Chapter 14
Discussion I of Part II

Representing and Meaning-Making: The Transformation of Transformation

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Abstract: This commentary simultaneously offers a broad and a narrow perspective. It is narrow in that it does not attempt to synthesize or digest the chapters in this section. It can be called broad as it attempts to sketch some salient features of the future development of and learning geometry, of the transformation of transformation. Four such strands of reasoning are discussed: the paramount significance of meaning-making, the role of artefacts as socially and culturally embedded, embodiment and enactment, and, finally, emotions, meaning-making, and triangulation.

All chapters in this section circle around the question how (technical) artefacts can be thought of as mediating mathematical meaning, especially how mathematical meaning emerges in the interaction of a subject with a (technical) artefact embedded in an educational situation, said shortly. In all chapters, geometry is the kind of representation that is investigated. I will not go into the question of the special kind of representations that geometry is incorporating. I will restrict my commentary to a discussion of multiple forms of psychosocial and semiotic triangulation that are salient in the teaching and learning of geometry, in meaning-making, emotion, and development. By triangulation I mean those meaning-making relationships that include three instances: a subject—an object—another subject, or a subject—an artefact—an object, or simply three subjects.

In my commentary, I will discuss the consequences of what has been called an "embodied" perspective on human activity and thinking following the seminal volume of Varela et al. (1991). They put this fundamentally different perspective on cognition as embodied action as follows:

> By using the term embodied we mean to highlight two points: first, that cognition depends upon the kinds of experience that come from having a body with various sensorimotor capacities, and second, that these individual sensorimotor capacities are themselves embedded in a more encompassing biological, psychological, and cultural context. By using the term action we mean to emphasize once again that sensory and motor processes, perception and action, are fundamentally inseparable in lived cognition. Indeed, the two are not merely contingently linked in individuals; they have also evolved together (Varela et al. 1991, p. 172–173).

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Reading this quotation again today, one is surprised to see that emotion is missing from this perspective—just as it is missing from the whole volume by Varela et al. It seemed easier then to think of cognition as having a bodily basis, leaving emotion unmentioned. It comes as no surprise that emotion is practically missing in a recent volume on the state of the art of mathematics education (Sriraman and English 2010), although today, emotion appears as the psychological function paradigmatic for a perspective of embodiment.

In a sense and finally, my commentary is also meant to be a discussion and critical reflection of the main concept of this volume, transformation.

14.1 Making Meaning

Meaning is the key. It has already been the key to the *New Math* movement that had been searching for an answer to fundamental changes in the society, the culture, and the life of the individual in the twentieth century. It has been central in the description of the ever growing faster and wider transformations of life. These transformations of the twentieth century had forced mathematics educators to respond, if they were followers of the *New Math* or not, and their common denominator was the problem of meaning. As Thom has expressed it:

> The real problem which confronts mathematics teaching is not that of rigour, but the problem of the development of 'meaning', of the 'existence' of mathematical objects (Thom 1973, p. 202).

Today, the problem in the center of mathematics education is still meaning—whatever we wish to call the societal, cultural, and psychosocial transformations under way in the twenty-first century. But now it is meaning with a discursive or dialogical and a distributed meaning (see Otte 2011).

In his famous speech on the equally famous 1972 *International Congress on Mathematical Education* in Exeter, Thom could still get away with a conception of meaning in a strictly object-related fashion. Forty years later, so it seems, the issue of meaning in mathematics education is not only related to the mathematical objects but also to the personal sense they make to students.

In this paper, I tend to use "meaning" in a rather unspecified way. Basically, most of the times when it is spoken of meaning, one could replace it with "sense". How are meaning and personal sense mutually related? A. N. Leont'ev (1981, 1978) has put it concisely in this way: "… sense is expressed in meanings (like motive in aim), but not meaning in senses" (Leont'ev 1981, p. 229). Meaning has a quasi-objective meaning, and meaning has a personal meaning, sense.

> Meaning is the reflection of reality irrespective of man's individual, personal relation to it. Man finds an already prepared, historically formed system of meanings and assimilates it just as he masters a tool, the material prototype of meaning (Leont'ev 1981, p. 227–228)

As it were, meaning has a general form mostly culturally organized; and meaning has a personal form: personal sense.

In what follows, I will begin to discuss the use of artefacts, not their creation and invention, that is, I will talk about technical artefacts "in use" and not "put down", as Stewart (2010, p. 19) has described the two modes of using tools and technical artefacts.

14.2 Artefacts as Socially and Culturally Embedded

The interaction with the artefact is not necessarily restricted to the interaction of an artefact and a person. The meaning of an artefact can only be accessed through the analysis of its socio-cultural (and historical) embeddedness. This is equally true for simple as for highly sophisticated technical artefacts.

Now, the fate of artefacts is that they "disappear" in the ongoing interaction as mastering the artefact is growing: when a human learns how to chop wood with an axe, the axe as technical artefact is the object of activity, when she or he has learned how to handle an axe with a sufficient degree of mastery, chopping wood, keeping a stove burning etc. is the object of activity and the artefact "disappears." Leont'ev (1978, p. 66) has described this very nicely as actions "sinking down" to operations as a very general case.

Now, this all is not true for artefacts that produce something automatically. Automata, in general, do not allow for any intervening actions of a human user: they produce a result that is either useful for a user or not. Correspondingly, the list is long with successful automata in mechanical systems governing and regulating output processes. However, the list of failed attempts to insert automata into human-machine-interactions or in user-supported processes is equally long (see, e.g., Latour 1993; Engeström and Escalante 1995).

It is interesting that the mastery of technical artefacts in any culture is often in itself an object—and here the artefact is not at all "disappearing", in the contrary. Gaining and showing mastery over the artefact is not only the core of the matter in wood-chopping tournaments, but in all kinds of sports like sailing, pole-jumping, motor-racing, darts tournaments, tennis, and so on and so forth. In many cultures, the mastery of artefacts has been and still is connected to weapons and other devices for survival. Often these tournaments have a religious embedding and ornamentation. But some, actually, also refer to mathematics and calculating like the *Soroban* contests held in Japan and the USA.

Learning to master a (technical) artefact, as it were, is a process deeply embedded into the culture. It is not the interaction with the artefact that determines the ultimate goal of the artefact-mediated activity, it is a goal and a motive that goes beyond this interaction. As Norbert Elias (1994) has shown for the cultural evolution of emotional and self-control in Europe from 800 to 1900, the control of artefacts is closely related to this process: mastering the artefact, like spoon and fork, entails mastering one's feeling and self-control.

In the interaction with the artefact, the cultural embeddedness is not something that is only "surrounding" the human-artefact interaction. It is, rather, the basic mechanism (see, e.g., Cole 1996; Hutchins 2010). In the learning sciences this has

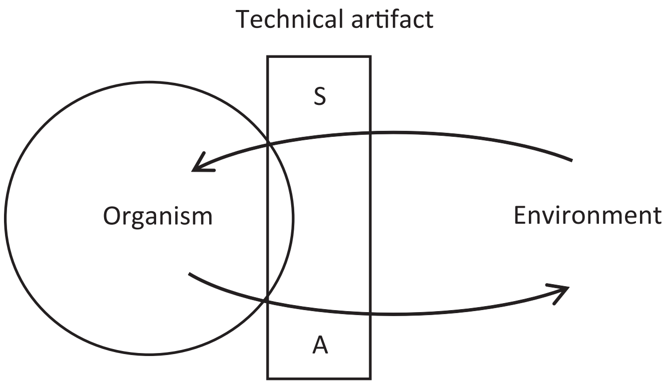

Fig. 14.1 Sensorimotor coupling of organism and environment mediated by the artefact (S = sensory processes; A = Actions; from Stewart 2010)

lead to an approach that focuses not on the direct teaching but on the indirect teaching that has its source in the culturally defined situation (see Lave 1988; Lave and Wenger 1991).

In order to make clearer what I am talking about, I will use the following figure taken from Stewart (2010) as an example (see Fig. 14.1).

Figure 14.1 is meant to give an idea of how an organism is related to an environment through artefacts. The artefact is not just "mediating" while remaining in a rather neutral position and status, the artefact is enacted into the organism as it is, in fact, constituting the world. It is of crucial importance that the processes of action and the sensory processes are of qualitative different nature. Neither can actions be explained through sensory processes nor can sensory processes be explained through actions. They remain in a sense incompatible. Only when looking at the complete circle as actions and the results of actions become sensory inputs do we realize that they belong together, much in the sense that Hutchins (2010, p. 446) has put it: "… perception is something we do, not something that happens to us."

Making a difference between sensory and motor processes is, of course, not a new idea. It has been a topic in the physiology of excitatory processes and in ethology as the difference between afference and efference including the principle of reafference (see, e.g., von Holst and Mittelstaedt 1950). However, already von Uexküll (2010) has elaborated in his theoretical model of environmental biology that the environment is not out there, outside the organisms, but that it is as well inside, as it is symbolically constructed. Insofar, the clear distinction of efference from afference is not tenable because it basically leads again to a great divide between organism and environment.

Uexküll's theoretical model made it clear that the unity, or complementarity, of organism and environment has to be grounded into a semiotic and symbolic approach. His attempt to resolve the uncomfortable duality of mind and body, of organism and environment, following from the Cartesian split, is reflected in current approaches to embodiment and enactment.

Thus, efference and afference will not be conceptually satisfactory to reveal the basic mechanisms of humans using technical artefacts. What would a satisfactory approach have to embrace that includes the role of the body in the use of technical artefacts?[1]

14.3 Embodiment and Enactment

As I said at the beginning, the role of the body appears of such great importance because in a unique way cognitive and emotional functions of human activity are no longer divided but can be seen as two fundamental features. In addition, this lens makes it absolutely essential to take record of what is happening at each and every moment of an ongoing activity, to understand it as a unity of lived experience and to try to reconstruct it accordingly.

As put in the initial quote by Varela et al., it is also essential to reconstruct the cultural embeddedness of such an ongoing activity. That is, to reconstruct the environment not as a loose collection of surrounding factors, but as a system of cultural meaning. In his paper *The problem of environment*, written in 1934 L. S. Vygotsky has discussed the issue why an identical learning environment does not "produce" identical learning results. The answer seems to suggest itself: different students experience the same situation differently. Vygotsky concludes that this means for educational research to look through a certain prism at the student and the situation:

> It ought to always be capable of finding the particular prism through which the influence of the environment on the child is refracted, i.e. it ought to be able to find the relationship which exists between the child and its environment, the child's emotional experience (*perezhivanie*), in other words how a child becomes aware of, interprets, and emotionally relates to a certain event (Vygotsky 1934/1994, p. 341).

This shift of focus on the concrete experience of the students entails some consequences in research on mathematics learning as attentional, cognitive-emotional, and social processes have to be considered in detail. Research has to highlight the interfaces where these different psychological functions meet and regulate the ongoing activity. One good example for such an interface is the research on *mathematical beliefs* as components that regulate student activity and action (see, e.g., Leder et al. 2002; Maas and Schlöglmann 2009; Goldin et al. 2011). Other examples

[1] In what follows, I will not discuss the specific details of embodied or enactment approaches. First, because these paradigms are still very much „under construction" and exhibit a great diversity; second, because space would not allow to go into the details. I will also not discuss the brain-focused approaches that sometimes label themselves as approaches to embodied cognition (for a short overview, see, e.g., Di Paolo and De Jaegher, 2012). Interesting and fascinating as these approaches may be, they reduce the role of the body to excitatory patterns in the brain that since recently can be traced with brain-scanners. For an overview and discussion of cognitive neuroscience see Campbell (2010). Dehaene, e.g., has presented an interesting tripartite model of the development of number (1992), but his numerous attempts to show how the components of this model are processed by and localized in the brain have not been very conclusive (see, e.g., Dehaene 1997).

of such interfaces could include the concepts of *student engagement* (see, e.g., Fredricks et al. 2004), *self-efficacy* (Bandura 1997, 2001) or *recognition* (see, e.g., Honneth 1995).

Another interface between the cognitive and the emotional, between thinking and communicating are gestures as movement and expression signs. Especially in mathematics education, gesturing has been found to be an interesting area of research (see, e.g., Maschietto and Bartolini Bussi 2009; Radford 2003; Radford et al. 2005; Robutti 2006; Sabena 2007; Roth 2001; Roth and Welzel 2001 as examples from other areas)[2].

14.4 Emotions, Meaning-Making, and Triangulation

In the center of meaning, body, and artefact we find emotions—or, to be more precise, emotions as they are related to cognition, and cognitions as they are related to emotions. Emotions have an important dual relation to body processes and to signs and symbols. In a unique way, emotions offer a possibility to access the two, now felt as basic, constituents of meaning-making: the embodiment of thinking and acting and the symbolic ground of meaning. Emotions control and express meaning and personal sense of one's own activity and actions and the activity and actions of others (Holodynski 2006). This, of course, brings the pivotal importance of emotions for learning right to the point.

We arrive at this pivotal point if we follow the trails of two great schools of thinking on the problem of meaning: the semiotic tradition after Peirce and the developmental approach to the ontogenesis of intersubjectivity, reciprocity, empathy, and cooperation—with L.S. Vygotsky somehow mediating the semiotic and the developmental approach. The development of meaning making under the perspective of reciprocity has to do with the complicated interplay between the social and the individual which has been the dominant theme in Vygotsky's developmental psychology[3].

Control and self-control have been an important motive for Peirce in formulating the pragmatic maxim. Equally, Vygotsky has taken great efforts in giving a vivid account of the transition from other-regulated to self-regulated control as it can be demonstrated in the development of volition and sign operations (see, e.g., Vygotsky 1997, 1999). To gain self-control is one of the great accomplishments

[2] This issue would deserve an extended discussion because the relation of gestures to language is still very much in need of clarifications. Gestures are often seen as developmental precursors to language (see, e.g., Tomasello 2008). At the same time, they seem so important because gestures potentially express what cannot be expressed through language. For an interesting account of gestural language see Sacks (1989). Goldin-Meadow (2003) has discussed intensely the gesture-speech mismatches, and Sinclair (2010) has elaborated the idea of overt and covert forms of knowing in mathematics education, gestures indicating covert knowing.

[3] For reasons of space, I will not present of Vygotsky's ideas in more detail, also because this has been done extensively elsewhere over the past years (see, e.g., van der Veer and Valsiner 1991, 1994; Daniels et al. 2007).

in human ontogeny. It is especially remarkable on the background of the fact that no infant and child can develop normally if attachment and secure base are not provided by the mother and other caretakers. It is as if we have again two paradoxically opposing poles where development has to find its way—the self being neither completely attached nor completely self-directed. It is quite clear that real self-determination and autonomy are not in a steady state but in a process of becoming, in a developmental process.

When we look at what research has found out about this development of self-control we find again that already at a very early age the infant is not a passive vessel controlled by the mother. Rather, the infant starts meaning making from birth on—and even earlier. What could be mistaken for a genetic predisposition turns out to be, at closer scrutiny, a result of interaction and preverbal communication. Beginning in the late 1960s, gradually intensified research has accumulated evidence of the fundamental nature of reciprocal interaction in early development (see, e.g., Bullowa 1979). The work of Hanus Papousek (Papousek and Papousek 1974, 1977, 1981), Andrew Meltzoff (Meltzoff 2002, 2007; Meltzoff and Moore 1977; Meltzoff et al. 2009), Colwyn Trevarthen (Trevarthen 1979, 1980, 1994; Trevarthen and Hubley 1978), Daniel Stern (1971, 1985), to name only a few, has paved the way to a new understanding of the "competent infant."[4]

This whole work in developmental psychology has been tremendously extended and amplified through developmental research in non-human primates. Here the work of Tomasello and his co-workers on the evolutionary transition field of the great apes and human infants (see, e.g., Tomasello 1999, 2005) has been immensely stimulating for theoretical advances in our understanding of the genesis of symbol-formation and meaning-making as the result of social-interactive processes of sharing attention, of pointing and gesturing, as semiotic exchange in a very general sense. And finally, as the result of some emotional grounding, of belonging, of sharing, of empathy—all lead into an enriched understanding of the sociogenesis of the self.

It is also remarkable that new research into the etiology and the development of autism has fundamentally added to our understanding of what it means to be human (see, e.g., Hobson 1993, 2002; Dornes 2005). It may seem odd to mention this here—but actually the mathematical experience has sometimes been described by outsiders as tending to be rather "autistic" and research indicates at least a certain tendency (Baron-Cohen et al. 1998, 2007). However, this tendency is noteworthy not from the epidemiological perspective. Rather, the study of autism reveals that the sometimes amazing capacities of autistic persons seem so isolated because they are not embedded into the natural art of relating to other persons, into empathy and cooperation, into the emotional experience. Wing has brought this perspective to the point: "The key to autism is the key to the essence of humanity" (Wing 1996, p. 225). The key feature of this essence is triangulation in the sociogenesis of the self.

Triangulation is in a particular way appropriate to capture the specific quality of human thinking and acting, of the human mind and human activity, be they used in

[4] A term coined already in a 1973 volume by Stone and others (Stone et al. 1973; see also Dornes 1993).

Table 14.1 Firstness, Secondness, and Thirdness according to Peirce (after Trevarthen 1994)

Firstness			
Sign as such	Quality	Icon	Emotion *in* subject
Secondness			
Sign and relation to Object	Actuality	Index	Object *of* subject in intended action
Thirdness			
Sign and relation to Interpretant	Potentiality	Symbol	Cooperation, self, and value *between* subjects

semiotics, in education, in psychoanalysis, in developmental psychology, or in the analysis of educational situations.[5]

One enduring issue in teaching and learning and the underlying assumptions on how learning can be organized is the credo that it has to proceed from the simple to the complex and complicated, from the concrete to the abstract, from trying out to planned and reflected learning, from unreflected drill to understanding, from emotional to rational regulation of action. Even though some of these principles certainly have their justification, it must be questioned whether they support meaning-making in mathematics education. It is not solely the question whether symbols do describe or construct reality as Steinbring (2005) has put it. It is the problem that human activity is situated within a universe of meaning that is discursive and distributed at the same time, it is the old problem of how meaning as generated between subjects and with the help of artefacts is becoming meaning within the self, making personal sense.

In Table 14.1, Trevarthen who has done seminal research in the genesis of intersubjectivity and interiorisation (see, e.g., Trevarthen and Hubley 1978), has tried to bring together a semiotic perspective after Peirce and a developmental perspective in order to show how *Firstness, Secondness, and Thirdness* could be understood in terms of concepts from developmental psychology. While this is in itself a good summary and presentation of the different approaches seen together, this table can also help to make the above idea much clearer about the progression of development and the progression of learning. It turns out a developmental perspective changes the place and function of the signs as they are presented in the classical semiotic model. The progression from Firstness to Secondness to Thirdness is prone to be interpreted as a developmental sequence from the assumed simple and unmediated to the complex and mediated. However, research in developmental psychology has amply demonstrated that it is, in fact, only the triangulation between these three forms of signs and meanings can account for the development of meaning. Thirdness, as the assumed highest form of meaning, being the result of interaction between subjects is actually the precondition for a sound development of emotion—as research has shown (see, e.g., Fonagy et al. 2002). Conversely, the quality of emotion as a potentiality seems like a late accomplishment of development that needs the relation to the object as well as the relation to other subjects. In addition, developmental research has demonstrated that the relation to the objects is fundamentally mediated through other persons, primarily through caregivers during infancy.

[5] Already Hegel expressed this specific feature of triangulation in his "Quadratum est lex naturae, triangulum mentis" (Hegel 1801, p. 533).

14.5 Transformation and Communication

In their path-breaking book on the linguistics and philosophy of meaning, Ogden and Richards (1923) chose the title “The meaning of meaning.” This nicely expressed that meaning is always reflexive and that the development of meaning is forming some sort of a never-ending spiral: one cannot get behind meaning and one cannot pin down some beginning, some starting point. Varela et al. (1991) have extended our understanding of this situation with the claim that in order to understand meaning-making one has to go beyond cognitive processes and include lived experience. So there is meaning that we make and meaning that is already there.

In a similar vein, there is also transformation that we make and transformation that appears to us as if it has always been there. It seems that we have largely lost the sense of authorship of the transformation going on. Looking at transformation from a global perspective, the current impression certainly reveals a picture that finds us largely alienated from the optimistic perspective on transformation: Gideon Rachman (2011) has presented a succession from the Age of *Transformation* (1978–1991), the Age of *Optimism* (1991–2008) to today’s Age of *Anxiety* into what he calls today’s Zero-Sum world. We do not have to take this analysis too seriously.[6] It is quoted only to show that transformation may not be a convenient motto for today’s mathematics education, and it indicates that we may have to lower our sights

In search of a new motto, a nice story told by George Dyson (2010) might be amusing and whetting the appetite to search for alternatives:

> In the North Pacific Ocean, there were two approaches to boatbuilding. The Aleuts (and their kayak-building relatives) lived on barren, treeless islands and built their vessels by piecing together skeletal frameworks from fragments of beach-combed wood. The Tlingit (and their dugout canoe-building relatives) built their vessels by selecting entire trees out of the rainforest and removing wood until there was nothing left but a canoe.
> The Aleut and the Tlingit achieved similar results—maximum boat/mini-mum material—by opposite means. The flood of information unleashed by the Internet has produced a similar cultural split. We used to be kayak builders, collecting all available fragments of information to assemble the framework that kept us afloat. Now, we have to learn to become dugout-canoe builders, discarding unnecessary information to reveal the shape of knowledge hidden within.
> I was a hardened kayak builder, trained to collect every available stick. I resent having to learn the new skills. But those who don’t will be left paddling logs, not canoes.

The problem for mathematics education seems to be that the *piecemeal* and the *dugout* style do coexist—and they coexist in different ways for students and for teachers at different moments of the teaching-learning process. Navigating the territory of meaning that is there and making meaning while we do this, certainly needs whatever works.

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[6] Rachman is a leading figure in the *Financial Times*. Although his intention is to present an account of today’s global situation, it must be doubted if he really can leave his fixation on the *investment* and money-making perspective behind.

References

Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York: Freeman.

Bandura, A. (2001). Social cognitive theory: An agentic perspective. *Annual Review of Psychology, 52,* 1–26.

Baron-Cohen, S., Bolton, P., Wheelwright, S., Scahill, V., Short, L., Mead, G., & Smith, A. (1998). Autism occurs more often in families of physicists, engineers, and mathematicians. *Autism, 2,* 296–301.

Baron-Cohen, S., Wheelwright, S., Burtenshaw, A., & Hobson, E. (2007). Mathematical talent is linked to autism. *Human Nature, 18,* 125–131.

Bullowa, M. (Ed.) (1979). *Before speech: The beginnings of human communication*. Cambridge: Cambridge University Press.

Campbell, S. R. (2010). Embodied minds and dancing brains: New opportunities for research in mathematics education. In B. Sriraman & L. English (Eds.), *Theories of mathematics education: Seeking new frontiers* (pp. 309–331). Heidelberg: Springer

Cole, M. (1996). *Cultural psychology. A once and future discipline*. Cambridge, MA: Harvard University Press.

Daniels, H., Cole, M., & Wertsch, J. V. (2007). *The Cambridge companion to Vygotsky*. New York: Cambridge University Press.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44,* 1–40.

Dehaene, S. (1997). *The number sense. How the mind creates mathematics*. New York: Oxford University Press.

Di Paolo, E., & De Jaegher, H. (2012). The interactive brain hypothesis. *Frontiers in Human Neuroscience*, 6, Article 163 (doi: 10.3389/fnhum.2012.00163)

Dornes, M. (1993). *Der kompetente Säugling*. [The competent infant]. Frankfurt/Main: Fischer.

Dornes, M. (2005). Die emotionalen Ursprünge des Denkens [The emotional origins of thinking]. *West-End. Neue Zeitschrift für Sozialforschung*, 2(1), 3–48

Dyson, G. (2010). Kayaks vs. canoes. http://www.edge.org/q2010/q10_2.html.

Elias, N. (1994). *The civilizing process*. Oxford: Blackwell (original 1934)

Engeström, Y., & Escalante, V. (1995). Mundane tool or object of affection? The rise and fall of the postal buddy. In B. Nardi (Ed.), *Context and consciousness:activity theory and human-computer interaction* (pp. 325–373). Cambridge: The MIT Press.

Fonagy, P,. Gergely, G., Jurist, E., & Target, M. (2002). *Affect regulation, mentalization, and the development of the self*. New York: Other Press.

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research, 74*(1), 59–109.

Goldin, G. A., Epstein, Y. M., Schorr, R. Y., & Warner, L. B. (2011). Beliefs and engagement structures: Behind the affective dimension of mathematical learning. *ZDM—The International Journal on Mathematics Education, 43,* 547–560.

Goldin-Meadow, S. (2003). *Hearing gesture—How our hands help us think*. Cambridge, MA: Harvard University Press.

Hegel, G. W. F. (1801). „Habilitationsthesen". In: *Georg Wilhelm Friedrich Hegel, Werke in Zwanzig Bänden. Band 2. Jenaer Schriften 1801-1807* (S. 533). Frankfurt am Main: Suhrkamp.

Hobson, R. P. (1993). *Autism and the development of mind*. Hillsdale: Lawrence Erlbaum.

Hobson, R. P. (2002). *The cradle of thought. Exploring the origins of thinking*. London: Macmillan.

Holodynski, M. (2006). *Die Entwicklung der Emotionen*. Berlin: Springer.

Honneth, A. (1995). *The struggle for recognition. The moral grammar of social conflicts*. Cambridge: Polity Press.

Hutchins, E. (2010). Enaction, imagination, and insight. In J. Stewart, O. Gapenne, & E. A. Di Paolo (Eds.), *Enaction. Toward a new paradigm for cognitive science* (pp. 425-450). Cambridge, MA: MIT Press.

Latour, B. (1993). *Aramis, or the love of technology*. Cambridge, MA: Harvard University Press.

Lave, J. (1988). *Cognition in practice. Mind mathematics and culture in everyday life*. Cambridge: Cambridge University Press.
Lave, J., & Wenger, E. (1991). *Situated learning—Legitimate peripheral participation*. Cambridge: Cambridge University Press.
Leder, G. C., Pehkonen, E., & Törner, G. (Eds.) (2002). *Beliefs: A hidden variable in mathematics education?* Dordrecht: Kluwer.
Leont'ev, A. N. (1981). *Problems of the development of the mind*. Moscow: Progress Publishers.
Leont'ev, A. N. (1978). *Activity, consciousness, and personality.* Englewood Cliffs, N.J.: Prentice-Hall.
Maschietto, M., & Bartolini Bussi, M. G. (2009). Working with artefacts: gestures, drawings and speech in the construction of the mathematical meaning of the visual pyramid. *Educational Studies in Mathematics, 70*, 143–157.
Maass, J., & Schlöglmann, W. (Eds.) (2009). *Beliefs and attitudes in mathematics education: New research results*. Rotterdam: Sense.
Meltzoff, A. N. (2002). Elements of a developmental theory of imitation. In A. N. Meltzoff &W. Prinz (Eds.), *The imitative mind* (pp. 19-41). Cambridge: Cambridge University Press.
Meltzoff, A. N. (2007). 'Like me': A foundation for social cognition. *Developmental Science, 10*(1), 126–134.
Meltzoff, A. N., Kuhl, P. K., Movellan, J., & Sejnowski, T. J. (2009). Foundations for a new science of learning. *Science, 325,* 284–288.
Meltzoff, A. N., & Moore, M. K. (1977). Imitation of facial and manual gestures by human neonates. *Science, 198,* 75–78.
Ogden, C. K., & Richards, I. A. (1923). *The meaning of meaning: A study of the influence of language upon thought and of the science of symbolism*. London: Routledge & Kegan Paul.
Otte, M. F. (2011). Evolution, learning, and semiotics from a Peircean point of view. *Educational Studies in Mathematics, 77,* 313–329.
Papousek, H., & Papousek, M. (1974). Mirror image and self-recognition in young infants. *Developmental Psychology, 7,* 149–157.
Papousek, H., & Papousek, M. (1977). Cognitive aspects of preverbal social interaction between human infants and adults. In R. Porter & M. O'Connor (Eds.), *Ciba Foundation Symposium 33: Parent-infant interaction* (pp. 241–269). New York: Wiley
Papousek, H., & Papousek, M. (1981). How human is the human newborn, and what else is to be done. In K. Bloom (Ed.), *Prospective issues in infancy research* (pp. 137–155). Hillsdale: Erlbaum.
Rachman, G. (2011). *Zero-sum future—American power in an age of anxiety*. New York: Simon & Schuster.
Radford, L. (2003). Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. *Mathematical Thinking and Learning,* 5(1), 37–70.
Radford, L., Bardini, C., Sabena, C., Diallo, P., & Simbagoye, A. (2005). On embodiment, artefacts, and signs: A semiotic-cultural perspective on mathematical thinking. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th International Group for the Psychology of Mathematics Education,* vol. 4, pp. 113–122. Melbourne, Australia
Robutti, O. (2006). Motion, technology, gesture in interpreting graphs. *International Journal of Computer Algebra in Mathematics Education, 13,* 117–126.
Roth, W.-M. (2001). Gestures: Their role in teaching and learning. *Review of Educational Research, 71*(3), 365–392.
Roth, W.-M., & Welzel, M. (2001). From activity to gestures and scientific language. *Journal of Research in Science Teaching, 38*(1), 103–136.
Sabena, C. (2007). *Body and signs: A multimodal semiotic approach to teaching-learning processes in early calculus.* Unpublished Doctoral Dissertation, Università degli Studi di Torino, Torino
Sacks, O. (1989). *Seeing voices—A journey into the world of the deaf*. Berkeley: University of California Press.
Sinclair, N. (2010). Knowing more that we can tell. In B. Sriraman & L. English (Eds.), *Theories of mathematical education: Seeking new frontiers* (pp. 595–612). Heidelberg: Springer

Sriraman, B., & English, L. (2010) (Eds.), *Theories of mathematical education: Seeking new frontiers*. Heidelberg: Springer
Steinbring, H. (2005). Do mathematical symbols serve to describe or to construct reality? In M. Hoffmann, J. Lenhard, & F. Seeger (Eds.), *Activity and sign* (pp. 91–104). New York: Springer.
Stern, D. (1971). A microanalysis of mother-infant interaction. *Journal of the American Academy of Child Psychiatry,* 10, 501–517
Stern, D. (1985). *The interpersonal world of the infant: A view from psychoanalysis and developmental psychology*. New York: Basic Books.
Stewart, J. (2010). Foundational issues in enaction as a paradigm for cognitive science: From the origin of life to consciousness and writing. In J. Stewart, O. Gapenne, & E. A. Di Paolo (Eds.), *Enaction. Toward a new paradigm for cognitive science* (pp. 1–31). Cambridge: MIT Press.
Stone, L. J., Smith, H. T., & Murphy, L. B. (Eds.) (1973). *The competent infant*. New York: Basic Books.
Thom, A. (1973), Modern mathematics: Does it exist? In A. G. Howson (Ed.), Developments in mathematical education. *Proceedings of the 2nd International Congress on Mathematical Education* (pp. 194–210). Cambridge: Cambridge University Press
Tomasello, M. (1999). *The cultural origins of human cognition*. Cambridge, MA: Harvard University Press.
Tomasello, M. (2005). *Constructing a language: A usage-based theory of language acquisition*. Cambridge, MA: Harvard University Press.
Tomasello, M. (2008). *Origins of human communication*. Cambridge, MA: MIT Press.
Tomasello, M. (2006). Why don't apes point? In N. J. Enfield & S. C. Levinson (Eds.), *Roots of human socialty: Culture, cognition and interaction* (pp. 506–524). Oxford: Berg.
Trevarthen, C. (1979). Communication and cooperation in early infancy: A description of primary intersubjectivity. In M. Bullowa (Ed.), *Before speech: The beginnings of human communication* (pp. 321–347). Cambridge: Cambridge University Press.
Trevarthen, C. (1980). The foundations of intersubjectivity: Development of interpersonal and cooperative understanding in infants. In D. Olson (Ed.), *The social foundations of language and thought: Essays in honor of J. S. Bruner* (pp. 316–342). New York: Norton.
Trevarthen, C. (1994). Infant semiosis. In W. Nöth (Ed.), *Origins of semiosis: Sign evolution in nature and culture* (pp. 219–252). Berlin: Mouton de Gruyter.
Trevarthen, C., & Hubley, P. (1978). Secondary intersubjectivity: Confidence, confiding and acts of meaningin the first year. In A. Lock (Ed.), *Action, gesture, and symbol: The emergence of language* (pp. 183–230). London: Academic.
Van der Veer, R., & Valsiner, J. (1991). *Understanding Vygotsky: A quest for synthesis*. Oxford: Blackwell.
Van der Veer, R., & Valsiner, J. (Eds.) (1994). *The Vygotsky reader*. Oxford: Blackwell.
Varela, F. J., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. Cambridge, MA: MIT.
Von Holst, E., & Mittelstaedt, H. (1950). Das Reafferenzprinzip. Wechselwirkungen zwischen Zentralnervensystem und Peripherie. *Die Naturwissenschaften*, 37, Heft 20, 464–476.
Von Uexküll, J. (2010). *A foray into the worlds of animals and humans; with a theory of meaning.* Minneapolis: University of Minnesota Press (original work published 1934).
Vygotsky, L.S. (1994). The problem of the environment. In van der Veer, R. & Valsiner, J. (Eds.), *The Vygotsky reader* (pp. 338–354). Oxford: Blackwell. (Original work 1934).
Vygotsky, L. S. (1997). The history of the development of higher mental functions. In R. W. Rieber (Ed.), *The collected works of L. S. Vygotsky, Vol. 4*. New York: Plenum. (Original work 1931).
Vygotsky, L. S. (1999). Tool and sign in the development of the child. In R. W. Rieber (Ed.), *The collected works of L. S. Vygotsky, Vol. 6* (pp. 1–68). New York: Kluwer. (Original work 1930).
Wing, L. (1996). *The autistic spectrum: A guide for parents and professionals*. London: Constable.