Chapter 1
How to Make a Decision

1.1 Introduction

ἱερά ἀρχή is the Greek word for hierarchy meaning holy origin or holy rule.[1] It is the ordering of parts or elements of a whole from the highest to the lowest. A hierarchy is the principle of control that secures the effective functioning of the organization.[2]

"You can't compare apples and oranges," so the saying goes. But is this true? Consider a hungry person who likes both apples and oranges and is offered a choice between a large, red, pungent, juicy looking Washington State apple and an even larger, old and shriveled, pale colored orange with a soft spot. Which one is that person more likely to choose? Let us reverse the situation and offer the same person on the next day a small, deformed, unripe apple with a couple of worm holes and a fresh colored navel orange from California. Which one is he or she more likely to choose now?

We have learned through experience to identify properties and establish selection criteria for apples and oranges and in fact we use that experience to make tradeoffs among the properties and reach a decision. We choose the apple or orange that yields, according to our preferences, the greater value across all the various attributes.

The Analytic Hierarchy Process (AHP) is a basic approach to decision making. It is designed to cope with both the rational and the intuitive to select the best from a number of alternatives evaluated with respect to several criteria. In this process, the decision maker carries out simple pairwise comparison judgments which are then used to develop overall priorities for ranking the alternatives. The AHP both allows for inconsistency in the judgments and provides a means to improve consistency.

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[1] Encyclopedia Catholica.
[2] The Great Soviet Encyclopedia, Moscow 1970.

T. L. Saaty and L. G. Vargas, *Models, Methods, Concepts & Applications of the Analytic Hierarchy Process*, International Series in Operations Research & Management Science 175,
DOI: 10.1007/978-1-4614-3597-6_1,

The simplest form used to structure a decision problem is a hierarchy consisting of three levels: the goal of the decision at the top level, followed by a second level consisting of the criteria by which the alternatives, located in the third level, will be evaluated. Hierarchical decomposition of complex systems appears to be a basic device used by the human mind to cope with diversity. One organizes the factors affecting the decision in gradual steps from the general, in the upper levels of the hierarchy, to the particular, in the lower levels. The purpose of the structure is to make it possible to judge the importance of the elements in a given level with respect to some or all of the elements in the adjacent level above. Once the structuring is completed, the AHP is surprisingly simple to apply.

In this chapter we show that there is a real and practical use for judgments and priorities in human affairs. This use is not contrived; we are led to them in a very natural way.

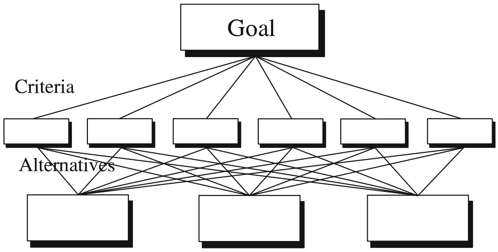
1.2 How to Structure a Decision Problem

Perhaps the most creative task in making a decision is deciding what factors to include in the hierarchic structure. When constructing hierarchies one must include enough relevant detail to represent the problem as thoroughly as possible, but not so thoroughly as to lose sensitivity to change in the elements. Considering the environment surrounding the problem, identifying the issues or attributes that one feels should contribute to the solution, and who are the participants associated with the problem, are all important issues when constructing a hierarchy. Arranging the goals, attributes, issues, and stakeholders in a hierarchy serves two purposes: It provides an overall view of the complex relationships inherent in the situation and in the judgment process, and it also allows the decision maker to assess whether he or she is comparing issues of the same order of magnitude.

The elements being compared should be homogeneous. The hierarchy does not need to be complete; that is, an element in a given level does not have to function as a criterion for *all* the elements in the level below. Thus a hierarchy can be divided into subhierarchies sharing only a common topmost element. Further, a decision maker can insert or eliminate levels and elements as necessary to clarify the task of setting priorities or to sharpen the focus on one or more parts of the system. Elements that are of less immediate interest can be represented in general terms at the higher level of the hierarchy and elements of critical importance to the problem at hand can be developed in greater depth and specificity. The task of setting priorities requires that the criteria, the subcriteria, the properties or features of the alternatives be compared among themselves in relation to the elements of the next higher level.

Finally, after judgments have been made on the impact of all the elements, and priorities have been computed for the hierarchy as a whole, sometimes, and with care, the less important elements can be dropped from further consideration because of their relatively small impact on the overall objective.

Fig. 1.1 A three level hierarchy



1.3 Philosophy, Procedure and Practice of the AHP

The Analytic Hierarchy Process is a general theory of measurement. It is used to derive ratio scales from both discrete and continuous paired comparisons in multilevel hierarchic structures. These comparisons may be taken from actual measurements or from a fundamental scale that reflects the relative strength of preferences and feelings. The AHP has a special concern with departure from consistency and the measurement of this departure, and with dependence within and between the groups of elements of its structure. It has found its widest applications in multicriteria decision making, in planning and resource allocation, and in conflict resolution [6, 8]. In its general form, the AHP is a nonlinear framework for carrying out both deductive and inductive thinking without use of the syllogism. This is made possible by taking several factors into consideration simultaneously, allowing for dependence and for feedback, and making numerical tradeoffs to arrive at a synthesis or conclusion (see Figs. 1.1 , 1.2).

For a long time people have been concerned with the measurement of both physical and psychological events. By physical we mean the realm of what is fashionably known as the tangibles in so far as they constitute some kind of objective reality outside the individual conducting the measurement. By contrast, the psychological is the realm of the intangibles, comprising the subjective ideas, feelings, and beliefs of the individual and of society as a whole. The question is whether there is a coherent theory that can deal with both these worlds of reality without compromising either. The AHP is a method that can be used to establish measures in both the physical and social domains.

In using the AHP to model a problem, one needs a hierarchic or a network structure to represent that problem, as well as pairwise comparisons to establish relations within the structure. In the discrete case these comparisons lead to dominance matrices and in the continuous case to kernels of Fredholm Operators [12], from which ratio scales are derived in the form of principal eigenvectors, or eigenfunctions, as the case may be. These matrices, or kernels, are positive and reciprocal, e.g., $a_{ij} = 1/a_{ji}$. In particular, special effort has been made to characterize these matrices [6, 16]. Because of the need for a variety of judgments, there has also been considerable work done to deal with the process of synthesizing group judgments [7].

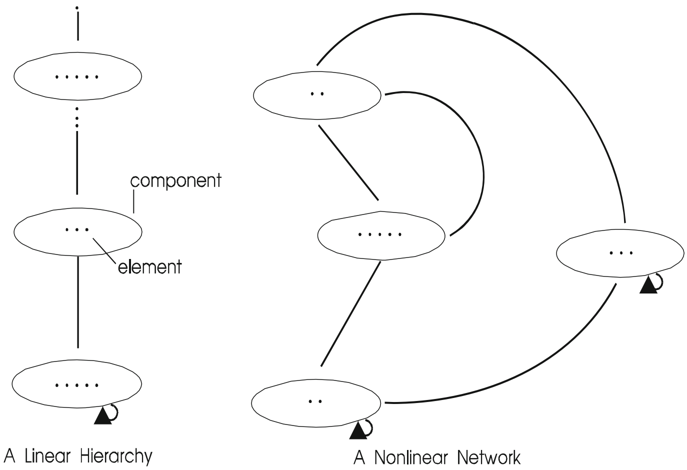


Fig. 1.2 Structural difference between a linear and a non linear network

For completeness we mention that there are four axioms in the AHP. Briefly and informally they are concerned with the reciprocal relation, comparison of homogeneous elements, hierarchic and systems dependence and with expectations about the validity of the rank and value of the outcome and their dependence on the structure and its extension [7].

1.4 Absolute and Relative Measurement and Structural Information

Cognitive psychologists have recognized for some time that there are two kinds of comparisons that humans make: absolute and relative. In absolute comparisons, alternatives are compared with a standard or baseline which exists in one's memory and has been developed through experience. In relative comparisons, alternatives are compared in pairs according to a common attribute. The AHP has been used with both types of comparisons to derive ratio scales of measurement. We call such scales absolute and relative measurement scales. Relative measurement w_i, $i = 1,\ldots, n$, of each of *n* elements is a ratio scale of values assigned to that element and derived by comparing it in pairs with the others. In paired comparisons two elements i and j are compared with respect to a property they have in common. The smaller i is used as the unit and the larger j is estimated as a multiple of that unit in the form $(w_i/w_j)/1$ where the ratio w_i/w_j is taken from a fundamental scale of absolute values.

Absolute measurement (sometimes called scoring) is applied to rank the alternatives in terms of either the criteria or the ratings (or intensities) of the criteria; for example: excellent, very good, good, average, below average, poor, and very poor; or A, B, C, D, E, F, and G. After setting priorities for the criteria (or subcriteria, if there are any), pairwise comparisons are also made between the ratings themselves to set priorities for them under each criterion and dividing each of their priorities by the largest rated intensity to get the ideal intensity. Finally, alternatives are scored by checking off their respective ratings under each criterion and summing these ratings for all the criteria. This produces a ratio scale score for the alternative. The scores thus obtained of the alternatives can in the end be normalized by dividing each one by their sum.

Absolute measurement has been used in a variety of applications. For example, it has been used to rank cities in the United States according to nine criteria as judged by six different people [13]. Another appropriate use for absolute measurement is in school admissions as in Chap. 22 [14]. Most schools set their criteria for admission independently of the performance of the current crop of students seeking admission. The school's priorities are then used to determine whether a given student meets the standard set for qualification. Generally, candidates are compared with previously set standard rather than with each other. In that case absolute measurement should be used to determine which students meet prior standards and qualify for admission.

1.5 The Fundamental Scale

Paired comparison judgments in the AHP are applied to pairs of homogeneous elements. The fundamental scale of values to represent the intensities of judgments is shown in Table 1.1. This scale has been validated for effectiveness, not only in many applications by a number of people, but also through theoretical justification of what scale one must use in the comparison of homogeneous elements.

There are many situations where elements are equal or almost equal in measurement and the comparison must be made not to determine how many times one is larger than the other, but what fraction it is larger than the other. In other words there are comparisons to be made between 1 and 2, and what we want is to estimate verbally the values such as 1.1, 1.2,…, 1.9. There is no problem in making the comparisons by directly estimating the numbers. Our proposal is to continue the verbal scale to make these distinctions so that 1.1 is a "tad", 1.3 indicates moderately more, 1.5 strongly more, 1.7 very strongly more and 1.9 extremely more. This type of refinement can be used in any of the intervals from 1 to 9 and for further refinements if one needs them, for example, between 1.1 and 1.2 and so on.

Table 1.1 The fundamental scale

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
2	Weak	
3	Moderate importance	Experience and judgment slightly favor one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation
Reciprocals of above	If activity *i* has one of the above nonzero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*	A reasonable assumption
Rationals	Ratios arising from the scale	If consistency were to be forced by obtaining *n* numerical values to span the matrix

The following two examples provide partial validation of the 1–9 scale used in the pairwise comparisons of homogeneous elements.

Which drink is consumed more in the US?

	A	B	C	D	E	F	G	Estimated	Actual
A:Coffee	1	9	5	2	1	1	1/2	0.177	0.18
B:Wine	1/9	1	1/3	1/9	1/9	1/9	1/9	0.019	0.01
C:Tea	1/5	3	1	1/3	1/4	1/3	1/9	0.042	0.04
D:Beer	1/2	9	3	1	1/2	1	1/3	0.116	0.12
E:Sodas	1	9	4	2	1	2	1/2	0.190	0.18
F:Milk	1	9	3	1	1/2	1	1/3	0.129	0.14
G:Water	2	9	9	3	2	3	1	0.327	0.33

C.R. = 0.022

Which food has more protein?

	A	B	C	D	E	F	G	Estimated	Actual
A:Steak	1	9	9	6	4	5	1	0.345	0.37
B:Potatoes	1/9	1	1	1/2	1/4	1/3	1/4	0.031	0.04
C:Apples	1/9	1	1	1/3	1/3	1/5	1/9	0.030	0.00
C:Soybeans	1/6	2	3	1	1/2	1	1/6	0.065	0.07
E:Whole wheat bread	1/4	4	3	2	1	3	1/3	0.124	0.11
F:Tasty cake	1/5	3	5	1	1/3	1	1/5	0.078	0.09
G:Fish	1	4	9	6	3	5	1	0.328	0.32

C.R. = 0.028

1.6 Comments on Benefit/Cost Analysis

Often, the alternatives from which a choice must be made in a choice-making situation have both costs and benefits associated with them. In this case it is useful to construct separate costs and benefits hierarchies, with the same alternatives on the bottom level of each. Thus one obtains both a costs-priority vector and a benefit-priority vector. The benefit/cost vector is obtained by taking the ratio of the benefit priority to the costs priority for each alternative, with the highest such ratio indicating the preferred alternative. In the case where resources are allocated to several projects, such benefit-to-cost ratios or the corresponding marginal ratios prove to be very valuable.

For example, in evaluating three types of copying machines, the good attributes are represented in the benefits hierarchy and the costs hierarchy represents the pain and economic costs that one would incur by buying or maintaining each of the three types of machines. Note that the criteria for benefits and the criteria for costs need not be simply opposites of each other but instead may be partially or totally different. Also note that each criterion may be regarded at a different threshold of intensity and that such thresholds may themselves be prioritized according to desirability, with each alternative evaluated only in terms of its highest priority threshold level. Similarly, three hierarchies can be used to assess a benefit/(cost $\times$ risk) outcome.

1.7 The Eigenvector Solution for Weights and Consistency

There is an infinite number of ways to derive the vector of priorities from the matrix (a_{ij}). But emphasis on consistency leads to the eigenvalue formulation $Aw = nw$. To see this, assume that the priorities $w = (w_1,\ldots,w_n)$ with respect to a single criterion are known, such as the weights of stones, we can examine what we have to do to recover them. So we form the matrix of ratio comparisons and multiply it on the right by w to obtain nw as follows:

$$\begin{pmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} = n \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}$$

If a_{ij} represents the importance of alternative i over alternative j and a_{jk} represents the importance of alternative j over alternative k and a_{ik}, the importance of alternative i over alternative k, must equal $a_{ij}a_{jk}$ or $a_{ij}a_{jk} = a_{ik}$ for the judgments to be consistent. If we do not have a scale at all, or do not have it conveniently as in the case of some measuring devices, we cannot give the precise values of w_i / w_j but only an estimate. Our problem becomes $A' \ w' = \lambda_{\max} w'$ where $\lambda_{\max}$ is the largest or principal eigenvalue of $A' = (a'_{ij})$ the perturbed value of $A = (a_{ij})$ with the reciprocal $a'_{ji} = 1 \ / \ a'_{ij}$ forced. To simplify the notation we shall continue to write $Aw = \lambda_{\max} w$ where A is the matrix of pairwise comparisons.

The solution is obtained by raising the matrix to a sufficiently large power, then summing over the rows and normalizing to obtain the priority vector $w = (w_1,\ldots, w_n)$. The process is stopped when the difference between components of the priority vector obtained at the kth power and at the (k+1)st power is less than some predetermined small value. The vector of priorities is the derived scale associated with the matrix of comparisons. We assign in this scale the value zero to an element that is not comparable with the elements considered.

An easy way to get an approximation to the priorities is to normalize the geometric means of the rows. This result coincides with the eigenvector for $n \leq 3$. A second way to obtain an approximation is by normalizing the elements in each column of the judgment matrix and then averaging over each row.

We would like to caution that for important applications one should use only the eigenvector derivation procedure because approximations can lead to rank reversal in spite of the closeness of the result to the eigenvector [10].

A simple way to obtain the exact value (or an estimate) of $\lambda_{\max}$ when the exact value (or an estimate) of w is available in normalized form is to add the columns of A and multiply the resulting vector by the priority vector w.

The problem now becomes, how good is the principal eigenvector estimate w? Note that if we obtain $w = (w_1,\ldots, w_n)^T$, by solving this problem, the matrix whose entries are w_i/w_j is a consistent matrix which is our consistent estimate of the matrix A. The original matrix itself A, need not be consistent. In fact, the entries of A need not even be transitive; i.e., A_1 may be preferred to A_2 and A_2 to A_3 but A_3 may be preferred to A_1. What we would like is a measure of the error due to inconsistency. It turns out that A is consistent if and only if $\lambda_{\max} = n$ and that we always have $\lambda_{\max} \geq n$.

It is interesting to note that $(\lambda_{\max} - n)/(n - 1)$ is the variance of the error incurred in estimating a_{ij}. This can be shown by writing $a_{ij} = (w_i \ / \ w_j)\varepsilon_{ij}$, $\varepsilon_{ij} > 0$,

Table 1.2 Average random consistency index (R.I.)

N	1	2	3	4	5	6	7	8	9	10
Random consistency index (R.I.)	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

$\varepsilon_{ij} = 1 + \delta_{ij}$, $\delta_{ij} > -1$, and substituting in the expression for λ_{max}. It is δ_{ij} that concerns us as the error component and its value $|\delta_{ij}| < 1$ for an unbiased estimator. The measure of inconsistency can be used to successively improve the consistency of judgments. The consistency index of a matrix of comparisons is given by C.I. = $(\lambda_{max} - n)/(n - 1)$. The consistency ratio (C.R.) is obtained by comparing the C.I. with the appropriate one of the following set of numbers (See Table 1.2) each of which is an average random consistency index derived from a sample of randomly generated reciprocal matrices using the scale 1/9, 1/8,…, 1,…, 8, 9. If it is not less than 0.10, study the problem and revise the judgments. The AHP includes a consistency index for an entire hierarchy. An inconsistency of 10 percent or less implies that the adjustment is small compared to the actual values of the eigenvector entries. A proof that the number of elements should be small to preserve consistency can be found in [6].

1.8 How to Structure a Hierarchy

Perhaps the most creative and influential part of decision making is the structuring of the decision as a hierarchy. The basic principle to follow in creating this structure is always to see if one can answer the following question: "Can I compare the elements on a lower level in terms of some or all of the elements on the next higher level?"

A useful way to proceed is to work down from the goal as far as one can and then work up from the alternatives until the levels of the two processes are linked in such a way as to make comparison possible. Here are some suggestions for an elaborate design.

1. Identify overall goal. What are you trying to accomplish? What is the main question?
2. Identify subgoals of overall goal. If relevant, identify time horizons that affect the decision.
3. Identify criteria that must be satisfied in order to fulfill the subgoals of the overall goal.
4. Identify subcriteria under each criterion. Note that criteria or subcriteria may be specified in terms of ranges of values of parameters or in terms of verbal intensities such as high, medium, low.
5. Identify actors involved.
6. Identify actor goals.
7. Identify actor policies.

8. Identify options or outcomes.
9. Take the most preferred outcome and compare the ratio of benefits to costs of making the decision with those of not making it. Do the same when there are several alternatives from which to choose.
10. Do benefit/cost analysis using marginal values. Because we are dealing with dominance hierarchies, ask which alternative yields the greatest benefit; for costs, which alternative costs the most.

The software program Expert Choice [2] incorporates the AHP methodology and enables the analyst to structure the hierarchy and resolve the problem using relative or absolute measurements, as appropriate.

1.9 Hierarchic Synthesis and Rank

Hierarchic synthesis is obtained by a process of weighting and adding down the hierarchy leading to a multilinear form. The hierarchic composition principle is a theorem in the AHP that is a particular case of network composition which deals with the cycles and loops of a network.

What happens to the synthesized ranks of alternatives when new ones are added or old ones deleted? The ranks cannot change under any single criterion, but they can under several criteria depending on whether one wants the ranks to remain the same or allow them to change. Many examples are given in the literature showing that preference reversal and rank reversal are natural occurrences. In 1990 Tversky et al. [18] concluded that the “primary cause” of preference reversal is the “failure of procedure invariance”. In the AHP there is no such methodological constraint.

In the distributive mode of the AHP, the principal eigenvector is normalized to yield a unique estimate of a ratio scale underlying the judgments. This mode allows rank to change and is useful when there is dependence on the number of alternatives present or on dominant new alternatives which may affect preference among old alternatives thus causing rank reversals (see phantom alternatives [7]). In the ideal mode of the AHP the normalized values of the alternatives for each criterion are divided by the value of the highest rated alternative. In this manner a newly added alternative that is dominated everywhere cannot cause reversal in the ranks of the existing alternatives [6].

1.10 Normative: Descriptive

All science is descriptive not normative. It is based on the notion that knowledge is incomplete. It uses language and mathematics to understand, describe and predict events with the object of testing the accuracy of the theory. Events involve two things: (1) controllable and uncontrollable conditions (e.g. laws) and (2) people or objects characterized by matter, energy and motion influenced by and sometimes influencing these conditions. A missile's path is subject to uncontrollable forces like gravity and controllable forces like the initial aim of the missile, its weight, perhaps the wind, and others. The conditions are not determined by the objects involved. The idea is to get the missile from A to B by ensuring that it follows its path with precision.

Economics is normative. It is based on expected utility theory and is predicated on the idea that the collective behavior of many individuals, each motivated by self interest, determines the market conditions which in turn influence or control each individual's behavior. In this case both the objects and the conditions are "up for grabs" because behavior is subject to rational influences that are thought to be understood. By optimizing individual behavior through rationality one can optimize the collective conditions and the resulting system, plus or minus some corrections in the conditions. But conditions are not all economic. Some are environmental, some social, some political and others cultural. We know little about their interactions. In attempting to include everything, normative theories treat intangible criteria as tangibles by postulating a convenient economic scale. It is hard to justify reducing all intangibles to economics in order to give the appearance of completeness. It is doubtful that economic theory can solve all human problems. To the contrary, some believe that it can create problems in other areas of human concern.

A normative theory is established by particular people external to the process of decision making. Experts often disagree on the criteria used to judge the excellence of a normative theory and the decision resulting from it. For example, a basic criterion of Utility Theory is the principle of rationality which says that if a person is offered more of that which he values, he should take it. In response to this dictum Herbert Simon [17] developed his idea of sufficiency (satisficing). Whenever we are saturated even with a highly valued commodity, there is a cutoff point where the marginal increase in total value is less than or equal to zero. A theory constructed to satisfy such an assumption would undoubtedly encounter difficulties in its applications. Rank reversals would be appropriate to overcome the disadvantages of oversaturation.

The AHP is a descriptive theory in the sense of the physical sciences. It treats people separately from the conditions in which they find themselves, because so far no complete integrated theory of socio-economic-political-environmental-cultural factors exists that would enable us to deduce optimality principles for people's behavior. The AHP is an instrument used to construct a complete order through which optimum choice is derived.

In the AHP approach a particular decision is not considered wrong merely because it does not follow a prescribed set of procedures. The purpose of the AHP is to assist people in organizing their thoughts and judgments to make more effective decisions. Its structures are based on observations of how influences are transmitted and its arithmetic is derived from psychologists' observations of how people function in attempting to understand their behavior.

In its simplest form, the AHP begins with the traditional concept of ordinal ranking to stratify a hierarchy and advances further into numerical paired comparisons from which a ranking of the elements in each level is derived. By imposing a multiplicative structure on the numbers ($a_{ij} \cdot a_{jk} = a_{ik}$), the reciprocal condition is obtained. Thus, the AHP infers behavioral characteristics of judgments (inconsistency and intransitivity) from its basic framework of paired comparisons. It begins by taking situations with a known underlying ratio scale and hence known comparison ratios, and shows how its method of deriving a scale uniquely through the eigenvector gives back the original scale. Then, through perturbation the AHP shows that a derived scale should continue (through the eigenvector) to approximate the original scale providing that there is high consistency.

1.11 Rationality

Rationality is defined in the AHP as:

- Focussing on the goal of solving the problem;
- Knowing enough about a problem to develop a thorough structure of relations and influences;
- Having enough knowledge and experience and access to knowledge and experience of others to assess the priority of influence and dominance (importance, preference or likelihood to the goal as appropriate) among the relations in the structure;
- Allowing for differences in opinion with an ability to develop a best compromise.

1.12 Examples

Relative Measurement: Choosing the Best House

To illustrate the ideas discussed above regarding relative measurement, consider the following example; a family of average income wants to purchase a house. They must choose from three alternatives. The family identifies eight factors to look for in a house. These factors fall into three categories: economic, geographic, and physical. Although one might begin by examining the relative importance of these categories, the family feels they want to prioritize the relative importance of all the factors without working with the categories to which they belong. The problem is to select one of three candidate houses. In applying the AHP, the first step is *decomposition*, or the structuring of the problem into a hierarchy (see Fig. 1.3). On the first (or top) level is the overall goal of *Satisfaction with House*. On the second level are the eight factors or criteria that contribute to

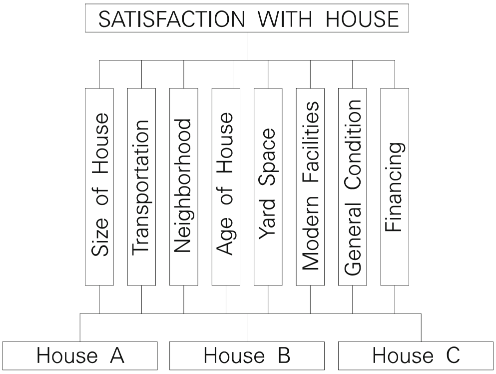


Fig. 1.3 Decomposition of the problem into a hierarchy

the goal, and on the third (or bottom) level are the three candidate houses that are to be evaluated in terms of the criteria on the second level. The definitions of the factor and the pictorial representation of the hierarchy follow.

The factors important to the family are:

1. *Size of House:* Storage space; size of rooms; number of rooms; total area of house.
2. *Transportation:* Convenience and proximity of bus service.
3. *Neighborhood:* Degree of traffic, security, taxes, physical condition of surrounding buildings.
4. *Age of House:* Self-explanatory.
5. *Yard Space:* Includes front, back, and side space, and space shared with neighbors.
6. *Modern Facilities:* Dishwasher, garbage disposal, air conditioning, alarm system, and other such items.
7. *General Condition:* Extent to which repairs are needed; condition of walls, carpet, drapes, wiring; cleanliness.
8. *Financing:* Availability of assumable mortgage, seller financing, or bank financing.

The next step is *comparative judgment*. The elements on the second level are arranged into a matrix and the family buying the house makes judgments about the relative importance of the elements with respect to the overall goal, *Satisfaction with House*.

The questions to ask when comparing two criteria are of the following kind: of the two alternatives being compared, which is considered more important by the family and how much more important is it with respect to family satisfaction with the house, which is the overall goal?

The matrix of pairwise comparisons of the factors given by the home buyers in this case is shown in Table 1.3, along with the resulting vector of priorities. The judgments are entered using the Fundamental Scale, first verbally as indicated in the scale and then associating the corresponding number. The vector of priorities is the principal eigenvector of the matrix. This vector gives the relative priority of the

Table 1.3 Pairwise comparison matrix for level 1

	1	2	3	4	5	6	7	8	Priority vector
1	1	5	3	7	6	6	1/3	1/4	0.173
2	1/5	1	1/3	5	3	3	1/5	1/7	0.054
3	1/3	3	1	6	3	4	6	1/5	0.188
4	1/7	1/5	1/6	1	1/3	1/4	1/7	1/8	0.018
5	1/6	1/3	1/3	3	1	1/2	1/5	1/6	0.031
6	1/6	1/3	1/4	4	2	1	1/5	1/6	0.036
7	3	5	1/6	7	5	5	1	1/2	0.167
8	4	7	5	8	6	6	2	1	0.333

$\lambda_{max} = 9.669$ C.R. $= 0.169$

factors measured on a ratio scale. That is, these priorities are unique to within multiplication by a positive constant. However, if one ensures that they sum to one they are always unique. In this case financing has the highest priority, with 33% of the influence.

In Table 1.3, instead of naming the criteria, we use the number previously associated with each.

Note for example that in comparing Size of House on the left with Size of House on top, a value of equal is assigned. However, when comparing it with Transportation it is strongly preferred and a 5 is entered in the (1, 2) or first row, second column position. The reciprocal value 1/5 is automatically entered in the (2, 1) position. Again when Size of House in the first row is compared with General Condition in the seventh column, it is not preferred but is moderately dominated by General Condition and a 1/3 value is entered in the (1, 7) position. A 3 is then automatically entered in the (7, 1) position. The consistency ration C.R. is equal to 0.169 and one needs to explore the inconsistencies in the matrix with the help of Expert Choice to locate the most inconsistent one and attempt to improve it if there is flexibility in the judgment. Otherwise, one looks at the second most inconsistent judgment and attempts to improve it and so on.

We now move to the pairwise comparisons of the houses on the bottom level, comparing them pairwise with respect to how much better one is than the other in satisfying each criterion on the second level. Thus there are eight 3×3 matrices of judgments since there are eight elements on level two, and three houses to be pairwise compared for each element. The matrices (Table 1.4) contain the judgments of the family involved. In order to facilitate understanding of the judgments, a brief description of the houses is given below.

House A: This house is the largest of them all. It is located in a good neighborhood with little traffic and low taxes. Its yard space is comparably larger than that of houses B and C. However, its general condition is not very good and it needs cleaning and painting. Also, the financing is unsatisfactory because it would have to be financed through a bank at a high interest.

House B: This house is a little smaller than House A and is not close to a bus route. The neighborhood gives one the feeling of insecurity because of traffic conditions. The yard space is fairly small and the house lacks the basic modern

Table 1.4 Pairwise comparison matrices for level 2

	A	B	C	Normalized priorities	Idealized priorities
Size of house[a]					
A	1	6	8	0.754	1.000
B	1/6	1	4	0.181	0.240
C	1/8	1/4	1	0.065	0.086
Transportation[b]					
A	1	7	1/5	0.233	0.327
B	1/7	1	1/8	0.005	0.007
C	5	8	1	0.713	1.000
Neighborhood[c]					
A	1	8	6	0.745	1.000
B	1/8	1	1/4	0.065	0.086
C	1/6	4	1	0.181	0.240
Age of house[d]					
A	1	1	1	0.333	1.000
B	1	1	1	0.333	1.000
C	1	1	1	0.333	1.000
Yard space[e]					
A	1	5	4	0.674	1.000
B	1/5	1	1/3	0.101	0.150
C	1/4	3	1	0.226	0.335
Modern facilities[f]					
A	1	8	6	0.747	1.000
B	1/8	1	1/5	0.060	0.080
C	1/6	5	1	0.193	0.258
General condition[g]					
A	1	1/2	1/2	0.200	0.500
B	2	1	1	0.400	1.000
C	2	1	1	0.400	1.000
Financing[h]					
A	1	1/7	1/5	0.072	0.111
B	7	1	3	0.650	1.000
C	5	1/3	1	0.278	0.428

[a] $\lambda_{max} = 3.136$ C.I. = 0.068 C.R. = 0.117
[b] $\lambda_{max} = 3.247$ C.I. = 0.124 C.R. = 0.213
[c] $\lambda_{max} = 3.130$ C.I. = 0.068 C.R. = 0.117
[d] $\lambda_{max} = 3.000$ C.I. = 0.000 C.R. = 0.000
[e] $\lambda_{max} = 3.086$ C.I. = 0.043 C.R. = 0.074
[f] $\lambda_{max} = 3.197$ C.I. = 0.099 C.R. = 0.170
[g] $\lambda_{max} = 3.000$ C.I. = 0.000 C.R. = 0.000
[h] $\lambda_{max} = 3.065$ C.I. = 0.032 C.R. = 0.056

facilities. On the other hand, its general condition is very good. Also an assumable mortgage is obtainable, which means the financing is good with a rather low interest rate. There are several copies of B in the neighborhood.

House C: House C is very small and has few modern facilities. The neighborhood has high taxes, but is in good condition and seems secure. The yard space

Table 1.5 Synthesis

	1 (0.173)	2 (0.054)	3 (0.188)	4 (0.018)	5 (0.031)	6 (0.036)	7 (0.167)	8 (0.333)		
Distributive mode										
A	0.754	0.233	0.754	0.333	0.674	0.747	0.200	0.072	=	0.396
B	0.181	0.055	0.065	0.333	0.101	0.060	0.400	0.650		0.341
C	0.065	0.713	0.181	0.333	0.226	0.193	0.400	0.278		0.263
Ideal mode										
A	1.00	0.327	1.00	1.00	1.00	1.00	0.500	0.111	=	0.584
B	0.240	0.007	0.086	1.00	0.150	0.080	1.00	1.00		0.782
C	0.086	1.00	0.240	1.00	0.335	0.258	1.00	0.428		0.461

is bigger than that of House B, but is not comparable to House A's spacious surroundings. The general condition of the house is good, and it has a pretty carpet and drapes. The financing is better than for A but not better than for B.

Table 1.4 gives the matrices of the houses and their local priorities with respect to the elements on level two.

The next step is to synthesize the priorities. There are two ways of doing that. One is the distributive mode. In order to establish the composite or global priorities of the houses we lay out in a matrix (Table 1.5) the local priorities of the houses with respect to each criterion and multiply each column of vectors by the priority of the corresponding criterion and add across each row, which results in the composite or global priority vector of the houses. The other way of synthesizing is the ideal mode. Here the priorities of the houses for each criterion are first divided by the largest value among them (Table 1.5). That alternative becomes the ideal and receives a value of 1. One then multiplies by the priority of the corresponding criterion and adds as before. House A is preferred if for example copies of B matter and hence the distributed mode is used. In a large number of situations with 10 criteria and 3 alternatives, the two modes gave the same best choice 92% of the time [7]. House B is the preferred house if the family wanted the best house regardless of other houses and how many copies of it there are in the neighborhood and hence the ideal mode is used.

1.13 Absolute Measurement

1.13.1 Evaluating Employees for Raises

Employees are evaluated for raises. The criteria are Dependability, Education, Experience, and Quality, Each criterion is subdivided into intensities, standards, or subcriteria as shown in Fig. 1.4. Priorities are set for the criteria by comparing them in pairs, and these priorities are then given in a matrix. The intensities are then pairwise compared according to priority with respect to their parent criterion

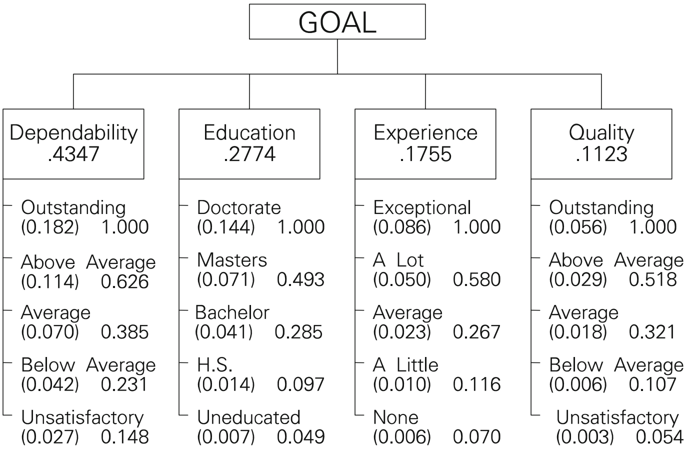


Fig. 1.4 Employee evaluation hierarchy

Table 1.6 Ranking Intensities

	Outstanding	Above average	Average	Below average	Unsatisfactory	Priorities
Outstanding	1.0	2.0	3.0	4.0	5.0	0.419
Above average	1/2	1.0	2.0	3.0	4.0	0.263
Average	1/3	1/2	1.0	2.0	3.0	0.630
Below average	1/4	1/3	1/2	1.0	2.0	0.097
Unsatisfactory	1/5	1/4	1/3	1/2	1.0	0.062

Inconsistency ratio = 0.015

(as in Table 1.6) and their priorities are divided by the largest intensity for each criterion (second column of priorities in Fig. 1.4). Finally, each individual is rated in Table 1.7 by assigning the intensity rating that applies to him or her under each criterion. The scores of these subcriteria are weighted by the priority of that criterion and summed to derive a total ratio scale score for the individual. This approach can be used whenever it is possible to set priorities for intensities of criteria, which is usually possible when sufficient experience with a given operation has been accumulated.

1.13.2 Organ Transplantation

The City of Pittsburgh has become a leader in the world in organ transplantations. Because there are more patients who need livers, hearts and kidneys than there are

Table 1.7 Ranking alternatives

	Dependability 0.4347	Education 0.2774	Experience 0.1775	Quality 0.1123	Total
1. Adams V	Outstanding	Bachelor	A little	Outstanding	0.646
2. Becker L	Average	Bachelor	A little	Outstanding	0.379
3. Hayat F	Average	Masters	A lot	Below average	0.418
4. Kesselman S	Above average	H.S.	None	Above average	0.369
5. O'Shea K	Average	Doctorate	A lot	Above average	0.605
6. Peters T	Average	Doctorate	A lot	Average	0.583
7. Tobias K	Above average	Bachelor	Average	Above average	0.456

available organs, it has become essential to assign priorities to the patients. The priorities shown in the figures are a result of several months of study by Alison R. Casciato and John P. O'Keefe in coordination with doctors and research scientists at a local hospital. Absolute measurement was used for this purpose and is shown in Fig. 1.5a–c. The hierarchy of Fig. 1.5 consists of the goal, the major criteria and subcriteria after which some of the subcriteria are further divided into yet smaller subcriteria or are divided into intensities for rating the patient. Figure 1.5a–c give further subdivision into intensities for those subcriteria in Fig. 1.5 that need to be further subdivide d into intensities. In general, one would use the intensities to score a patient. When there is no intensity, either the full value of the criterion is assigned, or a zero value otherwise. For example, Criminal has 0.033 priority and that value is awarded to a patient with no criminal record. A patient with a criminal record would receive a zero. The goal is divided into: emotionally dependent and financially dependent patients: Both are divided into single, married, and divorced with and without dependent and financially dependent patients. Then each of them is further divided into: medical history (time on donor list, degree of disability), physical history (degree of ability to endure rehabilitation, willingness to cooperate, etc.), and social status (criminal record, volunteer work). The priorities are indicated next to each factor and sum to one for each level. A patient is ranked according to the intensities under each criterion. The higher the total score the better the opportunity to receive a transplant.

1.14 Applications in Industry and Government

In addition to the many illustrations given in this book, the AHP has been used in the economics/management area in subjects including auditing, database selection, design, architecture, finance, macro-economic forecasting, marketing (consumer choice, product design and development, strategy), planning, portfolio selection, facility location, forecasting, resource allocation (budget, energy, health, project), sequential decisions, policy/strategy, transportation, water research, and performance analysis. In political problems, the AHP is used in such areas as arms control, conflicts and negotiation, political candidacy, security assessments, war games, and world influence. For social concerns, it is applied in education,

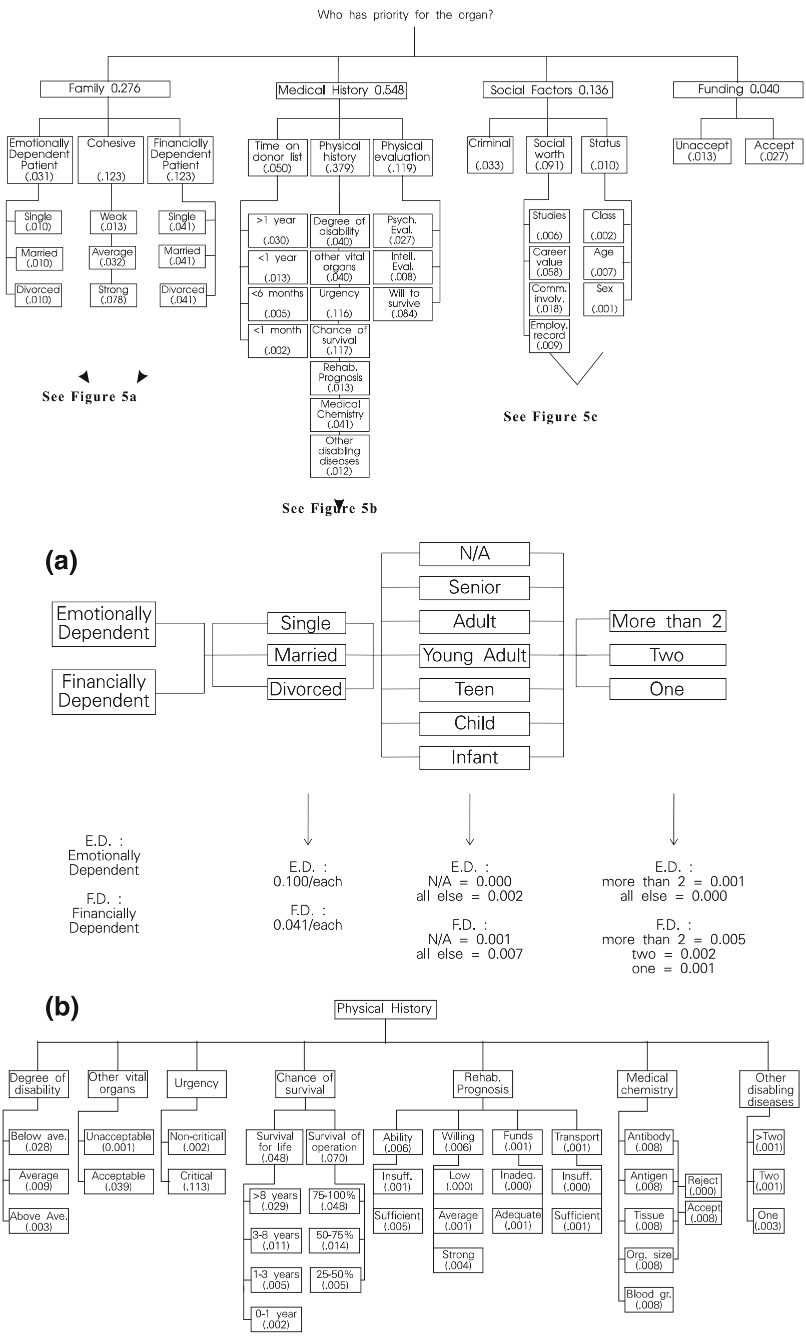
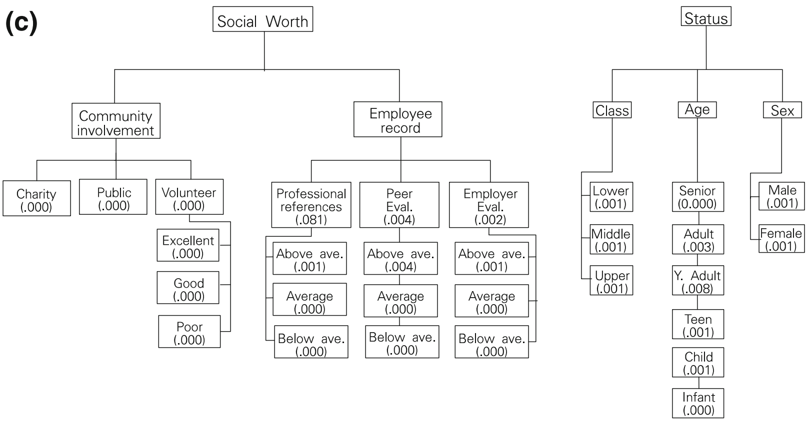


Fig. 1.5 **a** Organ transplantation–family factors. **b** Organ transplantation–medical history factors. **c** Organ transplantation–social factors

(c)



behavior in competition, environmental issues, health, law, medicine (drug effectiveness, therapy selection), population dynamics (interregional migration patterns, population size), and public sector. Some technological applications include market selection, portfolio selection, and technology transfer. Additional applications are discussed in Golden et. al. [3] and Dyer and Forman [1]. For a complete set of references see the bibliography at the end of Ref. [7].

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