Chapter 15
Profit Target Setting for Multiple Divisions: A Newsvendor Perspective

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Abstract Managers and firms often engage in decision making based on certain profit targets. Consequently, they may adopt the objective of maximizing the profit probability, namely, the probability of achieving those profit targets. However, there has been limited research on modeling profit target setting. In this chapter, we study analytic target setting under a common business scenario where a firm owns multiple divisions. The firm sets a profit target for each division, which then decides on production level and selling price to maximize the profit probability. We obtain the divisions' optimal profit targets in closed forms when the firm's objective is to maximize its expected profit. When the firm's own objective is also to maximize profit probability, the problem of profit target setting is more complicated. To gain more managerial insights, we focus on two specific cases. In the first case of fair target setting, we show that for most reasonable customer demand distributions, if a division has a relatively high (low) production cost, its assigned profit target decreases (increases) in its price elasticity. In the second case, if the firm is in control of two identical divisions, each division's optimal profit target is just half of the firm's profit target when the price elasticity is two or more, regardless of production cost and demand distribution. We hope that the managerial insights from this chapter help practitioners who are involved with target setting and target attainment.

Keywords Newsvendor • Pricing • Risk aversion • Target setting

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15.1 Introduction

A great deal of research in Operations Management assumes the objective of maximizing expected profit or, sometimes equivalently, minimizing expected cost. However, in business practice, employees, managers, and firms often engage in decision making based on profit targets (Bordley and Kirkwood 2004; Abbas and Matheson 2005; Abbas et al. 2009). That is, they either are assigned profit targets by some external forces such as the corporate headquarters and analysts or set profit targets themselves through participative budgeting process, and they are rewarded or penalized based on whether they meet those targets or miss them (Jensen 2003). As a result, they may adopt the objective of maximizing profit probability, namely, the probability of reaching those profit targets.

The difference between the objective of maximizing expected profit and that of maximizing profit probability is by no means trivial. Jensen (2003) provides examples of how the latter objective drives managerial behaviors that diverge from those driven by the former objective. In one case, a manager knows the target is far from reach and even if he tries his best, he is unlikely to reach the profit target. In the other case, a manager knows that he can easily beat the target and increasing profit will not bring him extra earnings. In both cases, the manager would not be motivated to maximize expected profit.

Assuming objective of maximizing profit probability has two advantages over assuming objective of maximizing expected profit. First, given a profit target exogenously set, the objective of maximizing profit probability assumes risk aversion by definition. To be more specific, it operationalizes risk aversion through a critical probability, specifically, the probability that the profit is no less than a certain threshold. Note that there are two closely related risk measures including Value-at-Risk and Conditional Value-at-Risk (Gan et al. 2005; Ozler et al. 2009). With those two measures, decision makers maximize expected profit while controlling for some critical probability. It is certainly worthwhile for future research to incorporating those more complicated measures when setting profits for divisions.

The second advantage is that under some situations, the objective of maximizing profit probability is more descriptive of how firms and managers make decisions. A big literature on public firms' earnings management behaviors (e.g., Burgstahler and Dichev 1997; Degeorge et al. 1999; Healy and Wahlen 1999; Dechow and Skinner 2000) indicates that meeting or beating various profit targets or thresholds (e.g., prior years' profit, analysts' forecasts, avoiding losses, and performance targets specified in executive compensation contracts) is the most important motive for firms to manipulate accounting numbers. Although such unlawful or unethical behavior is not carried out in most firms, it does indicate the extreme importance of meeting or beating profit targets at the firm level. One such example involves eBay. In the 4th quarter of 2004, eBay's reported profit of 33 cents per share missed the profit target of 34 cents, which was set by Wall Street analysts. Although the gap was only one cent, eBay's stock price fumbled by 12% right after the report (http://money.cnn.com/2005/01/19/news/fortune500/ebay/index.htm). For a more recent example, Swiss Life Holding, Switzerland's largest life insurer, missed its profit target of $1.6 billion for the year 2008. Consequently, its stock price fell 20% in Zurich trading (Giles 2008).

It has been well documented that the objective of maximizing profit probability is also common and important for divisional managers. As shown by Bouwens and Van Lent (2007), it is not uncommon for firms to evaluate their divisional managers based on their performance on profits. Paying managers based on their actual profits relative to a profit budget or target is also often seen in practice (Jensen 2003). In a recent paper, Brown and Tang (2006) interview six buyers from different firms and find that product profit and gross margin are ranked as the most important performance measures in their firms, and targets for profit and gross margin are often used. The authors use profit-target-based reward system to explain the irregularity they observe in their experimental setting, i.e., participants consistently select order quantities less than the newsvendor solution.

Limited research has been done on the objective of maximizing profit probability in the Operations Management literature. The earliest work includes Kabak and Schiff (1978) and Lau (1980). These two papers study a newsvendor with the objective of maximizing profit probability. Lau and Lau (1988) and Li et al. (1990) study the problem of a newsvendor selling two products with the objective of maximizing profit probability. These two studies focus on the special case where the customer demand for each product has a uniform distribution. Li et al. (1991) focus on the special case of exponential distribution. Parlar and Weng (2003) study the newsvendor model where the objective is to maximize the probability of achieving the expected profit, which is a function of order quantity. Recently, the study on the objective of maximizing profit probability is extended to the framework of supply chains. Shi and Chen (2007) study a basic supply chain where a single supplier sells to a newsvendor-type retailer and both adopt the objective of maximizing profit probability. Contrary to the result under the objective of maximizing expected profit, they show that a properly designed wholesale price contract can coordinate the supply chain.

It appears that the extant literature usually assumes that profit targets are exogenously set. However, it is of strategic importance to study profit target setting, i.e., how the values of profit targets are set. Too high a profit target provokes frustration and cynicism, whereas too low a profit target causes apathy and lacks motivational value. As has long been documented in the organizational behavior literature (for a review, see Locke 2001; Locke and Latham 1990, 2002), target or goal difficulty has a significantly positive effect on task performance until limits of ability are reached or individuals cease to be committed to the highly difficult target. As for how firms actually set their profit targets, Merchant and Manzoni's (1989) field study provides some interesting insights. They find that among firms with multiple divisions, 80% to 90% of the time they set their annual profit targets at achievable levels (vs. stretch goals). However, to best of our knowledge, little research has been done on modeling profit target setting.

There are three papers (Lau and Lau 1988; Li et al. 1990, 1991) that relate to analytic target setting indirectly. In these three papers, the authors study the two-product newsvendor problem where order quantity and profit target are related. However, their focus is on determining the optimal order quantities under the uniform and exponential demand distributions. Further, the authors assume that selling prices of the products are exogenously fixed. Finally, it is noted that there has been some research (see, e.g., Yang et al. 2009; Lovell and Pastor 1997) on target setting using the DEA, which is basically a nonparametric methodology based on deterministic linear programming. Consequently, DEA is not applicable to our research problem of profit target setting which involves stochastic customer demand.

In this chapter, we attempt to address the profit target setting issue for multiple divisions within a firm. To be more specific, we study a business scenario where a firm owns multiple autonomous divisions that have authority over operations and other decision making. Such a scenario is common in modern decentralized corporations to allow for rapid and flexible decision making (Aghion and Tirole 1997; Roberts 2004). When a firm sells a single product or service in different countries or continents, one division can be set for each country or continent. For example, Dell has Dell USA, Dell Germany, and Dell China as three of its divisions. When a firm sells different products within the same market, one division can be set for each product or product category. An example is Hewlett-Packard, which is organized into three divisions: the personal systems group, the imaging and printing group, and the technology solutions group (http://www.hp.com/hpinfo/abouthp/). In general, the roles of the firm include allocating resources (e.g., capital) across divisions and specifying outside vendors for economies of scale and/or quality assurance. It is also possible that the firm supplies some products directly to its divisions.

We model a single division as a newsvendor with the objective of maximizing profit probability. Furthermore, we extend the existing literature by considering a price-setting newsvendor. That is, given a profit target, each division decides on production level and selling price simultaneously to maximize the probability of achieving its profit target. We also examine how the firm should set profit targets for its divisions, all of which have the objective of maximizing profit probability. Our results are derived in two specific cases. In the first case of fair target setting, we follow prior studies (e.g., Bushman et al. 1995) and assume that the sum of all divisional profit targets equals the profit target for the firm. In the second case, we assume the firm only owns two divisions.

The rest of the chapter is organized as follows. In Sect. 15.2, we derive the optimal production and pricing decisions of a single division given its assigned profit target. We proceed on studying the problem of profit target setting from the firm's perspective when a firm owns multiple divisions, each of which has a profit target. In Sect. 15.3, we study the firm with the objective of maximizing expected profit. In Sect. 15.4, we study the firm with the objective of maximizing profit probability, where we focus on the special cases of fair target setting and two identical divisions. Finally, in Sect. 15.5, we summarize and discuss future research directions.

15.2 A Single Division Given a Profit Target

Before we study the problem of profit target setting for multiple divisions, we need to derive the optimal production level and selling price of a single division given a profit target. This is the focus of this section.

Suppose that a division has a unit production cost (or procurement cost) of c and a unit selling price of r. To simplify the presentation, both salvage value and loss-of-goodwill cost are assumed to be zero. Furthermore, instead of the traditional objective of maximizing expected profit, the division is assigned a profit target t and, hence, adopts the objective of maximizing profit probability. In other words, the division maximizes the probability of achieving the profit target t. For simplicity, the probability is called the profit probability.

It is worth noting that targets may not be always set to maximize profit. Under uncertain business environment (e.g., stochastic customer demand), only expected profit can be maximized. However, even if *expected* profit is maximized, it is likely that the *actual* profit is lower than the expected profit due to the variability in profit. This is particularly a concern because by definition, newsvendors make one-time and, hence, nonrepeatable decisions.

The market demand is random and is affected by selling price. In this research, the market demand is modeled as a multiplicative form:

$$D(r) = r^{-b}\varepsilon \tag{15.1}$$

where ε is a random variable taking positive values with cumulative distribution function $F(x)$ and probability density function $f(x)$. To avoid trivial situations, $F(x)$ is assumed to be increasing and differentiable. The price elasticity b represents the extent to which the customers are sensitive to price changes. To be more specific, if the price changes by 1%, the customer demand changes by $b\%$ in the opposite direction. Some examples of price elasticity for a product category include fresh green peas ($b = 2.8$), fresh tomatoes ($b = 4.6$), and Chevrolet automobiles ($b = 4.0$) (Gwartney 1976). In this chapter, it is assumed that each division is selling an individual product, which tends to have a greater price elasticity mainly due to the great availability of substitutes in the same product category (Gwartney 1976). Therefore, it is reasonable to limit ourselves to elastic goods or services ($b > 1$).

The multiplicative demand model is also called an isoelastic demand model or double-log-linear model. One limitation of such a model is that a price change results in a scale change, but *not* a location change, in the demand distribution. Despite this limitation, this demand model is the most frequently used demand specification among econometricians, market empiricists, and researchers in Operations Management. Monahan et al. (2004) summarize four reasons to explain the popularity of this multiplicative demand model:

1. It is consistent with consumer-utility-maximization theory; thus, it is a reasonable candidate for model building.

2. By explicitly accounting for the effects of price elasticity on demand, it has an unambiguous economic interpretation.
3. Its log-linearity is particularly amenable to empirical analysis because its parameters can be estimated using well-established linear regression techniques.
4. Perhaps most importantly, it typically provides a good statistical fit with available sales data.

Typical products whose demands may follow the multiplicative model include high fashion products or newly introduced products (Agrawal and Seshadri 2000).

The division's random profit function as a function of production level and selling price is given by:

$$\Pi(q,r) = (r-c)q - r(q-D(r))^{+}. \tag{15.2}$$

Now, the division is given a profit target t to achieve. For simplicity, the profit target t, once determined, is assumed to be independent of the market demand in this section. Of course, when a profit target is being set, a number of factors may be taken into account, including the market demand. This is the target-setting problem to be studied in later sections.

If selling price r is exogenous, the division's optimal production level and the maximal profit probability are given by (see, e.g., Kabak and Schiff 1978):

$$q(r) = \frac{t}{r-c} \tag{15.3}$$

$$P(r) = 1 - F_{D(r)}\left(\frac{t}{r-c}\right) = 1 - F\left(\frac{t}{(r-c)r^{-b}}\right), \tag{15.4}$$

where $F_{D(r)}(\cdot)$ denotes the cumulative distribution function of demand $D(r)$. To choose r to maximize the profit probability $P(r)$, it is equivalent to maximize $(r-c)r^{-b}$. It can be easily verified that the optimal selling price, denoted by $G(b,c)$, is given by:

$$G(b,c) = \frac{b}{b-1}c. \tag{15.5}$$

It can be seen from (15.5) that the greater the price elasticity, the lower the optimal selling price. More interestingly in the current context, the optimal selling price is independent of profit target t. This is because of the multiplicative demand model where pricing affects only the scale, but not the location, of the demand distribution. Finally, the optimal profit margin is:

$$\frac{G(b,c)-c}{G(b,c)} = \frac{1}{b}, \tag{15.6}$$

which is exactly the reciprocal of the price elasticity. Hence, a greater price elasticity always leads to a lower profit margin.

By substituting the optimal price $G(b,c)$ into (15.3) and (15.4), we have the optimal production level and the maximal profit probability as:

$$q^* = \frac{b-1}{c}t = H(b,c)t \tag{15.7}$$

$$\begin{aligned} P^* &= 1 - F(L(b,c)H(b,c)t) \\ &= 1 - F\left(\frac{c^{b-1}b^b}{(b-1)^{b-1}}t\right), \end{aligned} \tag{15.8}$$

where $H(b,c) = (b-1)/c$ and $L(b,c) = (bc/(b-1))^b$ are defined for notation simplicity.

It can be seen from (15.7) that the optimal production level q^*s increases with respect to the price elasticity. This is because a greater price elasticity leads to a lower profit margin. To achieve the same profit target, production level (and, hence, sales revenue) has to be larger. From (15.8), it can be seen that the higher the production cost and the higher the profit target, the smaller the maximal profit probability P^*. Both predictions make intuitive sense. To detail how price elasticity impacts the maximal profit probability, we have the following two propositions.

Proposition 1. *When production cost is relatively high, i.e., $c \geq (b-1)/b$, the term $L(b,c)H(b,c)$ increases with respect to the price elasticity b. Otherwise, the term $L(b,c)H(b,c)$ decreases with respect to the price elasticity b.*

Proof. It can be seen from (15.8) that:

$$L(b,c)H(b,c) = \frac{c^{b-1}b^b}{(b-1)^{b-1}}. \tag{15.9}$$

Differentiating $L(b,c)H(b,c)$ with respect to b, we have:

$$\frac{\partial L(b,c)H(b,c)}{\partial b} = \frac{c^{b-1}b^b}{(b-1)^{b-1}} \ln \frac{bc}{b-1}. \tag{15.10}$$

Therefore, if production cost is relatively high, i.e., $c \geq (b-1)/b$, we have $\ln(bc/(b-1)) \geq 0$ and, hence, the first derivative $\partial L(b,c)H(b,c)/\partial b \geq 0$. Otherwise, we have $\partial L(b,c)H(b,c)/\partial b < 0$. This concludes the proof. □

Proposition 2. *When production cost is relatively high, i.e., $c \geq (b-1)/b$, a greater price elasticity leads to a smaller maximal profit probability. When production cost is relatively low, i.e., $c < (b-1)/b$, a greater price elasticity leads to a larger maximal profit probability.*

Proof. It can be seen from (15.8) that the maximal profit probability P^* and the term $L(b,c)H(b,c)$ change with respect to the price elasticity b in *opposite* directions. Hence, Proposition 2 can be proved similarly to Proposition 1. □

Therefore, how maximal profit probability changes with respect to price elasticity depends on the value of production cost. The intuitions are as follows. To increase profit and, hence, profit probability, a division has two options. Option 1 is to increase selling price, which, however, reduces customer demand. Option 2 is to lower selling price, which increases customer demand. For a division with high price elasticity, Option 1 is unattractive because the benefit of price increase is more than offset by the cost of demand drop. Therefore, a division with high price elasticity should adopt Option 2. If the same division has relatively low production cost, the greater the price elasticity, the larger demand increase will result from the same level of price drop. Consequently, profit as well as profit probability will be higher. On the other hand, if the same division has a relatively high production cost, there is a limit in terms of price reduction because selling price has to be larger than production cost to have a positive profit. Therefore, with high price elasticity, the benefit of demand increase is insufficient to compensate for the cost of price drop. As a result, both profit and profit probability will be lower.

The discussion above implicitly assumes that the random variable ε, and, hence, the customer demand, is defined on $[0,+\infty]$, which implies any profit target t in theory could be achieved. However, we assume that only an achievable profit target will be assigned. We thus establish an upper bound on profit target where ε is defined on a limited interval. The result is given in the following proposition, which will be used in Sect. 15.4.

Proposition 3. *Suppose that the market demand is given by $D(r) = r^{-b}\varepsilon$ where the random variable ε is defined on $[\alpha,\beta]$. For a profit target t to be achievable at all, it is required that $t < \frac{1}{c^{b-1}}\frac{(b-1)^{b-1}}{b^b}\beta$.*

Proof. If the division sets selling price at r, then the maximal achievable profit is $(r-c)r^{-b}\beta$. This happens when the random variable ε is realized at its maximal possible value β. Furthermore, $(r-c)r^{-b}\beta$ is maximized at $G(b,c)$ (defined in (15.5)). Therefore, all feasible profit targets are less than $(G(b,c)-c)L^{-1}(b,c)\beta$, which is equivalent to $t < \frac{(b-1)^{b-1}}{c^{b-1}b^b}\beta$. This concludes the proof. □

15.3 A Firm with the Objective of Maximizing Expected Profit

Starting with this section, we will study the business scenario where a firm owns n divisions. Suppose each division's performance is evaluated based on whether or not it achieves a predetermined profit target. As a result, each division always adopts the objective of maximizing profit probability, i.e., the objective of maximizing the probability of achieving its profit target. On the other hand, the firm itself may adopt the objective of maximizing expected profit or the objective of maximizing profit probability, which will be addressed in this section and next section, respectively.

If the firm adopts the objective of maximizing expected profit, the firm is assumed to be risk neutral. This assumption of risk neutrality could be reasonable when the firm is able to diversify its risk through the n divisions, especially when n is relatively large.

Because of the objective of maximizing expected profit, maximizing expected profit of the firm is equivalent to maximizing the expected profit of each division assuming that intra-firm transactions and interdependence between divisions are negligible. This property holds true only because expected value is a linear operator. Since there are n divisions, the firm needs to assign n profit targets t_i, where subscript $i = 1\cdots n$ denotes division i. Division i has a unit production cost c_i and a price elasticity b_i. Its customer demand is given by $D_i(r_i) = r_i^{-b_i}\varepsilon_i$, where the random variables ε_i (with cumulative distribution function $F_i(\cdot)$) is independent of ε_j, $j \neq i$.

It can be shown from (15.2) that the expected profit of division i is given by:

$$E\Pi_i^*(t_i) = t_i - G_i(b_i,c_i)\int_0^{H_i(b_i,c_i)t_i} F_i(L_i(b_i,c_i)x)\mathrm{d}x. \tag{15.11}$$

We have the following theorem on setting the optimal profit targets for the divisions.

Theorem 1. *If the firm is risk neutral, the divisions' optimal profit targets are given by:*

$$t_i^* = \frac{1}{c_i^{b_i-1}}\frac{(b_i-1)^{b_i-1}}{b_i^{-b_i}}F_i^{-1}\left(\frac{1}{b_i}\right), \quad i = 1\cdots n \tag{15.12}$$

Proof. Based on (15.11), we have the following derivatives:

$$\frac{\partial E\Pi_i^*(t_i)}{\partial t_i} = 1 - b_iF_i(L_i(b_i,c_i)H_i(b_i,c_i)t_i), \tag{15.13}$$

$$\frac{\partial^2 E\Pi_i^*(t_i)}{\partial t_i^2} = -b_iL_i(b_i,c_i)H_i(b_i,c_i)f_i(L_i(b_i,c_i)H_i(b_i,c_i)t_i). \tag{15.14}$$

Because $\partial^2 E\Pi_i^*(t_i)/\partial t_i^2 < 0$, $\Pi_i^*(t_i)$ is concave in t_i. By setting the first derivative to zero, we can obtain (15.12). This concludes the proof. □

Based on (15.12), an immediate observation is that a division with a higher production cost should be assigned a lower profit target. Moreover, it can be seen from Proposition 1 that when production cost is relatively high, $L_i(b_i,c_i)H_i(b_i,c_i)$ increases in price elasticity and, hence, a division with a greater price elasticity will be assigned a lower profit target. However, when production cost is relatively low, because both the denominator $L_i(b_i,c_i)H_i(b_i,c_i)$ and the nominator $F_i^{-1}(1/b_i)$ decrease in b, the optimal profit target may increase or decrease with price elasticity.

15.4 A Firm with the Objective of Maximizing Profit Probability

In this section, we assume the firm itself has a profit target T to achieve, and the firm adopts the objective of maximizing profit probability, i.e., the objective of maximizing the probability of achieving the target T. Its decisions are to assign profit target t_i to division i, $i = 1\cdots n$.

Given a profit target t_i, division i chooses the optimal selling price as in (15.5) and the optimal production level as in (15.7). Substituting (15.5) and (15.7) into (15.2), we have the associated random profit of division i as:

$$\Pi_i(t_i) = t_i - G_i(b_i,c_i)(H_i(b_i,c_i)t_i - L_i^{-1}(b_i,c_i)\varepsilon_i)^+. \tag{15.15}$$

The random profit function of the firm is then given as:

$$\Pi(t_1,\cdots t_n) = \sum_{i=1}^n \left[t_i - G_i(b_i,c_i)(H_i(b_i,c_i)t_i - L_i^{-1}(b_i,c_i)\varepsilon_i)^+\right]. \tag{15.16}$$

It can be seen from (15.16) that the maximal achievable profit target for the firm is the sum of individual profit target, i.e., $\sum_{i=1}^n t_i$. This is because the function $(\cdot)^+$ only takes non-negative values. Therefore, to make sure the profit target is achievable for the firm, it is required that $\sum_{i=1}^n t_i \geq T$.

The firm's optimization problem is to select profit target t_i such that its own profit probability is maximized. Mathematically, it is formulated as follows:

$$\begin{aligned}\max_{t_i} P_T(t_1,\cdots t_n) &= \max_{t_i} P\{\Pi(t_1,\cdots t_n) \geq T\}\\ &= \max_{t_i} P\left\{\sum_{i=1}^n G_i(b_i,c_i)\left(H_i(b_i,c_i)t_i - L_i^{-1}(b_i,c_i)\varepsilon_i\right)^+ \leq \sum_{i=1}^n t_i - T\right\}.\end{aligned} \tag{15.17}$$

Unfortunately, this optimization problem in general does not have analytic solutions and may require the use of numerical simulation. To simplify the optimization problem and thus gain more managerial insights, we study two simplified cases, namely, the case of fair target setting and the case of two divisions in the following two sections, respectively. Further, for concision, we denote the functions $G_i(b_i,c_i)$, $H_i(b_i,c_i)$ and $L_i(b_i,c_i)$ by G_i, H_i, and L_i, respectively, when no confusion arises.

15.4.1 Fair Target Setting Case

In the case of fair target setting, the sum of the divisions' profit targets is equal to the profit target of the firm, i.e., $\sum_{i=1}^n t_i = T$ (see, e.g., Lau and Lau 1988). This is consistent with Bushman et al. (1995) who assumes that the sum of the multiple divisions' outputs is the firm's output.

Under the requirement of fair target setting, i.e., $\sum_{i=1}^n t_i = T$, the optimization problem (15.17) can be simplified to:

$$\max_{t_i} P\left\{\sum_{i=1}^n G_i\left(H_i t_i - L_i^{-1}\varepsilon_i\right)^+ \leq 0\right\}. \tag{15.18}$$

Because the function $(\cdot)^{+}$only takes nonnegative values and random variables ε_i's are independent of each other, the optimization problem (15.18) can be further simplified to:

$$\max_{t_i} P\left\{L_i^{-1}\varepsilon_i \geq H_i t_i,\ i=1,\cdots n\right\} = \max_{t_i} \bar{F}_1(L_1H_1t_1)\bar{F}_2(L_2H_2t_2)\cdots\bar{F}_n(L_nH_nt_n), \tag{15.19}$$

where $\bar{F}_i(x) = 1 - F_i(x)$. We further define $W_i(x) = f_i(x)/\bar{F}_i(x)$ as the failure rate (or the hazard rate) of random variable ε_i. A demand distribution exhibits the property of increasing failure rate if $\partial W_i(x)/\partial x > 0$. Examples of such distributions include most reasonable customer demand distributions such as uniform, exponential, normal, gamma, and Weibull distributions (Barlow and Proschan 1965; Lariviere 2006).

Theorem 2. *If each division's demand distribution has an increasing failure rate, under fair target setting, the optimal profit targets for the divisions can be solved from the following equations:*

$$W_1(L_1H_1t_1^*)L_1H_1 = \ldots = W_n(L_nH_nt_n^*)L_nH_n \text{ and } \textstyle\sum_{i=1}^{n} t_i^* = T. \tag{15.20}$$

Further, for a division with a relatively high (low) production cost, a greater price elasticity leads to a lower (higher) optimal profit target.

Proof. To maximize (15.19), it is equivalent to maximize

$$\textstyle\sum_{i=1}^{n} \ln\bar{F}_i(L_iH_it_i) \tag{15.21}$$

subject to the constraint $\sum_{i=1}^{n} t_i = T$. The Lagrangian function for this optimization problem is:

$$Z(t_1,\cdots t_n,\lambda) = \textstyle\sum_{i=1}^{n} \ln\bar{F}_i(L_iH_it_i) = -\lambda\left(\sum_{i=1}^{n} t_i - T\right). \tag{15.22}$$

The first-order derivatives are given by:

$$\frac{\partial Z(t_1,\cdots t_n,\lambda)}{\partial t_i} = -L_iH_i\frac{f_i(L_iH_it_i)}{\bar{F}_i(L_iH_it_i)} - \lambda = -L_iH_iW_i(L_iH_it_i) - \lambda, \quad i=1,\cdots n \tag{15.23}$$

$$\frac{\partial Z(t_1,\cdots t_n,\lambda)}{\partial \lambda} = \textstyle\sum_{i=1}^{n} t_i - T \tag{15.24}$$

The second-order derivatives with respect to t_i are given by:

$$\frac{\partial^2 Z(t_1,\cdots t_n,\lambda)}{\partial t_i^2} = -L_i^2H_i^2\frac{\partial W_i(L_iH_it_i)}{\partial t_i}. \tag{15.25}$$

Because the demand is modeled as $D_i(r_i) = r_i^{-b_i}\varepsilon_i$, the distribution of D_i has an increasing failure rate if and only if the distribution of ε_i has an increasing failure rate $W_i(x)$. Therefore, we have $\partial W_i(L_iH_it_i)/\partial t_i > 0$ and thus $\partial^2 Z(t_1,\cdots t_n,\lambda)/\partial t_i^2 < 0$.

Hence, $Z(t_1,\cdots t_n,\lambda)$ is concave in $(t_1,\cdots t_n)$. Setting the first-order derivatives (15.23) and (15.24) to zero, we can obtain (15.20).

Suppose division i has a relatively high production cost. It can be seen from Proposition 1 that the term $L_i(b,c)H_i(b,c)$ increases in the price elasticity b_i. Based on (15.20), the optimal profit target then decreases in b_i because $W_i(x)$ is an increasing function. Using similar arguments, we can show that the opposite is true for a division with a relatively low production cost. □

Example 1. Suppose that ε_i follows uniform distribution defined on interval $[0,\beta_i]$. So we have:

$$\bar{F}_i(x)=\frac{\beta_i-x}{\beta_i},\quad 0\le x\le\beta_i \text{ and } W_i(x)=\frac{1}{\beta_i-x}. \tag{15.26}$$

Based on (15.20), we have:

$$\frac{L_1H_1}{\beta_1-L_1H_1t_1^*}=\frac{L_2H_2}{\beta_2-L_2H_2t_2^*}=\ldots=\frac{L_nH_n}{\beta_n-L_nH_nt_n^*}, \tag{15.27}$$

which further gives:

$$t_j^*=t_i^*+\frac{\beta_j}{L_jH_j}-\frac{\beta_i}{L_iH_i},\quad i,j=1,\cdots n \text{ and } i\neq j. \tag{15.28}$$

Together with $\sum_{i=1}^n t_i^*=T$, we have the optimal profit target for each division:

$$t_i^*=\frac{T}{n}+\frac{1}{n}\sum\nolimits_{j=1,j\neq i}^{n}\left(\frac{\beta_i}{L_iH_i}-\frac{\beta_j}{L_jH_j}\right),\quad i=1,\cdots n \tag{15.29}$$

Therefore, the optimal profit target consists of two components. The first component is the firm's profit target divided by the number of divisions. The second component shows how the optimal profit target for each division is further adjusted by its market size β_i and the term $L_i(b,c)H_i(b,c)$, which increases (decreases) with respect to the price elasticity b_i when production cost c_i is relatively low (high).

15.4.2 Two Divisions Case

In this section, we focus on the case where the firm owns only two divisions. For the firm, the optimization problem is then to decide on profit target t_1 and t_2 for the two divisions, respectively. We first have the following proposition.

Proposition 4. *Suppose that the firm with profit target T owns two divisions. Under the following reasonable assumptions:*

$$t_1\le(b_2-1)t_2+T \text{ and } t_2\le(b_1-1)t_1+T, \tag{15.30}$$

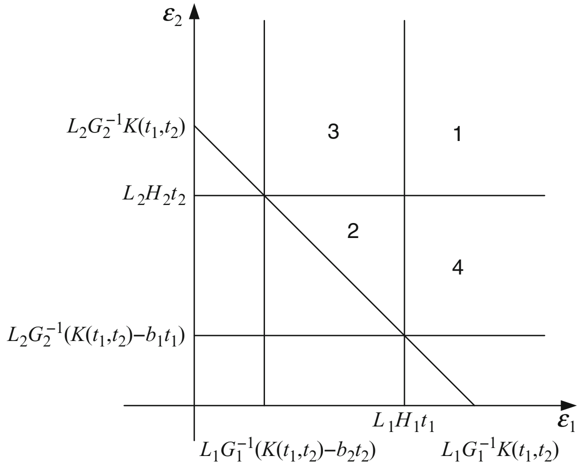

Fig. 15.1 The four possible scenarios where the firm achieves its profit target

the firm's profit probability is given by:

$$P_T(t_1,t_2) = P\{\varepsilon_1 \geq L_1G_1^{-1}(K(t_1,t_2)-b_2t_2), \varepsilon_2 \geq L_2G_2^{-1}(K(t_1,t_2)-b_1t_1),\ G_1L_1^{-1}\varepsilon_1+G_2L_2^{-1}\varepsilon_2 \geq K(t_1,t_2)\}, \quad (15.31)$$

where

$$K(t_1,t_2) = (b_1-1)t_1+(b_2-1)t_2+T. \quad (15.32)$$

Proof. Based on (15.17), it can be seen that the firm's profit target T will be achieved if and only if

$$\textstyle\sum_{i=1}^{2} G_i(H_it_i - L_i^{-1}\varepsilon_i)^+ \leq (t_1+t_2-T). \quad (15.33)$$

Of course, we should have the constraint $t_1+t_2 \geq T$ to guarantee that T is achievable at all. Depending on the possible realizations of ε_1 and ε_2, we have the following four possible scenarios:

Scenario 1: If $\varepsilon_1 \geq L_1H_1t_1$ and $\varepsilon_2 \geq L_2H_2t_2$, (15.33) becomes $t_1+t_2 \geq T$.
Scenario 2: If $\varepsilon_1 \leq L_1H_1t_1$ and $\varepsilon_2 \leq L_2H_2t_2$, (15.34) becomes $G_1L_1^{-1}\varepsilon_1 + G_2L_2^{-1}\varepsilon_2 \geq K(t_1,t_2)$.
Scenario 3: If $\varepsilon_1 \leq L_1H_1t_1$ and $\varepsilon_2 \geq L_2H_2t_2$, (15.35) becomes $\varepsilon_1 \geq L_1G_1^{-1}[K(t_1,t_2) -b_2t_2]$.
Scenario 4: If $\varepsilon_1 \geq L_1H_1t_1$ and $\varepsilon_2 \leq L_2H_2t_2$, (15.36) becomes $\varepsilon_2 \geq L_2G_2^{-1}[K(t_1,t_2) -b_1t_1]$.

The four scenarios are shown graphically in Fig. 15.1.

When plotting the graph, assumptions as in (15.30) are employed to guarantee that $L_1G_1^{-1}(K(t_1,t_2)-b_2t_2) > 0$ and $L_2G_2^{-1}(K(t_1,t_2)-b_1t_1) > 0$.

These assumptions are reasonable because a division's profit target is generally no more than the firm's profit target. Moreover, it can be verified that:

$$
\begin{aligned}
L_1H_1t_1 - L_1G_1^{-1}(K(t_1,t_2) - b_2t_2) &= L_1G_1^{-1}(t_1+t_2-T) \\
L_2H_2t_2 - L_2G_2^{-1}(K(t_1,t_2) - b_1t_1) &= L_2G_2^{-1}(t_1+t_2-T).
\end{aligned} \tag{15.34}
$$

Hence, the Area 2 in Fig. 15.1 directly depends on the difference between the firm's profit target and the sum of the divisions' profit targets. Finally, because the firm's profit probability $P_T(t_1,t_2)$ is the sum of the four areas in Fig. 15.1, (15.31) is true. This concludes the proof. □

Proposition 4 greatly simplifies the calculation of $P_T(t_1,t_2)$ for the case of two divisions. To obtain more concrete results, now we assume two identical divisions. This means that the two divisions have the same production cost c, the same price elasticity b, and ε_1 and ε_2 are independent and identically distributed. Given these two identical divisions, it is practical to assume that the two divisions should be assigned an identical profit target, denoted by t. The optimization problem for the firm is then to select a single profit target t for both divisions so that the firm's profit probability is maximized. We have the following theorem.

Theorem 3. *Suppose that the firm with profit target T is in control of two identical divisions. Then its profit probability is given by:*

$$P_T(t) = P\{\varepsilon_1+\varepsilon_2 \geq K_1(t), \varepsilon_1 \geq K_2(t), \varepsilon_2 \geq K_2(t)\}, \tag{15.35}$$

where

$$K_1(t) = LG^{-1}[(2b-2)t+T] \tag{15.36}$$

$$K_2(t) = LG^{-1}[(b-2)t+T]. \tag{15.37}$$

Furthermore, if $b \geq 2$, the optimal profit target for each division t^* is just half of the firm's profit target T, i.e., $t^* = T/2$.

Proof. Because this is the special case of two identical divisions, we can (15.35) readily from (15.31). In addition, the requirements in (15.30) become:

$$(2-b)t \leq T. \tag{15.38}$$

If $b \geq 2$, (15.38) is true. If $1 < b < 2$, (15.38) is true as well because $t \geq T/2$. Therefore, for the case of two for the case of two identical divisions, the assumption in (15.38) is satisfied automatically. To prove the second half of the theorem, we differentiate $P_T(t)$ with respect to t:

$$
\begin{aligned}
P_T'(T) = &-(b-2)f_{\varepsilon_1|\varepsilon_2\geq k_2(t),\varepsilon_1+\varepsilon_2\geq K_1(t)}(K_2(t))P\{\varepsilon_2 \geq K_2(t), \varepsilon_1+\varepsilon_2 \geq K_1(t)\} \\
&-(b-2)f_{\varepsilon_2|\varepsilon_1\geq K_2(t),\varepsilon_1+\varepsilon_2\geq K_1(t)}(K_2(t))P\{\varepsilon_1 \geq K_2(t), \varepsilon_1+\varepsilon_2 \geq K_1(t)\} \\
&-2(b-1)f_{\varepsilon_1+\varepsilon_2|\varepsilon_1\geq K_2(t),\varepsilon_2\geq K_2(t)}(K_2(t))P\{\varepsilon_1 \geq K_2(t), \varepsilon_2 \geq K_2(t)\}.
\end{aligned} \tag{15.39}
$$

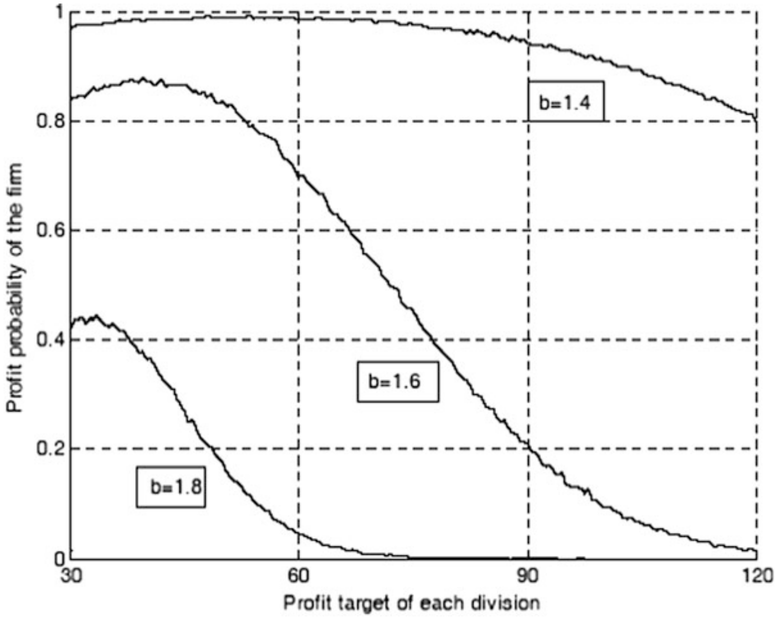

Fig. 15.2 The firm's profit probability as a function of each division's profit target when $b = 1.4, 1.6$, and 1.8

If the price elasticity $b \geq 2$, $P_T'(T) < 0$. Therefore, the firm's profit probability decreases in t and the optimal profit target t^* should be set at its minimum, i.e., $t^* = T/2$. This concludes the proof. □

It can be seen when the price elasticity of the product is reasonably large ($b \geq 2$), the optimal profit target for each division is always half of the firm's profit target. This result holds independent of all the other parameters including the underlying market demand distribution and production cost. It is also worth noting that for many individual products, it is common to have substitutes from competitors in the market (Gwartney 1976). As a result, $b \geq 2$ should hold for many business scenarios.

In the case of a relatively small price elasticity, in this case $b < 2$, it is difficult to obtain an analytic expression for the optimal profit target in general. In this case, the use of numerical simulation may be necessary. We present two examples here. In the first example, we conduct a computer simulation when the underlying market demand is normally distributed. In the second example, we are able to derive the closed-form expression of the optimal profit target for uniform distribution.

Example 2. Suppose that ε_i follows a normal distribution with mean 200 and standard deviation 50. Other parameters are: $c = 2$, $T = 60$, and $b = 1.4$, 1.6, and 1.8. Figure 15.2 shows the firm's profit probability as a function of the profit target of each division when $b = 1.4$, 1.6, and 1.8. The computer simulation is implemented using Matlab.

It can be seen from Fig. 15.2 that the firm's maximal profit probability ($P_T^*(t)$) is quite sensitive to the price elasticity b. As b changes from 1.4 to 1.8 (a change of 29%), $P_T^*(t)$ changes from almost 1.00 to approximately 0.45 (a change of 55%). Finally, the optimal target for each division (t^*) decreases with respect to b. As b approaches 2, the optimal target t^* approaches $T/2 = 30$.

Example 3. Suppose that ε_i follows a uniform distribution with lower bound 0 and upper bound β. To calculate the probability based on (15.31), we need to check the lower and upper bounds of $K_1(t)$ and $K_2(t)$ defined in (15.32) and (15.33), respectively. This is important because the random variable ε_i now is defined on a limited interval.

Because we have $T \le 2t$, we have $K_2(t) = LG^{-1}[(b-2)t+T] \le G^{-1}Lbt$. Further, based on Proposition 3 (see Sect. 15.2), we have $0 < K_2(t) < LG^{-1}b^*(g-c)$ $L^{-1}\beta = \beta$. Similarly, we can have $0 < K_1(t) < 2\beta$.

Based on (15.35), we can have:

$$\begin{aligned} P_T(t) &= P\{\varepsilon_1 \ge K_2(t), \varepsilon_2 \ge K_2(t)\} - P\{\varepsilon_1 \ge K_2(t), \varepsilon_2 \ge K_2(t), \varepsilon_1 + \varepsilon_2 \le K_1(t)\} \\ &= \frac{(\beta - K_2(t))^2}{\beta^2} - \frac{L^2}{2G^2}\frac{(2t-T)^2}{\beta^2}. \end{aligned} \tag{15.40}$$

The first- and second-order conditions are given by:

$$P_T'(t) = \frac{2L^2}{\beta^2 G^2}[(2-b)(L^{-1}G\beta + (2-b)t - T) - (2t - T)]. \tag{15.41}$$

$$P_T''(t) = \frac{2L^2}{\beta^2 G^2}\left[(2-b)^2 - 2\right] \tag{15.42}$$

When $1 < b < 2$, we have $(2-b)^2 - 2 < 0$ and, hence, $P_T''(t) < 0$. Therefore, the profit probability $P_T(t)$ is concave in the divisions' profit target t. The solution to $P_T'(t) = 0$ is given by:

$$t^o = \frac{(b-1)T + L^{-1}G(2-b)\beta}{2-(2-b)^2}. \tag{15.43}$$

Therefore, the optimal profit target for each division is given by $t^* = \max(t^o, T/2)$. Similar to Example 2, as the price elasticity approaches 2, t^o and, hence, the optimal target t^* approaches $T/2$.

15.5 Conclusions and Future Research

Existing studies have shown that the objective of maximizing profit probability leads to vastly different managerial insights than those based on the objective of maximizing expected profit. For example, Shi and Chen (2007) demonstrate that the simple wholesale price contract, when properly designed, can coordinate a supply chain with a single supplier and a single retailer, both of which adopt the objective of maximizing profit probability.

However, there has been little research on profit target setting under stochastic customer demand. We attempt to fill this gap with this chapter. To be more specific, we present analytic models on profit target setting under a common business scenario where a firm owns n divisions. While each division always has a profit target to achieve and, hence, adopts the objective of maximizing profit probability, the firm may adopt the objective of maximizing expected profit or the objective of maximizing profit probability. Given its assigned target, each division acts as a price-setting newsvendor and decides on divisional production level and selling price simultaneously.

We start by deriving the optimal behavior of a single division given its assigned profit target. We obtain the close-form expressions of the optimal production level, the optimal selling price, and the maximal profit probability. We show that for a division with a relatively higher (lower) production cost, a greater price elasticity always leads to a smaller (larger) profit probability.

We then study the problem of profit target setting from the firm's perspective. We first study the firm with the objective of maximizing expected profit, where we obtain closed-form expressions of the optimal profit target for each division. We then study the firm with the objective of maximizing profit probability. After deriving the firm's profit probability as a function of the divisions' profit targets, we proceed to study two special cases to gain more managerial insights.

In the first case of fair target setting, we derive the optimal profit targets when each division has a demand distribution with the property of increasing failure rate. We show that for a division with a relatively high (low) production cost, a greater price elasticity always leads to a (lower) higher profit target. In the second case where the firm has two divisions, we first derive the firm's profit probability as a function of the profit targets of the two divisions. If those divisions are identical, we show that each division's optimal target is just half of the firm's profit target when the price elasticity is two or more. This result is true regardless of the other parameters such as production cost and the demand distribution.

Our results in this chapter are helpful for practitioners who engage in the profit target setting and target attainment. However, much more work can be done in the area of profit target setting, which is both interesting and challenging. The first natural extension is to confirm the robustness of the results in this chapter by considering additive demand models. It is well documented that additive and multiple demand models may lead to different managerial insights (see, e.g., Choi 1991). The second natural extension is to see how risk seeking of firms impacts profit target setting for multiple divisions. Risk seeking can be modeled, for example, using the utility function approach where variance positively contributes to utility. Third, we can try to answer the following question: how does a firm set targets on multiple and potentially conflicting performance measures, such as profit, revenue, and market share? Fourth, we can study analytic target setting over multiple periods. For example, if a division of a firm has a yearly target to achieve, how can the division set quarterly targets and adjust them over time if necessary? Last but not least, it would be important to conduct empirical studies on profit target setting in business practice.

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