Chapter 7
Discussion of Part I

Transitions in Learning Mathematics as a Challenge for People and Institutions

Rolf Biehler

Abstract: The transition from school to university is a challenge for all students as the teaching-learning cultures and the types of mathematics are very different and require from students large efforts of adaptation. A deeper understanding and research into the features of this transition is necessary for informing institutions and their teachers to better support students in the transition phase. Vice versa, a backwards transition from university to school is part of every teachers' biography and includes particular challenges. On an institutional level, the backwards transition is concerned with updating school curricula by taking new developments of mathematics and science at university level into account. The paper elaborates these problems and provides an introduction into the set of papers that are concerned with transitions and transformations on a personal and institutional level.

7.1 Overview

The papers in this book are predominantly concerned with the transition between school and university. We can see this as a specific instance of the problem of transitions between different mathematical practices (Abreu et al. 2002).

When a student enters a university, he or she is already bearing a mathematical biography. The student has encountered mathematics as a subject to be learned in primary, middle and high school. The type of mathematics might have been heavily dependent on the respective school as an institution. Moreover, the student may have encountered implicit mathematics in other school subjects or everyday situations without being aware that he or she is encountering mathematics. Grevholm (Chap. 6) provides an illuminative case study of mathematical moments in the life of Lisa from her early childhood, where she encountered implicit mathematics in the vocational contexts of her parents, up to entering university. The mathematical development of a person can have discontinuities, partly due to discontinuities in

R. Biehler (✉)
Department of Mathematics, University of Paderborn
33098 Paderborn, Germany
e-mail: biehler@math.upb.de

S. Rezat et al. (eds.), *Transformation—A Fundamental Idea of Mathematics Education,* DOI 10.1007/978-1-4614-3489-4_18,

the mathematical practices of the different institutions a person moves through. Earlier levels can disappear as such and become replaced or integrated in later levels or can still coexist in the mind of a student. For instance, a university student will have an opinion about the differences and commonalities between school and university mathematics, although it may not be trivial for him or her to switch back to the practice of high school mathematics or even lower levels of mathematics. Obviously, this “switching-back ability” is a fundamental qualification of teachers. Grevholm’s Lisa seems to be capable of doing this as she is successfully coping with the challenge of acting as a private teacher of school students during her university studies.

When mathematics students have finished with their university studies they face a further transition, namely from academic university mathematics into vocational contexts where, as a rule, mathematics is practiced in context. For future mathematics teachers, this second transition is a very specific one. Teachers go “back” to school and therefore may suffer in their mathematical biography from a “double discontinuity,” namely in the transition school-university and in the transition university-school. Felix Klein has coined this term in the introduction to his book “Elementary Mathematics from an Advanced Standpoint” (1st edition 1908, see Klein 1932) and he is often quoted in recent movements to “overcome” the double discontinuity (Ableitinger et al. 2013). Most papers in this book discuss either the first discontinuity or both from various perspectives.

The ability to see a single mathematical topic in the context of different mathematical practices seems to be one of the qualifications good mathematics teachers should have. Hefendehl-Hebeker (1996) describes this as the ability to see mathematics with a high depth of focus. Discontinuities in the transitions have been a concern in education for long. Bruner’s (1960) idea of orienting the whole school curriculum according to fundamental ideas, whose source is the respective university discipline, can be interpreted as avoiding or reducing discontinuities between levels within the school system and between school and university level. Teachers’ knowledge has to reflect this; Loewenberg Ball and Bass (2009) and Hill et al. (2008) put forward the notion of “horizon knowledge” as part of teachers’ knowledge; that is knowledge in and about mathematics and the mathematical practice of the next (upper) level in the educational system. Vice versa however, it seems to be equally helpful for teachers to have knowledge about the mathematical practices below the level he or she is teaching. A question is with which attitude and respect should mathematical practices of lower levels be regarded and taken into account at higher levels? The case study of a student that enters university that Pepin (Chap. 4) is presenting in her paper to this book reports of university teachers that favor a “confrontation approach” and tell students “forget the mathematics you have learnt in school.” Confrontation could be an adequate measure if it is unavoidable in making students aware of a discontinuity. Devaluating previous mathematical knowledge and experiences however, seems hardly to be a reasonable pedagogical strategy.

Pepin' s paper (Chap. 4) as well as the papers by Deiser and Reiss (Chap. 3) and Vollstedt et al. (Chap. 2) in this book analyze the transition from school to university from various perspectives for *all* mathematics major students, not just for future mathematics teachers. This transition has many general features that makes it difficult not only for all students but also for future mathematics teachers, who however may face specific motivational problems and have the specific concern of how university mathematics relates to the future school mathematics they have to teach. Kaiser and Buchholtz (Chap. 5) report on a particular innovative German project. This project was particularly devoted to first semester students who want to become mathematics teachers and it offered mathematics in a different way in order to smooth out the first of Klein's discontinuities.

Some transitions from school to university and within university are not discussed in the set of papers of this book. The papers that are concerned with teachers are looking at teachers who will teach at university or college bound schools. The transition has to be analyzed in quite a different way, if we think of mathematics courses for future primary or lower secondary teachers. The TEDS-M study (Blömeke et al. 2010a, b) reinforces the need of a careful specific curriculum design for these audiences. The LIMA project (*L*ehr*I*nnovation in der Studieneingangsphase "*Ma*thematik im Lehramtsstudium"—Hochschuldidaktische Grundlagen, Implementierung und Evaluation) (Biehler et al. 2012c) is one of the projects particularly devoted to improving the mathematics education of lower secondary student teachers in the first year of their studies. Moreover, school students who enter university studies in the STEM subjects (science, technology, engineering, and mathematics) but who are not mathematics majors encounter a different type of mathematics, which may vary in content and style among the different STEM subjects such as physics, biology, and the engineering sciences. Mathematics (including statistics) is present also in non-STEM subjects such as psychology, social and economic sciences, where the mathematical practices as well as students' attitudes and mathematical competences differ from the STEM subjects. In a very rough approximation, one can say that mathematics is more considered as a tool and language, whereas proof and formalization, the most distinguishing feature of all courses of mathematics majors, does not play a central role in these courses. These domains themselves often suffer from a clash of mathematical practices and cultures. The mathematical practice in an engineering course is quite different from the mathematical practice in course "mathematics for engineers." This can be a source of tensions between departments and within students' minds. Mathematical courses for economy students may be taught either by lecturers of the mathematics department (mathematics as a service subject) or by lecturers from the economy department. Some departments teach their mathematics courses themselves because they are convinced that lecturers from the mathematics department would import a mathematical culture that makes the transition to the uses of mathematics in the respective domain more difficult. Seeing this wider domain of mathematical practices including mathematics in vocational settings and in industry is a perspective that Rudolf Sträßer (2000) put forward in his work.

7.1.1 Transforming Backwards—From Universities to Schools

The paper of Biermann and Jahnke (Chap. 1) in this book reminds us of several historical instances where a new stance of university mathematics was used as a basis of school curricula reforms. University mathematics served as a major input for updating school mathematics. The authors point out how the eighteenth century conception of "algebraic analysis" systematically influenced the curricula in Prussian Grammar schools in the nineteenth century. The Meran curriculum reform of 1905, which was inspired by Felix Klein's ideas, intended to update school mathematics from a very different perspective, taking up major developments in nineteenth century mathematics as Klein and his companions conceived them. "Functional thinking" became a keyword of the reform movement. The concept of function was regarded from inside mathematics and at the same time seen as a bridging concept that relates mathematics to its applications in natural and engineering sciences (see Krüger 2000 for a deeper analysis). Klein was also concerned about the gap between pure mathematics and its applications. Klein's books on Elementary Mathematics from an Advanced Standpoint (Klein 1932, 1939) were written as books for teachers for supporting this specific backwards transition into schools. In the 1950s and 1960s a very strong international movement under the well-known name of the "new math reform" tried again to update school mathematics based on a specific view of university mathematics. The gap between school and university mathematics was supposed to be narrowed by changing school mathematics accordingly and adapt it closer to mathematics as a (university) discipline. More than 50 years later, we know much more about the complexities of the transformation from scientific knowledge to knowledge to be taught. Chevallard's (1985) book was a milestone in research in mathematics education that looks at these processes of transformation with an analytical stance (see Seeger et al. 1989 for a review). Rudolf Sträßer was among those, who took Chevallard's work into account in his work, particularly in geometry and particularly arguing for regarding not only university mathematics as a source for school mathematics but also vocational contexts and other non-university uses of mathematics (Sträßer 1992). This more general approach for reconstructing sources of meanings for concepts in school mathematics can be also exemplified by the concept of function (Biehler 2005).

In recent years, mathematical curricula seem to have developed in the direction of a more student-centered, application-oriented, and visual, less formal kind of mathematics allowing much more types of reasoning and argumentation than just formal proof. Due to these developments, the gap between school mathematics and the mathematics taught in university courses for mathematics majors seems to have become wider again. There is no easy solution with regard to the questions in which sense schools could readapt to university mathematics. We can frame it differently: How can we redefine what it means to mathematically "prepare" university bound students at school level for university courses with mathematical content. We have to take into account the problems of the school to university transition for gaining more insight.

7.1.2 The School to University Transition

The secondary–tertiary transition has become the object of theoretical analyses such as Gueudet's (2008) and of practical measures such as creating "bridging courses" (such as Biehler et al. 2011, 2012b). Redesigning the introductory university courses are further measures. The papers in this book contribute to this research and development domain from various perspectives. The paper by Kaiser and Buchholtz reports on an innovative German project that redesigned the introductory courses in analysis and in geometry and linear algebra at tertiary level in order that they better fit the needs of future Gymnasium teachers (Beutelspacher et al. 2012). Overcoming Klein's double discontinuity was one of the objectives. The courses itself were to reflect the relation between school and university mathematics by adequate examples and activities. Hereby, the students should appreciate and understand the need of a different kind of mathematics at university level, while at the same time understanding how this new mathematics is related to school mathematics, why it is different, and why it nevertheless has the potential of contributing to the development of students' mathematical competences in a way to make them useful for a qualified mathematics teaching at school level. Working in this way, the first discontinuity was regarded as laying foundations for smoothing out the second discontinuity at the transition from university to school. This is a very interesting experiment as it aims at maintaining and cultivating two views of mathematical practices at school and at university level from the beginning. In other words, it is cultivating the "switching back ability" as I called it earlier. The paper of Buchholtz and Kaiser reports on an empirical study that evaluated this innovative project in comparison to students who participated in more standard programs at different German universities. A quantitatively oriented evaluation has to develop adequate measurement instruments for mathematical competence. Based on knowledge conceptualizations and instruments developed in the context of the IEA directed TEDS-M project (Blömeke et al. 2010a, b), instruments were newly developed that distinguish between academic mathematical knowledge, knowledge in mathematics from an advanced standpoint, and mathematical pedagogical content knowledge and beliefs about mathematics. The study presents some slightly positive results but shows at the same time how difficult sustained educational reforms at the tertiary level are. A further theoretical modeling of the *development* of mathematical competencies at the secondary–tertiary transition and developing measurement instruments on this basis is still a challenge for the future.

The other papers of this section analyze the secondary–tertiary transition from various perspectives. Grevholm and Pepin provide holistic insights on how students experience a totally new culture including new people, tools, and learning and mathematical practices. Grevholm's narratives on transformation focus on the mathematical biography of Lisa, beginning in her preschool age up to her first years at university. It shows the complexity of constituting a mathematical identity and coping with transformations of various kinds. Lisa's story is a success story in the sense that she mastered all the transformations and finally started her doctoral studies in mathematics. Many students give up their mathematical studies in the first year. Also, engineering students fail, among other reasons because they fail in mathematics. Dropout rates of 30–50 % are not an exception, at least in German universities. Dieter (2012) did a quantitative study concerning premature matriculation and its influencing factors. Qualitative studies similar to Grevholm's case study, done with first year university students may contribute to a deeper understanding of factors that affect a successful and a less successful mathematical biography in the process of transition.

Pepin's paper goes into this direction, based on the TransMath project at the University of Manchester. Pepin analyzes the fundamental differences between schools and universities with regard to providing feedback and the requirements of self-regulated learning. This general framework is also applicable for other subjects than mathematics and opens the perspective to general problems students of all subjects face when entering the secondary–tertiary transition. Specific problems related to mathematics as a subject can thus be placed into a broader perspective. Based on case studies, Pepin provides a very instructive detailed portrait of the mathematical teaching-learning culture with the elements of lecturers, tutorials, and self-study and the implicit values and views of students and lecturers, which may not fit well with each other. Innovative reforms have to address the whole teaching-learning system and must not focus on curriculum and mathematics alone. In line with this perspective, we changed the teaching-learning culture in the small group tutorials that are accompanying our lectures with big audiences by creating a specific program for supporting student mathematics tutors that improved the quality of the group tutorials. We focused on enabling the student tutors to present solutions of homework with a view to student difficulties and to the process of problem solving. Moreover, the quality of feedback given to the problem solutions that students submitted as part of their homework assignments was increased. A third domain was supporting the student tutors in moderating collaborative group work with minimal and strategic intervention types. As a further step, our study has revealed the need of directly supporting students' motivation and competence of dealing with feedback as not all of them make optimal use of the improved feedback provided to them (Biehler et al. 2012b, c)

The papers of Vollstedt et al. and of Deiser and Reiss are concerned with mathematical knowledge and with mathematical learning resources in the secondary–tertiary transition and open a further important dimension in studying the school–university transition. Vollstedt et al. focus on a comparison of mathematical textbooks for upper secondary level and for school level. Learning resources are an important part of the knowledge and learning practice (Sträßer 2009), therefore comparisons can contribute to a deeper understanding of the major differences. Based on research on text book structure and use (among others, see Rezat 2009), the authors develop a new instrument for comparative analysis of secondary and tertiary text books along the dimensions of motivation, structure and visual representation, development and understanding of concepts, development and understanding of theorems, presentation of the proving process and proofs, and type of tasks. Although the categories were as yet only applied in a feasibility study, the system of categories provides orientation for future comparative text book research.

The paper by Deiser and Reiss focuses on specific elements of mathematical knowledge. Key differences between school and the mathematics of mathematics majors can be related to the different conceptions of proofs and definitions: "The move from elementary to advanced mathematical thinking involves a significant transition: that from *describing* to *defining*, from *convincing* to *proving* in a logical manner based on definitions" (Tall 1991, p. 20). Deiser and Reiss' paper fits well into the tradition to analyze students' difficulties with definitions (Edwards and Ward 2004; Kintzel et al. 2011) as key part of the transition difficulties but also gives new insights into difficulties with seemingly basic definitions in a first semester analysis course. Deiser and Reiss provide first results of a larger research project that will study the development of mathematical competences of student teachers of mathematics within the first years of their university studies.

The papers of this book show that the transitions and transformations mathematical learners have to face have become the object of promising research and development studies in mathematics education, aiming at a deeper understanding and theoretical basis for educational innovations in the future.

References

Ableitinger, C., Kramer, J., & Prediger, S. (Eds.). (2013). Zur doppelten Diskontinuität in der Gymnasiallehrerbildung—Ansätze zu Verknüpfungen der fachinhaltlichen Ausbildung mit schulischen Vorerfahrungen und Erfordernissen. Heidelberg: Springer Spektrum

Abreu, G. d., Bishop, A. J., & Presmeg, N. C. (2002). *Transitions between contexts of mathematical practices*. Dordrecht: Kluwer.

Beutelspacher, A., Danckwerts, R., Nickel, G., Spies, S., & Wickel, G. (2012). *Mathematik Neu Denken—Impulse für die Mathematiklehrerbildugn an Universitäten*. Wiesbaden: Vieweg+Teubner.

Biehler, R. (2005). Reconstruction of meaning as a didactical task—the concept of function as an example. In J. Kilpatrick, C. Hoyles, O. Skovsmose, & P. Valero (Eds.), *Meaning in mathematics education* (pp. 61–81). New York: Springer.

Biehler, R., Fischer, P. R., Hochmuth, R., & Wassong, T. (2011). Designing and evaluating blended learning bridging courses in mathematics. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education* (pp. 1971–1980). Rzeszów: University of Rzeszów.

Biehler, R., Fischer, P. R., Hochmuth, R., & Wassong, T. (2012a). Self-regulated learning and self assessment in online mathematics bridging courses. In A. A. Juan, M. A. Huertas, S. Trenholm, & C. Steegman (Eds.), *Teaching mathematics online: Emergent technologies and methodologies* (pp. 216–237). Hershey: IGI Global.

Biehler, R., Hochmuth, R., Klemm, J., Schreiber, S., & Hänze, M. (2012b). Tutorenschulung als Teil der Lehrinnovation in der Studieneingangsphase "Mathematik im Lehramtsstudium" (LIMA-Projekt). In M. Zimmermann, C. Bescherer, & C. Spannagel (Eds.), *Mathematik lehren in der Hochschule—Didaktische Innovationen für Vorkurse, Übungen und Vorlesungen* (pp. 33–44). Hildesheim: Franzbecker.

Biehler, R., Hoppenbrock, A., Klemm, J., Liebendörfer, M., & Wassong, T. (2012c). Training of student teaching assistants and e-learning via math-bridge—Two projects at the German Centre for Higher Mathematics Education. In D. Waller (Ed.), *CETL-MSOR Conference Proceedings 2011* (pp. 21–27). The Maths, Stats & OR Network.

Blömeke, S., Kaiser, G., Lehmann, R., & International Association for the Evaluation of Educational Achievement. (2010a). *TEDS-M 2008: Professionelle Kompetenz und Lerngelegenheiten angehender Mathematiklehrkräfte für die Sekundarstufe I im internationalen Vergleich*. Münster [u.a.]: Waxmann.

Blömeke, S., Kaiser, G., Lehmann, R., & International Association for the Evaluation of Educational Achievement. (2010b). *TEDS-M 2008: Professionelle Kompetenz und Lerngelegenheiten angehender Primarstufenlehrkräfte im internationalen Vergleich*. Münster [u.a.]: Waxmann.

Bruner, J. S. (1960). *The process of education*. Cambridge: Harvard University Press.

Chevallard, Y. (1985). *La transposition didactique— du savoir savant au savoir enseigné*. Grenoble: La Pensée Sauvage.

Dieter, M. (2012). *Studienabbruch und Studienfachwechsel in der Mathematik: Quantitative Bezifferung und empirische Untersuchung von Bedingungsfaktoren*: Dissertation Universität Duisburg-Essen.

Edwards, B. S., & Ward, M. B. (2004). Surprises from mathematics education research: Student (mis)use of mathematical definitions. *The American Mathematical Monthly, 111*(5), 411–424.

Gueudet, G. (2008). Investigating the secondary–tertiary transition. *Educational Studies in Mathematics, 67*(3), 237–254.

Hefendehl-Hebeker, L. (1996). Geistige Ermutigung im Mathematikunterricht. In R. Biehler, H. W. Heymann, & B. Winkelmann (Eds.), *Mathematik allgemeinbildend unterrichten: Impulse für Lehrerbildung und Schule* (pp. 83–91). Köln: Aulis.

Hill, H. C., Loewenberg Ball, D., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.

Kintzel, M. T., Cavey, L. O., Walen, s. B., & Rohrig, K. L. (2011). How do mathematicians make sense of definitions? In S. Brown, K. Larsen, K. Marrongelle, & M. Oehrtman (Eds.), *Proceedings of the 14th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 163–174). Portland: Oregon.

Klein, F. (1932). *Elementary mathematics from an advanced standpoint. Arithmetic, algebra, analysis* (trans: Hedrick, E. R., Noble, C. A. from German). New York: The Macmillan company.

Klein, F. (1939). *Elementary mathematics from an advanced standpoint: Geometry* (trans: Hedrick, E. R., Noble, C. A. from German). New York: The Macmillan company.

Krüger, K. (2000). *Erziehung zum funktionalen Denken—Zur Begriffsgeschichte eines didaktischen Prinzips*. Berlin: Logos-Verlag.

Loewenberg Ball, D., & Bass, H. (2009). With an eye on the mathematical horizon: Knowing mathematics for teaching to learners' mathematical futures. In M. Neubrand, et al. (Ed.), *Beiträge zum Mathematikunterricht 2009*. Münster: WTM-Verlag http://www.mathematik.uni-dortmund.de/ieem/BzMU/BzMU2009/Beitraege/Hauptvortraege/BALL_Deborah_BASS_Hyman_2009_Horizon.pdf%5D.

Rezat, S. (2009). *Das Mathematikbuch als Instrument des Schülers: eine Studie zur Schulbuchnutzung in den Sekundarstufen*. Wiesbaden: Vieweg+Teubner.

Seeger, F., Steinbring, H., & Sträßer, R. (1989). Die didaktische Transposition. Rezension von Yves Chevallard: La Transposition didactique—du savoir savant au savoir enseigné. Grenoble: La Pensée Sauvage. 1985. *mathematica didactica, 12*(2/3), 157–177.

Sträßer, R. (1992). Didaktische Transposition—eine "Fallstudie" anhand des Geometrieunterrichts. *Journal für Mathematik-Didaktik, 13*(2/3), 231–252.

Sträßer, R. (2000). Mathematical means and models for vocational contexts—A German perspective. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. 65-80). Dordrecht: Kluwer.

Sträßer, R. (2009). Instruments for learning and teaching mathematics. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidids (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 67–81). Thessaloniki: PME.

Tall, D. (1991). The psychology of advanced mathematical thinking. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 3–21). Dordrecht: Kluwer.