Chapter 8
Dynamic Altitude Synchronization Using Graph Theory

8.1 Introduction

The Q-structure proposed in Chap. 7 provides a promising avenue for formation control of helicopters under a flexible and scalable framework. We presented a kinematic control scheme that does not consider the dynamics of the helicopters, and is useful for formation motion planning to determine the desired motion of the helicopters. Dynamic formation control using Q-structures in which the helicopter dynamics are directly taken into account in the formation control design can yield better flight performance, but it is an open and challenging problem. In this chapter, we take a different approach to solving the dynamic formation control problem, by combining graph theory with adaptive neural networks.

We focus on the synchronized tracking problem of helicopters in vertical flight, in which multiple helicopters track the same desired trajectory while the desired trajectory is not accessible to all the helicopters in the team. The vertical fight mode starts when the helicopter is at rest on the ground IGE (in ground effect). Then take-off is started and the helicopter climbs. Vertical descent precedes landing. Since the coupling between longitudinal and lateral-directional equations in this flight regime is weak, it can be presented by single-input–single-output (SISO) models with zero-dynamics to yield useful results [83, 102]. In the formation group, the desired trajectory is not available to all the helicopters in the team, synchronized tracking control is designed for each helicopter by using the information exchange with its neighbors. The main contributions of the work are as follows:

1. The extended formation graph Laplacian, which contains a spanning tree which the root helicopter can access for the desired trajectory, is proved to be positive definite.
2. The neural approximation based control is designed for the purpose of synchronized tracking of each helicopter by using the weighted average of its neighbors' states. All signals are proved to be bounded and the tracking errors of all helicopters will converge to a neighborhood of the origin.

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3. A high gain observer is employed for each helicopter to estimate the unaccessible derivation of the states of both itself and its neighbors. It is shown that in this case, the boundness of all the closed-loop signals are guaranteed.

8.2 Problem Formulation

8.2.1 Helicopter Dynamics

Consider the class of SISO helicopter systems described by

$$\begin{aligned} \dot{x}_j &= x_{j+1}, \quad j = 1, \ldots, \rho - 1 \\ \dot{x}_\rho &= f(\eta, x) + g(x, \eta)(u + d) \\ \dot{\eta} &= q(x, \eta) \\ y &= x_1 \end{aligned} \tag{8.1}$$

where $x = [x_1, \ldots, x_\rho]^{\mathrm{T}} \in \mathbb{R}^\rho$ and $\eta \in \mathbb{R}^{n-\rho}$ are the states of the system, $u,\ y \in \mathbb{R}$ the input and output, respectively, $f : \mathbb{R}^n \to \mathbb{R}$ an unknown smooth function, and $q : \mathbb{R}^n \to \mathbb{R}$ is a partially unknown vector field satisfying certain properties, which will be described shortly, $g : \mathbb{R}^n \to \mathbb{R}$ is an unknown function with certain properties, and d is the external disturbance in the input channel.

Assumption 8.1. The zero dynamics of system (8.1), given by $\dot{\eta} = q(x, \eta)$, are exponentially stable. In addition, $q(\xi, \eta)$ is Lipschitz in x, i.e., there exists positive constants a_q and a_x such that

$$\|q(x, \eta) - q(0, \eta)\| \leq a_x \|x\| + a_q \quad \forall (x, \eta) \in \mathbb{R}^n \tag{8.2}$$

Under the assumption that the zero dynamics are stable, by the converse Lyapunov theorem, there exists a Lyapunov function $V_0(\eta)$ which satisfies the following Lyapunov inequalities for $(x, \eta) \in \mathbb{R}^n$:

$$\gamma_1 \|\eta\|^2 \leq V_0(\eta) \leq \gamma_2 \|\eta\| \tag{8.3}$$

$$\frac{\partial V_0}{\partial \eta} q(0, \eta) \leq -\lambda_a \|\eta\|^2 \tag{8.4}$$

$$\left\| \frac{\partial V_0}{\partial \eta} \right\| \leq \lambda_b \|\eta\| \tag{8.5}$$

where γ_1 , γ_2, λ_a, and λ_b are positive constants.

Assumption 8.2. The external disturbance d is an uncertain bounded function $d \in L_\infty$. That is, there exists unknown positive constants ϱ such that $|d(t)| \leq \varrho < \infty$ where ϱ can be arbitrarily large.

Assumption 8.3. There exist smooth functions $\bar{g}(x,\eta)$ and a positive constant $\underline{g} > 0$, such that $\bar{g}(x,\eta) \geq g(x,\eta,u) > \underline{g} > 0$, $\forall(x,\eta) \in \bar{U}$. Without loss of generality, it is further assumed that the sign of $g(x,\eta,u)$ is positive $\forall(x,\eta) \in \bar{U}$.

Assumption 8.4. There exists a positive function $g_0(x,\eta)$ satisfying $|\dot{g}(x,\eta)/2g(x,\eta)| \leq g_0(x,\eta)$, $\forall(x,\eta) \in \bar{U}$.

Remark 8.1. The SISO representation considered in this chapter is valid for simple operations involving the regulation or tracking of a single degree of freedom, such as altitude tracking and pitch regulation, among others. The general nonlinear SISO helicopter model can be described in [48]

$$\begin{aligned}\dot{x} &= f(x,u)\\ y &= h(x)\end{aligned} \tag{8.6}$$

with some assumptions such as it can be input–output linearizable with strong relative degree $\rho < n$, which can be described as (8.1). In addition, we will show that the helicopter given in Sect. 4.5.2, which will be used in the subsequent simulation section, can be changed to (8.1) and satisfies the above assumptions.

8.2.2 Formation Control of Helicopters

We associate the helicopters with nodes in a graph and information exchange with the graph edges. The Following definitions are useful for describing the formation.

Definition 8.2. [24] A directed graph $\mathcal{G}'$ consists of a non-empty finite set $\mathcal{V}'$ of elements called nodes and a finite set $\mathcal{E}' \subset \mathcal{V}'^2$ of ordered pairs of nodes called arcs, where $e = (v_i, v_j) \in \mathcal{E}'$ and $v_i, v_j \in \mathcal{V}'$. The neighbors set of vertical v_i is defined as $\mathcal{N}_i' = \{v_j \in \mathcal{V}' |\ (v_j, v_i) \in \mathcal{E}'\}$.

For the multiple agents tracking problem, we introduce a virtual agent v_0, whose motion follows the desired trajectory restrictively. And we define a non-empty set $\mathcal{V}_0 \subset \mathcal{V}'$, in which the elements can access the desired trajectory, i.e., $v_0 \in \mathcal{N}_j$, iff $v_j \in \mathcal{V}_0$. Then the extended formation graph can be described as $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \mathcal{V}' \cup \{v_0\}$, and $\mathcal{E} = \mathcal{E}' \cup \{(v_0, v_j)|v_0 \in \mathcal{N}_j\}$. For all agents $v_j \in \mathcal{V}_0$, $\mathcal{N}_j = \mathcal{N}_j' \cup \{v_0\}$.

Definition 8.3. The weighted adjacency matrix of the extended formation graph $\mathcal{G}$, denoted as $A^*(\mathcal{G})$, is a square matrix of size $|\mathcal{V}|$, with its elements $A^*_{ij} > 0$ if $(v_j, v_i) \in \mathcal{G}$, and is zero otherwise. Define a diagonal matrix $\Delta(\mathcal{G})$ with its elements $\Delta_{jj} = \sum_k A^*_{jk}$, and the normalized Laplacian of the graph is defined as $L = I - A$ with $A = \Delta^{-1}A^*$.

Definition 8.4. A spanning tree of a directed graph $\mathcal{G}'$ is a directed tree formed by graph edges that connect all the nodes of the graph. We say that a graph has (or contains) a spanning tree if a subset of the edges forms a spanning tree.

Definition 8.5. A substochastic matrix is a square matrix with nonnegative entries such that every row adds up to at most 1.

Definition 8.6. A directed graph is called weakly connected if there exits a node which is globally reachable.

Definition 8.7. [92] If matrix $L = (\ell_{ij}) \in \mathbb{R}^{(n+1)\times(n+1)}$ satisfies the following three conditions:

1. $|\ell_{ii}| \geq \sum_{j\neq i} |\ell_{i,j}|, \quad (i = 0, 1, \ldots, n)$;
2. $J = \{k \in N|\ |\ell_{kk}| > \sum_{j=1, j\neq k}^{n} |\ell_{kj}|\} \neq \varnothing$, where $N = \{0, 1, \ldots, n\}$, it also means that there at least exists an i that satisfies $|\ell_{ii}| > \sum_{j\neq i} |\ell_{ij}|$; and
3. For each $i \notin J$ there exists a sequence of nonzero elements of L with the form $\ell_{ii_1}, \ell_{i_1 i_2}, \ldots, \ell_{i_s k}$ with $k \in J$.

Then we say L is a diagonally dominant matrix with nonzero elements chain.

Property 8.8. [92] For a diagonally dominant matrix with nonzero elements chain $L = (\ell_{ij})$, we have the following properties:

1. L is a nonsingular matrix;
2. If $B = I - D^{-1}L$, where $D = \mathrm{diag}\{\ell_{11}, \ldots, \ell_{nn}\}$, $\ell_{ii} \neq 0$, then $\rho(B) < 1$, where $\rho(B)$ is the spectrum of B; and
3. If L is real and $\ell_{ij} \leq 0$, $\ell_{ii} > 0$, then L is an M-matrix.

Theorem 8.9. *Consider the multiple agent synchronized tracking problem, if the formation graph $\mathcal{G}'$ contains a spanning tree with its root $v_j \in \mathcal{V}_0$. Then the normalized adjacent matrix A of the extended formation graph $\mathcal{G}$ is sub-stochastic, and $L = I - A$ is positive definite, which inverse is given by $L^{-1} = \sum_{l=0}^{\infty} A^l$.*

Proof. By introducing the virtual agent v_0, we know that $\mathcal{N}_0 = \varnothing$ in the extended formation graph $\mathcal{G}$, as it does not accept any other agents' information and follows the desired trajectory strictly, and we also know that all the elements of the first row of A are zero. Since $\mathcal{G}'$ has a spanning tree and $v_j \in \{\mathcal{V}_0\}$ is the root, this means that each agent has at least one neighbor, therefore the sum of any other row of A equals to 1. According to Definition 8.5, we know that A is a sub-stochastic matrix.

It is clear that all the diagonal elements of L are 1, and all the row sums of A are 1 except the first row, this means L is a diagonal dominant matrix with the $J = \{0\}$ (Since the virtual agent is added, we start the row number from 0 corresponding to the label of agents). Let us revisit that $\mathcal{G}'$ has a spanning tree with $v_j \in \mathcal{V}_0$ as the root, this also means that there is a path from v_0 to any agent $v_i \in \mathcal{V}$; therefore, in the matrix L, for every element $i \neq 0$, there exists a sequence of nonzero elements form $\ell_{ii_1}, \ell_{i_1 i_2}, \ldots, \ell_{i_s 0}$. Then L satisfies all the conditions of Definition 8.7. Since L is real and $\ell_{ij} < 0, i \neq j,\ \ell_{ii} = 1$, According to Property 8.8, L is a nonsingular M-matrix [92]. By using Gerschgorin disc theory, we also know that all

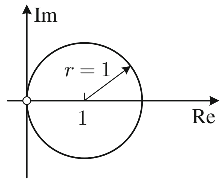
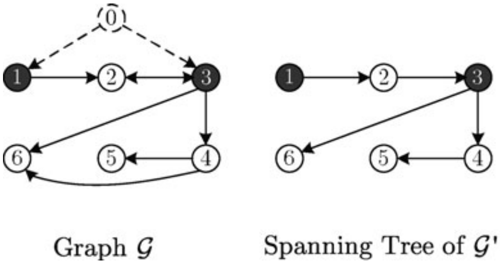


Fig. 8.1 Eigenvalue distribution of $L = (I - A)$



$$L = \begin{array}{c} \left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2/3 & 1 & -1/3 & 0 & 0 & 0 \\ -3/4 & 0 & -1/4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1/2 & -1/2 & 0 & 1 \end{array}\right] \end{array} \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Matrix L

Fig. 8.2 Sample graph and its Laplacian

the eigenvalues of L lie in the right part of the complex plane as shown in Fig. 8.1; therefore, we can conclude that L is positive definite. Furthermore, it follows from $\rho(A) < 1$ that $\lim_{l\to\infty} A^l = 0$. Then,

$$(I - A)(I + A + A^2 + \cdots) = (I + A + A^2 + \cdots) - (A + A^2 + A^3 + \cdots) = I \quad (8.7)$$

We obtain $L^{-1} = \sum_{l=0}^{\infty} A^l$. This completes the proof. □

Example 8.10. To demonstrate Theorem 8.9 clearly, take the sample graph shown in Fig. 8.2 for example. Both v_1 and v_3 can access the desired trajectory and $\mathcal{G}'$ contains a spanning tree with 1 as its root. Take the node 5 for example, we can find that in the Laplacian matrix L, there exists a sequence $\ell_{54}, \ell_{43}, \ell_{32}, \ell_{21}, \ell_{10} \neq 0$.

In this chapter, we studied the synchronized tracking problem of multiple unmanned helicopters as follows:

Considering a group of helicopters, the desired trajectory of the team $y_d(t)$ and its derivations up to ρ-th order are bounded, and are only available to the helicopters $v_j \in \mathcal{V}_0$. For each helicopter, design a control, (1) using its own full states and its neighbors' full states and (2) using its outputs and its neighbors' outputs, such that

$$\lim_{t\to\infty} |y_i(t) - y_d(t)| = \bar{\varepsilon}, \quad i = 1, \ldots, N \quad (8.8)$$

where $\bar{\varepsilon}$ is a small positive constant.

The desired trajectory $y_d(t)$ is generated by the following reference model:

$$\begin{aligned} \dot{x}_{dj} &= \dot{x}_{di+1}, \quad i = 1, \ldots, \rho - 1 \\ \dot{x}_{d\rho} &= f_d(x_d, t) \\ y_d &= x_{d1} \end{aligned} \tag{8.9}$$

where $\rho \geq 2$ is a constant index, $x_d = [x_{d1}, \ldots, x_{d\rho}]^{\mathrm{T}} \in \mathbb{R}^{\rho}$ are the states of the reference system, and $y_d \in \mathbb{R}$ is the system output.

Assumption 8.5. The reference trajectory $y_d(t)$ and its ρ-th derivatives remain bounded, i.e., $x_d \in \Omega_d \subset \mathbb{R}^{\rho}$, $\forall t \geq 0$.

Assumption 8.6. The formation graph $\mathcal{G}'$ of the helicopter group has a spanning tree which the root helicopter can access for the desired trajectory.

The following lemma is useful for analysis of the internal dynamics of the helicopter.

Lemma 8.11. *[35] Denote positive constants $a_1 = (\lambda_b a_x)/\lambda_a$ and $a_2 = (\lambda_b a_q)/\lambda_a$. If Assumptions 8.1 and 8.5 satisfied, there exists a positive constant T_0 such that the trajectories $\eta(t)$ of the internal dynamics satisfy*

$$\|\eta\| \leq a_1 \|x(t)\| + a_2 \tag{8.10}$$

8.3 Control with Full Information

In this section, we design the tracking control for each helicopter using the full information of itself and its neighbors. The adaptive NN control scheme is constructed for the synchronized tracking control. Since not all the helicopters can access the information of the desired trajectory, the tracking control is designed based on the relative states with its neighbors. Define the following error variables for the helicopters:

$$z_{i,1} = y_{i,1} - y_{ir}\,, \quad z_{i,2} := \dot{z}_{i,1} = x_{i,2} - \dot{y}_{ir}, \ldots, \quad z_{i,\rho} := z_{i,1}^{(\rho)} = x_{i,\rho} - y_{ir}^{(\rho)} \tag{8.11}$$

with

$$y_{ir}(t) = \sum_{j \in \mathcal{N}_i} a_{ij} y_j(t), \qquad y_{ir}^{(k)}(t) = \sum_{j \in \mathcal{N}_i} a_{ij} y_j^{(k)}(t), \quad k = 1, \ldots, \rho - 1 \tag{8.12}$$

where a_{ij} is the element of the normalized adjacent matrix A of the extended formation graph $\mathcal{G}$.

Remark 8.12. In (8.12), we defined that the reference state of each helicopter is the weighted average of its neighbors' states. If the helicopter v_i can access the desired trajectory, the virtual agent v_0 is viewed as one of its neighbors, and $1 \geq a_{i0} > 0$. While considering the pure tracking, we may choose $a_{i0} = 1$ for better tracking performance. In the synchronized tracking problem, if v_i has other neighbors $v_j \in \mathcal{N}_i$, we prefer to choose $a_{i0} < 1$ for better synchronization with its neighbors.

For each helicopter, we define vectors $\bar{z}_i$, and $\mathcal{Z}_i$ as

$$\begin{aligned}\bar{z}_i &= [z_{i,1}, \ldots, z_{i,\rho}]^{\mathrm{T}} \in \mathbb{R}^{\rho} \\ \mathcal{Z}_i &= [z_{i,1}, \ldots, z_{i,\rho-1}]^{\mathrm{T}} \in \mathbb{R}^{\rho-1}\end{aligned}$$

and the filtered tracking error as

$$s_i = [\Lambda^{\mathrm{T}} \;\; 1]\bar{z}_i \tag{8.13}$$

where $\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_{\rho-1}]^{\mathrm{T}}$ is an appropriately chosen coefficient vector so that $z_{i,\rho} \to 0$ as $s_i \to 0$, i.e., $p^{\rho-1} + \lambda_{\rho-1} p^{\rho-2} + \cdots + \lambda_1$ is Hurwitz. Then we have

$$\dot{\mathcal{Z}}_i = A_p \mathcal{Z}_i + b s_i \tag{8.14}$$

where $A_p = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \\ -\lambda_1 & -\lambda_2 & \cdots & -\lambda_{\rho-1} \end{bmatrix}$, and $b = [\underbrace{0, \ldots, 0}_{\rho-2}, 1]^{\mathrm{T}}$.

The dynamics of s_i are written as

$$\dot{s}_i = f_i(x_i, \eta_i) + g_i(u_i + d_i) + [0 \;\; \Lambda^{\mathrm{T}}]\bar{z}_i - y_{ir}^{(\rho)} \tag{8.15}$$

Consider the Lyapunov function candidate

$$V_i = \frac{1}{2g_i} s_i^2 + \frac{1}{2\gamma_2} \tilde{\theta}_i^{\mathrm{T}} \tilde{\theta}_i + \frac{1}{2\gamma_1} \tilde{\varphi}_i^2 \tag{8.16}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i^*$, and $\tilde{\varphi}_i = \hat{\varphi} - \varphi_i^*$ are the estimated errors of parameters and the error bounded, respectively.

Then,

$$\begin{aligned}\dot{V}_i &= -\frac{\dot{g}_i}{2g_i^2}s_i^2 + \frac{1}{g_i}s_i\dot{s}_i + \frac{1}{\gamma_2}\tilde{\theta}_i\dot{\tilde{\theta}}_i + \frac{1}{\gamma_1}\tilde{\varphi}_i\dot{\tilde{\varphi}}_i \\ &= -\left(g_0 + \frac{\dot{g}_i}{2g_i^2}\right)s_i^2 + s_i(u_i + d_i) + s_i\frac{f_i(x_i,\eta_i) + [0 \quad \Lambda^{\mathrm{T}}]\bar{z}_i - y_{ir}^{(\rho)} + g_i g_0 s_i}{g_i} \\ &\quad + \frac{1}{\gamma_2}\tilde{\theta}_i\dot{\tilde{\theta}}_i + \frac{1}{\gamma_1}\tilde{\varphi}_i\dot{\tilde{\varphi}}_i \end{aligned} \tag{8.17}$$

We use the parameter linearized NN to approximate the unknown nonlinear function $\bar{f}_i(x_i, \eta_i, \bar{z}_i, y_{ir}^{(\rho)}) = \frac{f_i(x_i,\eta_i)+[0 \quad \Lambda^{\mathrm{T}}]\bar{z}_i - y_{ir}^{(\rho)} + g_i g_0 s_i}{g_i}$, which can be described as

$$\bar{f}_i(Z_i) = \theta_i^{*\mathrm{T}}\varphi_i(Z_i) + \bar{\varepsilon}_i \tag{8.18}$$

where $Z_i = [x_i, \eta_i, \bar{z}_i, y_{ir}^{(\rho)}]^{\mathrm{T}}$.

Remark 8.13. The NN is constructed to approximate $\bar{f}_i(x_i, \eta_i, \bar{z}_i, y_{ir}^{(\rho)}) = \frac{f_i(x_i,\eta_i)+[0 \quad \Lambda^{\mathrm{T}}]\bar{z}_i - y_{ir}^{(\rho)} + g_i g_0 s_i}{g_i}$ on a whole, which avoids the possible singularity of the direct approximation of g_i.

Select the following control u_i for each helicopter

$$u_i = -\hat{\theta}_i^{\mathrm{T}}\psi_i - k_i s_i - \frac{1}{2}\hat{\varphi}_i s_i, \quad i = 1, \ldots, N \tag{8.19}$$

where $\hat{\varphi}_i$ and $\hat{\theta}_i$ denote the estimate of $\varphi_i^* = (\varrho_i + \bar{\varepsilon}_i)^2$ and θ_i^*, respectively.

The update law of parameters are designed as

$$\begin{aligned}\dot{\hat{\varphi}}_i &= -\gamma_1\left[-\frac{1}{2}(1 - \varpi_\varphi)s_i^2 + \sigma_1\hat{\varphi}_i\right] \\ \dot{\hat{\theta}}_i &= -\gamma_2\big(-\psi_i s_i + \sigma_2\hat{\theta}_i\big)\end{aligned} \tag{8.20}$$

By using the Using Young's inequality, we have

$$\begin{aligned}-\sigma_2\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i &\le -\frac{\sigma_2}{2}\|\tilde{\theta}_i\|^2 + \frac{\sigma_2}{2}\|\theta_i^*\|^2 \\ -\sigma_1\tilde{\varphi}_i\hat{\varphi}_i &\le -\frac{\sigma_1}{2}\tilde{\varphi}_i^2 + \frac{\sigma_1}{2}\varphi_i^{*2} \\ (\varrho_i + \bar{\varepsilon}_i)s_i &\le \frac{1}{2} + \frac{1}{2}s_i^2\varphi_i^*\end{aligned}$$

Considering (8.19) and (8.20), the time derivation of V_i in the closed-loop trajectory can be written as

$$
\begin{aligned}
\dot{V}_i &= -\left(g_0 + \frac{\dot{g}_i}{2g_i^2}\right) s_i^2 - k_i s_i^2 + s_i\left(\bar{\varepsilon}_i + d_i - \frac{1}{2}\hat{\varphi}_i\right) - \sigma_2 \tilde{\theta}_i^{\mathrm{T}} \hat{\theta}_i + \frac{1}{2}(1-\varpi_\varphi) s_i \tilde{\varphi}_i s_i \\
&\quad -\sigma_1 \tilde{\varphi}_i \hat{\varphi}_i \\
&\le -k_i s_i^2 + \frac{1}{2} + \frac{1}{2} s_i^2 \varphi_i^* - \frac{1}{2} s_i^2 \hat{\varphi}_i - \frac{\sigma_2}{2}\|\tilde{\theta}_i\|^2 + \frac{\sigma_2}{2}\|\theta_i^*\|^2 + \frac{1}{2}\tilde{\varphi}_i s_i^2 - \frac{\sigma_1}{2}\tilde{\varphi}_i^2 \\
&\quad + \frac{\sigma_1}{2}\varphi_i^{*2} \\
&= -k_i s_i^2 - \frac{\sigma_1}{2}\tilde{\varphi}_i^2 - \frac{\sigma_2}{2}\|\tilde{\theta}_i\|^2 + \frac{\sigma_2}{2}\|\theta_i^*\|^2 + \frac{\sigma_1}{2}\varphi_i^{*2} + \frac{1}{2}
\end{aligned}
\tag{8.21}
$$

Then,

$$\dot{V}_i \le -c_{1i} V_i + c_{2i} \tag{8.22}$$

$$c_{1i} = \min\{k_i, \gamma_2\sigma_2, \gamma_1\sigma_1\} \tag{8.23}$$

$$c_{2i} = \frac{\sigma_2}{2}\|\theta_i^*\|^2 + \frac{\sigma_1}{2}\|\varphi_i^*\|^2 + \frac{1}{2} \tag{8.24}$$

Now define

$$\Omega_{si} = \left\{ s_i \,\middle|\, |s_i| \le \sqrt{\frac{2c_{2i}}{c_{1i}}} \right\} \tag{8.25}$$

$$\Omega_{\theta_i} = \left\{ (\tilde{\theta}_i, \tilde{\varphi}_i) \,\middle|\, \|\tilde{\theta}_i\| \le \sqrt{\frac{2c_{2i}}{\sigma_2}}, |\tilde{\varphi}_i| \le \sqrt{\frac{2c_{2i}}{\sigma_1}} \right\} \tag{8.26}$$

$$\Omega_{ei} = \left\{ (s_i, \tilde{\theta}_i, \tilde{\varphi}_i) \,\middle|\, k_i s_i^2 + \frac{\sigma_2}{2}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \frac{\sigma_1}{2}\tilde{\varphi}_i^2 \le c_{2i} \right\} \tag{8.27}$$

Since c_{1i}, σ_1, σ_2, and k_i are positive constants, we know that Ω_{si}, Ω_{θ_i} and Ω_{ei} are compact sets. Equation (8.22) shows that $\dot{V}_i \le 0$ once the errors are outside the compact set Ω_{ei}. According to the standard Lyapunov theorem, we conclude that s_i, $\tilde{\theta}_i$, and $\tilde{\varphi}_i$ are bounded. From (8.22) and (8.25), it can be seen that V_i is strictly negative as long as s_i is outside the compact set Ω_{si}. Therefore, there exists a constant T_1 such that for $t > T_1$, the filtered tracking error s_i converges to Ω_{si}, that is to say, $s_i \le \beta_{si}(k_i, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \theta_i^*, \varphi_i^*, \varepsilon_i^*) = \sqrt{2c_{2i}/c_{1i}}$.

Now we will show that all the helicopters will track the desired trajectory although only some of them can access the desired trajectory. Define the error between i-th helicopter and the desired trajectory as $\tilde{y}_i(t) = y_i(t) - y_d(t) = y_i(t) - y_0(t)$, and the auxiliary states of each helicopter $\xi_i(t) = [\Lambda^{\mathrm{T}} \;\; 1]Y_i$ with

$Y_i = [y_i, y_i^{(1)}, \dots, y_i^{(\rho-1)}]^{\mathrm{T}}$. The filtered error is denoted as $\tilde{\xi}_i(t) = \xi_i(t) - \xi_d(t) = \xi_i(t) - \xi_0(t)$.

Using the fact that $s_i(t) = \xi_i(t) - \sum_{j \in \mathcal{N}_i} a_{ij}\xi_j(t)$, we have

$$\begin{aligned}\tilde{\xi}_i &= \xi_i - \xi_0 \\ &= \sum_{j \in \mathcal{N}_i} a_{ij}\xi_j + s_i - \xi_0, \quad i = 1, \dots, N\end{aligned}$$

and in the vector form

$$\tilde{\xi} = A\xi + s - \xi_0 \mathbf{1} \tag{8.28}$$

where $\mathbf{1} = [1, \dots, 1]^{\mathrm{T}}$, $s = [s_0, s_1, \dots, s_N]^{\mathrm{T}}$, and A is the normalized adjacency matrix of the extended formation graph. Note that the elements in the first row of A are all equal to 0, and the other row summations of the matrix A are 1, and we have $[0, 1, \dots, 1]^{\mathrm{T}} = A[0, 1, \dots, 1]^{\mathrm{T}}$. Then,

$$\begin{aligned}\tilde{\xi} &= A(\tilde{\xi} + \xi_0\mathbf{1}) + s + [1, 0 \dots, 0]^{\mathrm{T}}\xi_0 - \xi_0\mathbf{1} \\ &= A\tilde{\xi} + [0, 1, \dots, 1]^{\mathrm{T}}\xi_0 + s + [1, 0 \dots, 0]^{\mathrm{T}}\xi_0 - \xi_0\mathbf{1} \\ &= A\tilde{\xi} + s \end{aligned} \tag{8.29}$$

Under the Assumption 8.6, we know that $L = (I - A)$ is an invertible matrix, and we have

$$\tilde{\xi} = L^{-1}s \tag{8.30}$$

Define vectors

$$\begin{aligned}\mathcal{Y} &= [Y_0^{\mathrm{T}}, Y_1^{\mathrm{T}}, \dots, Y_N^{\mathrm{T}}]^{\mathrm{T}} \\ \tilde{\mathcal{Y}} &= [\tilde{Y}_0^{\mathrm{T}}, \tilde{Y}_1^{\mathrm{T}}, \dots, \tilde{Y}_N^{\mathrm{T}}]^{\mathrm{T}} \\ X &= [X_0^{\mathrm{T}}, X_1^{\mathrm{T}}, \dots, X_{\rho-1}^{\mathrm{T}}]^{\mathrm{T}} \\ \tilde{X} &= [\tilde{X}_0, \tilde{X}_1, \dots, \tilde{X}_{\rho-1}]^{\mathrm{T}}\end{aligned}$$

where $X_j = [X_{0,j}, X_{1,j}, \dots, X_{N,j}]^{\mathrm{T}}$, $\tilde{X}_j = X_j - X_{jd} = X_j - y_0^{(j)}\mathbf{1}$, $\tilde{Y}_i = Y_i - Y_d = Y_i - Y_0$. Then we have

$$\dot{\tilde{\mathcal{Y}}} = \bar{A}_p\tilde{\mathcal{Y}} + \bar{b}\tilde{\xi} \tag{8.31}$$

where $\bar{A}_p = I_{N+1} \otimes A_p$ and $\bar{b} = I_{N+1} \otimes b$.

Considering (8.29), the error dynamics can be written as

$$\dot{\tilde{\mathcal{Y}}} = \bar{A}_p\tilde{\mathcal{Y}} + \bar{b}\tilde{\xi} = \bar{A}_p\tilde{\mathcal{Y}} + \bar{b}L^{-1}s \tag{8.32}$$

Lemma 8.14. *Define* $s_{i,\max} = \sup_{0\le\tau\le t}|s_i(t)|$, $\beta_{s_i} = \sup_{t>T_1|s_i(t)|}$, *and* $s_{\max,i}(t) = \max_i \sup_{0\le\tau\le t}|s_i(t)|$, *then the following equations hold:*

$$\|\tilde{\mathcal{Y}}(t)\| \le k_0 \mathrm{e}^{-\lambda_0 t}\|\tilde{\mathcal{Y}}(0)\| + \frac{k_0}{\lambda_0}\left[N\lambda_{\max}(L^{-1}) + N - 1\right]s_{\max,i}(t)$$

$$\|\tilde{\mathcal{Y}}(t)\| \le k_0 \mathrm{e}^{-\lambda_0 t}\left(\|\tilde{\mathcal{Y}}(0)\| + \frac{\mathrm{e}^{\lambda_0 T_1}}{\lambda_0}\beta_s(T_1)\right) + \frac{k_0}{\lambda_0}\beta_{s_T}$$

where $\beta_s(t) = N\lambda_{\max}(L^{-1})s_{\max,i}(t)$ *and* $\beta_{s_T} = N\lambda_{\max}(L^{-1})\sup_{T_1\le t}s_{\max,i}(t)$ *with constants* $\lambda_0 > 0$ *and* $k_0 > 0$.

Proof. From (8.32) and the fact that A_p is Hurwitz, we have

$$\tilde{\mathcal{Y}}(t) = \tilde{\mathcal{Y}}(0)\mathrm{e}^{\bar{A}_p t} + \int_0^t \mathrm{e}^{\bar{A}_p(t-\tau)}\bar{b}L^{-1}s\,\mathrm{d}\tau$$

$$\|\mathrm{e}^{\bar{A}_p t}\| \le k_0\mathrm{e}^{-\lambda_0 t}$$

Then,

$$\begin{aligned}
\|\tilde{\mathcal{Y}}(t)\| &\le k_0\mathrm{e}^{-\lambda_0 t}\|\tilde{\mathcal{Y}}(0)\| + \int_0^t \mathrm{e}^{-\lambda_0(t-\tau)}\left\|\bar{b}L^{-1}s\right\|\mathrm{d}\tau \\
&\le k_0\mathrm{e}^{-\lambda_0 t}\|\tilde{\mathcal{Y}}(0)\| + k_0\mathrm{e}^{-\lambda_0 t}\left[N\lambda_{\max}(L^{-1})s_{\max,i}(t)\right]\int_0^t \mathrm{e}^{\lambda_0\tau}\,\mathrm{d}\tau \\
&\le k_0\mathrm{e}^{-\lambda_0 t}\|\tilde{\mathcal{Y}}(0)\| + k_0\mathrm{e}^{-\lambda_0 t}\left[N\lambda_{\max}(L^{-1})\right]s_{\max,i}(t)\frac{\mathrm{e}^{\lambda_0 t}-1}{\lambda_0} \\
&\le k_0\mathrm{e}^{-\lambda_0 t}\|\tilde{\mathcal{Y}}(0)\| + \frac{k_0}{\lambda_0}\left[N\lambda_{\max}(L^{-1})\right]s_{\max,i}(t)
\end{aligned} \tag{8.33}$$

where $\lambda_{\max}(\cdot)$ is the maximum eigenvalue of the matrix.

Noting the above equation and that

$$\begin{aligned}
\int_0^t \mathrm{e}^{-\lambda_0(t-\tau)}\left\|\bar{b}(L^{-1}s)\right\|\mathrm{d}\tau &= \int_0^{T_1}\mathrm{e}^{-\lambda_0(t-\tau)}\left\|\bar{b}(L^{-1}s)\right\|\mathrm{d}\tau \\
&\quad + \int_{T_1}^t \mathrm{e}^{-\lambda_0(t-\tau)}\left\|\bar{b}(L^{-1}s)\right\|\mathrm{d}\tau
\end{aligned}$$

We have (8.33) as follows:

$$
\begin{aligned}
\|\tilde{\mathcal{Y}}(t)\| &\leq k_0 \mathrm{e}^{-\lambda_0 t}\|\tilde{\mathcal{Y}}(0)\| + k_0 \mathrm{e}^{-\lambda_0 t}\frac{\mathrm{e}^{\lambda_0 T_1}-1}{\lambda_0}\beta_s(T_1) + k_0 \mathrm{e}^{-\lambda_0 t}\frac{\mathrm{e}^{\lambda_0 t_0}-\mathrm{e}^{\lambda_0 T_1}}{\lambda_0}\beta_{s_T} \\
&\leq k_0 \mathrm{e}^{-\lambda_0 t}\left(\|\tilde{\mathcal{Y}}(0)\| + \frac{\mathrm{e}^{\lambda_0 T_1}}{\lambda_0}\beta_s(T_1)\right) + \frac{k_0}{\lambda_0}\beta_{s_T} \qquad (8.34)
\end{aligned}
$$

This completes the proof. □

Now we will show that for a proper choice of the control parameters, the trajectories of each vehicle do remain in the compact set. From the fact that $L^{-1}s = ([\Lambda^{\mathrm{T}} \;\; 1] \otimes I_{N+1})\tilde{\bar{X}}$, where $\tilde{\bar{X}} = [\tilde{X}^{\mathrm{T}} \;\; \tilde{x}_\rho^{\mathrm{T}}]^{\mathrm{T}}$, we can see that $\tilde{x}_\rho = L^{-1}s - (\Lambda^{\mathrm{T}} \otimes I_N)\tilde{X}$. Therefore,

$$
\begin{aligned}
\|\tilde{\bar{X}}\| &\leq \|\tilde{X}\| + \|\tilde{x}_\rho\| \\
&\leq (1 + \|\Lambda\|)\|\tilde{X}\| + \|L^{-1}\|\|s\| \\
&\leq \big(1 + \|\Lambda\|)\|\tilde{\mathcal{Y}}\| + \lambda_{\max}(L^{-1})\|s\|
\end{aligned}
$$

It follows from (8.34) and the fact that s_i will converge to Ω_{si}, we know that $\|\tilde{\bar{X}}\| \leq k_a\|\tilde{\mathcal{Y}}(0)\| + k_b\beta_{s_T} + k_c, \forall t \geq T_1$, with $k_a = (1 + \|\Lambda\|)k_0$, $k_b = (k_a/\lambda_0) + 1$ and $k_c = k_a(\mathrm{e}^{\lambda_0 T_1}/\lambda_0)\beta_s(T_1)$. Hence,

$$
\begin{aligned}
\|\bar{X}(t)\| &\leq \|\tilde{\bar{X}}(t)\| + \|\bar{X}_d(t)\mathbf{1}\| \\
&\leq k_a\|\tilde{\mathcal{Y}}(0)\| + k_b\beta_{s_T} + k_c + c, \quad \forall t \geq T_1 \qquad (8.35)
\end{aligned}
$$

We now provide the conditions which guarantee $\bar{X} \in \Omega_{\bar{X}}, \forall t \geq 0$. Define the compact set

$$
\Omega_0 := \left\{\bar{X}(0) \,\middle|\, \{\bar{X} | \|\bar{X}(t)\| < k_a\|\tilde{\mathcal{Y}}(0)\|\} \subset \Omega_{\bar{X}}, \lambda_{\max}(L^{-1})\|s(0)\| < \beta_{s_T}\right\}
$$

and the positive constant

$$
c^* := \sup_{c \in \mathbb{R}+}\left\{c \,\middle|\, \left\{\bar{X} | \|\bar{X}\| < k_a\|\tilde{\mathcal{Y}}(0)\| + k_c + c, \; \bar{X}(0) \in \Omega_0\right\} \subset \Omega_{\bar{X}}\right\} \qquad (8.36)
$$

We summarize our results for the full-state feedback case in the following theorem.

Theorem 8.15. *Consider a group of helicopters dynamics (8.1) and the communication graph containing a spanning tree which the root helicopter can access for the desired trajectory, with Assumptions 8.1–8.5, under the action of the control (8.19) and parameters update law (8.20) for each helicopter. For initial conditions* $\bar{X}(0)$, $\eta(0)$, $\tilde{\theta}_i(0)$ *and* $\tilde{\varphi}_i(0)$ *starting in any compact set, and the desired trajectory* *with its derivations up to ρ-th bounded, all closed signals of the system are Semi-Globally Uniformly Ultimately Bounded (SGUUB), and the total tracking error of the helicopters $\tilde{\bar{X}}$ converges to a neighborhood of the origin.*

Proof. From (8.35), we know that the overall system state $\bar{X}(t)$ will stay in $\Omega_{\bar{X}}$ for all time. Furthermore, because the NN weights have been proven bounded for any bounded $\hat{\theta}_i(0)$ and $\hat{\varphi}_i(0)$, and due to Lemma 8.11, it can be seen that η_i is bound if x_i is bounded. As a result, the states of the internal dynamics of the helicopter will converge to the compact set $\Omega_{\eta_i} = \{\eta_i \in \mathbb{R}^p | \|\eta_i\| \le a_1(\sqrt{2c_2/c_1} + \|X_d\|) + a_2\}$, where $a_1 = \lambda_b a_x/\lambda_a$ and $a_2 = \lambda_b a_q/\lambda_a$ are positive constants. Because the control signal $u_i(t)$ is a function of the weights $\hat{\theta}_i$ and $\hat{\varphi}_i$, the states η_i, x_i, and the filtered tracking error s_i, we know that it is also bounded. Therefore, we know that all the closed-loop signals are SGUUB. This completes the proof. □

8.4 Control with Partial Information

From the definition (8.11) of reference states of each helicopter, we know that not all of the helicopters can access the desired altitude and its derivation. For each helicopter in the team, its reference output at time t is the weighted average of its neighbors' outputs at the same time, and in the control design, each helicopter needs to use its neighbors' states $y_{ir}^{(k)}(t)$, $k = 1, \ldots, \rho$, which are not easy for them to access. In this section, we assume that each helicopter can only access its neighbors' output information y_{ir}, and use high observer to estimate $y_{ir}^{(k)}(t)$, $k = 1, \ldots, \rho$.

In the following lemma, high gain observer used in [7] is presented, which will be used to estimate the neighbors' states.

Lemma 8.16. *[35][102] Consider the following linear system:*

$$\begin{aligned}\epsilon\dot{\pi}_i &= \pi_{i+1} \quad i = 1, 2, \ldots, \rho - 1\\ \epsilon\dot{\pi}_\rho &= -\bar{\gamma}_1\pi_\rho - \bar{\gamma}_2\pi_{\rho-1} - \cdots - \bar{\gamma}_{\rho-1}\pi_2 - \pi_1 + \chi(t)\end{aligned} \tag{8.37}$$

where ϵ is a small positive constant and the parameters $\bar{\gamma}_1$ to $\bar{\gamma}_{\rho-1}$ are chosen such that the polynomial $s^\rho + \bar{\gamma}_1 s^{\rho-1} + \ldots + \bar{\gamma}_{\rho-1}s + 1$ is Hurwitz. Suppose the states $\chi(t)$ and its first n derivatives are bounded, so that $\chi^{(k)} < \varpi_k$ with positive constants ϖ_k. Then the following property holds:

$$\tilde{\chi}^{(k)} := \frac{\pi_k}{\epsilon^{k-1}} - \chi^{(k)} = -\epsilon\zeta^{(k)}, \quad k = 1, 2, \ldots, \rho \tag{8.38}$$

where $\zeta := \pi_p + \bar{\gamma}_1\pi_{\rho-1} + \cdots + \bar{\gamma}_{\rho-1}\pi_1$ and $\zeta^{(k)}$ denotes the kth derivative of ζ. Furthermore, there exist positive constants h_k and t^* such that for all $t > t^*$ we have $|\zeta^{(k)}| \le h_k$, $k = 2, 3, \ldots, \rho$.

Note that π_{k+1}/ϵ^k asymptotically converges to $\zeta^{(k)}$, with a small time constant provided that ζ and its k derivatives are bounded. Hence, π_{k+1}/ϵ^k for $k = 1, \ldots, \rho$ is a suitable observer to estimate the output derivatives up to the ρ-th order.

To prevent peaking [52], saturation functions are employed on the observer signals whenever they are outside the domain of interest Ω as follows:

$$\pi^s_{i,j} = \bar{\pi}_{i,j}\phi\left(\frac{\pi_{i,j}}{\bar{\pi}_{i,j}}\right), \quad \bar{\pi}_{i,j} \geq \max_{(\tilde{y}_i, s_i, \tilde{\theta}_i, \tilde{\varphi}_i)\in\Omega}(\pi_{i,j})$$

$$\phi(a) = \begin{cases} -1, & \text{for } a < -1 \\ a, & \text{for } |a| < 1 \\ 1, & \text{for } a > 1 \end{cases} \tag{8.39}$$

Now, we revisit the control law (8.19) and adaption laws (8.20) for the full-state feedback case. Via the certainty equivalence approach, we modify them by replacing the partially available quantities with their estimates, which can be written as

$$u_i = -\hat{\theta}_i^{\mathrm{T}}\psi_i(\hat{Z}_i) - k_i\hat{s}_i - \frac{1}{2}\hat{\varphi}_i\hat{s}_i, \quad i = 1, \ldots, N \tag{8.40}$$

And the update law of parameters is designed as

$$\begin{aligned} \dot{\hat{\varphi}}_i &= -\gamma_1\left[-\frac{1}{2}(1-\varpi_\varphi)\hat{s}_i^2 + \sigma_1\hat{\varphi}_i\right] \\ \dot{\hat{\theta}}_i &= -\gamma_2\big(-\psi_i\hat{s}_i + \sigma_2\hat{\theta}_i\big) \end{aligned} \tag{8.41}$$

where γ_1, γ_2, σ_1 and σ_2 are positive constants, and

$$\varpi_{\varphi_i} = \begin{cases} 0, & \text{if } |\hat{\varphi}_i| \leq M_{\varphi_i} \\ 1, & \text{otherwise} \end{cases} \tag{8.42}$$

where M_{φ_i} is a designed positive constant.

Select Lyapunov function candidate

$$V_{ie} = \frac{1}{2}s_i^2 + \frac{1}{2\gamma_2}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \frac{1}{2\gamma_1}\tilde{\varphi}_i^2 \tag{8.43}$$

And the following lemma is useful for handling the terms containing the estimation errors.

Lemma 8.17. *There exist positive constants F_{ik} which are independent of ϵ_i, such that for $t > t^*$, the estimate $\hat{y}_{ir}^{(k)}$, $i = 1, \ldots, N$, $k = 1, \ldots, \rho$, satisfy the following inequalities:*

$$|\tilde{y}_{ir}^{(k)}| = |\hat{y}_{ir}^{(k)} - y_{ir}^{(k)}| \le \epsilon_i F_{ik} \tag{8.44}$$

Since s_i is the linear combination of Y_i and $Y_j,\ j \in \mathcal{N}_i$, we know that there exist positive constants G_{is} which are independent of ϵ_i such that $|\tilde{s}_i| \le \epsilon_i G_{is}$.

Taking the time derivative of V_i along the closed-loop trajectory and using the property $\psi_i(\hat{Z}_i) - \psi_i(Z_i) = \epsilon_i \psi_{ti}$, where ψ_{ti} is a bounded vector function [30], we have

$$\begin{aligned}
\dot{V}_{ie} &= -\left(\frac{\dot{g}_i}{2g_i^2} + g_0\right) s_i^2 - k_i s_i^2 - k_i s_i \tilde{s}_i - s_i \hat{\theta}_i^{\mathrm{T}} \psi_i(\hat{Z}_i) - \frac{1}{2}\hat{\varphi}_i s_i \hat{s}_i + s_i(d_i + \bar{\varepsilon}_i) \\
&\quad + s_i \theta^{*\mathrm{T}} \psi_i(Z_i) + \frac{1}{\gamma_2}\tilde{\theta}_i \dot{\tilde{\theta}}_i + \frac{1}{\gamma_1}\tilde{\varphi}_i \dot{\tilde{\varphi}}_i \\
&\le -\frac{k_i}{2} s_i^2 + \frac{k_i}{2}\tilde{s}_i^2 - \frac{1}{2}\hat{\varphi}_i s_i \hat{s}_i + \frac{1}{2}\varphi_i s_i^2 + \frac{1}{2}\tilde{\varphi}_i \hat{s}_i^2 - s_i \hat{\theta}_i^{\mathrm{T}} \psi_i(\hat{Z}_i) + s_i \theta^{*\mathrm{T}} \psi_i(Z_i) \\
&\quad + \hat{s}_i \tilde{\theta}_i^{\mathrm{T}} \psi_i(\hat{Z}_i) - \sigma_2 \tilde{\theta}_i^{\mathrm{T}} \hat{\theta}_i - \sigma_1 \tilde{\varphi}_i \hat{\varphi}_i + \frac{1}{2}
\end{aligned}$$

For the term $-s_i^2\tilde{\varphi}_i - s_i \tilde{s}_i \hat{\varphi}_i + \hat{s}_i^2 \tilde{\varphi}_i$, we have

$$\begin{aligned}
-s_i^2\tilde{\varphi}_i - s_i \tilde{s}_i \hat{\varphi}_i + \hat{s}_i^2 \tilde{\varphi}_i &= \tilde{s}_i(s_i \tilde{\varphi}_i + \tilde{s}_i \tilde{\varphi}_i - s_i \varphi_i) \\
&\le \epsilon G_{is}|s_i \tilde{\varphi}_i| + \epsilon^2 G_{is}^2 |\tilde{\varphi}_i| + \epsilon G_{is}|s_i \varphi_i| \\
&\le \frac{1}{2}(s_i^2 + \epsilon^2 G_{is}^2 |\tilde{\varphi}_i|^2) + \frac{1}{2}\epsilon^2 G_{is}^2 \tilde{\varphi}_i^2 + \frac{1}{2}\epsilon^2 G_{is}^2 + \frac{1}{2}s_i^2 \\
&\quad + \frac{1}{2}\epsilon^2 G_{is}^2 \varphi_i^2 \\
&= s_i^2 + \epsilon_i^2 G_{is}^2 \tilde{\varphi}_i^2 + \frac{1}{2}\epsilon_i^2 G_{is}^2 \varphi_i^2 \\
&\le s_i^2 + \epsilon_i^2 G_{is}^2 \left(\hat{\varphi}_i^2 + \frac{3}{2}\varphi_i^2\right)
\end{aligned} \tag{8.45}$$

For the term $-s_i \hat{\theta}_i^{\mathrm{T}} \psi_i(\hat{Z}_i) + s_i \theta^{*\mathrm{T}} \psi_i(Z_i) + \hat{s}_i \tilde{\theta}_i^{\mathrm{T}} \psi_i(\hat{Z}_i)$, we have

$$\begin{aligned}
&-s_i \hat{\theta}_i^{\mathrm{T}} \psi_i(\hat{Z}_i) + s_i \theta^{*\mathrm{T}} \psi_i(Z_i) + \hat{s}_i \tilde{\theta}_i^{\mathrm{T}} \psi_i(\hat{Z}_i) \\
&= -s_i \tilde{\theta}_i^{\mathrm{T}} \psi_i(\hat{Z}_i) - s_i \theta_i^{*\mathrm{T}} \psi_i(\hat{Z}_i) + s_i \theta^{*\mathrm{T}} \psi_i(Z_i) + \hat{s}_i \tilde{\theta}_i^{\mathrm{T}} \psi_i(\hat{Z}_i) \\
&= \tilde{s}_i \tilde{\theta}_i^{\mathrm{T}} \psi_i(\hat{Z}_i) - \epsilon_i s_i \theta_i^{*\mathrm{T}} \psi_{ti} \\
&\le \frac{1}{2}\tilde{\theta}_i^{\mathrm{T}} \tilde{\theta}_i + \frac{1}{2}\epsilon_i G_{is}^2 \|\psi_{ti}\|^2 + \frac{1}{2}s^2 + \frac{1}{2}\epsilon_i^2 \|\psi_{ti}\|^2 \|\theta_i^*\|^2
\end{aligned} \tag{8.46}$$

Then,

$$\dot{V}_{ie} \le -\frac{1}{2}(k_i-2)s_i^2 - \frac{\sigma_2-1}{2}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i - \frac{\sigma_1}{2}\tilde{\varphi}_i^2 + \frac{1}{2}\epsilon_i^2 G_{is}^2\hat{\varphi}_i^2 + \frac{1}{2}\epsilon_i G_{is}^2\|\psi_{ti}\|^2$$
$$+\frac{\epsilon_i^2\|\psi_{ti}\|^2+\sigma_2}{2}\|\theta_i^*\|^2 + \left(\frac{3}{4}\epsilon_i^2 G_{is}^2 + \frac{\sigma_1}{2}\right)\varphi_i^{*2} + \frac{k_i}{2}\epsilon_i^2 + \frac{1}{2} \tag{8.47}$$

Then,

$$\dot{V}_{ie} \le -c_{1ie}V_{ie} + c_{2ie} \tag{8.48}$$

$$c_{1ie} = \min\left\{\frac{1}{2}(k_i-2), (\gamma_2-1)\sigma_2, \gamma_1\sigma_1\right\} \tag{8.49}$$

$$c_{2ie} = \frac{1}{2}\epsilon_i^2 G_{is}^2\hat{\varphi}_i^2 + \frac{1}{2}\epsilon_i G_{is}^2\|\psi_{ti}\|^2 + \frac{\epsilon_i^2\|\psi_{ti}\|^2+\sigma_2}{2}\|\theta_i^*\|^2 \tag{8.50}$$
$$+\left(\frac{3}{4}\epsilon_i^2 G_{is}^2 + \frac{\sigma_1}{2}\right)\varphi_i^{*2} + \frac{k_i}{2}\epsilon_i^2 + \frac{1}{2}$$

Now define

$$\Omega_{sie} = \left\{ s_i \,\middle|\, |s_i| \le \sqrt{\frac{2c_{2ie}}{c_{1ie}}} \right\} \tag{8.51}$$

$$\Omega_{\theta_{ie}} = \left\{ (\tilde{\theta}_i, \tilde{\varphi}_i) \,\middle|\, \|\tilde{\theta}_i\| \le \sqrt{\frac{2c_{2ie}}{\sigma_2}}, |\tilde{\varphi}_i| \le \sqrt{\frac{2c_{2ie}}{\sigma_1}} \right\} \tag{8.52}$$

$$\Omega_{eie} = \left\{ (s_i, \tilde{\theta}_i, \tilde{\varphi}_i) \,\middle|\, k_i s_i^2 + \frac{\sigma_2}{2}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \frac{\sigma_1}{2}\tilde{\varphi}_i^2 \le c_{2ie} \right\} \tag{8.53}$$

Since c_{1ie}, σ_1, σ_2, and k_i are positive constants, we know that Ω_{sie}, $\Omega_{\theta_i e}$ and Ω_{eie} are compact sets. Equation (8.48) shows that $\dot{V}_{ie} \le 0$ once the errors are outside the compact set Ω_{ei}. According to the standard Lyapunov theorem, we conclude that s_i, $\tilde{\theta}_i$, and $\tilde{\varphi}_i$ are bounded. From (8.48) and (8.51), it can be seen that V_{ie} is strictly negative as long as s_i is outside the compact set Ω_{sie}. Therefore, there exists a constant T_1 such that for $t > T_1$, the filtered tracking error s_i converges to Ω_{sie}, that is to say, $s_i \le \beta_{sie}$, with $\beta_{sie}(k_i, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \theta_i^*, \varphi_i^*, \epsilon_i) = \sqrt{2c_{2ie}/c_{1ie}}$.

We can conclude the following theorem.

Theorem 8.18. *Consider a group of helicopters dynamics (8.1) and the communication graph containing a spanning tree with the leader as the root, with Assumptions 8.1–8.5, under the action of the control law (8.40), parameters update law (8.41), and the high gain observer (8.37), which is turned on at time t^* in advance. For initial conditions $\bar{X}(0)$, $\eta(0)$, $\tilde{\theta}_i(0)$ and $\tilde{\varphi}_i(0)$ starting from any compact set, and the desired trajectory with its derivations up to ρ-th bounded,* *all closed signals of the system are SGUUB, and the total tracking error of the helicopters $\tilde{\bar{X}}$ converges to a neighborhood of origin.*

Proof. We have concluded that s_i will converge to a compact set Ω_{sie}, then following Lemma 8.17, it can be concluded that $\|\tilde{\mathcal{Y}}\| \leq k_0 e^{-\lambda_0 t}\left(\|\tilde{\mathcal{Y}}(0)\| + \frac{e^{\lambda_0 T_1}}{\lambda_0}\beta_s(T_1)\right) + \frac{k_0}{\lambda_0}\beta_{s_T}$, and from (8.35), we can find that $\|\bar{X}\|$ is also bounded. Following the same procedure in the full-state feedback control, we can complete the proof. □

Remark 8.19. It is shown in (8.50) that the smaller c_{2ie} might be obtained by choosing a smaller σ_1 and σ_2, which may lead to a smaller tracking error. Nevertheless, from (8.52) it can be seen that the smaller σ_1 and σ_2 may cause large NN weight and disturbance compensation errors. If σ_1 and σ_2 are chosen to be very large, it will lead to a large tracking error. Hence, the parameters σ_1 and σ_2 should be adjusted carefully in practical implementations.

8.5 Simulation Study

In this section, we consider the synchronized altitude tracking of 6 X-cell 50 helicopters whose communication graph is shown in Fig. 8.2. The dynamics of the helicopter can be written as follows as in Sect. 4.5.2

$$\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= a_0 + a_1\zeta_2 + a_2\zeta_2^2 + \left(a_3 + a_4\zeta_4 - \sqrt{a_5 + a_6\zeta_4}\right)\zeta_3^2 \\
\dot{\zeta}_3 &= a_7 + a_8\zeta_3 + (a_9 \sin\zeta_4 + a_{10})\zeta_3^2 + a_{\mathrm{th}} \\
\dot{\zeta}_4 &= \zeta_5 \\
\dot{\zeta}_5 &= a_{11} + a_{12}\zeta_4 + a_{13}\zeta_3^2 \sin\zeta_4 + a_{14}\zeta_5 - K_1 u
\end{aligned} \tag{8.54}$$

where ζ_1 denotes altitude (m), ζ_2 the height rate of the altitude rate (m/s), ζ_3 the rotational speed of the rotor blades (rad/s), ζ_4 the collective pitch angle (rad), ζ_5 the collective pitch rate (rad/s), $a_{\mathrm{th}} = 111.69\,\mathrm{s}^{-2}$ the constant input to the throttle, and u the input to the collective servomechanisms.

Let y be the altitude ζ_1. By restricting the throttle input to be constant, we obtain a SISO in which u is the only input variable forcing the output y to track a desired trajectory y_d, which is generated by

$$y_d = \frac{150.056}{s^4 + 12.6s^3 + 64.19s^2 + 154.35s + 150.056} h_{\mathrm{ref}} \tag{8.55}$$

where

$$h_{\mathrm{ref}}(t) = 5.5 - 0.5\sin t \tag{8.56}$$

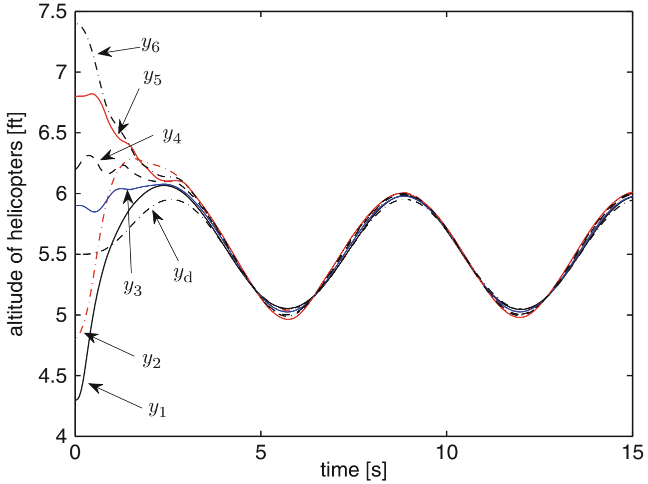


Fig. 8.3 Altitude of all helicopters with output feedback control

The nominal values for constants K_1 and a_i are given to be: $K_1 = 0.25397\text{s}^{-2}$, $a_0 = -17.67\text{m/s}^2$, $a_1 = a_2 = -0.1\,\text{s}^{-2}$, $a_3 = 5.31 \times 10^{-4}$, $a_4 = 1.5364 \times 10^{-2}$, $a_5 = 2.82 \times 10^{-7}$, $a_6 = 1.632 \times 10^{-5}$, $a_7 = -13.92\text{s}^{-2}$, $a_8 = -0.7\,\text{s}^{-2}$, $a_9 = a_{10} = -0.0028$, $a_{11} = 434.88\,\text{s}^{-2}$, $a_{12} = -800\,\text{s}^{-2}$, $a_{13} = -0.1$ and $a_{14} = -65\,\text{s}^{-2}$.

It can be shown that the system has strong relative degree 4 as in Sect. 4.5.2. The control parameters are chosen as $\Lambda = [64, 48, 12]^{\mathrm{T}}$, $k_i = 3$, $i = 1, \ldots, 6$, while the NN parameters for each helicopter are chosen as $\sigma_1 = 0.05$, $\gamma_1 = 1$, $\sigma_2 = 0.01$, $\gamma_2 = 100$. For high gain observer, we choose $\epsilon_i = 0.08$, $\bar{\gamma}_1 = 4$, $\bar{\gamma}_2 = 6$, $\bar{\gamma}_3 = 4$, $\bar{\pi}_2 = 0.1$, $\bar{\pi}_3 = 0.15$, $\bar{\pi}_4 = 0.025$. The saturation limits of the control are ± 400 mrad. The initial conditions are $\zeta_1(0) = [4.3, 0.0, 95.3567, 0.222, 0.0]^{\mathrm{T}}$, $\zeta_2(0) = [4.8, 0.0, 95.3567, 0.3, 0.0]^{\mathrm{T}}$, $\zeta_3(0) = [5.9, 0.0, 95.4, 0.22, 0.0]^{\mathrm{T}}$, $\zeta_4(0) = [6.2, 0.0, 95.3567, 0.3, 0.0]^{\mathrm{T}}$, $\zeta_5(0) = [6.8, 0.0, 95.3567, 0.22, 0.0]^{\mathrm{T}}$, $\zeta_6(0) = [7.4, 0.0, 95.4, 0.21, 0.0]^{\mathrm{T}}$, $\hat{\theta}_i(0) = 0$, and $\hat{\varphi}_i(0) = 0$ for each helicopter.

Simulation results are shown in Figs. 8.3–8.6. From Fig. 8.3, we can find that good tracking performance is achieved for each helicopter by the proposed control. The tracking performance for full-state and output feedback cases are similar for the choice of ϵ_i made. The initial errors of all helicopters are sufficiently reduced and the altitude trajectories all lie in close proximity of the desired sinusoidal trajectory. Meanwhile, the internal dynamics and the NN weights are all bounded, as shown in Figs. 8.5 and 8.6. From Fig. 8.4, we can find that the control input of the helicopters are bounded, both in the full-state feedback and the output feedback cases.

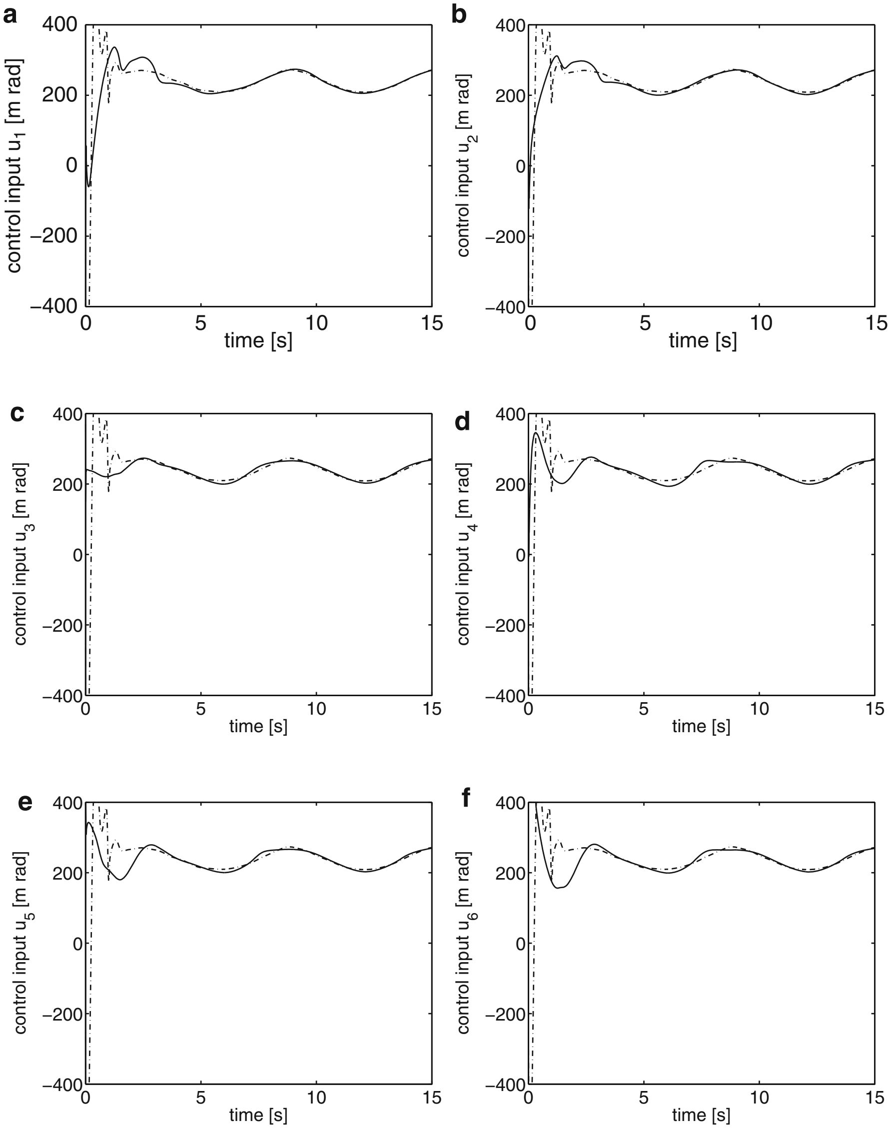


Fig. 8.4 Control input of helicopters under full-state (*solid*) and output (*dash-dot*) feedback control

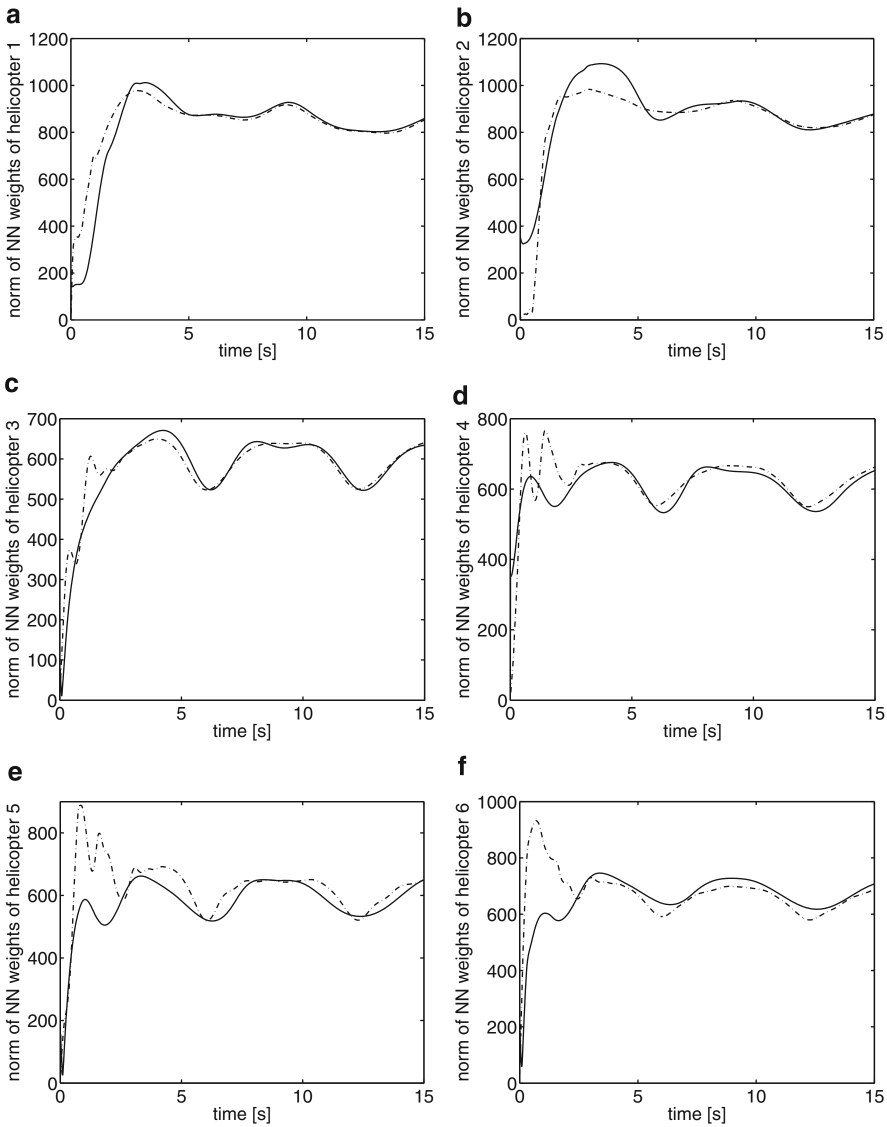


Fig. 8.5 Norm of neural weights under full-state (*solid*) and output (*dash-dot*) feedback control. Norm of neural weights (**a**) $\|\hat{\theta}_1\|$, (**b**) $\|\hat{\theta}_2\|$, (**c**) $\|\hat{\theta}_3\|$, (**d**) $\|\hat{\theta}_4\|$, (**e**) $\|\hat{\theta}_5\|$, (**f**) $\|\hat{\theta}_6\|$

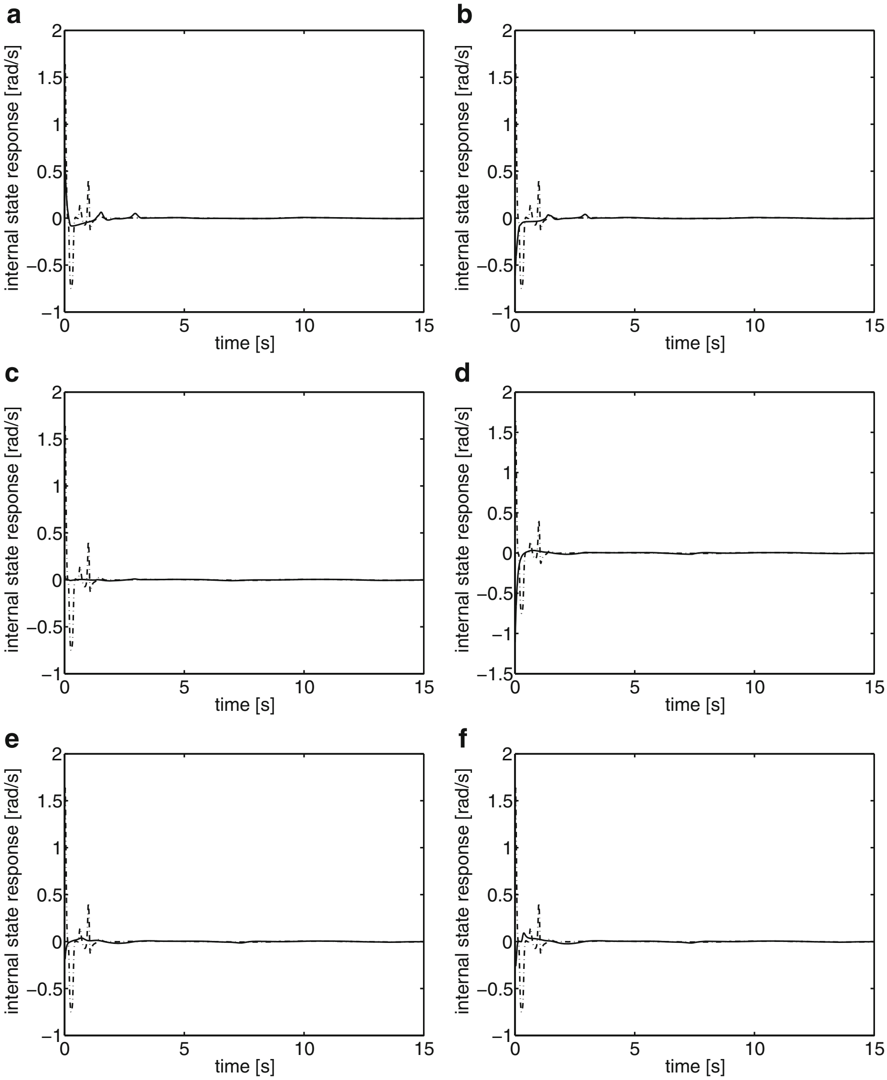


Fig. 8.6 Internal state response under full-state (*solid*) and output (*dash-dot*) feedback control. (**a**) Internal state of helicopter 1, (**b**) Internal state of helicopter 2, (**c**) Internal state of helicopter 3, (**d**) Internal state of helicopter 4, (**e**) Internal state of helicopter 5, (**f**) Internal state of helicopter 6

8.6 Conclusion

In this chapter, we studied the synchronized tracking problem of multiple helicopters in vertical flight mode. Under the condition that the Laplacian matrix of the extended formation graph, which contains a spanning tree which the root helicopter can access for the desired trajectory, by using the weighted average of its neighbors' states as its reference signal, through neural network based approximation, the adaptive tracking control law has been designed for each helicopter. By using high gain observer to reconstruct the unavailable states, an extension has been made to the output feedback case where both the helicopter's states and its neighbors' states are not available for control design. It has been shown that the tracking errors of each helicopter converge to adjustable neighborhoods of the origin for both cases, although some of them do not access the desired tracking trajectory. Simulation results have shown the effectiveness of the approach presented.