$x^*_{i,\mathrm{PC}}$	$y^*_{i,\mathrm{PC}}$
1	96	14	43	11	42	109	22	85.07	127.23	0.00	117.63
2	96	11	41	14	49	103	28	77.34	117.85	63.15	39.28
3	92	14	47	14	46	102	24	0.00	114.29	0.00	100.84
4	91	17	46	14	46	108	29	34.94	122.29	0.00	105.76
5	99	18	49	14	43	103	27	52.69	121.21	0.00	108.00
6	105	19	45	24	45	106	27	82.37	112.84	93.99	0.00
7	105	18	44	22	49	109	24	89.74	115.53	99.37	0.00
8	104	14	42	22	44	106	22	90.23	113.50	96.28	2.68
9	109	16	46	24	46	108	22	89.34	114.67	95.33	4.20
10	101	17	47	24	44	101	22	72.90	105.38	80.56	7.73
π^*								37,073.94	33,340.71	44,301.65	

linear constraint, i.e., $c_i > 0$, $i = 1,\dots,n$, for FC model, and $v_i > 0$, $i = 1,\dots,n$, for OC model; IMLCR requires that the ratio of marginal loss to marginal cost increases in order quantity, i.e. $\frac{\mathrm{d}[-\pi_{\mathrm{FC}}(x)]}{c_i \mathrm{d}x_i} = -\frac{p_i - c_i}{c_i} + \frac{p_i - s_i}{c_i} F_i(x_i)$ increases in x_i, $i = 1,\dots,n$, for FC model, and $\frac{\mathrm{d}[-\pi_{\mathrm{OC}}(y)]}{v_i \mathrm{d}y_i} = -\frac{p_i - w_i - v_i}{v_i} + \frac{p_i - w_i}{v_i} F_i(y_i)$ increases in y_i, $i = 1,\dots,n$, for OC model. Since FC and OC models satisfy PMC and IMLCR, the method with linear computation complexity developed by Zhang and Hua (2008) can be directly used to solve them.

It is worth noticing that the method proposed by Zhang and Hua (2008) is not available for solving the portfolio contract model proposed in this chapter, due to the fact that the objective function of problem P is nonseparable.

In the numerical experiment, the relative profit differences between FC, OC, and PC, i.e., $\Delta\pi^{\mathrm{PC}}_{\mathrm{FC}} = (\pi^*_{\mathrm{PC}} - \pi^*_{\mathrm{FC}})/\pi^*_{\mathrm{FC}} \times 100\%$, $\Delta\pi^{\mathrm{PC}}_{\mathrm{OC}} = (\pi^*_{\mathrm{PC}} - \pi^*_{\mathrm{OC}})/\pi^*_{\mathrm{OC}} \times 100\%$ are reported to show the benefit of portfolio contract model and to obtain some managerial insights through sensitivity analysis.

16.4.1 An Illustrative Example

In this example, demands of 10 products are all normally distributed, and there is a budget constraint $B = 30{,}000$. Table 16.1 shows the relevant information, where μ_i, σ_i, $i = 1,\dots,n$, are parameters of the mean and standard deviation of the normal demand $x^*_{i,\mathrm{FC}}$ and $y^*_{i,\mathrm{OC}}$ are the optimal solutions of the fixed-price contract model and the option contract model, respectively; $x^*_{i,\mathrm{PC}}$ and $y^*_{i,\mathrm{PC}}$ are the optimal solutions of the portfolio contract model and π^* stands for the optimal expected profits of the three different models. In order to investigate the general case, we set the parameters such that $\frac{p_i - w_i - v_i}{p_i - w_i} > \frac{p_i - c_i}{p_i - s_i}$ for $i = 1,\cdots,5$ and $\frac{p_i - w_i - v_i}{p_i - w_i} < \frac{p_i - c_i}{p_i - s_i}$ for $i = 6,\cdots,10$. The results in Table 16.1 show that the portfolio contract model is better than the fixed-price contract and option contract models.

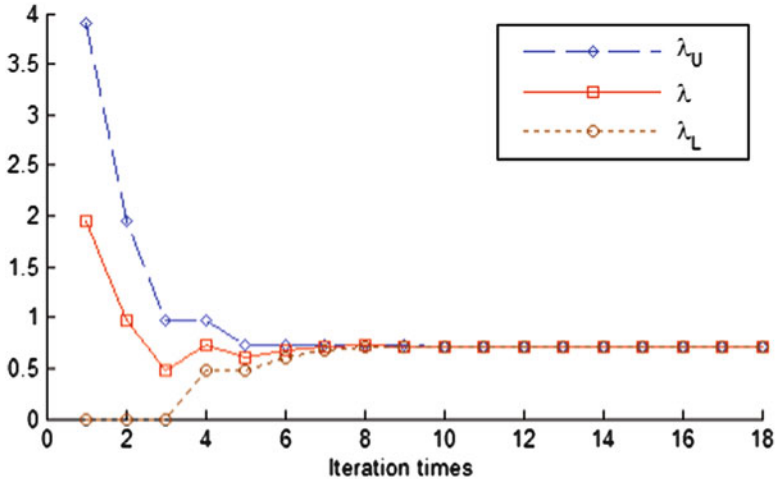

Fig. 16.2 The values of λ_L, λ_U, and λ in the iteration process

To show the efficiency of the proposed solution procedure, we plot the iterative solution process of this example in Fig. 16.2. In this figure, we report the values of λ_L, λ_U, and λ in the iteration process. From this figure, it can be observed that Algorithm 1 solves the optimal value $\lambda^* = 0.7059$ within only 18 iteration times.

16.4.2 Sensitivity Analysis

To investigate how the budget constraint affects the relative profit differences among three procurement strategies, we provide sensitivity analysis by changing B of the base example shown in Table 16.1 and keeping other parameters unchanged. The results of more cases with different B are presented in Tables 16.2–16.4. In Table 16.2, we report the optimal profits and the relative profit differences of different models, and the ratio of total budget used on option contracts, which is defined as $\Delta B_{\text{PC}}^{\text{O}} = \sum_{i=1}^{n} v_i y_{i,\text{PC}}^* / \sum_{i=1}^{n} \left(c_i x_{i,\text{PC}}^* + v_i y_{i,\text{PC}}^*\right) \times 100\%$. Tables 16.3 and 16.4 give the optimal order quantity of fixed-price contract and the optimal order quantity of option contract in the portfolio contract for different B, respectively.

From Table 16.2, we have the following observations:

(1) The optimal profits of three procurement strategies are all nondecreasing in the available budget. This observation is obvious since a larger B will provide a larger feasible domain of the optimization problem.

Table 16.2 The profit comparisons, shadow prices, and ΔB_{PC}^{O} for different B

B	π_{FC}^{*}	π_{OC}^{*}	π_{PC}^{*}	$\Delta\pi_{FC}^{PC}(\%)$	$\Delta\pi_{OC}^{PC}(\%)$	λ_{FC}^{*}	λ_{OC}^{*}	λ_{PC}^{*}	$\Delta B_{PC}^{O}(\%)$
100	147.62	390.91	390.91	164.81	0.00	1.48	3.91	3.91	100
1,100	1,623.74	4,072.63	4,072.63	150.82	0.00	1.48	3.00	3.00	100
2,100	3,096.56	6,943.82	6,943.82	124.24	0.00	1.46	2.45	2.45	100
3,100	4,520.40	9,272.62	9,272.62	105.13	0.00	1.39	2.28	2.28	100
4,100	5,906.56	11,517.79	11,517.79	95.00	0.00	1.39	2.21	2.21	100
5,000	7,150.73	13,453.37	13,453.37	88.14	0.00	1.38	2.05	2.05	100
10,000	13,934.67	21,831.71	21,831.71	56.67	0.00	1.33	1.54	1.54	100
15,000	20,472.33	28,993.59	28,993.59	41.62	0.00	1.24	1.30	1.30	100
20,000	26,527.46	33,198.01	34,978.14	31.86	5.36	1.15	0.27	1.13	60.37
25,000	32,091.06	**33,340.71**	40,180.57	25.21	20.52	1.02	**0.00**	0.91	36.03
30,000	37,073.94	33,340.71	44,301.65	19.50	32.88	0.97	0.00	0.71	21.98
35,000	41,808.94	33,340.71	46,892.01	12.16	40.64	0.91	0.00	0.41	15.77
40,000	45,933.08	33,340.71	48,692.49	6.01	46.05	0.72	0.00	0.31	9.78
45,000	48,838.10	33,340.71	49,983.31	2.34	49.92	0.43	0.00	0.21	5.37
50,000	50,185.62	33,340.71	50,581.42	0.79	51.71	0.11	0.00	0.01	2.71
55,000	**50,276.22**	33,340.71	**50,582.34**	**0.61**	**51.71**	**0.00**	0.00	**0.00**	**2.63**
60,000	50,276.22	33,340.71	50,582.34	0.61	51.71	0.00	0.00	0.00	2.63

Table 16.3 The optimal order quantity of fixed-price contract in the portfolio contract

	$x_{i,PC}^{*}$									
B	1	2	3	4	5	6	7	8	9	10
100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1,100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2,100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3,100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4,100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5,000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10,000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15,000	0.00	0.00	0.00	0.00	0.00	**0.00**	**0.00**	**0.00**	0.00	0.00
20,000	0.00	0.00	0.00	0.00	0.00	40.74	73.00	68.60	**0.00**	**0.00**
25,000	0.00	**0.00**	0.00	0.00	0.00	81.87	90.72	86.80	83.27	17.89
30,000	0.00	63.15	0.00	**0.00**	0.00	93.99	99.37	96.28	95.33	80.56
35,000	**0.00**	87.44	**0.00**	54.62	0.00	104.94	108.70	105.92	106.66	96.26
40,000	34.84	92.75	68.96	76.45	**0.00**	108.33	111.61	108.39	109.27	99.24
45,000	82.66	97.97	80.78	88.40	49.67	111.94	114.72	111.02	112.04	102.36
50,000	100.31	108.10	95.71	105.99	89.37	119.60	121.34	116.57	117.85	108.75
55,000	100.95	108.58	96.31	106.75	90.37	119.98	121.67	116.84	118.13	109.05
60,000	100.95	108.58	96.31	106.75	90.37	119.98	121.67	116.84	118.13	109.05

(2) The optimal profits of three procurement strategies do not change when the available budget exceeds the maximal active budgets, which are $\sum_{i=1}^{n} c_i\tilde{x}_{i,\mathrm{FC}}$, $\sum_{i=1}^{n} v_i\tilde{y}_{i,\mathrm{OC}}$, and $\sum_{i=1}^{n} c_i\tilde{x}_{i,PC} + v_i\tilde{y}_{i,PC}$ in FC, OC, and PC models, respectively. The budget constraint is inactive when the budget is larger than the maximal

Table 16.4 The optimal order quantity of option contract in the portfolio contract

	$y^*_{i,\mathrm{PC}}$									
B	1	2	3	4	5	6	7	8	9	10
100	9.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1,100	89.31	0.00	0.00	0.00	8.40	0.00	0.00	0.00	0.00	0.00
2,100	97.30	0.00	0.00	0.00	73.55	0.00	0.00	0.00	0.00	0.00
3,100	99.37	46.36	18.83	0.00	78.16	0.00	0.00	0.00	0.00	0.00
4,100	100.31	55.35	54.38	24.22	80.08	0.00	0.00	0.00	0.00	0.00
5,000	102.20	65.69	66.87	60.58	83.70	0.00	0.00	0.00	0.00	0.00
10,000	108.01	83.45	83.95	84.50	93.63	0.00	34.44	73.13	67.01	0.00
15,000	110.75	89.54	89.45	91.52	97.92	**68.17**	**77.83**	**83.87**	82.62	60.31
20,000	112.63	93.31	92.80	95.75	100.74	36.89	12.33	20.25	**88.47**	**73.03**
25,000	115.16	**98.07**	97.02	101.02	104.47	4.61	1.98	7.58	11.33	64.16
30,000	117.63	39.28	100.84	**105.76**	108.00	0.00	0.00	2.68	4.20	7.73
35,000	**121.38**	21.26	**106.33**	57.90	113.26	0.00	0.00	0.00	0.00	0.00
40,000	87.82	18.01	39.16	38.28	**115.02**	0.00	0.00	0.00	0.00	0.00
45,000	41.40	15.00	29.27	28.70	67.27	0.00	0.00	0.00	0.00	0.00
50,000	26.77	9.53	18.38	16.06	31.64	0.00	0.00	0.00	0.00	0.00
55,000	26.28	9.28	17.98	15.54	30.84	0.00	0.00	0.00	0.00	0.00
60,000	26.28	9.28	17.98	15.54	30.84	0.00	0.00	0.00	0.00	0.00

active budget. In these examples, the maximal active budgets are 51,706.19, 21,086.23, and 50,196.49 in FC, OC, and PC models, so π^*_{FC}, π^*_{OC}, and π^*_{PC} in Table 16.2 do not change when B are larger than the maximal active budgets, respectively.

(3) The relative profit difference between FC and PC decreases as the value of B increases. When the budget B becomes larger, the total order $x^*_{i,\mathrm{PC}} + y^*_{i,\mathrm{PC}}$ will be close to $\tilde{x}_{i,\mathrm{PC}} + \tilde{y}_{i,\mathrm{PC}} = \max\left\{F_i^{-1}\left(\frac{p_i - v_i - w_i}{p_i - w_i}\right), F_i^{-1}\left(\frac{p_i - c_i}{p_i - s_i}\right)\right\}$, $i = 1, \ldots, n$, and the optimal order $x^*_{i,\mathrm{FC}}$ of fixed-price contract will be close to $\tilde{x}_{i,\mathrm{FC}} = F_i^{-1}\left(\frac{p_i - c_i}{p_i - s_i}\right)$, $i = 1, \ldots, n$. When the budget B becomes smaller, the difference between $x^*_{i,\mathrm{PC}} + y^*_{i,\mathrm{PC}}$ and $x^*_{i,\mathrm{FC}}$ become smaller, then the relative profit difference between FC and PC also becomes smaller.

(4) The relative profit difference between OC and PC decreases with the decreasing of B. When the budget B becomes smaller, $\Delta B^{\mathrm{O}}_{\mathrm{PC}}$ becomes larger, i.e., most of the budget will be spent on the option contracts, since option contracts occupy small unit procurement costs (i.e., $v_i < c_i$, $i = 1, \ldots, n$). Thus, the result of PC is close to that of OC as B decreases.

According to Tables 16.3 and 16.4, we know that: (1) The optimal order quantity of fixed-price contract in the portfolio contract increases as B increases, and it will become zero when B is small enough; (2) The optimal order quantity of option contract in the portfolio contract initially increases and then decreases with the increasing of B, and the turning point is $x^*_{i,\mathrm{PC}} > 0$, which is indicated in bold in

Table 16.5 Statistical comparison of the three different procurement models

Problem size n		10		50		100	
Relative profit difference (%)		$\Delta\pi_{FC}^{PC}$	$\Delta\pi_{OC}^{PC}$	$\Delta\pi_{FC}^{PC}$	$\Delta\pi_{OC}^{PC}$	$\Delta\pi_{FC}^{PC}$	$\Delta\pi_{OC}^{PC}$
Mean		21.86	25.35	22.09	25.64	21.89	25.82
Std. Dev.		3.47	3.54	1.61	1.56	1.17	1.07
95% C.I.	Lower	21.17	24.65	21.77	25.33	21.65	25.60
	Upper	22.55	26.06	22.41	25.95	22.12	26.03

Table 16.4. It will reach the minimal value $F_i^{-1}\left(\frac{p_i-w_i-v_i}{p_i-w_i}\right) - F_i^{-1}\left(\frac{w_i+v_i-c_i}{w_i-s_i}\right) > 0$ for the case of $\frac{p_i-w_i-v_i}{p_i-w_i} > \frac{p_i-c_i}{p_i-s_i}$ as B becomes large enough, and it will be zero for the case of $\frac{p_i-w_i-v_i}{p_i-w_i} < \frac{p_i-c_i}{p_i-s_i}$ as B becomes large enough.

From our theoretical results and the above observations, we come to the following insights: (1) managers should attempt to find FC strategy when the available budget is large and PC strategy is not available; (2) OC strategy should be paid more attention to when the available budget is too small and PC strategy cannot be used.

16.4.3 Strategies Comparison

In this section, the three procurement strategies, i.e., FC, OC, and PC, are compared by using randomly generated problems. In these examples, demands of all products are all normally distributed, and the total budget is $B = 3{,}500 \times n$. Let μ_i, σ_i, $i = 1,\ldots,n$, are parameters of the mean and standard deviation of the normal demand. We use the notation $x \sim U(\alpha,\beta)$ to denote that x is uniformly generated over $[\alpha,\beta]$. The problem parameters are generated as follows: $\mu_i \sim U(101,110)$, $\sigma_i \sim U(21,30)$, $p_i \sim (91,100)$, $c_i \sim U(41,50)$, $s_i \sim U(11,20)$, $w_i \sim U(41,50)$, $v_i \sim U(11,20)$, $i = 1,\ldots,n$. Note that the generated parameters satisfy the assumptions made in this chapter.

In this numerical study, we set $n = 10$, 50, 100, respectively. For each problem size n, 100 test instances are randomly generated. The statistical results of relative profit difference $\Delta\pi_{FC}^{PC}$ and $\Delta\pi_{OC}^{PC}$, are reported in Table 16.5, and the statistical results of computation time and number of iterations for searching λ^* in the portfolio contract model, are reported in Table 16.6. In these tables, 95% C.I. stands for 95% confidence interval.

From Table 16.5, we verify that the portfolio contract model outperforms the fixed-price contract and option contract models. This suggests that the retailer should pay more attention to portfolio contract when managing multi-product newsvendor problem with budget constraint if portfolio contracts are available. Additionally, the relationship between the two pure contract models depends on the problem parameters, e.g., in the 100 test instances for the case of $n = 10$, 34 option contract models outperform fixed-price contract models.

Table 16.6 Computation times and number of iterations of the solution method

Problem size n		Computation time			Number of iterations		
		10	50	100	10	50	100
Mean		10.92	29.17	52.27	30.10	31.81	32.88
Std. Dev.		1.91	2.35	3.90	3.08	2.66	2.61
95% C.I.	Lower	10.54	28.71	51.50	29.49	31.28	32.36
	Upper	11.30	29.64	53.04	30.71	32.34	33.40

According to Table 16.6, we know that our solution method can solve the problems quickly in limited iterations. The standard deviations of number of iterations and computation times are quite low in Table 16.6, reflecting the fact that our solution method is quite effective and robust. Robustness of our method should be attributed to the effectiveness of binary search procedure.

16.5 Conclusion

In this chapter, we investigate a portfolio approach to multi-product newsvendor problem with budget constraint, in which the procurement strategy for the newsvendor products is designed as portfolio contract. By establishing the structural properties of optimal solution, we develop an efficient solution method for the studied problem. The proposed algorithm has two main advantages: (1) it has linear computation complexity; (2) it is applicable to general continuous demand distribution.

In comparison with fixed-price contract and option contract models, the portfolio contract model generates significant improvement when managing multi-product newsvendor problem with budget constraint. Through sensitivity analysis, we come to the following insights: (1) The performance difference between fixed-price contract and portfolio contract models will become smaller as the available budget increases; (2) the performance gap between option contract and portfolio contract models increases when the available budget becomes larger. These insights suggest that managers with large budgets should pay more attention to fixed-price contract if the portfolio contract is not available, and that managers with small budgets should attempt to seek an option contract if the portfolio contract cannot be used.

There are several ways to extend this research. At first, this work can be directly extended to consider the scenario where different procurement strategies are available for different products. Secondly, another area of the future research is the consideration of an environment with supply uncertainty in sourcing and to investigate the effect of portfolio contract on managing supply uncertainty. Thirdly, one extension of this chapter might be to study portfolio contract model with demand updating in multi-stage settings. In addition, the demands for the multiple products are independent of one another in our study, an interesting and challenging extension is to consider the model in which the multiple products are substituted to some extent and thus the respective demands are correlated. Finally, it will be a significant issue to consider the model with the horizontal and/or vertical competition in the supply chain, and some topics of this extension have been investigated in a working paper of ours.

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Appendix:

Proof of Proposition 2. To prove this proposition, we consider two cases, respectively:

(1) $\lambda > \min\left\{\frac{p_i}{c_i}-1, \frac{p_i-w_i}{v_i}-1\right\}$, (2) $\lambda \le \min\left\{\frac{p_i}{c_i}-1, \frac{p_i-w_i}{v_i}-1\right\}$.

Case (1): The condition $\lambda > \min\left\{\frac{p_i}{c_i}-1, \frac{p_i-w_i}{v_i}-1\right\}$ holds only if $p_i - c_i < \lambda c_i$ or $p_i - w_i - v_i < \lambda v_i$.

If $p_i - c_i < \lambda c_i$, then we have $(p_i - s_i)F_i(x_i) < \alpha_i$ from (16.5). $(p_i - s_i)F_i(x_i) < \alpha_i$ and $\alpha_i x_i = 0$ implies $\tilde{x}_i^\lambda = 0$.

If $p_i - w_i - v_i < \lambda v_i$, then we have $(p_i - w_i)F_i(y_i) < \beta_i$ from (16.6). $(p_i - w_i)F_i(y_i) < \beta_i$ and $\beta_i y_i = 0$ implies $\tilde{y}_i^\lambda = 0$.

If $p_i - c_i < \lambda c_i$ and $p_i - w_i - v_i \ge \lambda v_i$, substituting $\tilde{x}_i^\lambda = 0$ into (16.6), we have $(p_i - w_i)F_i(y_i) \ge \beta_i$. $(p_i - w_i)F_i(y_i) \ge \beta_i$ and $\beta_i y_i = 0$ implies $\tilde{\beta}_i^\lambda = 0$. Then we have $\tilde{y}_i^\lambda = F_i^{-1}\left(\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}\right)$ from (16.6).

If $p_i - w_i - v_i < \lambda v_i$ and $p_i - c_i \ge \lambda c_i$, substituting $\tilde{y}_i^\lambda = 0$ into (16.5), we have $(p_i - s_i)F_i(x_i) \ge \alpha_i$. $(p_i - s_i)F_i(x_i) \ge \alpha_i$ and $\alpha_i x_i = 0$ implies $\tilde{\alpha}_i^\lambda = 0$. Then we have $\tilde{x}_i^\lambda = F_i^{-1}\left(\frac{p_i-(1+\lambda)c_i}{p_i-s_i}\right)$ from (16.5).

Thus, $\tilde{x}_i^\lambda = F_i^{-1}\left(\left(\frac{p_i-(1+\lambda)c_i}{p_i-s_i}\right)^+\right)$, and $\tilde{y}_i^\lambda = F_i^{-1}\left(\left(\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}\right)^+\right)$ if $\lambda > \min\left\{\frac{p_i}{c_i}-1, \frac{p_i-w_i}{v_i}-1\right\}$.

Case (2): According to (16.6), we have $x_i + y_i = F_i^{-1}\left(\frac{p_i-w_i-(1+\lambda)v_i+\beta_i}{p_i-w_i}\right)$. Substituting it into (16.5), we have $x_i = F_i^{-1}\left(\frac{w_i-(1+\lambda)(c_i-v_i)+\alpha_i-\beta_i}{w_i-s_i}\right)$. The condition $\lambda \le \min\left\{\frac{p_i}{c_i}-1, \frac{p_i-w_i}{v_i}-1\right\}$ implies $p_i - c_i \ge \lambda c_i$ and $p_i - w_i - v_i \ge \lambda v_i$.

In this case, we consider three subcases:

(2.1) $$\frac{p_i-(1+\lambda)c_i}{p_i-s_i}<\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i},$$

(2.2) $$\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}<\frac{p_i-(1+\lambda)c_i}{p_i-s_i},$$

(2.3) $$\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}=\frac{p_i-(1+\lambda)c_i}{p_i-s_i}.$$

Subcase (2.1): In this case, we have $\frac{w_i-(1+\lambda)(c_i-v_i)}{w_i-s_i}<\frac{p_i-(1+\lambda)c_i}{p_i-s_i}<\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}$. By combining (16.5) and (16.6), we have $w_i-(1+\lambda)(c_i-v_i)-\beta_i=(w_i-s_i)F_i(x_i)-\alpha_i$.

If $w_i-(1+\lambda)(c_i-v_i)<0$, then we have $(w_i-s_i)F_i(x_i)<\alpha_i$. $(w_i-s_i)F_i(x_i)<\alpha_i$ and $\alpha_i x_i=0$ implies $\tilde{x}_i^{\lambda}=0$. Substituting $\tilde{x}_i^{\lambda}=0$ into (16.6), we have $(p_i-w_i)F_i(y_i)>\beta_i$. $(p_i-w_i)F_i(y_i)>\beta_i$ and $\beta_i y_i=0$ implies $\tilde{\beta}_i^{\lambda}=0$. Then we have $\tilde{y}_i^{\lambda}=F_i^{-1}\left(\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}\right)$ from (16.6).

If $w_i-(1+\lambda)(c_i-v_i)\geq 0$, then $w_i-(1+\lambda)(c_i-v_i)=(w_i-s_i)F_i(x_i)-\alpha_i+\beta_i\geq 0$. Since $p_i-w_i-(1+\lambda)v_i>0$, there must be $x_i+y_i>0$. According to $\alpha_i x_i+\beta_i y_i=0$, we know $\alpha_i\beta_i=0$. If $\beta_i=0$, then $(w_i-s_i)F_i(x_i)-\alpha_i\geq 0$ and $\alpha_i x_i=0$ implies $\alpha_i=0$. If $\alpha_i=0$, then $x_i+y_i>x_i$ implies $y_i>0$, and then $\beta_i=0$. Thus $\tilde{\alpha}_i^{\lambda}=\tilde{\beta}_i^{\lambda}=0$, and

$$\tilde{x}_i^{\lambda}=F_i^{-1}\left(\frac{w_i-(1+\lambda)(c_i-v_i)}{w_i-s_i}\right),$$
$$\tilde{y}_i^{\lambda}=F_i^{-1}\left(\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}\right)-F_i^{-1}\left(\frac{w_i-(1+\lambda)(c_i-v_i)}{w_i-s_i}\right).$$

These results in subcase (2.1) can be rewritten as:

$$\tilde{x}_i^{\lambda}=F_i^{-1}\left(\left(\frac{w_i-(1+\lambda)(c_i-v_i)}{w_i-s_i}\right)^+\right),$$
$$\tilde{y}_i^{\lambda}=F_i^{-1}\left(\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}\right)-F_i^{-1}\left(\left(\frac{w_i-(1+\lambda)(c_i-v_i)}{w_i-s_i}\right)^+\right),$$

if $\lambda\leq\min\left\{\frac{p_i}{c_i}-1,\frac{p_i-w_i}{v_i}-1\right\}$ and $\frac{p_i-(1+\lambda)c_i}{p_i-s_i}<\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}$.

Subcase (2.2): In this case, we have $\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}<\frac{p_i-(1+\lambda)c_i}{p_i-s_i}<\frac{w_i-(1+\lambda)(c_i-v_i)}{w_i-s_i}$. $x_i+y_i\geq x_i$ requires $\tilde{\beta}_i^{\lambda}>0$, and, hence, $\tilde{y}_i^{\lambda}=0$. Since $\tilde{x}_i^{\lambda}=F_i^{-1}\left(\frac{p_i-w_i-(1+\lambda)v_i+\tilde{\beta}_i^{\lambda}}{p_i-w_i}\right)$ $>0, \alpha_i x_i=0$ implies $\tilde{\alpha}_i^{\lambda}=0$. $\frac{w_i-(1+\lambda)(c_i-v_i)-\tilde{\beta}_i^{\lambda}}{w_i-s_i}=\frac{p_i-w_i-(1+\lambda)v_i+\tilde{\beta}_i^{\lambda}}{p_i-w_i}$ implies

$$\tilde{\beta}_i^\lambda = \frac{(p_i - w_i)(w_i - s_i)}{p_i - s_i}\left(\frac{w_i - (1+\lambda)(c_i - v_i)}{w_i - s_i} - \frac{p_i - w_i - (1+\lambda)v_i}{p_i - w_i}\right).$$

Substituting it into $\tilde{x}_i^\lambda$, we have $\tilde{x}_i^\lambda = F_i^{-1}\left(\frac{p_i-(1+\lambda)c_i}{p_i-s_i}\right)$.

Subcase (2.3): In this case, we have $\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i} = \frac{p_i-(1+\lambda)c_i}{p_i-s_i} = \frac{w_i-(1+\lambda)(c_i-v_i)}{w_i-s_i}$. If $\alpha_i > 0$ and $\beta_i = 0$, then $x_i + y_i < x_i$, it is in contradiction with $x_i + y_i \geq x_i$. If $\alpha_i = 0$ and $\beta_i > 0$, then $x_i < x_i + y_i$, and, hence, $y_i > 0$; It is in contradiction with $\beta_i y_i = 0$. If $\alpha_i > 0$ and $\beta_i > 0$, then $x_i + y_i > 0$, and, hence, $\alpha_i x_i + \beta_i y_i \neq 0$, which violates the slackness condition. Thus, there must be $\tilde{\alpha}_i^\lambda = \tilde{\beta}_i^\lambda = 0$, then $\tilde{x}_i^\lambda + \tilde{y}_i^\lambda = \tilde{x}_i^\lambda$, and $\tilde{y}_i^\lambda = 0$, $\tilde{x}_i^\lambda = F_i^{-1}\left(\frac{p_i-(1+\lambda)c_i}{p_i-s_i}\right)$.

Thus, the results in subcases (2.2) and (2.3) are both $\tilde{x}_i^\lambda = F_i^{-1}\left(\frac{p_i-(1+\lambda)c_i}{p_i-s_i}\right)$, and $\tilde{y}_i^\lambda = 0$, if $\lambda \leq \min\left\{\frac{p_i}{c_i} - 1, \frac{p_i-w_i}{v_i} - 1\right\}$ and $\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i} \leq \frac{p_i-(1+\lambda)c_i}{p_i-s_i}$.

In summary, all these results in the three subcases can be generalized as the equations in Proposition 2. □

Proof of Proposition 4. To prove this proposition, we first cite the following results from the proof of Proposition 2:

(a) If $\lambda > \min\{\frac{p_i}{c_i} - 1, \frac{p_i-w_i}{v_i} - 1\}$, then $\tilde{x}_i^\lambda = F_i^{-1}((\frac{p_i-(1+\lambda)c_i}{p_i-s_i})^+)$, and $\tilde{y}_i^\lambda = F_i^{-1}((\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i})^+)$;
(b) If $\lambda \leq \min\left\{\frac{p_i}{c_i} - 1, \frac{p_i-w_i}{v_i} - 1\right\}$ and $\frac{p_i-(1+\lambda)c_i}{p_i-s_i} < \frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i}$, then

$$\tilde{x}_i^\lambda = F_i^{-1}\left(\left(\frac{w_i - (1+\lambda)(c_i - v_i)}{w_i - s_i}\right)^+\right), \quad \tilde{y}_i^\lambda = F_i^{-1}\left(\frac{p_i - w_i - (1+\lambda)v_i}{p_i - w_i}\right) - F_i^{-1}\left(\left(\frac{w_i - (1+\lambda)(c_i - v_i)}{w_i - s_i}\right)^+\right);$$

(c) If $\lambda \leq \min\left\{\frac{p_i}{c_i} - 1, \frac{p_i-w_i}{v_i} - 1\right\}$ and $\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i} \leq \frac{p_i-(1+\lambda)c_i}{p_i-s_i}$, then

$$\tilde{x}_i^\lambda = F_i^{-1}\left(\frac{p_i - (1+\lambda)c_i}{p_i - s_i}\right), \quad \tilde{y}_i^\lambda = 0.$$

Under case (a) or (c), the result in this proposition is obvious. Under case (b), we have $\tilde{x}_i^\lambda = F_i^{-1}((\frac{w_i-(1+\lambda)(c_i-v_i)}{w_i-s_i})^+)$ and $\tilde{x}_i^\lambda + \tilde{y}_i^\lambda = F_i^{-1}(\frac{p_i-w_i-(1+\lambda)v_i}{p_i-w_i})$. Thus $c_i\tilde{x}_i^\lambda + v_i\tilde{y}_i^\lambda = (c_i - v_i)\tilde{x}_i^\lambda + v_i(\tilde{x}_i^\lambda + \tilde{y}_i^\lambda)$ is nonincreasing in λ. □

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