Chapter 2
Design of Uncertain Prestressed Space Structures: An Info-Gap Approach

Aurélien Hot, Scott Cogan, Emmanuel Foltête, Gaetan Kerschen, Fabrice Buffe, Jérôme Buffe, and Stéphanie Behar

Abstract Uncertainty quantification is an integral part of the model validation process and is important to take into account during the design of mechanical systems. Sources of uncertainty are diverse but generally fall into two categories: aleatory uncertainties due to random processes and epistemic uncertainty resulting from a lack of knowledge or erroneous assumptions. This work focuses on the impact of uncertain levels of prestress on the behavior of solar arrays in their stowed configuration. In this context, snubbers are inserted between two adjacent panels to maintain contact and absorb vibrations during launch. However, under high excitation loads, a loss of contact between the two panels may occur. This results in impacts that can cause extensive damages to fragile elements.

In practice, the specific load configuration for which the separation of the two panels occurs is difficult to determine precisely since the exact level of prestress applied to the structure is unknown. An info-gap robustness analysis is applied to study the impact of this lack of knowledge on the prestress safety factor required to avoid loss of contact. The proposed methodology is illustrated using a simplified model of a solar array.

2.1 Introduction

In the field of structural dynamics, the mathematical model must not only be validated against tests, it must also be validated with respect to its final performance goal in the presence of uncertain parameters. Indeed, the required performance may be drastically affected even by a slight perturbation. Moreover, real environmental conditions and loads on a structure are sometimes unknown or can not be reproduced during tests. It is thus important to take these uncertainties into account in the design process.

One can distinguish two sources of uncertainty. The first class of uncertainties, called aleatory uncertainty, is due to the intrinsic randomness of the structure parameters such as variability of its physical properties, of the assembling process, of its service conditions (temperature, hydrometry, etc.), or again the variability of the measuring process. The second class pertains to epistemic uncertainties. They result from a lack of knowledge or erroneous assumptions, such as unknown values of stiffnesses, linearity assumptions, omission of contact elements or other nonlinear elements, unknown probability distributions, among others.

A. Hot • S. Cogan (✉) • E. Foltête
FEMTO-ST Institute – Applied Mechanics Department, 24 rue de l'Epitaphe, 25000 Besançon, France
e-mail: aurelien.hot@univ-fcomte.fr; scott.cogan@univ-fcomte.fr; emmanuel.foltete@univ-fcomte.fr

G. Kerschen
Department of Aerospace and Mechanical Engineering, University of Liege, Chemin des Chevreuils 1 (B52), 4000 Liége, Belgium
e-mail: g.kerschen@ulg.ac.be

F. Buffe
Centre National d'Etudes Spatiales, 18 Avenue Edouard Belin, 31401 Toulouse, Cedex 9
e-mail: fabrice.buffe@cnes.fr

J. Buffe • S. Behar
Thales Alenia Space, 100bd du Midi – BP99 – 06156, Cannes la Bocca, Cedex
e-mail: jerome.buffe@thalesaleniaspace.com; stephanie.behar@thalesaleniaspace.com

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Dealing with aleatory uncertainties, it is common practice to use probabilistic methodologies [1, 2] to quantify and propagate them through the system model. A classical approach consists in using a deterministic finite element model for the mechanical system and then adding the uncertainty through either parametric or non-parametric methods. When using the parametric approach, the uncertain physical parameters, e.g. Young's modulus or plate thickness, are characterized by their mean value, their standard deviation or by probability density functions, e.g. Gaussian or uniform distribution. However, the difficulty in quantifying the level of stochastic uncertainty in physical parameters has motivated the development of alternative non-parametric approaches. Soize proposes in [3] an overview of these methodologies, for which the uncertainties are no longer applied to physical parameters but directly introduced in the global randomized matrices through dispersion parameters.

However, using a stochastic approach for all types of problems would be too restrictive and inappropriate. Indeed, for epistemic uncertainties, where a large amount of information is missing, the probabilistic approach is unsuitable. Thus, to study these cases, one can use a second class of uncertainty models, based on non-probabilistic methods such as the interval arithmetic theory [4], the Fuzzy sets approach [5] or the Transformation method [6]. Moreover, to deal with problems subject to severe uncertainty, Ben-Haim proposed in [7] the Info-gap theory. The basic idea of this theory is to investigate the degree of lack of knowledge that can be tolerated while satisfying a critical level of performance.

In this paper, only the case of epistemic uncertainty due to lack of knowledge is tackled using the Info-gap theory. This method will be first briefly described and then applied to an uncertain prestressed structure.

2.2 The Info-Gap Theory

The Info-gap theory proposed by Ben-Haim [7] helps to quantify the robustness of model-based decisions to lack of knowledge in the system model. This approach provides a useful tool since the trade-off between performance and robustness can be investigated and different design configurations can be compared. The methodology has already been applied in many different fields. In structural dynamics analysis, studies concerning model updating [8], reliability [9], optimal sensors and exciters positioning [10] and design of civil structures in seismically active regions [11] have already been carried out. Moreover, applications of the Info-gap theory can also be found in medical [12] and management [13] applications, as well as elsewhere.

An Info-gap model quantifies the difference—the gap—between the information that is known, for example the nominal value of a parameter, and the information that has to be known in order to insure a critical level of performance. This non-probabilistic approach to uncertainty quantification is briefly described in the following sections.

An Info-gap robustness analysis is composed of four components:

- The *system model* defines the relation between the system inputs and outputs. In the case of structural dynamics, the system model consists in the equation of motion that links the inputs, the excitation force, to the outputs, displacement, velocity and acceleration, through the finite element model. The system model is denoted $S(\nu)$ and depends on various unknown parameters ν.
- The *uncertainty model* $\mathcal{U}$ represents the uncertainty in the variables ν. It consists in a collection of nested sets of uncertain events whose size is controlled by a horizon of uncertainty h. The larger this horizon, the more inclusive is the uncertainty model. A simple model could be an interval within which ν varies around a nominal value ν_0:

$$\mathcal{U}(\nu_0, h) = \{\nu \quad : \quad |\nu - \nu_0| \leq h\}, \quad h \geq 0 \tag{2.1}$$

- The *performance requirement* is a specification of a critical level, denoted P_{crit}, of the system performance $P(S(\nu))$, e.g. a maximum tolerable displacement or stress in the structure:

$$P(S(\nu)) \leq P_{crit} \tag{2.2}$$

- The *robustness to uncertainty* $\hat{h}$ is defined as the greatest value of the uncertainty parameter h for which the critical performance requirement is still insured. This can be expressed by:

$$\hat{h} = max\left\{h \quad : \quad \max_{\nu \in \mathcal{U}(\nu_0, \alpha)} P(S(\nu)) \leq P_{crit}\right\} \tag{2.3}$$

2.3 Application to a Simplified Solar Array System

2.3.1 Presentation

A specific difficulty encountered on solar arrays concerns the impacts that occurs between two adjacent panels. In order to limit this phenomenon, which can lead to significant damage, dedicated snubbers are inserted between the panels. The presence of the snubbers has two consequences. First, they lead to an increase in the local stiffness, thus limiting the relative displacements. Second, the induced prestress forces insures that contact is maintained between the two panels. However, for high input levels, loss of contact between the snubbers and the panels may arise and impacts still occur.

The Info-gap approach is used here to assess the robustness of the solar array system performance to epistemic uncertainties due to lack of knowledge in the induced prestress forces. The performance criterion here is the non-separation of two adjacent panels that will guarantee the integrity of the structure. The four components of an Info-gap analysis described above are presented in the following paragraphs, namely:

- System model: Simplified solar array system prestressed due to the presence of snubbers
- Performance requirement: Non-separation of the two panels in order to avoid impacts
- Uncertainty model associated with uncertain prestress values
- Robustness: What maximum level of uncertainty in the prestress can be tolerated while insuring that the panels do not impact one another?

2.3.2 System Model

The finite element model of the simplified solar array is depicted in Fig. 2.1a. The two plates are meshed with 3072 NASTRAN CQUAD4 elements. The stacking points are modeled using three CBAR beams that are linked to the plates through rigid elements RBE2. Only the masses of the two hinges are taken into account in this model by concentrated masses CONM2, also linked to the plates with rigid elements. The element properties are listed in Table 2.1, where E, ν, ρ, T, d and m are the Young's modulus, the density, the Poisson's ratio, the plates thickness, the beams cross section dimension and the

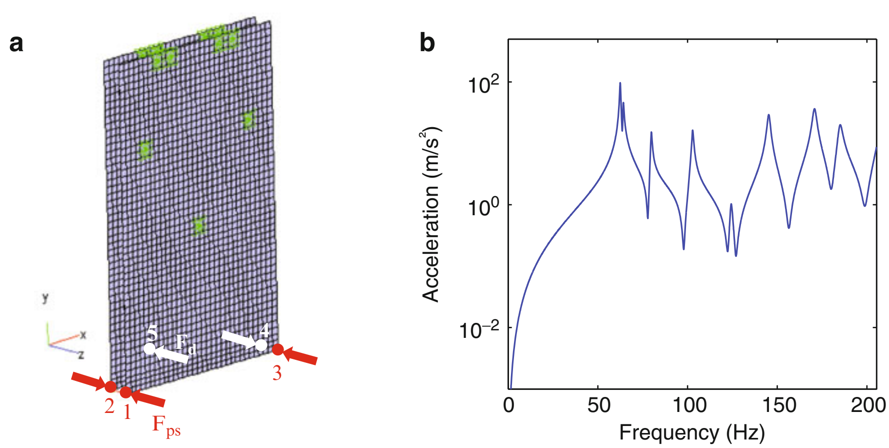

Fig. 2.1 System model. (**a**) Simplified solar array model FEM; (**b**) Frequency response of the system at node 1

Table 2.1 Element property of solar array model

CQUAD4	$E = 6.52e10$ Pa	$\rho = 2{,}761$ kg.m^{-3}	ν=0.34	T= 5 mm
CBAR	$E = 5.18e10$ Pa	$\rho = 3{,}453$ kg.m^{-3}	ν=0.34	d= 16 mm
CONM2	$m = 191$ g			

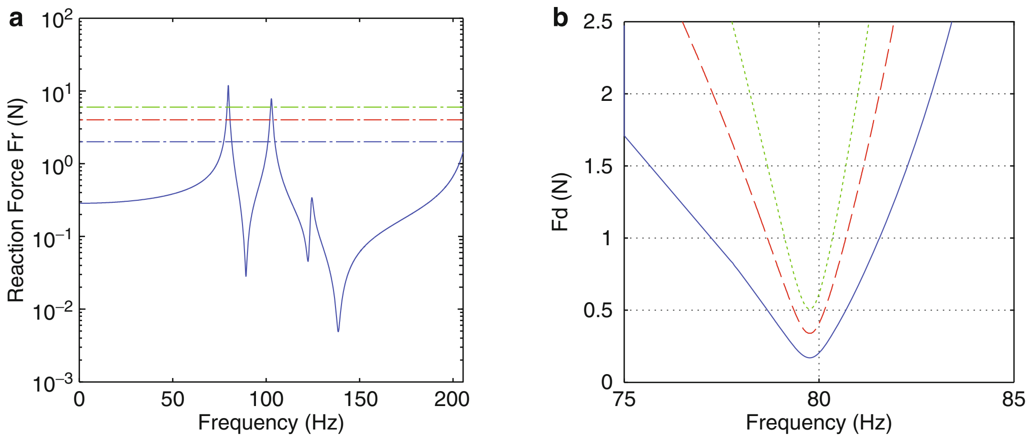

Fig. 2.2 Model performance. (**a**) Reaction force F_r at node 1 — $F_{ps} = 2$ N, - - - $F_{ps} = 4$ N, $\cdots$ $F_{ps} = 6$ N; (**b**) Separation zone $\mathcal{V}(\omega, F_d)$ for prestress values — $F_{ps} = 2$ N, - - - $F_{ps} = 4$ N, $\cdots$ $F_{ps} = 6$ N

concentrated masses value, respectively. The whole model contains 19,548 dofs. Two linear springs that model the snubbers behavior are inserted between 1 and 2 and 3 and 4 whose stiffnesses are denoted k_s. In a first step, $k_s = 1e5$ N/m.

A 1 N sine excitation F_d is applied at node 5 in the z-direction. The corresponding displacement frequency response of node 1 is plotted in Fig. 2.1b.

2.3.3 *Performance Requirement*

The performance requirement in this study consists in avoiding the separation of the two panels. Indeed this requirement insures that no impact can occur between the panels, so that the system integrity is preserved. This statement is expressed as a condition on local forces: separation occurs when the reaction force F_r of the snubber on the panel exceeds the applied static prestress load F_{ps}:

$$F_r \leq s_f F_{ps} \tag{2.4}$$

where s_f is a safety factor that enables to allow variations on the separation condition. A small safety factor guaranties a safe behavior but has very strict requirements, whereas a large value is easier to realize but is dangerous regarding the performance requirement. Unless otherwise mentioned, this s_f value remains equal to 1.

Figure 2.2a plots the reaction force F_r at node 1 obtained using the MPCF NASTRAN card, as well as three given threshold prestress force values. One remarks that two modes are liable to lead to the performance failure. We will focus on the one around 80 Hz, the highest amplitude one. However, the extension to a frequency band containing several modes can be readily made.

With the performance requirement defined, one can seek all realizable configurations (frequency, dynamic input force, prestress loads) that lead to failure. The set $\mathcal{V}(\omega, F_d)$, expressed in (2.5), corresponds to the points that bounds the performance. In other words, all the configuration points inside the parabola do not satisfy the performance requirement. Figure 2.2b shows these sets for the three prestress loads. For a given prestress load, the lowest dynamic force F_d for which the panels separate occurs for the resonance frequency. Indeed, with the increasing relative displacement of the two corners of the panels, the reaction force increases and exceeds the prestress threshold value. Moreover, one can also remark that, as expected, the higher the prestress load, the narrower the $\mathcal{V}(\omega, F_d)$ set and thus the smaller the opportunity of failure.

$$\mathcal{V}(\omega, F_d) = \{\{\omega, F_d\} \quad : \quad |F_r(\omega)| = s_f F_{ps}\} \tag{2.5}$$

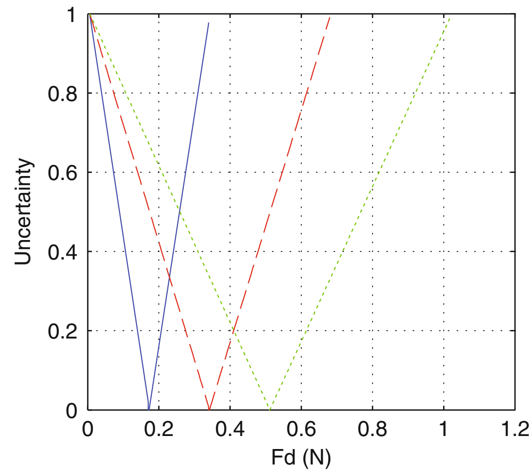

Fig. 2.3 Evolution of the critical separation point with uncertainty — $F_{ps} = 2$ N, - - - $F_{ps} = 4$ N, $\cdots$ $F_{ps} = 6$ N

2.3.4 Uncertainty Model

As mentioned in the presentation paragraph, the source of epistemic uncertainty in this example is the prestress load value F_{ps}. Indeed, it is assumed here that the level of prestress introduced by the snubbers is difficult to assess due to structural uncertainties (geometric defects, interface stiffness, etc).

Let $\widetilde{F_{ps}}$ be the best estimate of F_{ps}. This $\widetilde{F_{ps}}$ quantity is a known but perhaps unreliable estimate of the corresponding coefficient. In fact, we do not know by how much the estimated value is wrong. The uncertainties on F_{ps} are modeled here using an fractional-error Info-gap model:

$$\mathcal{U}(h) = \left\{ F_{ps} \quad : \quad \frac{|F_{ps} - \widetilde{F_{ps}}|}{\widetilde{F_{ps}}} \leq h \right\}, \quad h \geq 0 \tag{2.6}$$

The inequality states that the fractional error of the estimate $\frac{|F_{ps} - \widetilde{F_{ps}}|}{\widetilde{F_{ps}}}$ is bounded by the horizon of uncertainty h. Since we do not know the magnitude of error, no realistic worst case is known, h is unbounded. When $h = 0$, there is no uncertainty and the estimation is correct. Whereas, as h increases the uncertainty set $\mathcal{U}(h)$ is more and more inclusive. Thus the Info-gap model is an unbounded family of sets of possible realizations of F_{ps}.

Figure 2.3 plots the evolution of the failure point in an uncertainty vs dynamic force diagram. For each curve, the first decreasing part corresponds to the worst case of performance failure, and the second increasing branch corresponds to the best case. As engineering structures are usually designed in the worst case, the study focuses on the curves left hand part.

For a null uncertainty h, the F_{ps} value is the nominal one and the failure point corresponds to the one seen in Fig. 2.2b. Regarding the nominal value of 2 N prestress load example, the separation occurs for a dynamic force of 0.17 N, which actually corresponds to the minimum of the corresponding curve in Fig. 2.2b. Then, the more the amount of uncertainty increases, the lower the dynamic force required for failure.

These curves also enable to evaluate the allowed uncertainty that still avoids failure. For example, if the system is excited with a dynamic force of 0.17 N, no uncertainty is tolerable if the prestress load is equal to 2 N. Thus, the prestressed force value must be exactly known. However, if $F_{ps} = 0.4$ N, a 0.51 uncertainty value is now tolerable, and even 0.68 in the last case.

2.3.5 Robustness

We now define the robustness of the system model to uncertainty in the prestress load value. The robustness $\hat{h}$ is the greatest horizon of uncertainty h up to which the performance requirement is satisfied for all realizations of the uncertain quantity:

$$\hat{h} = max \left\{ h \quad : \quad \left(\max_{F_{ps} \in \mathcal{U}(h)} [F_r - s_f F_{ps}] \right) \leq 0 \right\} \tag{2.7}$$

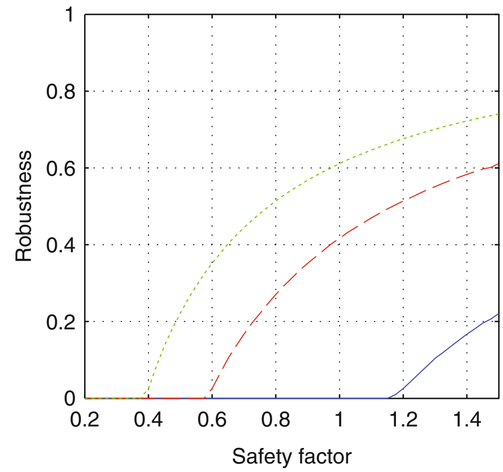

Fig. 2.4 Robustness evaluation for three different prestress loads — $F_{ps} = 2$ N, - - - $F_{ps} = 4$ N, $\cdots$ $F_{ps} = 6$ N

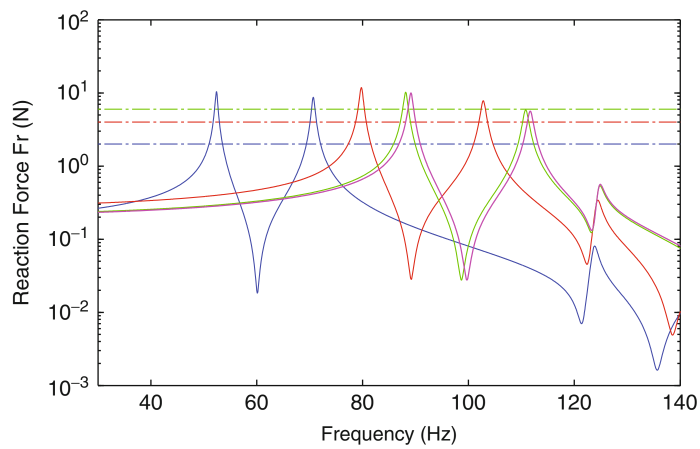

Fig. 2.5 Reaction force F_r at node 1 for different snubber stiffnesses values — $k_s = 1e4$ N/m, — $k_s = 1e5$ N/m, — $k_s = 1e6$ N/m, — $k_s = 1e7$ N/m - · - $F_{ps} = 2$ N, - · - $F_{ps} = 4$ N, - · - $F_{ps} = 6$ N

Figure 2.4 displays the robustness $\hat{h}$ vs the safety factor s_f, that remained equal to 1 up till now. Several conclusions can be drawn from these curves.

First, for low safety factors, the robustness remains null. This means that, in these cases, what ever the prestress configuration, failure of the performance requirement is certain.

Then, for the 6 N prestressed configuration, the robustness curve presents two different behaviors. For a safety factor below 0.8, the slope is quite steep, which means that for a small effort in increasing the safety factor the gain in robustness is important. Whereas, for safety factors above 0.8, the slope is now flat: a large variation of s_f only leads to a small profit of the robustness.

Finally comparing the three curves, one can note that, for all safety factor values, the higher the prestress load value, the greater the robustness to the sought performance, that is to say, avoiding separation between the panels.

2.3.6 Comparison of Different Snubber Stiffnesses

The previous paragraph compared the robustness of the system for different prestress loads and safety factor values. Now a third parameter is introduced: the snubber stiffness k_s. The previous results were obtained using a unique value: $k_s = 1e5$ N/m. Let us now assess the influence of k_s on the robustness function.

The reaction forces on the snubbers nodes are first recomputed for four different k_s values: $1e4$, $1e5$, $1e6$ and $1e7$ N/m. These results are plotted in Fig. 2.5 and one can observe, as expected, a softening or stiffening effect depending on the snubber stiffness value.

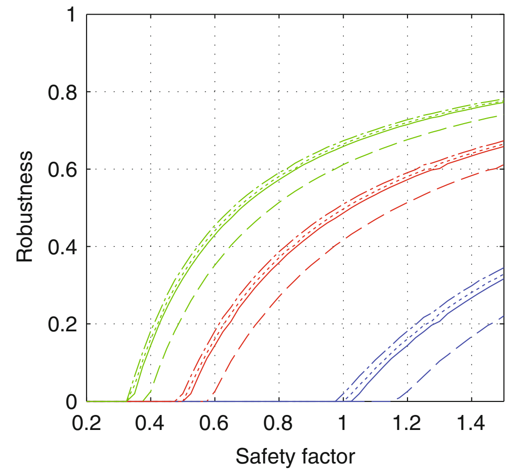

Fig. 2.6 Robustness evaluation for different snubber stiffnesses values — $k_s = 1e4$ N/m, - - - $k_s = 1e5$ N/m, ··· $k_s = 1e6$ N/m, - · - $k_s = 1e7$ N/m — $F_{ps} = 2$ N, — $F_{ps} = 4$ N, — $F_{ps} = 6$ N

The same procedure as above is then followed to estimate the robustness of the different designs. The corresponding results are displayed in Fig. 2.6. For all k_s values, the obtained curves present the same shape, which enables to directly compare the different robustness levels. It appears that, in this specific example, the case $k_s = 1e7$ N/m leads to the most robust design. However, the greater stiffness does not always means the more robust. Indeed, here the $k_s = 1e4$ N/m configuration is more robust than the $k_s = 1e5$ N/m one.

2.4 Conclusions

The objective of this work was to develop a model-based decision indicator for designing the prestress level required between the panels of a stowed solar generator in order to avoid damage due to impacts during launch. Given the presence of severe uncertainty in the prestress values induced by the introduction of dedicated snubbers, an info-gap approach was used to study the robustness of the critical design performance, namely the absence of separation, to lack of knowledge in the prestress states at each snubber location. Moreover, the impact of snubber stiffness on the robustness function was also investigated and the existence of an optimal value was demonstrated. The proposed methodology was illustrated using a simplified solar array model.

Before applying this methodology to an industrial structure, several limitations in the current approach must be addressed. Future work will consist in building a more accurate nonlinear model of the contact phenomena between the snubbers and the panels. Moreover, an estimation of how much prestress uncertainty must be tolerated in practice would be very useful for design purposes. This question will be examined by studying the impact of lack of knowledge in the mechanical properties of the structure on the prestress forces. Finally, an experimental validation of the methodology will be carried out on a simplified prototype structure.

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