Chapter 13
Analysis of the Single-Period Problem under Carbon Emissions Policies

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Abstract We investigate the classical single-period (newsvendor) problem under carbon emissions policies including the mandatory carbon emissions capacity, the carbon emissions tax, and the cap-and-trade system. Specifically, under each policy, we find a firm's optimal production quantity and corresponding expected profit, and draw analytic managerial insights. We show that, in order to reduce carbon emissions by a certain percentage, the tax rate imposed on the high-margin firm should be less than that on the low-margin firm for the high-profit perishable products, whereas the high-margin firm should absorb a high tax than the low-margin firm for the low-profit products. Under the cap-and-trade policy, the emissions capacity should be set to a level such that the marginal profit of the firm is less than the carbon credit purchasing price. We also derive the specific (closed-form) conditions under which, as a result of implementing the cap-and-trade policy, the firm's expected profit is increased and carbon emissions are reduced.

Keywords Cap-and-trade • Carbon emissions • Carbon tax • Single-period model

13.1 Introduction

The past three decades have clearly witnessed an increasingly serious impact of carbon dioxide on the environment. Carbon dioxide has been regarded as the main

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pollutant that is warming the Earth. It is a greenhouse gas that is emitted through transport, land clearance, and the production and consumption of food, fuels, manufactured goods, materials, wood, roads, buildings, and services CO2List.org (2006). For the purpose of environmental protection, many governments and organizations have been contributing to carbon emissions reduction with a common goal that carbon emissions should be reduced by at least half by 2050, as reported by, e.g., the International Energy Agency (2008).

In practice, a great number of governments have implemented some policies to control carbon emissions. In Congress of the United States (2008), the Congressional Budget Office (CBO) of the Congress of the United States provided a comprehensive study on the policy options for reducing CO_2 emissions. We find from the CBO's study that there are four *major* carbon emissions policies as follows: (i) a mandatory capacity on the amount of carbon emitted by each firm; (ii) a tax imposed to each firm on the amount of carbon emissions; (iii) a cap-and-trade system implemented to allow the emission trading; and (iv) an investment made by each firm in the carbon offsets to meet its carbon capacity requirement. In Sect. 13.2.2, we shall specify these four major policies, and show that the fourth policy can be per se regarded as a special case of the third Policy and it should be thus necessary, and interesting, to investigate the first, the second, and the third policies.

In this paper, we analyze the impact of the three policies on a profit-oriented firm's production quantity decision. We note that many profit-oriented firms have also observed the importance of the carbon emissions reduction, and responded by developing low-carbon technologies and adopting new and renewable energy resources. Furthermore, the Barloworld Optimus—the logistics arm of the multinational corporation "Barloworld"—reported that, even though over 80% of carbon savings are usually achieved at the product design stage, each firm can reduce carbon emissions by optimizing its operations in production, inventory, and transportation; see, for example, Benjaafar et al. (2010) and BuySmart network (2008). A survey by Accenture.com (2009) indicated that more than 86% supply chain executives have undertaken at least one green initiative in the areas such as recycling, lighting management, and energy-efficient systems. We also learn from Accenture.com (2009) that 10% of companies have actively modeled their supply chain carbon footprints and implemented successful sustainability initiatives.

For our analysis of carbon emissions policies, we focus on the optimal quantity decision of a firm making a perishable item with a short lifespan. The production of the item results in carbon emissions. It is realistic to consider the perishable item for the firm. For example, in the Huber Group (2003), the Huber Group—which provides facility services to commercial, industrial, educational, medical, retail, government, and institutional customers—released a technical information regarding the impact of newspaper printing with the carbon-based ink on the environment. In addition, as reported in Environmental News Energies Correspondent (2009), Carbon Trust, a British governmental organization, suggests that consumers should use real Christmas trees instead of artificial equivalents, because the carbon footprint left by artificial trees is at least ten times greater than real Christmas trees.

However, in today's market, the demand for artificial Christmas trees is still very high; for example, Tesco—the largest British supermarket chain—sold 300,000 artificial Christmas trees in December 2009.

To examine how each carbon emissions policy affects the firm's production quantity decision, we shall involve a corresponding parameter into the classical single-period model, and address the following questions:

1. What are the firm's optimal production quantity decision and corresponding maximum expected profit under each carbon emissions policy?
2. How does the implementation of a policy influence the carbon emissions reduction and the expected profits of the low-margin, the moderate-margin, and the high-margin firms?
3. Does there exist a "win–win" scenario in which the carbon emissions are decreased while the firm's expected profit is not reduced?

Our paper contributes to the literature by analyzing the single-period problem under carbon emissions policies and presenting managerial discussion on the incentive of the firm on the carbon emissions reduction. Even though our discussions on the policies are motivated by the practice of the U.S., our analytic approach and results should be useful to any government who intends to choose a proper policy to reduce carbon emissions. The remainder of this paper is organized as follows: In Sect. 13.2, we briefly review the relevant literature in Sect. 13.2.1, which shows the originality of this paper; and we present our discussion on existing carbon emissions policies in Sect. 13.2.2. In Sect. 13.3, we consider three policies, and for each policy analyze the single-period model to find the corresponding optimal quantity decision. Numerical study with sensitivity analysis are provided in Sect. 13.4. This paper ends with a summary of our results in Sect. 13.5. In addition, a list of major notations used in this paper is given in Table 13.1.

13.2 Preliminaries: Literature Review and Carbon Emissions Policies

In this section, we briefly review major relevant publications and discuss four carbon emissions policies, which are preliminaries to our analysis of the single-period problem under carbon emissions policies.

13.2.1 Brief Literature Review

We now review major publications that are closely related to this paper where we analyze the classical single-period model in the presence of carbon emissions policies. For a detailed description of the classical model, see, e.g., Hadley and Whitin (1963). The single-period model has been widely used to investigate a

Table 13.1 A list of major notations that are used in this paper

Notation	Definition
α	Unit purchasing price of the carbon credits
β	Unit selling price of the carbon credits
c	Unit acquisition cost of the perishable product
c_o	Unit overage cost
c_u	Unit underage cost
C	Fixed carbon capacity
e	Average carbon emissions per unit of the perishable product
κ	Percentage of the reduction in carbon emissions
Q	Order/production quantity
Q_c	Mandatory capacity for carbon emissions
s	Shortage (stockout) cost for each unsatisfied demand
τ	Tax amount paid by the firm for each unit of the perishable product
v	Salvage value per unit of the unsold perishable product
X	Aggregate demand, which is assumed to be a random variable with the probability density function (p.d.f.) $f(x)$ and the cumulative distribution function (c.d.f.) $F(x)$.

variety of problems in the operations management (OM) area. Khouja (1999) proposed a literature review of various single-period problems. In today's OM area, many scholars still extend the classical model to incorporate different objectives and utility functions, address different pricing policies, analyze the value of the demand information, etc.

Starting from the middle of 1990s, the carbon emissions-related issues have been attracting the OM scholars' attention. As a seminal publication, Penkuhn et al. (1997) considered the emission taxes and developed a nonlinear programming model for a production planning problem. Letmathe and Balakrishnan (2005) constructed two analytic models to determine a firm's production quantities under different environmental constraints. Kim et al. (2009) investigated the relationship between transportation costs and CO_2 emissions using the multi-objective optimization method. Cachon (2009) discussed how a reduction in carbon footprints affects supply chain operations and structures.

In recent two years, an increasing number of OM scholars examine some carbon emissions-related issues. For example, Hoen et al. (2010) investigated the effects of two regulation mechanisms on the decision on the transportation mode selection. Benjaafar et al. (2010) discussed how the carbon emissions concerns could be involved into the operational decision-making models with regard to procurement, production, and inventory management. They also provided insights that highlight the impact of operational decisions on the carbon emissions and the importance of the operational models in assessing the benefits of investments in more carbon-efficient technologies. Hua et al. (2010) investigated how firms manage the carbon emissions in their inventory control under the carbon emissions- trading mechanism. They derived the EOQ model, and analytically examined the impact of carbon trade, carbon price, and carbon capacity on order decisions, carbon emissions, and total cost.

Table 13.2 Four major carbon emission policies discussed by the Congressional Budget Office of the Congress of the United States

Policy		Brief Description
Policy 1:	Mandatory carbon emissions capacity	A firm's production quantity Q of the items that emit the carbon cannot exceed the mandatory capacity Q_c.
Policy 2:	Carbon emissions tax	A firm absorbs the tax τ for each unit of the produced item that emits the carbon.
Policy 3:	Cap-and-trade	A firm—with carbon credits prescribed by the policy-maker to allow the firm to make at most Q_c units of the items—can sell its unused credits at the sale price $\$\beta$ per item or buy other firms' extra credits at the purchasing price $\$\alpha$ per item.
Policy 4:	Investment in the carbon offsets	A firm is allowed to invest for the reduction in carbon emissions to meet the requirement of the mandatory capacity Q_c.

In this paper, we consider the classical single-period problem under three carbon emissions policies, which significantly distinguishes our analysis and those by, e.g., Benjaafar et al. (2010) and Hua et al. (2010). Moreover, we quantify the impact of different policies on the emissions reduction and the expected profit of the firm. This further shows the originality of our paper.

13.2.2 Description of Carbon Emissions Policies

We now describe four major carbon emissions policies that are discussed by the Congressional Budget Office of the Congress of the United States (2008). We begin by presenting a summary of these four policies as given in Table 13.2, where Q denotes a firm's production quantity of the items that emit the carbon, and Q_c means the mandatory capacity of the production that results in carbon emissions. Moreover, in Table 13.2, τ represents the tax amount paid by the firm for each unit of the item that emits the carbon; and, β and α denote the firm's unit sale price and unit purchasing price of the carbon credits in the cap-and-trade system, respectively.

Next, we discuss the four policies listed in Table 13.2 to determine which policies shall be later used to analyze the single-period problem. For our single-period problem under Policy 1 ("mandatory carbon emissions capacity"), the firm's optimal decision is subject to the mandatory capacity. That is, the firm needs to determine an optimal production quantity that maximizes its profit under the constraint that the firm's production quantity Q is smaller than or equal to the mandatory capacity Q_c, i.e., $Q \leq Q_c$. Note that, to simplify our analysis and facilitate our managerial discussion, we measure the carbon emissions-related parameters and constraints on the product-unit basis throughout the paper. This is justified as follows: In reality, carbon emissions can be generated from production, transportation, inventory, etc. Letting e denote the average carbon emissions generated by making one unit of product over the single period, we find that, when the firm has to adhere to a fixed carbon capacity C, he cannot produce more than $Q_{\mathrm{c}} = C/e$ products (that is, $Q \leq Q_{\mathrm{c}}$). This implies that it is reasonable to use Q_{c} instead of C for our analysis of the single-period problem.

For our problem under Policy 2 ("carbon emissions tax"), there is no carbon emissions constraint; but, the firm absorbs the tax on the amount of carbon emissions. Specifically, denoting by τ the carbon tax charged for one unit of product, we can calculate the firm's total tax payment as τQ. Under Policy 3 ("cap-and-trade"), the firm has prescribed carbon credits from the policy-maker, which allow the firm to produce at most Q_{c} units of products. However, the firm can trade extra (unused) carbon credits through a cap-and-trade system to vary its carbon capacity. This means that, in the cap-and-trade system, the firm can buy and sell the "right to emit."

Under Policy 4 ("investment in the carbon offsets"), the firm can invest in the carbon emissions-reduction projects to offset emissions in excess of the capacity Q_{c}. We note that the investment under Policy 4 is *per se* the same as the credit purchase in a cap-and-trade system under Policy 3 with $\beta = 0$. That is, if the firm's unused carbon credits cannot be sold, i.e., $\beta = 0$, then Policy 3 is equivalent to Policy 4 because α can be assumed to be the unit investment cost. Hence, Policy 4 can be regarded as a special case of Policy 3. For generality, we do not analyze our single-period problem under Policy 4 in this paper.

According to the above, we subsequently investigate the impact of Policies 1, 2, and 3 on the optimal decision in the single-period problem.

13.3 Analysis of the Single-Period Problem Under Carbon Emissions Policies

In this section, we analyze the classical single-period inventory model under three carbon emissions policies—i.e., Policies 1, 2, and 3 in Table 13.2. Our analytic results are also compared to investigate the impact of the three policies on the reduction in carbon emissions and the firm's expected profit. Next, we start with the firm's single-period inventory problem under Policy 1.

13.3.1 The Single-Period Problem Under Policy 1 (Mandatory Carbon Emissions Capacity)

For our analysis of the classical single-period problem, we let X denote the aggregate demand, which is assumed to be a random variable with the probability density function (p.d.f.) $f(x)$ and the cumulative distribution function (c.d.f.) $F(x)$.

In addition, p is the selling price per unit of the perishable product; c is the firm's unit acquisition cost; s is the shortage (stockout) cost for each unsatisfied demand; and v is the salvage value per unit of the unsold product. Then, $c_o \equiv c - v$ is the unit overage cost, and $c_u \equiv p + s - c$ represents the unit underage cost. Note that Q denotes the firm's order quantity, as defined in Table 13.1.

Using the above, we write the firm's expected profit function as,

$$J(Q) = (p - v)\int_0^Q xf(x)\,dx + (p + s - c)\int_Q^\infty Qf(x)\,dx$$
$$- s\int_Q^\infty xf(x)\,dx - (c - v)\int_0^Q Qf(x)\,dx. \tag{13.1}$$

We learn from our discussion in Sect. 13.2.2 that, in order to find optimal quantity Q^* under Policy 1 ("mandatory carbon emissions capacity"), the firm should maximize its expected profit $J(Q)$ in (13.1) under the constraint that $Q \leq Q_c$, where Q_c is the mandatory capacity. That is, the firm's maximization problem under Policy 1 is written as follows: $\max_{Q \leq Q_c} J(Q)$.

Theorem 1. *For the single-period problem under Policy 1 (mandatory carbon emissions capacity), the optimal quantity decision is found as* $Q_1^* = \min(Q^*, Q_c)$, *where* Q^* *is optimal solution of the classical single-period problem, i.e.,*

$$Q^* = F^{-1}(w), \quad \text{where } w \equiv \frac{c_u}{c_u + c_o} = \frac{p + s - c}{p + s - v}. \tag{13.2}$$

Proof. For a proof of this theorem and the proofs of all subsequent theorems, see 13.5. □

From the above theorem, we note that Policy 1 is effective only when the mandatory capacity Q_c does not exceed the Q^*, i.e., $Q_c \leq Q^*$. Otherwise, if $Q_c > Q^*$, then the firm always determines its optimal solution as Q^* for any value of Q_c, which means that the firm's optimal solution under Policy 1 is the same as that with not any policy. It thus follows that, in order to *effectively* reduce carbon emissions generated by the firm, the policy-maker needs to set the mandatory capacity as a value lower than the firm's optimal decision under no policy constraint.

Theorem 1 also indicates that we can compute Q_1^* when the c.d.f. $F(x)$ is explicitly given. For simplicity, we hereafter assume that the aggregate demand X for the perishable product is normally distributed with mean μ and standard deviation σ, i.e., $X \sim N(\mu, \sigma)$. We thus have,

$$J(Q^*) = \mu(p - c) - \sigma(c_u + c_o)\,\phi(z^*), \tag{13.3}$$

where $z^* \equiv (Q^* - \mu)/\sigma$, and ϕ is the p.d.f. of the standard normal distribution.

13.3.2 The Single-Period Problem Under Policy 2 (Carbon Emissions Tax)

Under the policy, the firm needs to pay the tax $\$\tau$ for each unit of product, as discussed in Sect. 13.2.2. This means that the firm incurs the per unit cost $\$\tau$ in addition to its acquisition cost c. Thus, we can easily write the firm's corresponding profit function, by replacing c in $J(Q)$ given in (13.1) with $c+\tau$. As a result, the optimal production quantity under Policy 2 is given as,

$$Q_2^* = F^{-1}\left(\frac{p+s-c-\tau}{p+s-v}\right) = F^{-1}(w_2). \tag{13.4}$$

Next, we discuss the effect of the carbon tax τ on the reduction in carbon emissions. More specifically, we need to consider the following question: what should be the value of τ if we desire to reduce the firm's carbon emissions by a certain percentage. Note that, if Policy 2 does not apply, then the firm's optimal quantity decision is Q^*, as given in (13.2); and, if this policy applies, then the optimal decision is Q_2^* as in (13.4). Therefore, the reduction in carbon emissions can be calculated as $\kappa \equiv (Q^* - Q_2^*)/Q^*$.

In addition, we should also consider the impact of the profitability-related attributes of the perishable product on the policy-maker's tax decision. As discussed by Schweitzer and Cachon (2000), in the single-period problem, the perishable product with $w_2 \geq 0.5$ and that with $w_2 < 0.5$—where w is defined as in Theorem 1—are called a high-profit product and a low-profit product, respectively. Noting that the aggregate demand X follows a normal distribution, we find from (13.2) that $Q^* \geq \mu$ for the high-profit products with $w_2 \geq 0.5$, and $Q^* < \mu$ for the low-profit products with $w_2 < 0.5$. Furthermore, it should be interesting to investigate whether or not the firm selling a high-profit product and the firm selling a low-profit product should have the same tax payment if they desire to achieve a same emission-reduction percentage κ.

Theorem 2. *If the firm makes a high-profit perishable product (i.e., $w_2 \geq 0.5$), then the carbon emissions-reduction percentage κ is* decreasing *in c, i.e., $\partial\kappa/\partial c < 0$. But, if the firm makes a low-profit perishable product (i.e., $w_2 < 0.5$), then the carbon emissions-reduction percentage κ is* increasing *in c, i.e., $\partial\kappa/\partial c > 0$.*

As the above theorem indicates, for a high-profit and a low-profit products under Policy 2 with a fixed value of the carbon tax τ, we find that, *ceteris paribus*, the carbon emissions-reduction percentage κ varies in *different* manners as the unit cost is changed. For a high-profit product, the reduction decreases as c increases, whereas, for a low-profit product, the reduction increases as c increases. The result implies an important insight from the perspective of the policy-maker, as given in the following remark.

Remark 1. The policy-maker should consider the attributes of the perishable product and the unit acquisition cost of the firm, in order to achieve the emissions reduction at a certain desired level. Specifically, for a given value of κ, if the perishable product belongs to the high-profit category, then the tax rate τ imposed on the high-margin firm (i.e., its unit acquisition cost c is small) should be less than that on the low-margin firm (i.e., c is high). On the other hand, for the low-profit product, the high-margin firm should absorb a high tax than the low-margin firm.

13.3.3 The Single-Period Problem Under Policy 3 (Cap-and-Trade)

Under the policy, the firm has to buy the carbon credits at the per unit price α if it produces more than the prescribed capacity Q_{c}. We thus calculate the purchasing cost of carbon credits as $\alpha\,(Q-Q_{\text{c}})^{+}$, where,

$$(Q-Q_{\text{c}})^{+} = \max(Q-Q_{\text{c}},0) = \begin{cases} Q-Q_{\text{c}}, & \text{if } Q \geq Q_{\text{c}}, \\ 0, & \text{otherwise.} \end{cases} \tag{13.5}$$

Note that, if $Q_{\text{c}} \leq Q$, then $\alpha\,(Q-Q_{\text{c}})^{+} = 0$, which implies that the firm makes no payment if it does not need any extra carbon credits. However, the firm may benefit from emitting less than the capacity Q_{c} by selling its unused carbon credits in the trading market. In fact, for the single-period problem where the unused credits should be salvaged, the firm has to sell unused credits and thus obtain the revenue as $\beta\,(Q_{\text{c}}-Q)^{+}$.

Therefore, the firm's expected profit under the cap-and-trade policy can be written as,

$$J_3\,(Q) = J\,(Q) + \alpha(Q-Q_{\text{c}})^{+} + \beta(Q_{\text{c}}-Q)^{+}, \tag{13.6}$$

where $J\,(Q)$ is given as in (13.1); and as discussed above, the second and third terms can be regarded as the firm's "penalties" and "rewards" generated by transferring carbon credits under the cap-and-trade policy, respectively. The firm should maximize $J_3\,(Q)$ in (13.6) to find the optimal quantity Q_3^* under Policy 3.

Theorem 3. *When Policy 3 ("cap-and-trade") is implemented, we find the firm's optimal quantity decision Q_3^* as given in Table 13.3, where Q^* is the optimal solution for the classical single-period problem, as given in (13.2); and,*

$$w_\alpha \equiv \frac{c_{\text{u}}-\alpha}{c_{\text{u}}+c_{\text{o}}}, \quad w_\beta \equiv \frac{c_{\text{u}}-\beta}{c_{\text{u}}+c_{\text{o}}}, \quad \gamma \equiv \left.\frac{\mathrm{d}J(Q)}{\mathrm{d}Q}\right|_{Q=Q_{\text{c}}}. \tag{13.7}$$

Note that γ in (13.7) means the firm's marginal *profit at the point that $Q = Q_{\text{c}}$. Moreover, the firm's corresponding expected profit is also calculated as in Table 13.3.* ■

Table 13.3 The firm's optimal quantity decision Q_3^* under Policy 3 ("cap-and-trade"). Note that w_α, w_β, and γ are defined as in (13.7)

Condition	$Q_c < Q^*$	$Q_c \geq Q^*$
$\beta \geq c_u$	$Q_3^* = 0$; $J_3(Q_3^*) > J(Q^*)$	$Q_3^* = 0$; $J_3(Q_3^*) > J(Q^*)$
$c_u > \beta > \gamma$	$Q_3^* = F^{-1}(w_\beta) < Q_c$	$Q_3^* = F^{-1}(w_\beta) \leq Q_c$ $J_3(Q_3^*) > J(Q^*)$
$\beta \leq \gamma \leq \alpha$	$Q_3^* = Q_c$; $J_3(Q_3^*) < J(Q^*)$	
$\alpha < \gamma$	$Q_3^* = F^{-1}(w_\alpha) > Q_c$; $J_3(Q_3^*) < J(Q^*)$	

We learn from Theorem 3 that, if Q_c is sufficiently high such that $Q_c \geq Q^*$, then the firm's optimal production quantity should be smaller than the capacity Q_c and the firm should sell its unused carbon credits under the cap-and-trade policy. For this case, the trade-off between reducing the production quantity and selling unused carbon credits is that the revenue reduction generated by decreasing Q from Q^* to Q_3^* should be compensated by selling the increments in the unused carbon credits (i.e., $Q^* - Q_3^*$).

Remark 2. Theorem 3 indicates that the firm's carbon emissions could be reduced when a *proper* cap-and-trade policy is implemented. Specifically, the amount of the carbon-emissions reduction depends on the values of α, β, c_u, and γ. In order to assure that the firm's carbon emissions are reduced to Q_c or less, the policy-maker should set the unit carbon-credit purchasing cost α no less than γ, i.e., $\alpha \geq \gamma$; otherwise, Policy 3 may not be effective in reducing carbon emissions that are generated by the firm.

We find from Theorem 3 that $Q_3^* = 0$ when $\beta \geq c_u$. This implies that the firm can profit more from selling carbon credits than from selling perishable products, when the price for carbon credits is *extremely* high. In practice, the policy-maker should effectively "manage" the cap-and-trade market to prevent the firm from acting as a carbon credit "dealer" instead of as a product "manufacturer."

Corollary 1. *When $Q^* > Q_c$ and $c_u > \beta > \gamma$, we find that*

$$\begin{cases} J_3(Q_3^*) \geq J(Q^*), \text{ if } \beta \geq \beta_0 \equiv c_u - (c_u + c_o) F(2Q_c - Q^*); \\ J_3(Q_3^*) < J(Q^*), \text{ if } \beta < \beta_0. \end{cases}$$

Proof. For a proof of this corollary, see 13.5.

From the above corollary, we note that, if $Q^* > Q_c$, $c_u > \beta > \gamma$, and $\beta \geq \beta_0$, then, as a result of implementing Policy 3, the firm's profit is increased (i.e., $J_3(Q_3^*) \geq J(Q^*)$) and its carbon emissions are decreased (i.e., $Q_3^* < Q_c$). That is, under the conditions that $Q^* > Q_c$, $c_u > \beta > \gamma$, and $\beta \geq \beta_0$, the firm should be willing to reduce its production quantity under Policy 3 and the policy is thus effective.

13.4 Numerical Study

In this section, we provide numerical examples to illustrate our analysis in Sect. 13.3. Since the analysis under Policy 1—which is provided in Sect. 13.3.1—is simple, we next compute the firm's optimal production quantities and expected profits under Policy 2 (carbon emissions tax) and Policy 3 (cap-and-trade). For simplicity, we assume that the firm does not incur a shortage cost (i.e., $s = 0$) and does not have a salvage value (i.e., $v = 0$). In addition, $X \sim N(500, 150)$, and $p = 100$. We consider several scenarios that differ in the values of other parameters including the unit acquisition cost c, the carbon tax τ, the unit carbon-credit purchasing cost α, the unit carbon-credit selling price β, and the prescribed emissions capacity Q_c.

13.4.1 Numerical Example for Policy 2

We now provide an example to illustrate our analysis for Policy 2 in Sect. 13.3.2. In this example, we use four different values of the unit cost c to represent four types of products, which include two high-profit products ($c = 15$ and $c = 35$) and two low-profit products ($c = 65$ and $c = 85$). For each product, we consider three scenarios, and for each scenario, we compute the corresponding optimal quantity for the firm.

In the first scenario, we assume that there is no capacity constraint. Accordingly, we calculate Q^* and $J(Q^*)$. In the second scenario, we assume that the carbon tax τ is equal to 10, and we calculate Q_2^* and $J_2(Q_2^*)$, which are then compared with Q^* and $J(Q^*)$ in the first scenario, respectively. We also compute the emissions reduction percentage $\kappa = (Q^* - Q_2^*)/Q^*$ and find the profit decrease percentage $\omega \equiv [J(Q^*) - J_2(Q_2^*)]/J(Q^*)$. In the third scenario, assuming that the firm desires to reduce carbon emissions by a specific percentage κ (e.g., $\kappa = 10\%$), we calculate Q_2^*, $J_2(Q_2^*)$, and ω; and also compute the corresponding tax rate τ in order to achieve the emissions reduction percentage κ. Our numerical results are presented in Table 13.4.

As Table 13.4 indicates, the firm's optimal production quantity is reduced as a result of implementing the carbon tax policy. From Scenario 2, we find that, if the per unit tax rate is 10, then the carbon emissions reduction for the high-profit products decreases as the profit margin $(p - c)$ decreases, whereas the reduction for the low-profit products significantly increases (from 9.73% to 26.38%) as the profit margin declines. We also note that the profit reduction percentage ω is strictly increasing in c; that is, if the profit margin is reduced, then the profit reduction percentage is increased.

In Scenario 3, when the carbon-emissions reduction percentage κ is equal to 10% for all products, the tax rate τ imposed on the high-profit product with $c = 35$ should be higher than that imposed on the high-profit product with $c = 15$. On the other hand, for the two low-profit products, the tax rate τ should be higher for the

Table 13.4 The firm's optimal quantities and corresponding expected profits in three scenarios

	High-profit		Low-profit	
c	15	35	65	85
Scenario 1: No carbon emissions policy				
Q^*	656	558	442	345
$J(Q^*)$	39,009	26,954	11,958	4,017
Scenario 2: Policy 2 with $\tau = 10$				
Q_2^*	601	519	399	254
$J_2(Q_2^*)$	32,741	21,377	7,748	965
$\kappa(\%)$	8.34	6.99	9.73	26.38
$\omega(\%)$	16.07	20.69	35.21	75.98
Scenario 3: Policy 3 with $\kappa = 10\%$				
Q_3^*	590	502	398	310
$J_2(Q_3^*)$	31,254	19,279	7,688	2,443
$\omega(\%)$	19.88	28.47	35.71	39.18
τ	12.5	14.5	10.2	4.8

Table 13.5 The numerical results when $Q^* \leq Q_c$

	High-profit product		Low-profit product	
c	15	35	65	85
c_u	85	65	35	85
Q_c	706	608	492	395
β	10	10	10	10
Q_3^*	601	519	399	254
$J_3(Q_3^*)$	39,799	27,652	12,666	4,914
$\kappa(\%)$	8.34	6.99	9.73	26.38
$\omega(\%)$	−2.03	−2.59	−5.92	−22.33

product with a smaller value of c. We also find that, even though the profit reduction percentage ω increases as the profit margin decreases, the increases for the four products are not as significant as those in Scenario 2.

13.4.2 Numerical Example for Policy 3

We now consider two examples to illustrate our analysis for Policy 3 in Sect. 13.3.3. From Theorem 3, we find that the firm's optimal quantity decision depends on the comparison between Q^* and Q_c. Next, we first present an example for the case that $Q_c \geq Q^*$, using the values of the unit acquisition cost c for four products as in Sect. 13.4.1. Setting the specific values of Q_c and β for each product, we present our calculation results in Table 13.5, where we find that, for each product, carbon emissions are decreased but the firm's expected profit is increased.

Next, we present another example to illustrate our analysis for the case that $Q_c < Q^*$. We set $\alpha = 12.5$ and $\beta = 10$, and we select three different values of Q_c for each product, as given in Table 13.6, where we find the following results.

Table 13.6 The numerical results when $Q^* > Q_c$

	High-profit product						Low-profit product					
c	15			35			65			85		
c_u	85			65			35			15		
Q^*	656			558			442			345		
$J(Q^*)$	39,009			26,954			11,958			4,017		
α	12.5			12.5			12.5			12.5		
β	10			10			10			10		
Q_c	640	595	550	550	515	480	435	390	345	340	230	180
β_0	5.42	26.01	49.53	4.01	22.44	39.36	3.57	21.09	30.42	1.45	14.53	14.98
γ	2.58	11.37	17.08	1.99	11.06	20.35	1.80	11.88	19.97	0.74	11.45	13.40
Q_3^*	601	595	590	519	515	510	399	390	386	254	230	207
$J_3(Q_3^*)$	39,139	38,685	38,125	27,072	26,720	26,287	12,096	11,637	11,076	4,363	3,246	2,632
$\kappa(\%)$	8.38	9.30	10.06	6.99	7.71	8.60	9.73	11.76	12.67	26.38	33.33	40.00
$\omega(\%)$	−0.33	0.83	2.27	−0.44	0.87	2.47	−1.15	2.68	7.38	−8.61	19.19	34.48

For each product, Q_3^* is reduced as Q_c is smaller; and, $J_3(Q_3^*)$ is greater than $J(Q^*)$ as long as $\beta > \beta_0$. We also find that Q_c more significantly impacts Q_3^* and $J_3(Q_3^*)$ for the low-profit products than for the high-profit products. In addition, if the profit margin is lower, then the impact of the carbon capacity on carbon emissions and the firm's expected profit are more significant.

13.5 Summary and Concluding Remarks

In this paper, we investigated the single-period problem under three carbon emissions policies including the mandatory carbon emissions capacity, the carbon emissions tax, and the cap-and-trade system. Under each policy, we obtained the optimal production quantity and calculated the corresponding expected profits for the firm. From our analysis, we draw some important *analytic* managerial insights. For example, we showed that, in order to reduce carbon emissions by a certain percentage, the tax rate τ imposed on the high-margin firm should be less than that on the low-margin firm for the high-profit perishable products, whereas the high-margin firm should absorb a higher tax than the low-margin firm for the low-profit products.

We also found that, from the perspective of the policy-maker, the emissions capacity should be set to a level such that the marginal profit of the firm is less than the carbon credit purchasing price, because, otherwise, the firm would produce more than the emissions capacity. We also derived the specific conditions under which, as a result of implementing the cap-and-trade policy, the firm's expected profit is increased and carbon emissions are reduced. The conditions assure the firm's and the policy-maker's incentives on the cap-and-trade policy.

The research problem discussed in this paper could be extended in several directions. In future, we may relax the single-period assumption and consider the quantity decisions of nonperishable products in multiple periods. In another possible research direction, we may also consider pricing decision for the firm, assuming the price-dependent aggregate demand in an additive and a multiplicative function form. In addition, from the policy-maker prospective, it would be nice if one could propose a way for a firm to select the best policy. The method of choosing the best carbon emission reduction policy for a given managerial situation likely has critical business implications for manufacturers.

Appendix A: Proofs of Theorems

Proof of Theorem 1. Temporarily ignoring the constraint that $Q \leq Q_c$, we can solve the classical single-period problem to find that

$$F(Q^*) = w \equiv \frac{p+s-c}{p+s-v}. \tag{13.8}$$

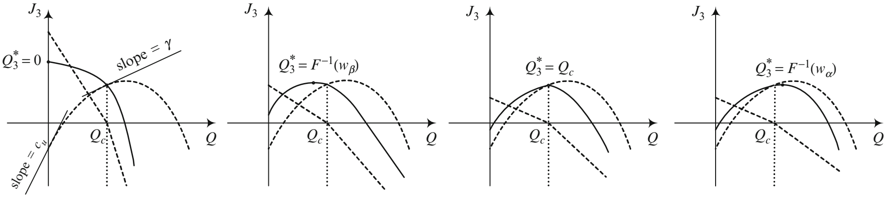

Fig. 13.1 The analysis of $J_3(Q)$ in four scenarios: (1) $\beta > c_u$, (2) $c_u > \beta > \gamma$, (3) $\beta < \gamma < \alpha$, and (4) $\alpha < \gamma$

Taking the constraint into consideration, we can easily obtain the result in this theorem. □

Proof of Theorem 2. To discuss the impact of w on the effectiveness of the carbon tax policy, we assume that the unit cost c in (13.4) takes two different values, e.g., c_1 and c_2 (w.l.o.g., $c_1 < c_2$); and then, ceteris paribus, the corresponding optimal quantities given by (13.2) are $\hat{Q}_2^*$ and $\tilde{Q}_2^*$, respectively.

Using (13.2) and (13.4), we find that, replacing c with $c+\tau$, the optimal production quantity is changed from Q^* to Q_2^*. If $\tau \to 0^+$, then $Q^* - Q_2^* = -\mathrm{d}Q^*/\mathrm{d}c$. Differentiating both sides of (13.8) once w.r.t. c, we have, $\mathrm{d}Q^*/\mathrm{d}c = -1/[(p+s-v)f(Q^*)]$. It thus follows that, as $\tau \to 0^+$, $\kappa = (Q^* - Q_2^*)/Q^* = 1/[(p+s-v)f(Q^*)]$, which is easily shown to be strictly increasing in Q^* when $Q^* \geq \mu$ but strictly decreasing in Q^* when $Q^* < \mu$. Therefore, for a high-profit product, $\hat{Q}_2^* > \tilde{Q}_2^*$, and $\hat{\kappa} \equiv (\hat{Q}^* - \hat{Q}_2^*)/\hat{Q}^* > \tilde{\kappa} \equiv (\tilde{Q}^* - \tilde{Q}_2^*)/\tilde{Q}^*$, whereas, for a low-profit product, $\hat{\kappa} < \tilde{\kappa}$. This theorem is thus proved. □

Proof of Theorem 3. We find from (13.6) that $J_3(Q)$ is a continuous, piecewise function in Q. We next consider two cases: $Q_\mathrm{c} < Q^*$ and $Q_\mathrm{c} \geq Q^*$; and for each case, we compute the corresponding optimal decision Q_3^*.

When $Q_\mathrm{c} < Q^*$, we depict four scenarios as shown in Fig. 13.1; and, for each scenario, we compute the optimal solution Q_3^* as follows: If $\beta \geq c_\mathrm{u}$, then we find from Fig. 13.1(1) that $J_3(Q)$ is strictly decreasing in Q over $[0,+\infty)$; and thus, the optimal quantity maximizing J_3 is $Q_3^* = 0$, and $J_3(Q_3^*) = \beta Q_\mathrm{c} - s\mu$. If $c_\mathrm{u} > \beta > \gamma$, then as Fig. 13.1(2) indicates, Q_3^* can be obtained as $Q_3^* = F^{-1}(w_\beta)$, which is in the range $(0, Q_\mathrm{c})$. If $\beta \leq \gamma \leq \alpha$, then, as Fig. 13.1(3) indicates, $J_3(Q)$ is increasing in $Q \in [0, Q_\mathrm{c}]$ but decreasing in $Q \in (Q_\mathrm{c}, +\infty)$. The optimal solution Q_3^* is thus determined as $Q_3^* = Q_\mathrm{c}$. If $\alpha < \gamma$, then $Q_3^* = F^{-1}(w_\alpha) \in (Q_\mathrm{c}, +\infty)$, as shown in Fig. 13.1(4).

When $Q_\mathrm{c} \geq Q$, we find from (13.6) that $J_3(Q) = J(Q) + \beta(Q_\mathrm{c} - Q)$, which is a concave function of Q. Similarly, we can show that $J_3(Q)$ is a decreasing, concave function of Q in the range $(Q_\mathrm{c}, +\infty)$. Thus, the optimal solution Q_3^* must exist in the range $[0, Q_\mathrm{c}]$. If $J_3(Q)$ is also strictly decreasing in $Q \in [0, Q_\mathrm{c}]$, then $Q_3^* = 0$. Otherwise, Q_3^* should be obtained by solving $\mathrm{d}J_3(Q)/\mathrm{d}Q = 0$; that is, $Q_3^* = F^{-1}(w_\beta)$. Noting that $\mathrm{d}J_3(Q)/\mathrm{d}Q|_{Q=0} < 0$ *only* if $\beta > c_\mathrm{u}$, we find that $Q_3^* = 0$

if $\beta > c_{\mathrm{u}}$; $Q_3^* = F^{-1}\left(w_\beta\right)$ otherwise. In addition, $Q_3^* \leq Q_{\mathrm{c}}$ because $w_\beta \leq w$; and, $J_3\left(Q_3^*\right) \geq J\left(Q^*\right) + \beta\left(Q_{\mathrm{c}} - Q^*\right) \geq J\left(Q^*\right)$. □

Appendix B: Proof of Corollary 1

We learn from Theorem 3 that, if $c_{\mathrm{u}} > \beta > \gamma$, then $Q_3^* = F^{-1}\left(w_\beta\right)$ and $\Phi\left(z_3^*\right) = w_\beta$. Hence, z_3^* is dependent on β, and $\phi\left(z_3^*\right)$ can be written as $\phi\left(z_3^*\right) = -[1/(p+s-v)] \times (\mathrm{d}\beta/\mathrm{d}z_3^*)$. Using (13.3), we have,

$$\begin{aligned} J_3\left(Q_3^*\right) - J\left(Q^*\right) &= \beta\left(Q_{\mathrm{c}} - \mu\right) + \sigma\left(c_{\mathrm{u}} + c_{\mathrm{o}}\right)\left[\phi\left(z^*\right) - \phi\left(z_3^*\right)\right] \\ &= \beta\left(Q_{\mathrm{c}} - \mu\right) + \sigma\beta' + \sigma\left(c_{\mathrm{u}} + c_{\mathrm{o}}\right)\phi\left(z^*\right). \end{aligned} \tag{13.9}$$

Equating $J_3\left(Q_3^*\right)$ to $J\left(Q^*\right)$ and solving the resulting equation for β, we find that

$$\beta = \frac{\sigma\left(c_{\mathrm{u}} + c_{\mathrm{o}}\right)\phi\left(z^*\right)}{Q_{\mathrm{c}} - \mu}\left[e^{(Q_{\mathrm{c}}-\mu)\left(z^*-z_3^*\right)/\sigma} - 1\right]. \tag{13.10}$$

Substituting β in (13.9) into (13.10), we obtain z_3^* as $z_3^* = z^* = 2\left(Q_{\mathrm{c}} - \mu\right)/\sigma - z^*$. It is easy to show that the corresponding value of β for z_3^* is $\beta_0 = c_{\mathrm{u}} - \left(c_{\mathrm{u}} + c_{\mathrm{o}}\right)F\left(2Q_{\mathrm{c}} - Q^*\right)$. We also find that $J_3\left(Q_3^*\right) - J\left(Q^*\right) > 0$ for $\beta > \beta_0$, but $J_3\left(Q_3^*\right) - J\left(Q^*\right) < 0$ for $\beta < \beta_0$.

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