

# Chapter 1

## Introduction

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**Abstract** This introductory chapter gives a brief overview of power system coherency and model reduction literature. This survey focuses on both the early results and some more recent developments, and organizes power system model reduction techniques into two broad categories. One category of methods is to use coherency and aggregation methods to obtain reduced models in the form of nonlinear power system models. The other category is to treat the external system or the less relevant part of the system as an input–output model and obtain a lower order linear or nonlinear model based on the input–output properties. This chapter also provides a synopsis of the remaining chapters in this monograph.

### 1.1 Introduction

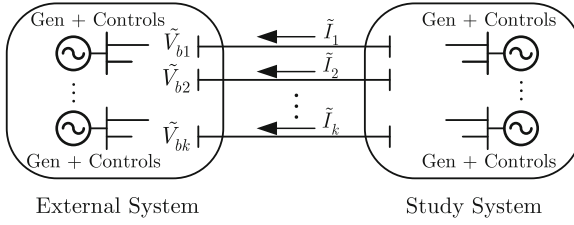
In the simulation of power system dynamics for stability analysis on a digital computer, a prudent approach is to develop the most comprehensive power system models so that the relevant dynamics can be accurately simulated given the computing resources and a desired simulation completion time.

One of the decisions that a power system engineer has to make is the geographic extent of the power system data set. Although the purpose of a stability investigation is to determine the dynamic response of generators and control systems in a study region due to disturbances inside the region, because of the interconnected nature of large power systems, these disturbances will impact the neighboring and other areas, the so-called external system, which in turn will impact the study region (Fig. 1.1).<sup>1</sup> For example, a short-circuit fault cleared by a line trip will redistribute the pre-fault

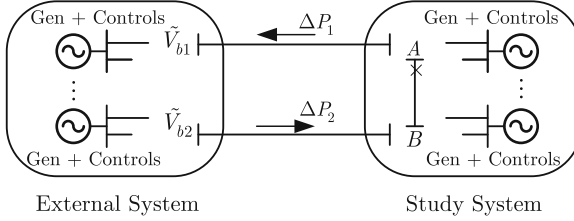
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<sup>1</sup> This discussion can readily be extended to multiple external systems, as well as some buffer or boundary systems between the study system and the external system.



**Fig. 1.1** Separation of a power system into a study system and an external system



**Fig. 1.2** Loop flow through the external system

flow on the tripped line to other paths, some of which may circulate through the external system. This situation is illustrated in Fig. 1.2, in which tripping the line from Bus A to Bus B causes a portion of the pre-fault line flow to be exported into the external system at one boundary bus and to return via another boundary bus. These uncontrolled loop flows need to be modeled accurately in order for the stability simulation to be valid. On the other hand, disturbances in the study region will most likely not excite significantly the internal dynamics of the external system. Thus a less detailed model of the external system can be used. As a result, for system studies with a strict turnaround time requirement, such as contingency analysis, or limited computing resources, power engineers commonly construct low-order models with the study region modeled in full detail and a reduced external system. For practical applications in large power grids, the low-order models, after reducing the size of the external system, may still have upwards of 5,000 buses and 1,000 generators.

To facilitate the discussions in the rest of this monograph, we express the power system model in the nonlinear differential-algebraic equation form for the study system as

$$\dot{x}_s = f_s(x_s, V_s, u_s, t), \quad V_s = g_s(x_s, x_e, u_s, u_e, t) \quad (1.1)$$

and the external system as

$$\dot{x}_e = f_e(x_e, V_e, u_e, t), \quad V_e = g_e(x_s, x_e, u_s, u_e, t) \quad (1.2)$$

where  $x$  denotes the state vector,  $V$  is a vector of the real and imaginary parts of the voltage phasors  $\tilde{V}$ ,  $u$  is the control,  $t$  denotes time,  $f$  represents the dynamic equations such as the swing equation and excitation system models,  $g$  contains the

algebraic network equations, and the subscripts  $s$  and  $e$  denote “study” and “external”, respectively. Note that  $f_s$  and  $f_e$  depend only on the bus voltage phasors within the study area and the external system, respectively. The bus voltage phasors are, in general, functions of all the states of the interconnected study and external systems.

Thus the objective of the model reduction task is to obtain a model

$$\dot{x}_s = f_s(x_s, V_s, u_s, t), \quad V_s = g_s(x_s, \bar{x}_e, u_s, \bar{u}_e, t) \quad (1.3)$$

$$\dot{\bar{x}}_e = \bar{f}_e(\bar{x}_e, \bar{V}_e, \bar{u}_e, t), \quad \bar{V}_e = \bar{g}_e(x_s, \bar{x}_e, u_s, \bar{u}_e, t) \quad (1.4)$$

in which the external system has been reduced. Here  $\bar{x}_e$ ,  $\bar{V}_e$ , and  $\bar{u}_e$  are the variables,  $\bar{f}_e$  is the dynamic equation, and  $\bar{g}_e$  is the network equation for the reduced external system. Often the boundary bus voltages  $\bar{V}_{bi}$ ,  $i = 1, \dots, k$ , are kept in the reduced external system, so that the interface currents  $\bar{I}_i$ ,  $i = 1, \dots, k$ , between the study system and the reduced external system can still be computed individually. The reduced external system may also be a linear model.

Many model reduction techniques for general applications have been proposed and advanced techniques are still being developed. We will limit the model reduction overview here to those methods that have been applied to power systems. To provide some structure to the overview, we group the methods under coherency and aggregation, linear and nonlinear input–output methods, and the relatively new phasor measurement-based method.

## 1.2 Coherency

The practical method most commonly used by power utilities to derive reduced models of large power systems is based on the concept of coherency and aggregation. This method uses the inherent properties, such as line admittances and loading, and machine inertias, in a practical power system to derive a reduced nonlinear external system that retains the relevant dynamics (1.4). The method consists of two main steps: (1) identifying coherent groups of machines, and (2) aggregating each coherent group of machines into a single equivalent machine.

### 1.2.1 Time Simulation

Coherency means that some machines exhibit similar rotor angle swings after a disturbance. Podmore, deMello, and Germond proposed using linear time simulation to identify the coherent groups of machines [1–3]. The advantage is that linear time simulation can be computed much more quickly than a full nonlinear simulation. System disturbances can be modeled as equivalent injections so that the admittance matrix does not need to be rebuilt and refactorized. From the time response of the

synchronous machines, a grouping algorithm with a specific tolerance value for coherency can be used to identify the coherent machines.

### ***1.2.2 Modal Coherency***

Following Podmore's coherency result and Undrill's modal equivalent result (see Sect. 1.4), Schlueter and coworkers [4] proposed a modal-coherency approach in identifying coherent groups that can approximately capture the modal frequencies. The key idea is the use of a zero-mean, independent, and identically distributed disturbance to compute a rms coherency measure. By letting the time period of simulation approach infinity, an analytical formula without numerical integration can be developed. The success of the method was partly attributed to the rms measure being determined by the synchronizing torque coefficients, which depends on transmission line stiffness and generator inertias.

### ***1.2.3 Slow Coherency***

Shortly after the work of [2, 3], a group of singular perturbations researchers from the University of Illinois, Urbana-Champaign and the General Electric Company, Schenectday, New York, investigated the use of singular perturbations for power system model reduction. One of the significant results is establishing the connection between slow coherency and weak connections in power systems [5, 6]. Slow coherency is coherency arising from the slower interarea modes, which are oscillatory modes due to groups of machines oscillating against each other across power transfer interfaces. These interarea modes, if negatively damped, can lead to system separation and extensive loss of load [7].

From the slow coherency theory, the eigen-subspace of the interarea modes can be used to identify the slow coherent machines. To apply this grouping method, only the slowest electromechanical modes and their mode shapes represented by eigenvectors need to be computed. For large power systems, sparsity-based computation methods for calculating only the slow eigenvalues and their corresponding eigenvectors are preferred. An early attempt using the Lanczos method was documented in [8]. More successful methods include the inverse iteration method [9], the S-matrix method [10], and the Arnoldi method [11].

### ***1.2.4 Weak-Link Methods***

The grouping algorithm proposed in [5] requires the computation of eigenvalues, which can be time consuming for large power systems, despite the availability of

sparsity-based partial eigenvalue and eigenvector computation routines as mentioned earlier. It may be desirable to develop methods to identify the coherent areas without eigensubspace computation. The weak-connection and slow-coherency relationship in [5] points to exploring weak connections to find the coherent groups. In [12], a clustering algorithm based on the state matrix derived from the synchronizing torque coefficients and the machine inertias has been proposed to find the weakly connected machine clusters. A reduced incidence matrix is constructed by setting rows of off-diagonal entries of the state matrix whose sum is less than a threshold to zero. The connectivity that remains in the incidence matrix defines the slow coherent groups of machines.

The results in [13] extend the search of weak links to form weakly coherent areas and strongly coherent areas. The identification procedure starts by iteratively computing a coupling factor derived from the synchronizing torque coefficients, through a sequential search of the machines. Then relative changes in the coupling factors define the weakly coherent groups, similar to the slow coherent groups from the grouping algorithm [5]. In addition, relative changes in the second variation of the coupling factors can be used to determine the strongly coherent areas. The technique has been demonstrated on a 50-machine model of the northern India power grid.

## 1.3 Aggregation

The second part of the coherency approach is to aggregate each coherent group of machines into a single equivalent machine, followed by eliminating load buses in the external system that are not needed.

### 1.3.1 Generator Aggregation

The technique proposed by Podmore and Germond [3] is to connect all the coherent generators to a common bus. All the generators are then aggregated into a single generator. This equivalent generator construction preserves the power flow in the power system as well as the power system model and data structure. Furthermore, exciter and governing capability of the equivalent generator can be obtained from an aggregate frequency response approximation.

### 1.3.2 Singular Perturbation Models

An advantage of the singular perturbations method is that it generates asymptotic series expansion terms to improve the slow subsystem [14]. For power system model reduction, it is possible to improve the reduced model by aggregating the

coherent generators at the generator internal node, rather than at the generator bus as in the Podmore aggregation [15, 16]. The inertia aggregation method and the Podmore method both stiffen the power network by adding connections with infinite admittances. This stiffening causes the reduced model to have higher interarea mode frequency. Another term can be taken from the asymptotic series expansion to reduce the stiffening effect using only parameters from within the coherent area [15, 16].

## 1.4 Linear Equivalent Input–Output Models

One of the premises of reducing the external system is that it is not perturbed significantly by a disturbance in the study area. As a result, the external system can be represented by a linear model. To derive the linear model, one can detach the external system from the study system, in which the tielines to the study system are represented by current injections, as shown in Fig. 1.3.

The input–output model of the external system can be represented by

$$\dot{\bar{x}}_e = \bar{f}_e(\bar{x}_e, \bar{V}_e, u_e, \bar{I}, t), \quad \bar{V}_e = \bar{g}_e(\bar{x}_e, u_e, \bar{I}, t), \quad \bar{V}_b = \bar{g}_{be}(\bar{x}_e, u_e, \bar{I}, t) \quad (1.5)$$

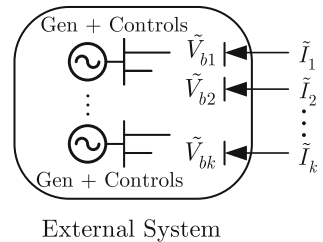
in which the tieline current injections  $\bar{I}$  are the input to the model and the boundary bus voltages  $\bar{V}_b$  are the output. The dynamic equation  $\bar{f}_e$  is driven by the bus voltages  $\bar{V}_e$  and the input  $\bar{I}$ . Note that the network equation  $\bar{g}_e$  is no longer dependent on the study network variables. In particular, the current injection  $\bar{I}$  is an independent variable, and no longer a function of both the study and external systems.

Note that it is also possible to use a formulation with the boundary bus voltages as input variables and the currents injected into the study system as the output variables [17]. This formulation is equivalent to (1.5) if the boundary buses are extended into the study system.

One of the first model reduction ideas is to simply linearize the external model (1.5) as

$$\Delta \dot{\bar{x}}_e = A \Delta \bar{x}_e + B \Delta I, \quad \Delta \bar{V}_b = C \Delta \bar{x}_e + D \Delta I \quad (1.6)$$

**Fig. 1.3** Input–output model of the external system



at the pre-fault power flow condition. The network equation and any feedback control have been included in (1.6) without showing them explicitly. Expressions for linearized models can be found in [18]. Alternatively, linearized models can be obtained via a numerical derivative process [19].

Representing the external system with a linear model would decrease the computation needed for simulating a disturbance in the study system. The linear model will be able to account for the loop flow shown in Fig. 1.2. In addition, a linear network is solved in the external system. To achieve further computation reduction, various approaches have been proposed for reducing the linear model of the external system. A recent survey of linear model reduction approaches can be found in [20].

### 1.4.1 Modal Truncation

In the early 1970s, Undrill, Price, and others developed a modal truncation approach [17, 21–23] following such work as Davison [24] by determining the modes to be retained in the reduced external systems. In this approach, (1.6) is represented by a reduced model in the modal form

$$\Delta \dot{\bar{x}}_m = A_m \Delta \bar{x}_m + B_m \Delta I, \quad \Delta \bar{V}_b = C_m \Delta \bar{x}_m + D \Delta I \quad (1.7)$$

in which only the dominant modes, including the electromechanical modes, are retained. The dimension of  $\Delta \bar{x}_m$  is less than that of  $\Delta \bar{x}_e$ .<sup>2</sup> Furthermore, if the state matrix  $A_m$  is expressed as a diagonal matrix, in which  $2 \times 2$  diagonal blocks are used for complex eigenvalues, the computation needs during simulation can be further reduced. Special computer code needs to be developed to interface the reduced linear model of the external system to the study system in a nonlinear power system simulation program.

One of the concerns with the modal truncation approach is that it does not preserve steady-state values [20]. This shortfall can be overcome using the singular perturbations technique [14]. However, the modes kept by the reduced model, the so-called slow subsystem, will no longer be identical to the modes of the full external system model.

### 1.4.2 Selective Modal Analysis

The modal truncation method [24] and the singular perturbations method [14] eliminate the modes with fast decaying transients and as such are less important.

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<sup>2</sup> Note that (1.7) can be shown to be equivalent to the formulation in [17] using the study system boundary buses as inputs and current injected into the study system as the outputs, and maintaining a linearized model of the external system power network.

The selective modal analysis technique designates the modes with high modal participating factors as relevant modes which will be retained, and those modes with lower modal participating factors as less relevant modes which will be eliminated [25, 26]. From a state-space model form, the relevant reduced model can be computed iteratively. The reduced models are also suitable for designing damping controllers, such as power system stabilizers.

### 1.4.3 Krylov and Balanced Model Reduction Methods

Another modal elimination approach is based on balanced truncation methods, which eliminate modes that are less controllable or observable [27, 28]. With balanced truncation, the reduced linear model of the external system will have the model

$$\Delta \dot{\bar{x}}_{br} = A_{br} \Delta \bar{x}_{br} + B_{br} \Delta I, \quad \Delta \bar{V}_b = C_{br} \Delta \bar{x}_{br} + D \Delta I \quad (1.8)$$

where the reduced matrices ( $A_{br}$ ,  $B_{br}$ ,  $C_{br}$ ) are obtained using controllability and observability Gramians. Such methods do not keep the external system modes exactly but provide a better frequency response approximation of the input–output model compared to the modal truncation method. These balanced truncation methods are supported by efficient computation algorithms [29].

A third class of linear model reduction methods is based on the Krylov method [30], in which the Markov parameters of the linear input–output model (1.6) are preserved up to a certain index. This matching part of the controllability–observability subspace is also known as moment matching. The Krylov is less computationally expensive than the balanced truncation method. The method does not provide any error bounds but, in general, seems to work well.

## 1.5 Nonlinear Equivalent Input–Output Models

Besides reduced linear models, an alternative approach for capturing the nonlinear dynamics of the input–output model (1.5) shown in Fig. 1.3 is to use reduced nonlinear models. In general, such nonlinear model reduction methods have to be tailored to the characteristics of the physical processes. Power systems have many different types of nonlinearities, such as sinusoidal functions, deadbands, saturation functions, and limits, to name a few. The coherency and aggregation method discussed earlier attempts to preserve these nonlinearities. A few methods have been proposed for developing reduced nonlinear external systems, including the two methods described next.



### ***1.5.1 Singular Perturbations Methods***

Once slow coherency has been identified for a power system, one can obtain the quasi-steady-state approximation from singular perturbations as presented in [31]. In this approach, the intraarea modes within each coherent area are assumed to be fast and have settled to their quasi-steady-state values. As a result, the differential equations describing the intra-area modes are solved as algebraic equations. This approach would require the development of special computer code to perform the quasi-steady solution in a conventional power system simulation program [31].

### ***1.5.2 Computational Intelligence Methods***

Computation intelligence methods have been used to capture nonlinear model dynamics [32]. One such non-model based method is the artificial neural network (ANN) approach [32]. In an ANN, neurons represented by selected nonlinear functions are arranged in layers connected by weights. These weights are trained from input–output data using a variety of tuning methods. Upon convergence, the method provides a reduced nonlinear model. The ANN can be used to represent all or part of an external system.

## **1.6 Measurement-Based Reduced Models**

Most of the model reduction techniques developed until recently are all based on power system load flow and dynamic data sets. With the advent of synchrophasor measurement technology [33], time-synchronized voltage and current phasor measurements across wide areas can be obtained. These synchrophasor measurements, obtained from phasor measurement units (PMUs), are particularly useful for extracting interarea oscillatory modes and their mode shapes [34]. Such a capability opens up the possibility of using synchrophasor measurements to construct simple interarea models. The results in [35] show that by monitoring voltage phasors at both ends of a power transfer path, it is possible to develop a simple two-area model to emulate the oscillatory modes and establish an energy function. Interestingly, the results were derived from examining the voltage magnitude oscillation of the interarea mode along the transfer path such that the effective impedance connecting the two areas can be computed. This property has been overlooked in coherency and other power system dynamics literature. Research is ongoing to extend this interarea model estimation method to develop simple models for multiple interarea modes.

## 1.7 Applications

In this section, we discuss some of the impacts of the research on power system model reduction. This list is by no means exhaustive.

### *1.7.1 Dynamic Model Reduction Programs*

Model reduction programs are available for practicing power engineers to reduce data sets of upwards of 30,000 buses to a more manageable 5,000–10,000 buses, which can be handled by power system analysis programs with functions such as transient stability simulation, voltage stability analysis, and optimal power flow. The main steps of the model reduction programs still follow the original coherency and aggregation approach [2] proposed more than 30 years ago.

### *1.7.2 Interarea Mode Analysis and Damping Control Design*

The development of coherency has contributed to the understanding of interarea mode oscillations. Interarea mode damping is an operational concern for systems with heavily loaded long distance transmission lines. Recommendations for interarea mode damping enhancement with power system stabilizers can be found in [36] and for flexible AC transmission systems in [37, 38]. An interarea damping controller applied to a Thyristor-Controlled Series Compensator (TCSC) is critical in the operation of the Brazil North–South Intertie [39]. With the deployment of synchrophasor measurement systems [33], the center of angle or speed of a coherent area, can be measured precisely. Hence, the interarea modes can be computed accurately in real-time and be used to improve the performance of wide-area damping control.

### *1.7.3 Islanding*

System separation or islanding is often a last but necessary resort to prevent a cascading blackout by preserving viable islands of generation and load. This is a difficult task because it needs to balance a number of factors, such as the load and generation balance in each island and the subsequent generator swings after separation. An adaptive out-of-step relay-based islanding strategy using synchrophasors and coherency on a radial-like system (US Florida-Georgia interface) is described in [40]. Another islanding strategy [41, 42] for a more complex situation is to identify the tielines connecting multiple slow coherent groups as cutsets. Such islands have the advantage

of strong internal connections such that after islanding, the sum of the synchronizing torques and hence the potential energy [43] in the islands are high.

### ***1.7.4 Dynamic Security Analysis***

Power system dynamic security analysis concerns transient, voltage, and small-signal stability [44] and normally gives a yes-or-no answer for a given set of contingencies. An important issue that is not well studied is how interarea modes travel through the various coherent areas for each of the contingencies and the voltage fluctuations on the transfer paths. A practical example of such an application is that the power import on a HVDC system is limited by the voltage stability of a transfer path in a neighboring area; that is, a bipolar fault on the HVDC system would cause voltage collapse on that transfer path. Such wide-area stability issues will become more important with higher power transfer levels between operating regions. Some investigations of the interarea modes in power networks can be found in [45].

## **1.8 Chapter Guide**

In this monograph, we selectively cover three of the model reduction concepts and approaches discussed earlier in this chapter.

The first topic is on coherency and aggregation and is covered in the following chapters:

1. Chapter 2 describes the coherency and aggregation ideas.
2. Chapter 3 describes the slow coherency concept and algorithms.
3. Chapter 4 describes a method to obtain equivalent nonlinear exciter models.
4. Chapter 7 describes the practical application of the dynamic model reduction program (DYNRED) on a large power system.

The second topic is on input–output models for external systems. Chapter 5 describes an approach using ANN to model the external system as a nonmodel-based nonlinear system. The ANN is successfully trained to show an improvement in the time response of the reduced-order model. There are also two chapters on linear reduced-order models:

1. Chapter 6 is on the Krylov method and the balanced truncation method.
2. Chapter 9 describes the selective modal analysis method.

In Chap. 9, the use of selective modal analysis technique for control design is also discussed.

The third topic is on using synchrophasor measurements to develop simple interarea models. Chapter 8 describes the Interarea Model Estimation method and uses measured PMU data from several WECC disturbances to develop two-area models for two transfer paths in the WECC system.

Many of the model reduction methods are based on analyzing interarea modes. Although coherency depends only on the mode shapes of the generators, there are many other interesting and important issues related to interarea modes. The last chapter, Chap. 10, in this monograph discusses the use of synchrophasor data to track the propagation of interarea mode oscillations resulting from large disturbances.

## 1.9 Conclusions

In this chapter, we have provided a brief survey of some of the fundamental approaches and results in equivalencing and model reduction of dynamic power system models. Methods using similar concepts or techniques are grouped together to provide a reader with a better perspective of the field. In the remainder of this monograph, various authors will share their many years of research and development results in this important field.

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