Chapter 5
Overcoming the Gap Between University and School Mathematics

The Impact of an Innovative Program in Mathematics Teacher Education at the Justus-Liebig-University in Giessen

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5.1 Introduction

Prospective teachers, starting to study in order to become a mathematics teacher, contribute in a very special way to transformation processes, which mathematics as a science is actually undergoing in learning and teaching situations. We use the term *transformation* hereby to describe the adequate modification of mathematical content according to situation, intention, and cognition in educational settings. So, transformation is not an oversimplification or trivialization of content but an adequate adaptation of the learning material to the learner's perspective. On the one hand, in university mathematics courses at the beginning of their studies, starting out from their *learner*'s perspective, prospective teachers experience to be taught mathematical content that clearly differs from school mathematics, not only in range but also in formality and stringency. For this reason university teachers, who impart mathematical content, need to be sensitive to transformation processes in order to impart mathematical content in a comprehensible manner. On the other hand, later in their professional life, also the prospective teachers need to be able to *teach* mathematical content and make it accessible to their students in a didactically well-prepared manner. Therefore, they independently need to undertake didactical transformations of mathematical content in the scope of their everyday preparation of teaching as well. In the 1950s, Klafki formed the idea of a didactical reduction of the complexity of the scientific content that has to be learned in school, a transformation of the content via exemplifying, carving out fundamental ideas, and concentrating on elementary aspects. The choice of adequate curricula

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that is known as *didactical analysis* (Klafki 1958) or *didactical transformation* (Aschersleben 1993), its legitimation and the validation of its educational substance as well as the associated reduction of extent and difficulty in order to take into account the learners' cognitive abilities, is considered to be one of the essential requirements for teachers even prior to the choice of adequate methodical approaches. Traditionally prospective teachers are not provided with enough learning opportunities in order to sufficiently acquire the knowledge and abilities that are taught at university for didactically bridging the gap between the academic mathematics and the student-oriented elementary mathematics. High dropout rates among the first-year students indicate that already within the first semester imparting mathematical content in a comprehensible way only partially succeeds. Currently, at the universities there often is a large gap between the specific training on subject-based content and its realization in terms of teaching methodology. It is especially criticized that the link between the separate parts of teacher training, i.e., mathematics, didactics of mathematics, and pedagogy, is insufficient (for an overview cf. Blömeke 2004).

The teacher training program has been criticized to be lacking in practical relevance already for a century. It was at the beginning of the twentieth century that Felix Klein already described the phenomenon known as *double discontinuity*: "The young university student found himself, at the outset, confronted with problems which did not suggest, in any particular, the things with which he had been concerned at school. Naturally he forgot these things quickly and thoroughly. When, after finishing his course of study, he became a teacher, he suddenly found himself expected to teach the traditional elementary mathematics in the old pedantic way; and, since he was scarcely able, unaided, to discern any connection between this task and his university mathematics, he soon fell in with the time honoured way of teaching, and his university studies remained only a more or less pleasant memory which had no influence upon his teaching." (Klein 1932, p. 1).

Even today the double discontinuity is in the center of discussion concerning the relation between school and university (see Biermann and Jahnke 2013, this volume; Deiser and Reiss 2013, this volume; Pepin 2013, this volume). In the past couple of years there have been several approaches to subtend the discontinuity by altering the conditions of studying; one of them at the University of Giessen was sponsored by the *Deutsche Telekom Stiftung* (German Telekom foundation). This program focuses strongly on the entrance phase of the mathematics teacher training program for those who are becoming mathematics teachers for the higher track of the German tripartite school system (so-called *Gymnasium*). A central assumption of this program is that the discontinuity between school and university can partly be overcome by an adequate teacher-oriented transformation of the mathematical content in the academic lectures. In the following, we report about this program and the achieved changes.

The German Telekom foundation intends to support projects within the teacher training program that helps avoiding breaks in biographic transition periods such as starting the teacher training program in mathematics. In the course of this, the University of Giessen, in cooperation with the University of Siegen, elaborated a research and development program called *Mathematik Neu Denken* (*Thinking Mathematics in a New Way*) that reorientates the teacher training program (Beutelspacher et al. 2011). This project seeks a long-term improvement in the quality of the education of future mathematics teachers for the higher track schools, and, associated with these changes, improvement of mathematics education at schools is intended. The program was realized from 2005 to 2009, respectively, to 2010 with state funding. The students, who become mathematics teachers, were introduced to a new combination of courses that separated them from those who are aiming for a diploma in mathematics during their first two semesters. The subproject of the University of Giessen focuses on the rearrangement of the lecture on linear algebra/analytical geometry to an introductory course that is adapted to school requirements emphasizing the relation to mathematics subject matters and that relies on the vividness and the primacy of geometry. On the one hand, this approach is standing in the tradition of Felix Klein who laid an emphasis on the need of the sensualization of ideal constructs by the use of drawings and models (Klein 1939). On the other hand, it is trying to realize the idea of a didactical transformation of the mathematical content by consciously taking the student teachers' existing preknowledge up and strongly connecting it to extramathematical applications of mathematics with regard to the student teachers' future professional life. Furthermore, laying an emphasis on this kind of transformation offers an opportunity to learn an application-oriented way of teaching.

This chapter is based on data from the evaluation study TEDS-Telekom. The main purpose of this study, which was funded by the German Telekom foundation as well, is the evaluation of the funded project *Thinking Mathematics in a New Way*. In TEDS-Telekom these innovative approaches were evaluated from an external point of view with regard to the impact that was achieved in the area of the development of mathematical, didactical, and pedagogical competences of the students, together with the development of the corresponding beliefs. Among others, it has been drawn on approaches of the international comparative study "Teacher Education and Development Study—Learning to Teach Mathematics" (TEDS-M; Blömeke et al. 2010a, b; Blömeke and Delaney 2012). This IEA study for the efficiency of the education of mathematics teachers presents an external reference framework that allows for specific statements about the innovation potential of the pilot project, funded by the German Telekom foundation, in terms of strengths and weaknesses of the teacher education at the university. Control groups at other universities, which agreed to evaluate their teacher training program too, set another external benchmark. All in all, first-year student cohorts at five universities (Giessen, Siegen, Bielefeld, Essen, and Paderborn) were analyzed. For reasons of confidentiality, the results of the universities that additionally took part in the study will be anonymously communicated throughout the chapter.

5.2 Theoretical Framework and Study Design of the TEDS-Telekom Study

The presented study attempts to answer the question of how far the innovative efforts at the University of Giessen actually influenced the development of the local students' competences. The TEDS-Telekom study is restricted to the examination of influence of institutional educational factors on the individual development of competences with the help of quantitatively oriented written tests. For that reason, the approach of the study, which is mainly able to capture the development of the students' professional competence, is enriched. Qualitatively oriented and problem-focused interviews with prospective teachers of the involved universities were used in order to have an additional perspective. In doing so, the qualitative approach enables us to gain insight into the impact of didactical concepts of the universities and different teaching and learning conditions on students and on their individual internal perception and acceptance of particular components of the university teacher education. The so-called mixed-method design—a qualitative–quantitative mixed study design (cf. Kelle 2008) that has been chosen to be applied—is supposed to compensate "blind spots" in the methods of a single research paradigm and customize a broader range of results.

The term professional competence has been conceptualized in various ways within the scope of empirical studies, such as the international studies MT21—"Mathematics Teaching in the twenty-first Century" (Blömeke et al. 2008) and TEDS-M (Blömeke et al. 2010a, b; Blömeke and Delaney 2012), the German COACTIV study—"Professionswissen von Lehrkräften, kognitiv aktivierender Mathematikunterricht und die Entwicklung mathematischer Kompetenz" (Kunter et al. 2011) or the Michigan LMT-Project "Learning Mathematics for Teaching" (Hill et al. 2008). The evaluation study presented below is based on the conceptualization of professional competence of prospective mathematics teachers as a multidimensional construct, like it has been developed in general by Weinert (1999) and Bromme (1992, 1997) and which forms the theoretical basis of the TEDS-M study too. According to this approach, professional competence includes subject-related and interdisciplinary cognitive dispositions of performance, as well as affective-motivational beliefs as part of a teacher's personality. In addition to that, in his topology of teacher's professional knowledge, Bromme (1992, 1997) underlines the impact of the teachers' personality on the professional competence by describing the knowledge on the philosophy of the school subject and its contained perspective of valuating. Further, elaborately discussed questions in the field of scientific research on the assessment of teachers' competences are about the integration of acting into models of professional competence of teachers and the measurability of competence for action.

The evaluation study TEDS-Telekom is restricted to the analysis of the cognitive components of professional competence (professional knowledge of teachers) and focuses in the area of personality features on *beliefs* concerning the subject and the teaching and learning of the respective subject. The study owes its restriction to the fact that students at the beginning of their studies cannot gain much action

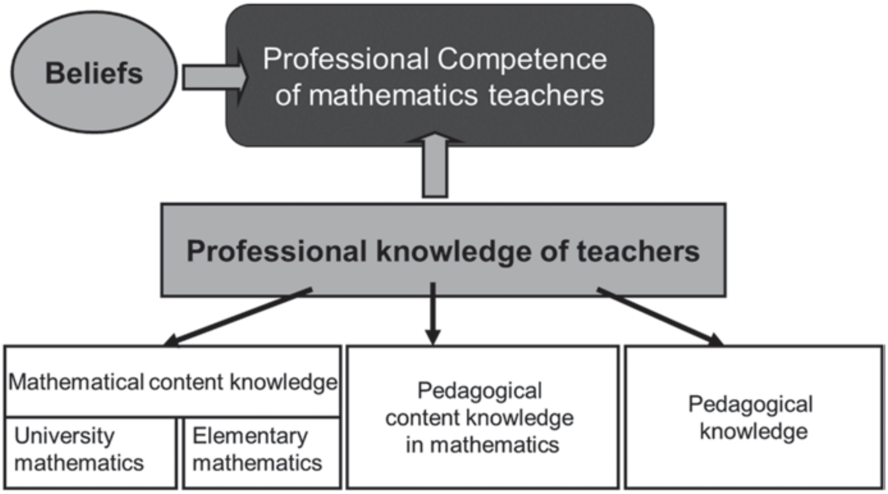

Fig. 5.1 Model about professional competence in the evaluation study TEDS-Telekom

competence, because up to that time most of them did not have the opportunity to get professional experience and, regarding the impact of personality traits on the characteristics of professional competence, it is widely believed that these influences tend to be less obvious at the beginning of the university study time than later in professional practice. For this reason, the evaluation study focuses on the central aspects of the knowledge in mathematics and didactics of mathematics of the first two academic years for future mathematics teachers for lower and upper secondary levels, including the related beliefs referring to the fundamental aspects of professional knowledge of teachers as outlined by Shulman (1986) and Bromme (1992, 1997) (see Fig. 5.1).

For the evaluation study the dimensions of professional competence have been subdivided and operationalized as follows:

- Academic mathematical knowledge in the area calculus and linear algebra/analytic geometry
- Elementary mathematics from an advanced standpoint
- Pedagogical content knowledge of mathematics or didactics of mathematics referring to upper secondary level
- Pedagogical knowledge focusing on action-related aspects, such as the structuring of teaching, motivation, classroom management, assessment and dealing with heterogeneity
- Beliefs on mathematics as a science and on learning and teaching of mathematics

In this connection it must be noted that elementary mathematics from an advanced standpoint is a subarea of mathematics, but, simultaneously, it also creates basic elements for an interlocking of academic mathematical knowledge and didactics of mathematics in the meaning of the approaches of Klein (1932) which have been developed further by Kirsch (1987). These subdomains then are differentiated further with respect to cognitive aspects, by evaluating the respective declarative knowledge and the repertoire of pedagogical acting. In order to identify the different qualities of cognitive requirements to be met by the prospective teachers for solving the test items, Bloom's taxonomy of cognitive processes, as revised and extended by Anderson and Krathwohl (2001), was applied in connection with the test items. The focus was on three dimensions of cognitive processes: memorizing, understanding/analyzing, and creating (see Blömeke et al. 2011).

These mathematical and mathematics-didactical-related items of the study have successfully been developed in accordance with two interrelated content-based frameworks of reference: on the one hand, the largely canonical contents of the lectures at the beginning of the university study courses for mathematics teacher education for the upper secondary level and, on the other hand, the respective recommendations for the structure and design of the study from the *Standards for mathematics teacher education* as suggested by the German Society of Mathematics (DMV), the German Society for Didactics of Mathematics (GDM) and the Union for the Advancement of Mathematics and Science Teaching (MNU) (DMV, GDM & MNU 2008) by considering the central ideas and approaches of the innovative concept of mathematics teacher education of the universities of Giessen and Siegen.

As far as the items are not taken from the TEDS-M study, mathematical and mathematics didactical items which had been created for TEDS-Telekom were developed further by the mathematics didactical working group at the University of Hamburg guided by Gabriele Kaiser in cooperation with Hans-Dieter Rinkens from the University of Paderborn and then refereed in workshops by further experts of mathematics didactics from universities which are also participating in the study. Then, based on that expertise, the items were revised again. The items related to pedagogy have been developed by the working group Systematic Didactics and Instructional Research at Humboldt University of Berlin directed by Sigrid Blömeke in cooperation with Johannes König.

The test also contained items from the TEDS-M study so that later the results of the evaluation study can be evaluated and interpreted with reference to an external standard. Like the initial development of items, the TEDS-M items were selected with respect to the above-described content-based frameworks of reference (canonical contents of the respective university courses and DMV–GDM–MNU suggestions).

To illustrate the items used in the study, we describe in the following one of the TEDS-M 2008 items that has been used in the TEDS-Telekom study with the respective solution frequencies. But attention should be given to the fact that performance on the level of individual items can vary due to chance and thus should not be over-interpreted.

The task US25 (see Fig. 5.2) comes from the content area of the academic mathematical knowledge of linear algebra and analytic geometry and requires basic knowledge of the geometry of the plane and the space. The amount of points that satisfies the equation $3x=6$ in the plane is a straight line, but in space it is a plane.

US25) We know that there is only one point on the number line that satisfies the equation $3x = 6$, namely $x = 2$.

Let us now transfer the equation to a plane with coordinates x and y, and then to space, with coordinates x, y and z. What is the set of points that satisfy the equation there?

Tick one box per row.

		A point	**A straight line**	**A plane**	**Else**
A)	The solution of $3x = 6$ in the plane	☐	☐	☐	☐
B)	The solution of $3x = 6$ in space	☐	☐	☐	☐

Fig. 5.2 TEDS-M 2008-item

Seventy-two percent of the German prospective teachers in TEDS-M 2008 were able to solve item A correctly; for item B of the figure, the proportion is still 68 %. The student teachers of the University of Giessen solve both items with 75 % at approximately the same height, as well as the comparative teacher training group (71.4 % for item A and 61.9 % for item B). For being able to make a statement about the achievements and the achievement development of first-year-student cohorts, the TEDS-Telekom study is designed as a real longitudinal study. The evaluation of the students by means of a 90-minute paper-and-pencil test took place at the beginning of the first semester (December 2008), the end of the second semester (July 2009), and at the end of the fourth semester (July 2010). Central assumptions for the evaluation of the test results were measurable success of achievements from the first to the third point of measurement, thus from the beginning of the first semester until the end of the fourth semester, as well as that the degree of success of achievement varies depending on the level of achievement at the beginning, the students' learning preconditions, and the learning opportunities provided by the universities—thus the innovative potential of the study programs (integration of domains of knowledge, extent of learning opportunities, etc.). Meanwhile, the results of the longitudinal measurements from all three evaluations are available (Buchholtz and Kaiser 2013, in print).

Going beyond the borders of the project *Thinking Mathematics in a New Way*, in order to investigate the influence of various aspects of institutional conditions and aspects of didactics of higher education on the individual acquisition of competence from a different, more qualitatively oriented point of view, additional problem-centered guided interviews according to Witzel (1982) were carried out with 19 prospective teachers from all participating universities. Within the scope of these interviews, the prospective teachers were asked about their perceptions and their estimations about learning opportunities and aspects of didactics of higher education in connection with their studies. Among these 19 prospective teachers, who participated voluntarily and were chosen randomly, there were four students from the Justus-Liebig-University of Giessen. The interviews were conducted by using a guideline which contains the following aspects of perception and estimation of university teaching within the introductory phase of their experienced university studies:

- Integration of visualization, examples and example-bound argumentations, and real-world applications in mathematical lectures
- Integration of elementary mathematics from an advanced standpoint in mathematical lectures
- Interweaving of mathematical and mathematics didactical content in university courses
- Beliefs about teaching and learning of mathematics

Currently, the interviews are systematically evaluated by means of the method of qualitative content analysis of Mayring (2000) so that now only preliminary results are available. A mixed-method design has been chosen due to the fact that empirical studies dealing with the efficiency of teacher education are mostly either grounded in the qualitative or the quantitative paradigm (as example see Blömeke et al. 2010a, b; Eilerts 2009; Schwarz 2013). The decisive advantage of a combination of qualitative and quantitative methods is that in this way characteristic weaknesses of one tradition of methods can be balanced by the strengths of others (Tashakkori and Teddlie 2003, p. 16). Johnson and Turner (2003, p. 299) even call it a fundamental principle "that has complementary strengths and non-overlapping weaknesses."

5.3 Development of Cognitive Dispositions of Achievement

Over all three measurement points, after the sample had been revised, altogether 128 students participated in the TEDS-Telekom study. Thirty-two students of them are from the Justus-Liebig-University. Because the subsamples of the control universities decreased over the study time of four semesters, and for particular methodical reasons and reasons of confidentiality, the results of the universities of Bielefeld, Paderborn, and Essen will not be compared to those of the University of Giessen

Table 5.1 Comparison of average *Abitur* grades; the grades can differ from 1.0 (best grade) to 4.0 (worst grade)

Group	*Abitur* grade M1	*Abitur* grade M2	*Abitur* grade M3	Deviation
University of Giessen	2.20	2.20	2.15	−0.05
Control group teaching	2.37	2.26	2.24	−0.13
Control group nonteaching	2.21	2.05	2.01	−0.20

on the level of universities. Instead of comparing small samples and heterogeneous groups, the groups of the students studying to become teachers and students not studying for the teaching profession will be treated separately, whereat these groups will be aggregated as "control groups," or "reference groups," containing students from all three universities: Bielefeld, Paderborn, and Essen. For this reason the composition of the control groups is extending across locations and consists of 39 prospective teachers (*Control Group Teaching*) and 30 nonteacher students (*Control Group Non-Teaching*). The results of the group of prospective teachers from the University of Siegen, in which also a subproject of the German Telekom foundation was funded, are not mentioned.

To give an overview about the individual preconditions of the students participating in the study, at each point of measurement the average grade of the general qualification for university entrance (the so-called *Abitur)* of the students still remaining in the sample was compared to the average grade at the beginning. At the beginning, at measurement point M1 the groups did nearly not show any differences in comparison, but at measurement point M3 the control group of nonteacher students had significantly better average grades of the general qualification for university entrance (the so-called *Abitur* grades) than all groups of prospective teachers. This points to possible selection processes, e.g., dropout of weaker students with lower grades in their *Abitur* grades during the first four semesters. However, it cannot be excluded that students with lower grades could not be reached anymore by the tests.

Likewise, the two universities promoted by the program of the German Telekom Foundation show a similar characteristic improvement of the average *Abitur* grade. However, the grade of improvement of the University of Giessen is the lowest. This suggests that the introductory selection procedure is less noticeable in this group (to compare see Table 5.1).

Further, data were collected about the kind of courses that were attended by students at school in the upper secondary level. For this, the answer options *basic course* (courses at basic mathematical level), *advanced course* (courses at higher mathematical level), and *neither basic nor advanced course* (optional in some federal states of Germany) were given. The comparison of the results of the samples of M1 and M3 is shown in Table 5.2. First, it shows that at the control universities the percentage of students—prospective teachers and nonteacher students as well—who had attended advanced mathematics courses during their schooltime increases

Table 5.2 Comparison of school-related preconditions

M	Kind of course	University of Giessen (%)	Control group teaching (%)	Control group nonteaching (%)
1	Advanced course	71.9	68.5	89.3
	Basic course	28.1	29.2	5.4
	Neither basic nor advanced course	0	2.2	5.4
3	Advanced course	71.9	76.9	96.7
	Basic course	28.1	20.5	3.3
	Neither basic nor advanced course	0	2.6	0

from measurement point 1 (M1) to measurement point 3 (M3), for which there might be a connection with effects from the introductory selection. However, at the University of Giessen the percentage remains the same. These results might indicate that the University of Giessen was more successful than the control universities in keeping students with less good preconditions over a longer period of time, at a minimum during the first four semesters.

The collected data from the tests have been scaled by IRT models (see Rost 2004), for which scales had to be distinguished according to the domains of knowledge as described in Sect. 5.2. For the estimation and presentation of individual abilities, *weighted likelihood estimates* (WLEs, Warm 1989) were applied. Each scaling has been executed by using the software ConQuest (see Wu and Adams 2007), a software for fitting item response and latent regression models. As the second and third collection of data were intended to measure development, it was necessary to "anchor" the various kinds of tests at all three measurement points in all domains of knowledge with a respective number of items. This means that the same items had to be calculated over all kinds of tests and measurement points by referring to the same parameters of difficulty. As it is very difficult to equalize item parameters in ConQuest, an approach has been chosen, which also Hartig and Kühnbach (2006) reverted to. For an estimation of the item difficulties, first a one-dimensional scaling for all items with so-called "virtual persons" has been carried out.

Then, for an estimation of the person's abilities the item difficulties of the anchor items of measurement point 1 (M1) have been imported into a three-dimensional scaling of all items, for which the single measurement points are indicating the three latent dimensions. For this, the difficulty parameters of the anchor items from the scaling with virtual persons have been taken and been fixed for all three measurement points. Thus the anchor items show the same difficulty at each measurement point. Then, based on this model, the person parameters have been estimated.

The scales' reliability in the three-dimensional model ranges from sufficient to good at all three measurement points (scale reliability from 0.63 to 0.83).

In the following, the ability parameters are presented graphically and subdivided according to the different domains of knowledge that were tested. The WLEs of all

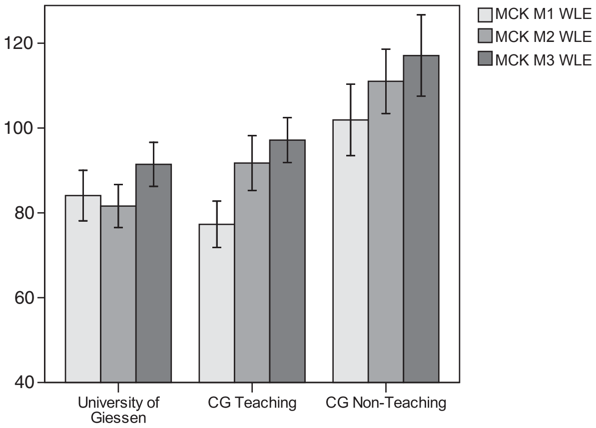

Fig. 5.3 Ability parameters (WLE) in the area content knowledge of calculus and linear algebra/analytic geometry

measurement points were transformed to an average value of M = 100 and a standard deviation of SD = 20. Finally, all results will be interpreted separately according to the respective domains.

5.4 Development of Performance in Cognitive Domains

5.4.1 Academic Mathematical Content Knowledge (Calculus & Linear Algebra/Analytic Geometry) (Fig. 5.3)

At all three measurement points, significant differences between the groups of the prospective teachers and the control group of nonteacher students exist with the group of the nonteacher students performing significantly better. However, altogether the achievements of all first semester prospective teachers increase significantly too. Therefore, the expectations of observable learning success have clearly been met by this study. After four semesters (M3) the prospective teacher groups have reached a similar level as the means of the two groups do not differ significantly, although they developed differently.

The stagnation between the first two measurement points can be balanced by the University of Giessen up to the third measurement point. The group reached the same level of ability as the control group “Teaching” by means of higher learning success from the second to the third testing, while the control group “Teaching” showed strong success from the first to the second testing. The stagnation of the University of Giessen between the first and second measurement point can easily be explained. According to their study program it was not planned that the prospective teachers of the sample attend the calculus courses. Instead of that they attended courses on linear algebra and analytic geometry. Therefore, no increase in the area

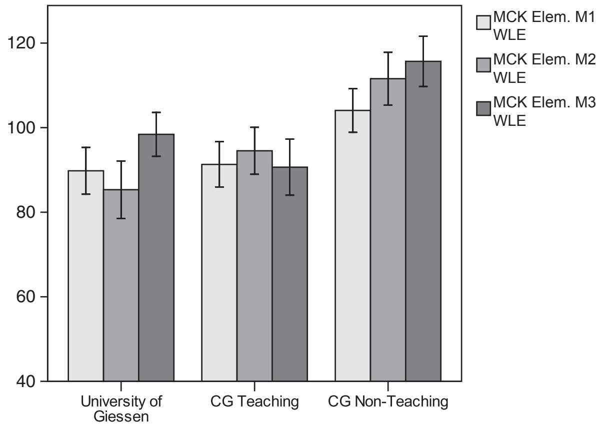

Fig. 5.4 Ability parameters (WLE) in the area content knowledge elementary mathematics from an advanced standpoint

of content knowledge could be expected, because many items of the tests refer to calculus which at the University of Giessen is part of the curriculum for the third and fourth semester. The high success of the Giessen group in the last testing can be explained by this different organization of the study structure and the timing of the offered learning opportunities.

5.4.2 *Elementary Mathematical Content Knowledge (Elementary Mathematics from an Advanced Standpoint) (Fig. 5.4)*

Best performances in the area of elementary mathematics from an advanced standpoint at the third measurement point are achieved again by the control group of the students not aiming for teaching profession (nonteacher students). In contrast to that, the group of the prospective teachers performs significantly lower. The control group of prospective teachers shows, at no point of measurement, any significant change of knowledge in the area of elementary mathematics, whereas the control group of nonteacher students improved their performance from an average level at measurement point M1 to a significantly higher level at measurement point M3. Among the prospective teachers, only the group of students from the University of Giessen achieved significantly better results at the third measurement point compared to that before, so that the University of Giessen holds the leading position among all participating groups of the prospective teachers.

At first sight, the outstanding performance of the nonteacher students in the area of elementary mathematics from an advanced standpoint is surprising, as especially one would assume the elementary mathematics from an advanced standpoint to be a domain where prospective teachers carve out their leading role in mathematics. A possible explanation might be that nonteacher students do not have problems with the school-related university mathematics on which the questions are based on. Further, generally this cohort of students does not study another academic subject, so that they can spend more hours per semester studying mathematics than prospective teachers. Compared to all subsample groups, the control group of nonteacher students (Non-Teaching Group) contains the biggest share of students who had attended advanced mathematics courses at school. Another reason may be the test structure and the test itself. The respective test differs from the tests for the other domains of knowledge insofar as the major part consists of TEDS-M 2008 items that aim at testing mathematical school knowledge. Only a few new and more difficult items are added in order to test an increase of knowledge. Altogether, the items of TEDS-M 2008 are structured in a slightly different way, sometimes more narrow in their questioning or stronger oriented to declarative knowledge. For this reason, the result suggests that these items tend to test other facets of knowledge which are structured differently compared to the other test parts, possibly more reproductive abilities. Anyhow, this predominant advantage of the group of students studying for nonteaching professions especially in this domain and the stagnation of the control group of prospective teachers compared to the students of the University of Giessen at all measurement points is a surprising and quite unexpected result.

The increase of performance of students from the University of Giessen is surprising because of the time it happens, namely between the second and third testing. As most of the activities in Giessen, which are supported by the German Telekom foundation, concentrate on the area of linear algebra, one would have expected an increase of performance in the area of elementary mathematics from an advanced standpoint between the first and second measurement point. But at that time the results stagnated. This result might be explained as a delayed effect of the support programs or caused by specific learning opportunities determined by the characteristics of the curricular content. In the third and fourth semester the students attend calculus courses and the items from the domain of knowledge of elementary mathematics from an advanced standpoint are more algebraic and algorithm oriented, which runs quite contrary to the more visual-based orientation of the mathematical content-based courses of the first two semesters at the University of Giessen, so that the numeric orientation of calculus might play a key role for understanding elementary mathematics.

5.4.3 Pedagogical Content Knowledge (Fig. 5.5)

The distinct pedagogical content knowledge of nonteacher students at M1 has been developed further by this group so that also until the third measurement point, they showed the best results in this field of knowledge. This indicates that the items which are mainly subject-matter based could obviously be solved well using mathematical-content knowledge. As expected, from all first-year students the students from the nonteaching group produced the lowest increase of performance. The prospective teacher groups showed generally a higher, sometimes even a clearly higher increase of performance. This result also refers to the fact that pedagogical content knowledge is an independent domain of knowledge.

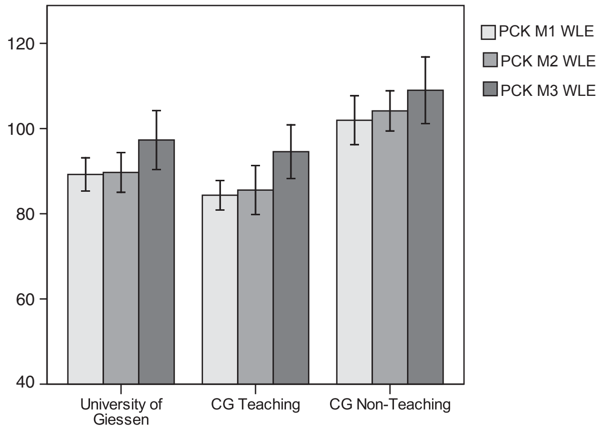

Fig. 5.5 Ability parameters (WLE) in the area pedagogical content knowledge

Students of the control group of prospective teachers and of the group of the University of Giessen showed a significant development of performance in pedagogical content knowledge from the second to the third measurement point. According to the study, regulations of the University of Giessen students must attend one pedagogical content course during the first four semesters, but not at a fixed time. This result indicates that the majority of the prospective teachers tend to choose a later point of time for attending these courses (Fig. 5.5).

5.5 Learning Opportunities

To get an overview about the students' learning opportunities, it is necessary to examine the specific university curricula for teacher education, concerning type and scope of courses of the participating universities (intended curriculum) among others. These data then need to be crosschecked with data about the students' perception about the recommended courses (implemented curriculum) (for differentiation of intended and implemented curriculum see McDonnell 1995). In TEDS-Telekom the analysis of the intended curriculum was based on the official study and examination regulations for upper secondary-level teacher education valid at that time at the participating universities. Besides that, module manuals and study plans taken from the internet which are more or less regulating the students' course of study were considered. The area of content knowledge is determined by a canonical uniformity of the study structure of the first four semesters (according to traditional teacher education terminology called "basic studies" or Grundstudium). During the first four semesters, students have to attend the classic introductory courses, which in the first two semesters traditionally are courses on calculus, and in the third and fourth semester on linear algebra. In contrast to the other universities which are participating in the study, at the University of Giessen the introductory courses are

Table 5.3 Number of students having attended the listed courses

Group	University of Giessen				n	CG-Teaching				n	CG-Non-Teaching				n
Semester	1	2	3	4		1	2	3	4		1	2	3	4	
course of MCK															
Linear Algebra 1	32				32	27	5	1		33	18	3	2		23
Linear Algebra 2		32			32		29	3	1	33		18		2	20
Calculus 1			30		30	28	2	4	1	35	23	2	4		29
Calculus 2				22	22		28	2		30		21	1	4	26
Calculus 3								10		10			11	2	13
Proseminar								1	1	2				1	1
Number theory							2	4	4	10			4		4
Numerics				1	1				1	1			5	1	6
Stochastics							1	11	10	22			7	3	10
Algebra								6	1	7			4	2	6
Geometry				9	9			5	2	7				2	2
Computers	6	2	2	4	14				8	8		1		2	3
else						1		3	6	10	2	1	8	6	17
course of PCK															
Didactics for low. Secondary		1	3		4	1	1	11	1	14	1				1
Didactics of Calculus								1	2	3					
Didactics of Linear Algebra	1	1	21		23				1	1					
Didactics of Geometry		2	21		23				1	1					
Didactics of Algebra			3		3			1	3	4					
Didactics of Arithmetic								1	2	3					

arranged in the reverse order by giving priority to linear algebra/analytic geometry. Some universities suggest students to attend calculus and linear algebra courses simultaneously in one and the same semester. These regulations for the first four semesters are completed by a compulsory mathematical elementary or an advanced seminar, or at some universities by a course on the usage of computers in mathematics, and in some cases even by more advanced courses on calculus such as differential equations, stochastics, numerics, number theory, or algebra. The amount of pedagogical content courses is relatively small, generally only up to maximal 20 % of mathematics courses. In the TEDS-Telekom study, data on the attendance of these courses referring to study regulations were sampled. As the range of courses concerning mathematical content and didactical content differs on a large scale between the participating universities and the instrument could not be changed for local deviance, the study had to restrict to a common nomenclature of elementary and advanced courses, which differs from the local nomenclature of the respective university. The suggested courses were presented to the students in a list in which they had to set the corresponding crosses, but the students also had the opportunity to note further courses they had attended. Table 5.3 gives an overview on the answers of the students, indicating courses which had been attended by the students.

Compared to the University of Giessen, the courses attended by students of the control universities appear broadly dispersed. Of course, this is caused by the fact that the control group consists of groups from three universities. However, as a part of the students of the control group attend courses on calculus and linear algebra in the same semester, they tend to take offered advanced courses already in the third and fourth semester, while students of the University of Giessen are limited to attend the courses in the strictly predetermined order. In addition, it is striking that over the whole time of four semesters, content courses on computer-application training are multiply mentioned by the students, which does not occur with students from the other universities. Reasons for high attendance of the geometry course in the fourth semester at the University of Giessen, which is not part of the studies' curriculum, lie probably in the overall high relevance of geometry at the University of Giessen.

Starting out from this background, the picture of the knowledge acquirement in various areas of knowledge gives a totally different impression. Concerning the acquisition of knowledge within the framework of the introductory courses, it obviously does not make any difference whether students take introductory courses simultaneously or one after another, because at the end of the fourth semester the prospective teachers have reached nearly the same ability level. Only the process of the development of content knowledge is different, and therefore no advantages or disadvantages of the study concept at the University of Giessen become evident. Effects might occur more strongly in the area of a working overload of the students and their perception of it.

Likewise, in the area of pedagogical content knowledge the groups differ significantly. Generally, students in Giessen attend during the second and third semester one or two pedagogical content courses which are focusing on geometry and linear algebra. Courses on pedagogical content knowledge were attended only sporadically at the control universities, but for most of the prospective teachers a kind of an introductory and overview-providing lecture exists. As expected, courses on pedagogical content knowledge were nearly not at all chosen by nonteacher students.

By looking at the increase of performance, the same effects from attending the respective course become evident. The performance of prospective teachers increases significantly from the second to the third measurement point. A differentiation of effects from attending introductory courses on pedagogical content knowledge and from specialized courses cannot be detected on the testing level.

5.6 First Results on the Evaluation of the Influence of Institutional Conditions

In the interviews the prospective teachers were asked to make estimations about the general institutional conditions. They should describe from which kind of learning opportunities they had gained most during their study time, and in addition which issues need to be addressed in order to ensure a more comprehension-oriented university teaching. In the following, only the aspect *integration of visualization in mathematical lectures* tackled in the guided interviews will be discussed, because the interviews are actually still being evaluated further, but this aspect is deeply related to the transformation of mathematics within academic teaching as Gustav Grüner formulated in his work on didactical transformation in general the importance of analogies, metaphors, and examples for illustrating a scientific statement in order to concretize it (Grüner 1967). In addition, especially in the work of Felix Klein we find the important role of models for visualizing abstract mathematical concepts (cf. Klein 1939). Below, we refer to the statements of the four prospective teachers from the University of Giessen, who—anonymously—give an insight into the impact of the project *Thinking Mathematics in a New Way*, which aims at a new orientation of teacher education for mathematics teachers for the upper secondary level at the University of Giessen. Within the statements, the students describe their involvement in the teacher training program and their perceptions about the mathematical courses on a very personal level. Biographical aspects as well as the individual development of professional knowledge for teaching form the background for these statements. (For a related approach to analyze professional identity of a student teacher going through teacher education and building up professional experience on a narrative way, see Grevholm 2013, this volume.)

The prospective teachers have repeatedly perceived the discontinuity described at the beginning of this chapter as a problem they are also struggling with during their studies. In Giessen, most of the statements refer to the calculus course which is taken only in the third semester and has not become part of the restructuring measures of mathematics teacher education: that means the calculus course is taught in the traditional abstract way attended by the prospective teachers and the mathematics students not aiming for the teaching profession.

> If now I‘m thinking back on school, [in calculus] we had a bit the evaluation of functions, a bit derivations. And actually in calculus, the time at school, now I do not remember anymore, but I think there were hardly any proofs. That is precisely the opposite at the university, there are definitions, proofs … And calculus in the university context actually consists only of proofs and definitions. […] No, there were clearly dropouts. Due to calculus there were clearly dropouts, but I can imagine, if we have had that in the first semester, the dropout rate would have been even higher. (Prospective teacher, female, 26)

This shows especially that a higher grade of abstraction causes obstacles for understanding and that what has been learned will be forgotten immediately, which shows that pure transmission of factual knowledge is not sustainable.

> The terminology of calculus remained abstract, 75 % of it I would even not know what to do with, what does it mean, for what I am actually doing that … That was just stupefying learned by heart and simply written down, that what the professor wanted to hear. (Prospective teacher, male, 21)

> It extends, the learning process extends. At home I sit down and work upon it by exemplification, so that I can understand it by myself. And so it takes much longer. If that would have been contributed by the lecture, one would not need to work on exemplification afterwards on one’s own. (Prospective teacher, female, 26)

The prospective teachers depend on an illustrative way of teaching mathematical content, which much too often cannot be realized in university courses which are attended by prospective teachers together with mathematics students not aiming for teaching profession.

> At the end actually, because there are so many prospective teachers, I personally think it would be wonderful if it would really be possible to separate Bachelor [(i.e. non-teacher) students] and prospective teachers, completely, and not only for selected courses. And the Bachelor [students] do not need these references of reality, the exemplification as strongly as prospective teachers need it. I think, because, the Bachelor [students] do not teach that later. (Prospective teacher, female, 26)

Obviously, the Justus-Liebig-University was successful with a course on Linear Algebra for students aiming for the teaching profession and succeeded to overcome, at least partly, comprehension problems through embedding exemplification into the mathematical content courses.

> In algebra, concerning vector spaces, it was beautifully made clear, that a vector is not just an arrow which is just drawn, but that it has a direction and which properties it has. Because, one has quasi developed an imagination of it, how it looks like. And therefore later it is good for the students, one can better explain it. (Prospective teacher, male, 21)

The students describe that indeed the exemplification is given a key role for their own understanding. The idea of transformation of mathematics in university mathematics courses via analogies, metaphors, and examples for illustrating and visualizing is even recognized in its exemplary function for the later demands of the teaching profession.

> I now also try to apply the exemplification in the private tutoring center, where I work. I try to put this also into the foreground. Because the experience, the short experience, that I could make now, has shown that the more exemplifying the beginning is, the more the pupils are willing to get to work on theory. (Prospective teacher, male, 21)

The teaching of mathematical content in an understanding-oriented way is on the one hand fostering learning, but on the other hand very time-consuming because the pace of learning may be reduced. But students do not consider that as impairing.

> Yes, exemplification I think is quite important, in order to have reference, so that one knows what one is doing there. If you have an image right in front of your eyes, then the theory remains more rooted in your head, later it is like this at school. And yes, then it is okay for me, if then in only one week lecture can be worked on only the half, but one knows: the students do understand it now. (Prospective teacher, female, 20)

> Yes, I just say, I personally think it makes more sense to work on less content, but to understand it, instead of working on more content of which one does not know anything at the end after having struggled through. (Prospective teacher, male, 20)

5.7 Conclusions and Further Prospects

The described analyses about the development of the average *Abitur* grades and the kind of the attended mathematics course at school of the students of each cohort demonstrate that at the University of Giessen during the first four semesters obviously no strong selection is prevailing. With the programs supported by the German Telekom foundation, the University of Giessen succeeds in keeping the grade of selectivity low, whereas a high grade of selectivity is characteristic for mathematics teacher education at the beginning of their study, which means that they succeed to keep students from the lower-achieving sector in their studies. Nevertheless, prospective teachers of the University of Giessen achieved comparably high results in the areas of calculus, linear algebra and analytic geometry, elementary mathematics, and pedagogical content knowledge. Concerning the learning increase, the students' performance stagnates between the first and second measurement point in the domains of content knowledge and pedagogical content knowledge, due to the structure of the curriculum of the University of Giessen. At the third measurement point, the students of the University of Giessen showed a remarkably high increase in the area of academic mathematical content knowledge, elementary mathematical, and pedagogical content knowledge.

We assume, and not at last based on the results of the interviews, that the special lectures for prospective teachers in Giessen separated from mathematics students not aiming for the teaching profession have a strong influence on the knowledge development of prospective teachers. One reason might be that a slower pacing and the empathic, exemplifying way of teaching applied in the courses has developed a "teacher-specific self-efficacy," which on the one hand might explain the high grade of identification with the project and on the other hand might have a positive influence on the acquisition of knowledge. Equally from the students' perspective it has been affirmed that this style of teaching in the mathematics courses has a motivating effect. Therefore, the students do not regard the sometimes slower pace of learning as an obstacle, but on the contrary, as a strengthening of their efforts of learning. Nevertheless, in the area of mathematical knowledge they do not show significant performance deficits. The results of this study give reason to ponder whether the improvement of teacher training can be achieved by restructuring the mathematical courses. Should teacher training in the future be more based on future practices, the integration of elements promoting understanding in mathematical lectures such as visualization or applications makes sense. The transformation of the mathematical content in the sense of being directed at understanding is related to curriculum changes and may, but not necessarily, need to be associated with a reduction of the teaching content. It lies in the responsibility of the universities to realize teacher-specific teaching, for example, by systematically building on the knowledge of elementary mathematics. According to the expressions of the students, in many mathematical lectures still a form of didactics is supposed to be dominating that conveys the systematics of the science subject on the learning process of student teachers in a rather unreflective way, which means without considering the perspective and the learning of student teachers. If the teacher education really should be improved, this thought of didactics should be discarded. Following Werner Jank and Hilbert Mayer (1991) in the general understanding of such an image didactics as a concept in which the professional scientific structures are transferred without changes to the process of selecting, structuring, and justifying the curriculum, the idea of transformation of academic content here means the exact opposite. The idea of transforming mathematical content as done by the Justus-Liebig University within the scope of the project *Thinking Mathematics in a New Way* can make a fruitful contribution to the sustainable improvement of teacher education.

References

Anderson, L. W., & Krathwohl, D. R. (2001). *A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives*. New York: Addison—Wesley.

Aschersleben, K. (1993). *Welche Bildung brauchen Schüler? Vom Umgang mit dem Unterrichtsstoff*. Bad Heilbrunn: Klinkhardt.

Beutelspacher, A., Danckwerts, R., Nickel, G, Spieß, S., & Wickel, G. (2011). *Mathematik Neu Denken. MathematikImpulse für die Gymnasiallehrerausbildung an Universitäten*. Wiesbaden: Vieweg+Teubner Verlag.

Biermann, H. R., & Jahnke, H. N. (2013). How 18th century mathematics was transformed into 19th century school curricula. (This volume).

Blömeke, S. (2004). Empirische Befunde zur Wirksamkeit der Lehrerbildung. In S. Blömeke et al. (Eds.), *Handbuch Lehrerbildung* (p. 59–91). Bad Heilbrunn: Klinkhardt.

Blömeke, S., Bremerich-Vos, A., Haudeck, H., Kaiser, G., Lehmann, R., Nold, G., Schwippert, K., & Willenberg, H. (Eds.). (2011). *Kompetenzen von Lehramtsstudierenden in gering strukturierten Domänen. Erste Ergebnisse aus TEDS-LT*. Münster: Waxmann.

Blömeke, S., & Delaney, S. (2012). Assessment of teacher knowledge across countries: A review of the state of research. *ZDM—The International Journal on Mathematics Education, 44*(3), 223–247.

Blömeke, S., Kaiser, G., & Lehmann, R. (Eds.). (2008). *Professionelle Kompetenz angehender Lehrerinnen und Lehrer. Wissen, Überzeugungen und Lerngelegenheiten deutscher Mathematik-Studierender und -referendare – Erste Ergebnisse zur Wirksamkeit der Lehrerausbildung*. Münster: Waxmann.

Blömeke, S., Kaiser, G., & Lehmann, R. (Eds.). (2010a). *TEDS-M 2008– Professionelle Kompetenz und Lerngelegenheiten angehender Primarstufenlehrkräfte im internationalen Vergleich*. Münster: Waxmann.

Blömeke, S., Kaiser, G., & Lehmann, R. (Eds.). (2010b). *TEDS-M 2008– Professionelle Kompetenz und Lerngelegenheiten angehender Mathematiklehrkräfte für die Sekundarstufe I im internationalen Vergleich*. Münster: Waxmann.

Bromme, R. (1992). *Der Lehrer als Experte: Zur Psychologie des professionellen Wissens*. Bern: Huber.

Bromme, R. (1997). Kompetenzen, Funktionen und unterrichtliches Handeln des Lehrers. In F. E. Weinert (Ed.), *Enzyklopädie der Psychologie: Psychologie des Unterrichts und der Schule*. (Vol. 3, pp. 177–212). Göttingen: Hogrefe.

Buchholtz, N., & Kaiser, G. (2013, in print). Improving mathematics teacher education in Germany: Empirical results from a longitudinal evaluation of innovative programs. *International Journal of Science and Mathematics Education*.

Deiser, O., & Reiss, K. (2013). *Knowledge transformation between secondary school and university mathematics*. (this volume).

DMV, GDM & MNU. (2008). Standards für die Lehrerbildung im Fach Mathematik. Empfehlungen von DMV, GDM, MNU. http://www.math.uni-sb.de/ag/lambert/LAHLAR/ StandardsLehrerbildungMathematik.pdf. Accessed 06 Nov. 2013.

Eilerts, K. (2009). *Kompetenzorientierung in der Mathematik-Lehrerausbildung. Empirische Untersuchung zu ihrer Implementierung. In: Paderborner Beiträge zur Unterrichtsforschung und Lehrerbildung, 14*. Münster: Lit Verlag.

Grevholm, B. (2013). *Mathematical moments in a human life: Narratives on transformation*. (This volume).

Grüner, G. (1967). Die didaktische Reduktion als Kernstück der Didaktik. *Die deutsche Schule, 59*(7/8), 414–430.

Hartig, J., & Kühnbach, O. (2006). Schätzung von Veränderung mit Plausible Values in mehrdimensionalen Rasch-Modellen. In A. Ittel & H. Merkens (Eds.), *Veränderungsmessung und Längsschnittstudien in der der Erziehungswissenschaft* (pp. 27–44). Wiesbaden: Verlag für Sozialwissenschaften.

Hill, H. C., Loewenberg Ball, D., & Schilling, S. G. (2008). Unpacking Pedagogical Content Knowledge: Conceptualising and Measurement Teachers' Topic-specific Knowledge of Students. *Journal for Research in Mathematics Education, 39*(4), 372–400.

Jank, W., & Meyer, H. (1991). *Didaktische Modelle* (3rd edn.). Frankfurt: Cornelsen.

Johnson, B., & Turner, L. A. (2003). Data collection strategies in Mixed Methods Research. In: A. Tashakkori & C. Teddlie (Eds.), *Handbook of Mixed Methods in Social & Behavioral Research*. Thousand Oaks: Sage.

Kelle, U. (2008). *Die Integration qualitativer und quantitativer Methoden in der empirischen Sozialforschung. Theoretische Grundlagen und methodologische Konzepte*. Wiesbaden: Verlag für Sozialwissenschaften.

Kirsch, A. (1987). *Mathematik wirklich verstehen: eine Einführung in ihre Grundbegriffe und Denkweisen*. Köln: Aulis Verlag Deubner.

Klafki, W. (1958). Didaktische Analyse als Kern der Unterrichtsvorbereitung. *Die deutsche Schule, 50*(10), 450–471.

Klein, F. (1932). *Elementary mathematics from an advanced standpoint. Arithmetic, algebra, analysis*. (Translated from the third German edition by E. R. Hedrick and C. A. Noble). New York: Macmillan.

Klein, F. (1939). *Elementary mathematics from an advanced standpoint. Geometry*. (Translated from the third German edition by E. R. Hedrick and C. A. Noble). New York: Macmillan.

Kunter, M., Baumert, J., Blum, W., Klusmann, U., Krauss, S., & Neubrand, M. (Eds.). (2011). *Professionelle Kompetenz von Lehrkräften. Ergebnisse des Forschungsprogramms COACTIV*. Münster: Waxmann.

Mayring, P. (2000). *Qualitative Inhaltsanalyse. Grundlagen und Techniken*. Weinheim: Deutscher Studien Verlag.

McDonnell, L. M. (1995). Opportunity to learn as a research concept and a policy instrument. *Educational Evaluation and Policy Cal-culus, 17*(3), 305–322.

Pepin, B. (2013). *'It's like a kind of journey'- 'Transformation' at transition from school to university mathematics education*. This volume.

Rost, J. (2004). *Lehrbuch Testtheorie – Testkonstruktion* (2nd Ed.). Bern: Verlag Hans Huber.

Schwarz, B. (2013). *Professionelle Kompetenz von Mathematiklehramtsstudierenden – Eine Analyse der strukturellen Zusammenhänge*. Wiesbaden: Springer Spektrum.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Tashakkori, A., & Teddlie, C. (2003). *Handbook of mixed methods in social & behavioral research*. Thousand Oaks: Sage.

Warm, T. A. (1989). Weighted likelihood estimation of ability in item response models. *Psychometrika, 54,* 427–450.

Weinert, F. E. (1999). *Kozepte der Kompetenz. Gutachten zum OECD-Projekt "Definition and Selection of Competencies: Theoretical and Conceptual Foundations (DeSeCo)". Neuchatel*. Schweiz: Bundesamt für Statistik.

Witzel, A. (1982). *Verfahren der qualitativen Sozialforschung. Überblick und Alternativen*. Frankfurt/New York: Campus Verlag.

Wu, M., & Adams, R. (2007). *Applying the Rasch Model to Psycho-Social Measurement. A practical approach*. Melbourne: Educational Measurement Solutions.