Chapter 15
Discussion II of Part II

Digital Technologies and Transformation in Mathematics Education

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Abstract: This chapter is a commentary on a collection of chapters that focus on the transformational potential of digital technologies for learning mathematics. I suggest that the theoretical perspectives represented within the collection cohere around theories that predominantly derive from sociocultural theory, with a focus on the mediating role of technologies in human activity. All of the chapters acknowledge the role of the teacher, and the importance of designing activities to exploit the semiotic potential of digital technologies for learning mathematics. However I argue that the chapters do not adequately take into account students' out-of-school uses of digital technologies which are likely to impact on their in-school use of 'mathematical' technologies, and also the societal and institutional factors that structure the use of technologies in schools. I also argue for the importance of scaling-up the design based studies represented in the collection and developing a model of professional development that exploits the potential of networked communities of mathematics teachers in order to initiate large-scale transformation in mathematics classrooms.

15.1 Introduction

This chapter is a commentary on a collection of chapters entitled Transformations Related to Representations of Mathematics, within the book Transformation—a Fundamental Idea of Mathematics Education. All of the chapters focus on the transformational potential of digital technologies as representational systems, and demonstrate how dynamic digital technologies both add to the available mathematical representational systems, and augment existing static representational systems. Dynamic representational systems offer the potential for transforming and democratising the teaching and learning of mathematics (Kaput et al. 2008), and the chapters in this book have provoked me to re-examine this potential in order to understand why changes at the level of the classroom have not been as dramatic as many of us had predicted.

My own involvement in mathematics education research started in 1983 with a research project that investigated the potential of Logo programming for learning

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mathematics and in particular algebra (Hoyles and Sutherland 1989; Sutherland 1989), and developed into a more general interest in the potential of computers and technology for learning mathematics (for example Sutherland and Rojano 1993; Sutherland 2007). More recently I have focused on approaches to professional development as I became aware that teachers need support to take the risk of experimenting with using digital technologies in the classroom (Sutherland et al. 2009). I mention this history because it feels as if I have lived through many "waves of optimism" about how "digital technologies" will transform mathematics education, yet despite extensive research in this area (see for example Hoyles and Lagrange 2010) it is widely recognized that teachers are generally not exploiting the potential of digital technologies for the teaching and learning of mathematics (Assude et al. 2010).

Over and over again it is the newest technology which excites teachers in schools, provoking them to think that the latest wave of technology will make a difference to teaching and learning. For example many schools in my local area are buying class sets of ipads, accompanied by a belief that the mere introduction of this technology into the classroom, together with the use of the internet is all that is needed to transform teaching and learning. It is difficult not to go along with this enthusiasm and the confidence that simply by making a technological system available, people will more or less automatically take advantage of the opportunities that it offers. It is a challenge to find ways of convincing school leaders and teachers that it is how the technology is used that is important, and that a seemingly "mathematical" technology can be used for non mathematical purposes. The theoretical ideas that are raised in this collection of chapters address this issue, providing frameworks for understanding the use of digital technologies and the role of the teacher in orchestrating such use for mathematical purposes.

Within this chapter I start by explaining why I believe it is important to consider the policy and institutional constraints on innovation at the level of the classroom. I then discuss the theoretical perspectives represented within this collection of chapters. I go on to argue that young people's out-of-school uses of digital technologies are likely to impact on their classroom learning of mathematics. I claim that whereas technologies can potentially be used to transform mathematics education, teachers and students have to learn to use them in mathematically purposeful ways. Finally I discuss why I believe that professional development is key to transformation in mathematics education.

15.2 Constraints on Innovation in Mathematics Classrooms

The research represented in this collection relates to what could be called bottom-up change at the level of the classroom. For example, the project by Bessot (Chap. 13) which designed and evaluated a computer-based mathematical simulator for vocational construction students to learn about geometry-related aspects of their professional practice, or the longitudinal study carried out by Geiger (Chap. 12) which investigated the dynamics of classroom interaction in which 16–17-year-old mathematics students had unrestricted access to a wide range of digital technologies. Both of these studies are design-based studies, the former influenced by the work of Brousseau (1997), and the later drawing on Sträßer's (2009) tetrahedral model for teaching and learning mathematics.

For most of my research career, I have also been involved in bottom-up research and development projects. However more recently I led a research project (the InterActive Education project) which examined learning at both the level of learner and classroom, as well as taking into account the institutional and societal factors which structure learning (Sutherland et al. 2009). If you take such an holistic perspective you begin to understand the challenges that teachers face when considering using digital technologies in the classroom. For example as we reported in the InterActive Education project, the mandate for ICT in education (in England) has overwhelmingly been interpreted by schools as a license to acquire equipment. Such a focus on acquiring equipment detracts from an emphasis on the professional development that teachers need in order to change established practices of teaching and learning.

When we examine the societal and institutional factors that structure the use of technologies in schools, we can begin to appreciate why mathematics teachers might not be embracing digital technologies for teaching and learning. For example in England many schools have recently invested in Virtual Learning Systems (VLEs) and this widespread adoption of VLEs is getting in the way of bottom-up innovation at the level of the classroom:

> "……far from being a source of enabling 'bottom-up' change, these institutional technologies appear to be entwined in a multiplicity of 'top down' relationships related to the concerns of school management and administration. It could be argued that the use of these systems is shaped more often by concerns of institutional efficiency, modernisation and rationalisation, rather than the individual concerns of learners or teachers. Indeed despite the connotations of the 'Learning Platform' and 'virtual learning environment' it would seem that the primary concern of these technologies is – at best – with a limited bureaucratic 'vision of academic success' based around qualifications and grades (Pring 2010, p. 84). With these issues in mind, we therefore need to approach institutional technologies in terms of enforcing the bureaucratic interests of the institution rather than expanding the educational interests of the individual" (Selwyn 2011, p. 477).

As Selwyn suggests it is important to understand the policy and institutional context in which digital technologies are being introduced into schools and this is likely to vary from country to country and change over time (Assude et al. 2010). Without such an understanding we may attribute lack of change in classrooms to, for example, lack of training of teachers, or to teachers' resistance to change, whereas there may be more complex and interrelated factors that need to be understood if we are going to be able to use digital technologies to innovate at the level of the classroom.

Engaging with the chapters in this book reminds me that there tends to be a divide in the education literature between those who focus on the more sociological aspects of learning in schools and those who focus on the more psychological aspects of learning. With notable exceptions (for example, Chevallard 1992) and the more recent work of Cobb (Cobb and McClain 2011), there is very little mathematics education research that situates teachers' classroom practices within the institutional and policy contexts in which they work. However, as Selwyn (2011) has pointed out digital technologies have not only been introduced into schools for educational purposes, with "many countries perceiving a close relationship between success in global economic markets and the increased use of technology in educational institutions" (p. 60).

Engaging with the political realities of schooling is a long way from the focus of the chapters in this collection, which are all concerned with classroom-based research that expands the potential of students to learn mathematics. I agree with the views of the authors, namely that digital technologies can potentially transform classroom mathematical practices. I also agree with the authors that transformation of learning mathematics needs to be informed by theory and evidence-informed research, and in the next section I discuss the theoretical perspectives represented in this collection.

15.3 Theory as a Way of Seeing

"Humans are irrepressible theorisers. We cannot help but note similarities among diverse experiences, to see relationships among events, and to develop theories that explain these relationships (and that predict others)" (Davis et al. 2000, p. 52).

The introductory chapter to this book starts by raising the issue of the diverse theoretical approaches that have evolved within the mathematics education community (Introduction). In this respect new researchers and practicing teachers could easily become confused by the plethora of theories related to the use of digital technologies for learning mathematics. However, it seems to me that many of the perspectives represented in this collection cohere around theories that predominantly derive from sociocultural theory and the work of Vygotsky (1978).

Sociocultural theory is predicated on the view that humans as learning, knowing, reasoning, feeling subjects are situated in social and cultural practices. Participation in these practices provides the fundamental mechanism for learning and knowing. Furthermore, human activity and practices must be understood as products of history, with artefacts and tools being fundamental parts of this history. A key concept within socio-cultural theory is the idea that all human activity is mediated by tools. These tools, invented by people living in particular cultures, are potentially transformative, that is they enable people to do things which they could not easily do without such tools. Within this framework the idea of person-acting-with-mediational-means (Wertsch 1991) expands the view of what a person can do and also suggests that a person will be constrained by their situated and mediated actions as they take place in various kinds of settings. In this respect as discussed in the previous section, learning events in school have to be understood as embedded in institutions, linked to the historical and political dynamics of the classroom.

The theoretical focus on tools is relevant when considering the role of the digital in mediating and potentially transforming mathematical activity. Both Mariotti (Chap. 9) and Geiger (Chap. 12) draw on the theory of instrumental genesis (Rabardel 2001) which derives in part from the work of Vygotsky. This conceptual approach allows us to understand more about the ways in which people interact differently with the same tool, and over time learn how to use it in different ways. This framework distinguishes between two aspects of a tool—the artefact and the instrument, separating what relates to the intention of the designer and what occurs in practice. From this perspective the instrument is made up of both artefact—type components and schematic components, associated with both the object/artefact and the subject/person. The instrument is constructed by the individual and relates to the context of use (utilisation process), which relates to the mathematical task to be solved as well as other contextual, institutional and policy-related factors. The particular instrument constructed by a student with respect to a particular artefact or technology (for example dynamic geometry software) may not be consistent with the intention of the teacher. To make the situation even more complex the instrument constructed by the student may not be consistent with the intentions of the designer. The theory of instrumental genesis has been used to explain the discrepancy between the students' behavior and the teacher's intentions with respect to the use of technology.

Acknowledging the role of the teacher in guiding instrumental genesis, Drijvers et al. (2010) have developed the idea of instrumental orchestration. This is defined as the intentional and systematic organisation and use of the various tools available to the teacher in a given mathematical situation, in order to guide students' instrumental genesis. This includes decisions about the way a mathematical task is introduced to students and worked on in the classroom, decisions about which tools to use (both digital and non digital), and on the schemes and techniques to be developed by the students. Mariotti also emphasises that the transformation process is not spontaneous and has to be "fostered by the teacher, through organizing specific social activities, designed to exploit the semiotic potential of the artefact" (Mariotti, Chap. 9). Bartollini-Bussi and Mariotti (2008) use the phrase "didactical cycle" to refer to the organisation of classroom activity incorporating the use of technologies. From a different theoretical antecedent Laborde and Laborde (Chap. 11) also emphasise the importance of designing mathematical teaching and learning situations, discussing the idea of the adidactical milieu which derives from the theory of Brousseau (1997).

Laborde and Laborde also discuss the perspective of the designer in terms of designing dynamic geometry environments and in particular Cabri 3D. They suggest that "the dragging facility in dynamic geometry environments illustrates very well the transformation technology can bring in the kind of representations offered for mathematical activity and consequently for the meaning of mathematical objects. A diagram in DGE is no longer a static diagram representing an instance of a geometrical object, but a class of drawings: representing invariant relationships among variable elements" (Chap. 11). They also emphasise that although the designer (Jean-Marie Laborde is the designer of Cabri) has clear intentions, the ways in which the technology is used may not relate to such intentions.

Whereas, appreciating that in the section above I have very much oversimplified the perspectives of the authors, I suggest that there are more similarities than differences in the theoretical perspectives represented in this collection of chapters. Work has already begun to connect these theoretical frames (Artigue and Cerulli 2008) and in the future more work could be carried out to develop an accessible framework that could inform mathematics teachers about the complex issues involved in using digital technologies to transform mathematics education.

15.4 Mathematics and Out-Of-School Use of Digital Technologies

Sociocultural theory recognises that a student's history of learning, what they learn out-of-school and what they have learned in previous schooling impacts on their ongoing learning experiences in school. From this perspective all students actively construct and make sense of a particular mathematical activity in terms of their previous learning, developing their own personal theories, or theories in action (Vergnaud 1994). In order to illustrate this I present an example from an interview with a 15-year-old student who was struggling with school mathematics. When interviewed about the meaning she gave to the use of letters in mathematics she told the interviewer that the value of a letter related to its position in the alphabet. When probed further she provided the following explanation:

Int: Does L have to be a larger number than A?
Eloise: Yes because A starts off as 1 or something.
Int: What made you think that [L has to be a larger number than A?]
Eloise: Because when we were little we used to do a code like that...in junior school...A would equal 1, B equals 2, C equals 3.....there were possibilities of A being 5 and B being 10 and that lot.....but it would come up too high a number to do it.........it was always in some order...

Eloise had developed her own theory about the meaning of letters, which derived from her work in primary school, and made sense to her in the context of the problems she was solving at the time. This personal knowledge had not been intentionally taught by the teacher and was no longer appropriate (or correct) in the context of secondary school mathematics. Eloise's theory about the role of letters in mathematics, influenced how she made sense of letters when she encountered them in secondary school algebra. What this example illustrates is that each student brings to the classroom his/her own history of learning and when faced with a new situation makes sense of this from his/her own particular experience and way of knowing.

Another example derives from an interview with Anthony when he was a 10 year old primary school student. Anthony had not met algebraic symbols in school mathematics, yet when asked the question:

Which is larger, 2n or n + 2? He responded:

> "You can't say that because it wouldn't always be right….if n was 6 that would be 12…. and that would be 8 so that would be right….but if n was one then 2n would be 2 and n = 2 would be 3."

This response was surprising given that research has shown that this question is only answered correctly by 6% of 14 year olds (Küchemann 1981). When asked why he was able to answer the interview questions correctly he said:

> "It might be partly because of BASIC, where I've learned to use things like variables and things….like p is a number and you can use any letter for a number…."

This is an example in which a primary school student learned from out-of-school computer programming ideas that are related to the "scientific concepts" of school mathematics. The idea of "scientific concepts" draws attention to the importance of a systematic organised body of knowledge, knowledge that can be separated from the community that produced it. Vygotsky discussed the difference between informal and scientific concepts, and claimed that there is a dialectic relationship between the development of informal and scientific concepts:

> "the dividing line between these two types of concepts turns out to be highly fluid, passing from one side to the other in an infinite number of times in the actual course of development. Right from the start it should be mentioned that the development of spontaneous and academic concepts turn out as processes which are tightly bound up with one another and which constantly influence one another" (Steiner and Mahn 1996, p. 365).

In my research I continue to find examples of young people's out-of-school use of digital technologies impacting on their learning of mathematics in schools. For example, the following is an interview with two 8-year olds from the InterActive Education Project.

Int: Do either of you use Excel at home (Alan shakes head)?
Ray: Sometimes. My Dad uses it for his paper work.
Int: And when you use it what do you use it for?
Ray: Umm, he uses it, cos when he's got paper calculations and some are hard like for him, he puts it in Excel and then he puts, he circles it and then presses the equal button and it tells him what the sums are.
Int: What do you use it for?
Ray: Maths homework.
Alan: Cheat.

From sitting alongside his father at home Ray had observed him using a spreadsheet for his work. Ray's explanation shows that he understands how a spreadsheet can carry out "hard" calculations which are related to mathematics. Interestingly until this interview was carried out by a researcher the class teacher was not aware of this "fund of knowledge" (Moll and Greenberg 1990), illustrating the way in which home learning out-of-school is often not recognised by teachers at school.

Nowadays, the vast majority of young people engage with digital technologies in their lives outside school, and these experiences can impact negatively or positively on their mathematical learning with digital technologies in school. For example young people's experience of playing games out-of-school can impact on the ways in which they make sense of digital technologies in school and this can detract from the intended or "scientific" learning (Sutherland et al. 2009).

One of the research results from the InterActive Education project was that teachers often underestimate the impact of students' past experiences on their learning in the classroom, and in particular their out-of-school experiences of using digital technologies. The theory of instrumental genesis discussed earlier explains why such out-of-school learning is likely to impact on the student's construction of a particular digital instrument, that is how they make sense of the potential of the digital technology for learning mathematics. It is perhaps surprising therefore, that none of the authors in this collection of chapters appear to take such factors into account in their research. I suggest that mathematics education researchers tend to underestimate the impact of students' out-of-school uses of ICT on their in-school learning of mathematics with digital technologies. As out-of-school uses of mobile devices become ubiquitous it will be even more important to consider the interrelationships between young people's construction of the digital from their learning out-of-school and the mathematical concepts which teachers intend them to learn in school. Raising such issues presents a challenge to the use of digital technologies for transforming the teaching and learning of mathematics. In the next section of the paper, I explain why I believe that professional development is the way forward.

15.5 A Way Forward: Transformation Through Professional Development

As I argued earlier, a sociocultural approach to learning enables us to see the potential transformative nature of tools and artefacts that have been designed to enable us to do things that would be difficult to do without them. For example, the long multiplication algorithm enables us to perform calculations that would be difficult to perform mentally, dynamic geometry software enables us to visualise the invariant and variable properties of geometrical figures that are difficult to see in paper-and-pencil constructions, spreadsheets enable us to construct financial models that would be very difficult or impossible to develop on paper. However a focus on the transformative potential of digital technologies can fall into the trap of deterministic thinking, that is a belief that the mere use of such tools is sufficient for transformation to occur, and as I have discussed already the authors of this collection of chapters provide ample evidence for why this is not the case. Such deterministic thinking gets in the way of the productive use of ICT for teaching and learning, because from such a perspective there is no acknowledgement of the complexities and challenges involved in embedding digital technologies into mathematical classroom practices.

In our everyday lives we learn about the transformative potential of a particular digital technology through experimentation and discussion with colleagues and friends. However, as academics we are also aware that within the institutional setting of the University we may be resistant to using a technology that is being imposed on us to transform our everyday work practices. For example, I am resisting using the digital calendar that I am supposed to be using, and continue to use a paper diary which I argue is more transformative for me personally than a digital diary.

In order to start to use a digital diary to transform my time-management practices I would have to learn to use it in a transformative way. Similarly, teachers have to learn how to use "mathematical" digital technologies in a transformative way. Here the challenge is much greater than the challenge for me personally of learning how to use a digital diary. Teachers firstly have to learn how to use the chosen digital technology to transform both their own mathematical practices and their teaching of mathematics. Teachers then have to "teach" students to learn how to use digital technologies in transformative and mathematically appropriate ways.

Most of the authors of this collection of chapters carry out what could be called design-based research (Brown 1992). In my opinion the challenge is to scale-up such design-based (or didactical engineering) approaches through processes of professional development. The InterActive project showed that a successful model for professional development is to create networked communities in which teachers and researchers work in partnership to design and evaluate learning initiatives which use digital technologies as a tool for transforming learning. We argue that such professional development requires people to break out of set roles and relationships in which researchers are traditionally seen as knowledge generators and teachers as knowledge translators or users. For meaningful researcher-practitioner communities to emerge, trading zones are needed where co-learning and the co-construction of knowledge take place (Triggs and John 2004). Within such communities design can be informed by: theory and research-informed evidence; the craft knowledge of teachers; curriculum knowledge; policy and management constraints and possibilities and young people's use of digital technologies in their everyday lives. The focus is on iterative design and evaluation and a dynamic record of classroom activity and learning can be created from video and audio recording, screen-capture, observation, student interviews, and students' work.

Such design-based professional development should also pay attention to areas of tension that emerge through the process of classroom-based innovation (Sutherland et al. 2012). For example, as discussed earlier, there may be an area of tension around the ways in which senior management in a school intend to use technology to improve the qualifications and grades of students and the ways in which mathematics teachers intend to use digital technology to transform students' understanding of mathematical concepts.

In summary, we know from research on the use of digital technologies in schools that there is a dominant belief that simply by making a technological system available, teachers and students will more or less automatically take advantage of the opportunities it offers. We also know that despite many years of investing in technology in schools mathematics teachers are not taking advantages of the opportunities such technology offers for transforming the teaching and learning of mathematics. Whereas, theories of teaching and learning mathematics are a necessary part of opening up new ways for teachers to see what is possible, I suggest that the way forward is to focus attention on developing a model of networked communities of mathematics teachers that can be scaled-up in order to initiate large-scale transformation in mathematics classrooms.

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